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ANALYSIS OF RADIATIVE HEAT TRANSFER IN THE PRESENCE OF OBSCURATIONS

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ABSTRACT

Numerical simulation of radiative heat transfer problems in general axisymmetric geometry in the presence of an active gas is considered. Such simulation requires subdivision of the radiating surfaces into discrete elements, which are in the present case radiating rings. While the effect of a participating medium is easily taken into account by integration along the lines of vision between the surface elements, the calculation of the different obscurations poses the main difficulty. We have written a closed expression which formulates the problem exactly, and then developed a systematic and compact computational approach to the obscuration problem in complex configurations. The present procedure is particularly suited to computer calculations associated with engineering applications in the aircraft and furnace industries.

I. Introduction

The radiative heat transfer problem in axisymmetric geometry frequently occurs in the thermal design of aircrafts and spacecrafts. As such it was considered by many authors (see eg.[1-7]) and formulated in terms of radiative transfer between a single simple axisymmetric surface and a surface element. In a non-participating medium, an important part of this calculation reduces to the shape (view) factor. Morizumi[2] used the double projection method to calculate the shape factor between a surface element and a paraboloid. Bobco[3] expressed the shape factor between a surface element and a cone in terms of a single integral. The solution of this particular problem was completed by Urquhart[4], who derived an analytic expression for the shape factor between a surface element and a cone, using the contour integration method. Urquhart's derivation was repeated several years later by Minning[7].

For complicated and/or multilayered axisymmetric bodies the approaches mentioned above are of limited practical value, since the problem must then be solved numerically. This requires a subdivision of the whole body into a large number of surface elements, and the calculation of the radiative heat interchange between each pair of them. One is immediately faced with the complication introduced by the presence of partial or full obscurations produced by the various parts of the body. Though it is often the most time-consuming part of the whole calulation, it did not seem to have received a systematic treatment. Robbins[1], e.g., treated the obscurations by an ad hoc ray tracing technique, which is difficult to apply to complicated bodies.

The next chapter outlines the method of computatio of the radiative heat transfer between surface elements in a general axisymmetric body with a possibly participating medium (we treat here only one spectral component of the thermal radiation). Its main purpose is introduction of concepts and notation and an exact formulation of the obscuration problem. The obscuration problem itself is treated in detail in chapter III.

II. The radiative heat transfer between surface elements in axisymmetric bodies

Consider the surface formed by the revolution of a profile $r^j(x)$, around the x axis, as depicted in Fig. 1. Let S_ℓ be a fixed ring cut out from this surface by two planes perpendicular to the x axis, and dx_ℓ apart. The abscissa of the ring is x_ℓ, its radius is $r_\ell = r^j(x_\ell)$, the elementary length of the ring's profile is $ds_\ell = \sqrt{dx_\ell^2 + dr_\ell^2}$, and $dS_\ell = r d\phi ds_\ell$ in an elementary area of S_ℓ.

Let S_m be another ring formed by revolution of the (possibly different) profile $r^h(x)$, and let dS_m be an elementary area at the bottom of S_m (see Fig. 1). We assume that the rings are opaque and diffusive (Lambertian), and that they are separated by a possibly participating medium. Let further $i_{\ell m}(\Phi)$ be the monochrometric intensity of radiation coming from the direction of dS_ℓ and incident on dS_m. If dS_m can see the whole of S_ℓ, then the flux of radiant energy, $dF_{\ell m}$, per unit time from S_ℓ to dS_m is[1]

$$dF_{\ell m} = dS_m \cdot 2\int_0^\pi i_{\ell m}(\Phi) \frac{|\cos\theta_\ell \cdot \cos\theta_m|}{R_{\ell m}^2} r_\ell d\Phi ds_\ell \qquad (2.1)$$

$R_{\ell m}$ is here the distance between dS_ℓ and dS_m, θ_ℓ, θ_m are the angles between the line of sight and the respective normals to dS_ℓ and dS_m, and Φ is the axial angle between dS_ℓ and dS_m. If opaque bodies of revolution (formed e.g. by such profiles as $r^g(x)$, or $r^j(x)$ and $r^h(x)$ themselves) obscure the full vision of S_ℓ from the observation point at dS_m, then the domain of Φ , in eqn. (1), should be reduced to the set of angular intervals of unobscured vision, say to (Φ_1, Φ_2), $(\Phi_3, \Phi_4), \ldots, (\Phi_{2N-1}, \Phi_{2N})$.

It is obvious that N as well as $\Phi_1, \ldots, \Phi_{2N}$ may depend on x_ℓ and x_m (or shortly - on ℓ and m), and that

$$0 \leq \Phi_1(\ell,m) < \Phi_2(\ell,m) < \ldots < \Phi_{2N}(\ell,m) \leq \pi \qquad (2.2)$$

(since if e.g. $\Phi_1 = \Phi_2$ then both Φ_1 and Φ_2 could be omitted).

The most general form of eqn. (2.1) is thus

$$dF_{\ell m} = 2dS_m r_\ell ds_\ell \left(\int_{\Phi_1}^{\Phi_2} + \int_{\Phi_3}^{\Phi_4} + \ldots + \int_{\Phi_{2N-1}}^{\Phi_{2N}} \right) i_{\ell m}(\Phi) \frac{|\cos\theta_\ell \cdot \cos\theta_m|}{R^2_m} d\Phi \qquad (2.3)$$

It remains to determine the angles $\Phi_1 \ldots \Phi_{2N}$, and to specify the explicit dependence of the integrand on Φ. The following geometrical relations are evident from Fig. 1.

$$R^2_{\ell m}(\Phi) = (x_m - x_\ell)^2 + r_m^2 + r_\ell^2 - 2r_m r_\ell \cos\Phi$$

$$K_\ell^2 = n_m^2 + R^2_{\ell m} - 2n_m R_{\ell m} \cos\theta_m \qquad (2.4)$$

and by symmetry

$$K_m^2 = n_\ell^2 + R^2_{m\ell} - 2n_\ell R_{m\ell} \cos\theta_\ell$$

where n_m and K_ℓ are respectively the segments joining the centers of

dS_m and dS_ℓ with the intersection point of the axis and the normal to dS_m. The angles between n_m, n_ℓ and the axis of symmetry are known. They are given respectively by $tg^{-1}\frac{dr^h(x)}{dx}$ and $tg^{-1}\frac{dr^j(x)}{dx}$. $R_{\ell m}$, θ_ℓ and θ_m can thus be explicitly calculated from eqns. (2.4).

We turn now to $i_{\ell m}$, and consider the equation satisfied by the intensity of radiation $i(z)$, along a beam propagating from dS_ℓ to dS_m:

$$di(z) = - i(z)\cdot\alpha(z)dz + i^b(z)\varepsilon(z)dz \qquad (2.5)$$

The monochromatic parameters of the participating medium, α, ε and i^b, are respectively the volumetric coefficients of absorption and emission, and the intensity of black body radiation ($\varepsilon=\alpha$ by Kirchoff's law). We integrate the linear ordinary differential eqn. (2.5) from z=0 at dS_ℓ to $z=R_{\ell m}$ at dS_m, and get $i_{\ell m}$, the intensity of radiation arriving at dS_m.

$$i_{\ell m}(\Phi) = i_\ell \exp\{-\int_0^{R_{\ell m}(\Phi)} \alpha(z)dz\} + \int_0^{R_{\ell m}(\Phi)} i^b(z)\varepsilon(z)\exp\{-\int_z^{R_{\ell m}(\Phi)} \alpha(z')dz'\}dz \qquad (2.6)$$

i_ℓ is here the (angle independent) radiant intensity leaving the diffusive surface of S_ℓ. The specification of the integrand in eqn. (2.3) is thus complete.

For a nonparticipating medium the indefinite integral of eqn. (2.3) is known in closed form[1] (see also the related results in references 3, 5,6). The main difficulty, however, is not the integration itself but the determination of the integration limits $\Phi_1,\Phi_2,\ldots,\Phi_{2N}$. This subject is considered in detail in the next chapter.

III. *Obscurations in axisymmetric bodies*

We start by analyzing the elementary situation shown in Fig. 2. The circles $S(x_\ell)$ and $S(x_m)$ are sections of a radiating surface at $x=x_\ell$ and $x=x_m$, with radii R_ℓ and R_m respectively. The ring $S(x)$ is a cross section at x of an opaque layer formed by revolution of two profiles around the x axis : $r_i(x)$ is the running radius of the inner profile, while $r_e(x)$ is the running radius of the exterior profile.

Consider the line of sight between a point P_m of $S(x_m)$, and P_ℓ, located at the bottom of $S(x_\ell)$. Let P be the point of intersection of $P_\ell P_m$ with the plane of $S(x)$, and let R be the distance of P to the axis. Denote by $\underline{i},\underline{j},\underline{k}$ three mutually orthogonal coordinate unit vectors, and by $\underline{u},\underline{u}_m$ and $\underline{\omega}$ - unit vectors, pointing respectively in the xP, $x_m P_m$ and $P_\ell P$ directions. Let further q and q_m be the lengths of $P_\ell P$ and of $P_\ell P_m$. The vision between P_ℓ and P_m is obstructed by S(x) when P is located within the bounds of S(x), that is, when R satisfies

$$r_i(x) < R < r_e(x) \tag{3.1}$$

To make this condition explicit we have to express R in terms of available parameters. To this end one starts from the following equations, deducible from Fig. 2.

$$\underline{i}(x-x_\ell) + \underline{u}R = -\underline{j}R_\ell + \underline{\omega}q \tag{3.2}$$

$$\underline{i}(x_m-x_\ell) + \underline{u}_m \cdot R_m = -\underline{j}R_\ell + \underline{\omega}q_m \tag{3.3}$$

Projecting the equations on the x axis we get

$$\frac{x-x_\ell}{x_m-x_\ell} = \frac{q}{q_m} \tag{3.4}$$

Next we multiply eqn. (3.2) by (x_m-x_ℓ), eqn. (3.2) by $(x-x_\ell)$, subtract, and employ eqn. (3.4)

$$\underline{u}R\cdot(x_m-x_\ell) = \underline{u}_m\cdot R_m\cdot(x-x_\ell) - \underline{j}R_\ell\cdot(x_m-x)$$

Finally, we substitute $\underline{k}\cos(\phi-\frac{\pi}{2}) + \underline{j}\cos(\pi-\phi)$ for $\underline{u}_m$, and square the equation

$$[R\cdot(x_m-x_\ell)]^2 = [R_m\cdot(x-x_\ell)]^2 + [R_\ell\cdot(x_m-x)]^2 + 2[R_m\cdot(x-x_\ell)]\cdot[R_\ell\cdot(x_m-x)]\cos\phi$$

This we use in the inequalities (3.1). For x, between x_ℓ and x_m, the product $(x-x_\ell)$ (x_m-x) is positive, consequently

$$\frac{[r_i(x)\cdot(x_m-x_\ell)]^2 - [R_\ell\cdot(x_m-x)]^2 - [R_m\cdot(x-x_\ell)]^2}{2[R_\ell\cdot(x_m-x)]\cdot[R_m\cdot(x-x_\ell)]} < \cos\phi < \qquad (3.5)$$

$$< \frac{[r_e(x)\cdot(x_m-x_\ell)]^2 - [R_\ell\cdot(x_m-x)]^2 - [R_m\cdot(x-x_\ell)]^2}{2[R_\ell\cdot(x_m-x)]\cdot[R_m\cdot(x-x_\ell)]}$$

The result is valid for $x_\ell \lessgtr x_m$, since we could equally well point the x axis in the opposite direction. Angles ϕ , satisfying both inequalities correspond to lines of sight $P_\ell P_m$, obscured by the intermediate ring S(x).

With the notation:

$$\mu = \cos\phi$$

$$\mu_{-1}(x) = \max\{-1;\ \frac{[(x_m-x_\ell)r_i(x)]^2 - [(x_m-x)R_\ell]^2 - [(x-x_\ell)R_m]^2}{2[(x_m-x)R_\ell]\cdot[(x-x_\ell)R_m]}\} \qquad (3.6)$$

$$\mu_1(x) = \min\{1;\ \frac{[(x_m-x_\ell)r_e(x)]^2 - [(x_m-x)R_\ell]^2 - [(x-x_\ell)R_m]^2}{2[(x_m-x)R_\ell]\cdot[(x-x_\ell)R_m]}\}$$

inequalities (3.5), become

$$\mu_{-1}(x) < \mu < \mu_1(x) \tag{3.7}$$

The interval $(\mu_{-1}(x),\ \mu_1(x))$ will be referred to as the interval of obscuration between $S(x_\ell)$ and $S(x_m)$, produced by $S(x)$. We can extend now the above argument to the whole opaque layer, which is assumed to occupy some interval $a \leq x \leq b$, between x_ℓ and x_m. Since the ring $S(x)$ is a section of the layer at x, it is evident that the domain of obscuration between $S(x_\ell)$ and $S(x_m)$, produced by the whole layer, is equal to the union of the intervals $(\mu_{-1}(x),\ \mu_1(x))$, defined above.

As x sweeps out (a,b), the corresponding interval of obscuration $(\mu_{-1}(x),\ \mu_1(x))$, changes continuously. It follows that the union of $(\mu_{-1}(x),\ \mu_1(x))$ is a single interval, say (μ_{-1},μ_1). Thus

$$\begin{aligned} \mu_{-1} &= \min_{a \leq x \leq b} [\mu_{-1}(x)] \\ \mu_1 &= \max_{a \leq x \leq b} [\mu_1(x)] \end{aligned} \tag{3.8}$$

are the limits of the interval of obscuration produced by the whole layer.

Two special cases are of frequent occurrence:

1) <u>An envelope enclosing the radiating system</u>. The envelope has the same effect on the transport of radiation in the system as a layer with an infinite exterior radius, that is $r_e(x) = \infty$. It follows from eqns. (3.6) and (3.8), that $\mu_1 = 1$ in this case.
2) <u>An inner body of the system</u>. An inner body has the same effect on transport of radiation as a layer with $r_i(x) = 0$. Eqns. (3.6) and (3.8), imply now $\mu_{-1} = -1$.

We turn to the general case of a multi-layered axisymmetric body separating two thin rings $S(x_\ell)$ and $S(x_m)$. The domain of obscuration produced by the body is equal to the union of obscuration intervals (μ^k_{-1},μ^k_1) produced by its constituent envelopes, layers and innerbodies. This domain will generally consist of several separate intervals. The domain of vision between $S(x_\ell)$ and $S(x_m)$, is defined as the complement of the domain of obscuration to the interval (-1,1). The domain of vision will therefore generally consist of several separate intervals. However, if only envelopes and innerbodies are present, and proper layers (both $r_i \neq 0$ and $r_e \neq \infty$) are absent, the domain of vision between $S(x_\ell)$ and $S(x_m)$, reduces to a <u>single interval of vision</u>. It can be obtained as the intersection of the intervals of vision produced by its constituent envelopes and innerbodies. The main steps involved in numerical calculation of the interval of obscuration, produced by an axisymmetric body of arbitrary complexity can now be summarized as follows:

1. Divide the part of the body between x_ℓ and x_m into layers,
2. For each layer calculate (μ_{-1},μ_1), using eqns. (3.6) and (3.8),
3. Form the union of all the intervals calculated in step 2; the interval of vision is the complementary of this interval to (-1,1).

The main practical difficulty in this scheme is the search for the extrema of the functions $\mu_{-1}(x)$ and $\mu_1(x)$ in step 2. These extrema can be estimated by calculating the values of $\mu_{-1}(x)$ and $\mu_1(x)$ at a sufficiently dense set of points in the interval [a,b]. Evidently, such calculation of the extrema is unnecessarily laborious and, moreover, the error cannot be easily evaluated. The other alternative is to restrict the various $r^j(x)$ to simple functions, and to locate the extrema by analytical methods. It turns out, that unless one restricts himself to piecewise linear profiles, numerous particular cases have to be considered and the scheme of computation becomes impractical. On the other hand, an approximation of arbitrary profiles by piecewise linear functions results in a simple, accurate and fast algorithm. Moreover, since piecewise-conical surfaces (piecewise-linear profiles) are frequent in engineering applications, the method is frequently exact.

From now on we shall assume that all the radiating and/or obscuring surfaces consist of piecewise-conical parts ($r_e(x)$ and $r_i(x)$ are piecewise-linear in x). We shall refer to any conical piece as a "zone". The zone is called internal if it is formed by some $r_i(x)$, and is called external if it is formed by some $r_e(x)$. The internal zones determine μ_{-1}, and the external determine μ_1. By considering all zones, between x_ℓ and x_m, the whole interval of obscuration of the layer can be determined, thus completing step 2 of our scheme.

The explicit formulas for μ_{-1} and μ_1, that is the limits of the interval of obscuration between $S(x_\ell)$ and $S(x_m)$ for $x_\ell \neq x_m$, produced by a given zone will be now derived. We specialize eqns. (3.6), to cylindrical or conical surfaces and consider

$$C = \frac{[(x_m-x_\ell)\cdot r(x)]^2 - [(x_m-x)\cdot R_\ell]^2 - [(x-x_\ell)R_m]^2}{2[(x_m-x)\ R_\ell]\cdot[(x-x_\ell)R_m]} \tag{3.9}$$

Here $r(x)$ is the current radius of the (internal or external) zone. Next we restrict the zone to its "obscuring part", that is the part located between x_ℓ and x_m, and denote the corresponding domain of the restricted zone $a \leq x \leq b$.

At this stage we shall limit ourselves to the case $x_\ell < a < b < x_m$. Since $r(x)$ is a linear function of x, it can be written in the form

$$r(x) = \frac{q_\ell}{x_m - x_\ell}(x_m - x) + \frac{q_m}{x_m - x_\ell}(x - x_\ell) \qquad (3.10)$$

The parameters q_ℓ and q_m may be of any sign. Their meaning is obvious, e.g., $|q_\ell|$, is the radius of the conical surface (extended if necessary) at x_ℓ. The following form of eqn. (3.9) simplifies further considerations:

$$2C = (p_m^2 - 1)z + (p_\ell^2 - 1)z^{-1} + 2p_m p_\ell \qquad (3.11)$$

where

$$z = \frac{R_m}{R_\ell} \frac{x - x_\ell}{x_m - x}$$
$$p_\ell = q_\ell / R_\ell \, , \; p_m = q_m / R_m \qquad (3.12)$$

For values of x between a and b, $\frac{dz}{dx} > 0$, and the correspondence between z and x is a one-to-one. We can therefore use z as the free variable instead of x. The domain of z is $[z_a, z_b]$, where $z_a \geq \varepsilon \geq 0$ and $z_b \leq \frac{1}{\varepsilon} < \infty$, for some sufficiently small ε.

Turning to the computation of μ_{-1} and μ_1, we recall that they were defined in eqns. (3.8), as extremal values of C. We start therefore with

$$2\frac{dC}{dz} = (p_m^2 - 1) - (p_\ell^2 - 1)z^{-2}$$

and note, that $z > 0$ implies, that for

$$(p_\ell^2 - 1)(p_m^2 - 1) \leq 0 \qquad (3.13)$$

C is a monotonic function of z or a constant. The extremal values are thus reached at the ends z_a and z_b, of the relevant interval of z. By equation (3.12)

$$z_a = \frac{R_m}{R_\ell} \frac{a-x_\ell}{x_m-a}$$

$$z_b = \frac{R_m}{R_\ell} \frac{b-x_\ell}{x_m-b} \tag{3.14}$$

On the other hand, for $(p_m^2-1)(p_\ell^2-1) > 0$, the equation $\frac{dC}{dz} = 0$ has a unique positive solution

$$z^* = +\sqrt{\frac{p_\ell^2-1}{p_m^2-1}} \tag{3.15}$$

which, upon substitution into eqn. (3.11), gives

$$C(z^*) = \mathrm{sgn}(p_\ell^2-1)\cdot\sqrt{(p_\ell^2-1)(p_m^2-1)} + p_\ell p_m \tag{3.16}$$

It is a maximum for $p_\ell^2-1<0$, and a minimum for $p_\ell^2-1>0$.

The extremum provided by eqn. (3.16) is not always the relevant one. First, the condition $z_a \le z^* \le z_b$ must be satisfied, to ensure that the critically obscuring section lies within the boundaries of the given zone. Second, if we are considering an internal zone and $p_\ell^2-1<0$, the extremum is irrelevant since it is a maximum, while we are looking for a minimum. Finally, if we are considering an external zone, and $p_\ell^2-1>0$, the extremum is again irrelevant, since it is a minimum, and we are looking for a maximum of C. In all these disabling cases, the desired solution is either $C(z_a)$ or $C(z_b)$. A simple scheme (to be referred as scheme a) of computation of μ_{-1} and μ_1 for $x_\ell \neq x_m$, can now be stated as follows:

1. Compute the parameters p_ℓ, p_m and

$$A = p_\ell^2 - 1, \quad B = p_m^2 - 1$$

2. If $B \neq 0$ $z^* = \left|\frac{A}{B}\right|^{1/2}$; else $z^* = 0$.

3. If the zone is internal, then:

   3.1 If $A>0$ and $B>0$ and $z_a < z^* \leq z_b$,

   then: $\mu'_{-1} = (AB)^{1/2} + p_\ell p_m$

   else: $\mu'_{-1} = \min[C(z_a), C(z_b)]$

   3.2 Set $\mu_{-1} = \max[-1, \mu'_{-1}]$

   3.3 Set $\mu_1 = 1$.

4. If the zone is external, then:

   4.1 If $A<0$ and $B<0$ and $z_a \leq z^* \leq z_b$

   then: $\mu'_1 = -(AB)^{1/2} + p_\ell p_m$

   else: $\mu'_1 = \max[C(z_a), C(z_b)]$

   4.2 Set $\mu_1 = \min[1, \mu'_1]$

   4.3 Set $\mu_{-1} = -1$.

The assignment of 0 to the value of z^* when $B=0$ at step 2 is arbitrary. This value is used only in the 'if condition' evaluations in steps 3 and 4.

The algorithm presented above is incomplete since it does not treat the cases $x_\ell = a$, $x_m = b$ or $x_\ell = x_m$. In addition it suffers from a kind of numerical instability which originates from a genuine physical situation and thus cannot be eliminated simply by algebraic manipulations. Consider, for instance, the case in which both circles S_ℓ and S_m lie on the same internal zone. If the radius of one of the circles is virtually increased by any amount, the obscuration by the common zone would be complete; on the other hand, if the radius is decreased, there will be no obscurations at all. This can be formalized as follows:

if the value of A is close to zero, small variations of A (even such that are due to round-off errors) can induce significant variations in the obscuration interval. The same is true for B.

This proposition turns out to be true, but its rigorous verification requires the consideration of a number of different geometrical configurations. We shall restrict ourselves to the verification in a particular but typical case of an envelope ($r_e=\infty$). Let $x_\ell<a<b<x_m$ and $q_\ell = R_\ell$, $q_m>R_m$. We then have (Fig. 3)

$$p_\ell = q_\ell/R_\ell = 1, \quad p_m = q_m/R_m>1$$

$$A = p_\ell^2-1 = 0 , \quad B = p_m^2-1>0$$

$$z^* = \sqrt{\left|\frac{A}{B}\right|} = 0$$

Suppose that a small round-off error δ was committed in the computation of A, and let $A = -\delta$ ($\delta>0$). According to step 3.1 of our algorithm: $\mu'_{-1} = \min[C(z_a), C(z_b)]$, where (see eqns (3.11) and (3.14))

$$C(z) = \frac{1}{2} B z - \frac{1}{2}\frac{\delta}{z} + p_m p_\ell , \text{ and}$$

$$z_a = \frac{R_m}{R_\ell}\ \frac{a-x_\ell}{x_m-a}$$

$$z_b = \frac{R_m}{R_\ell}\ \frac{b-x_\ell}{x_m-b}$$

Since $z_a<z_b$

$$\mu'_{-1} = C(z_a) = \frac{1}{2} B z_a - \frac{1}{2}\frac{\delta}{z_a} + p_m p_\ell \qquad (3.17)$$

The domain of z_a is $(0, +\infty)$, consequently $\frac{\delta}{z_a} > 0$, and its value is indefinite for small values of z_a. Also $\frac{1}{2} B z_a + p_m p_\ell > 1$, it follows then that the range of values of μ'_{-1}, computed from eqn. (3.17), may be larger than the entire interval (-1, +1) while the correct value is greater than 1. This confirms our proposition for the exemplified case. With the exception of A or B = 0 exactly, the chances that a round-off error will affect the calculations is negligible. In order to eliminate the round-off error in the case A or B=0 we adopted the following procedure: whenever the value $|A|$ or $|B| < \varepsilon_R$, we set the corresponding value exactly to zero. The small quantity ε_R should be chosen to be slightly larger than the maximal possible round-off error, and much smaller than the accuracy of the data.

According to the above discussion, we compute C(z) not from eqn. (3.11) but from the equivalent expression

$$C(z) = \frac{1}{2} Bz + \frac{1}{2} A/z + p_\ell p_m \qquad (3.11)'$$

So far we have assumed that $x_\ell < a$ and $b < x_m$. This eliminated infinite values of z and C(z). It occurs, however, in practice, that $x_\ell = a$ and/or $x_m = b$, and we should be able to incorporate these cases in the general scheme of calculation. This can be done by the replacement of a by $a+\varepsilon$, and b by $b-\varepsilon$ whenever $a=x_\ell$ or $b=x_m$ occurs. Again, the small quantity ε should be chosen to be much smaller than the accuracy of the data. It is simpler, however, to change a to $a+\varepsilon$, and b to $b-\varepsilon$, whenever $a<b$, regardless of $a=x_\ell$ and/or $b=x_m$.

It remains to consider the case $x_\ell = x_m$. Figure 4 reveals that only four cases of obscuration are now possible, and all are caused solely by the external radius $r_e(x)$. We may thus assume $r_i = 0$, and consequently get $\mu_{-1} = -1$. μ_1 is then easily computed from Fig. 4.

Consider, e.g., the case illustrated in Fig. 4(a). Here $R_\ell = R_m > r_e$ and $\cos\alpha = r_e/R_m = q_m/R_m = p_m = p_\ell < 1$. Consequently $\cos 2\alpha = 2\cos^2\alpha - 1 = 2p_m^2 - 1$. This is to say $\mu_1 = 2p_m^2 - 1$. We observe now that the same value for μ_1 could be obtained if the then clause of step 3.1 in our algorithm were used. This result is generally true, and is easily verified for the remaining cases b, c and d of Fig. 4. Before presenting the final version of the algorithm we note that steps 2 and 3 of the former version (scheme a) can be unified, by the introduction of a zone parameter J, whose value is -1 for an internal zone, and +1 for an external zone. Before executing the algorithm the zone parameters p_ℓ, p_m, z_a, z_b, J, and the coordinates of the circles x_ℓ and x_m have to be defined.

The final version of the algorithm is as follows:

1. Compute:

$$A = p_\ell^2 - 1, \quad B = p_m^2 - 1$$

2. If $|A| < \varepsilon_R$ then $A=0$.

3. If $|B| < \varepsilon_R$ then $B=0$.

4. If $x_\ell = x_m$, then:

   4.1 $\mu_{-1} = -1$,

   4.2 $\mu_1 = (AB)^{1/2} + p_\ell p_m$.

5. If $x_\ell \neq x_m$ then:

   5.1 replace a by $a+\varepsilon$ and b by $b-\varepsilon$,

   5.2 if $B\neq 0$ then $z^* = \left|\frac{A}{B}\right|^{1/2}$; else $z^* = 0$,

   5.3 if $JA<0$ and $JB<0$ and $z_a \leq z^* \leq z_b$,

   then $\mu'_J = -J(AB)^{1/2} + p_\ell p_m$;

   else $\mu'_J = J\max[JC(z_a), JC(z_b)]$;

5.4 let $\mu_J = J \cdot \min[1, J\mu_J']$,

$$\mu_{-J} = -J.$$

We stress again the importance of replacing eqn. (3.11) by eqn. (3.11)' in the evaluation of the function C(z) in step 5.3 above.

There is still one subtle point left to consider. It often occurs that radiating surfaces which are perpendicular to the axis are present. Such surfaces were apparently omitted from our treatment (their profile cannot be represented by eqn. (3.10)). They were, however, tacitly taken into account. Indeed, any vertical surface can be considered as an opaque ring created by a vertical section at the edge of some layer, and all constituent rings were taken into account in our scheme.

IV. Conclusion

In a discrete simulation of a radiative transfer problem, an obscuration calculation has to be done for any pair of the discrete surface elements. An obscuration calculation has thus to be invoked $\frac{1}{2}$ N(N+1) times where N is the number of such elements. An obscuration calculation itself requires consideration on the average of $\frac{1}{2}$ M obscuring surfaces. In a straightforward approach M=N. In our approach M = number of zones, which is usually much smaller than N.

Our method is conceptually new, being based on the idea that an opaque layer can be represented by a continuum of opaque rings. This idea allowed the construction of a compact and general algorithm, with the help of which one can mechanize the computation of obscurations in axisymmetric bodies of arbitrary complexity. This method was indeed utilized in a comprehensive computer program.

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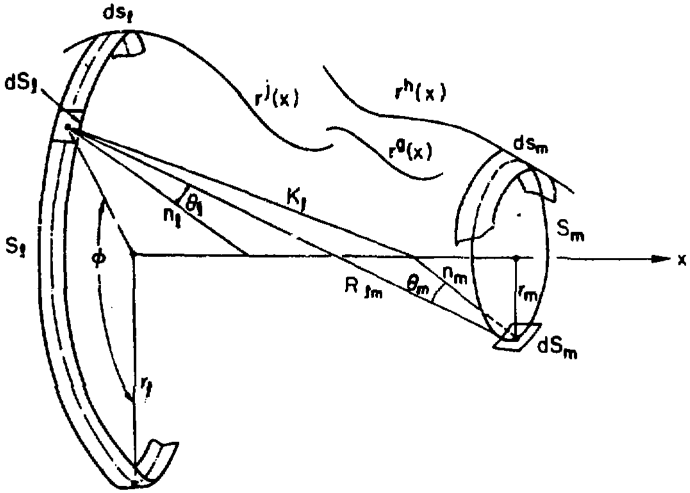

Fig. 1

Calculation of the shape factor between two rings.

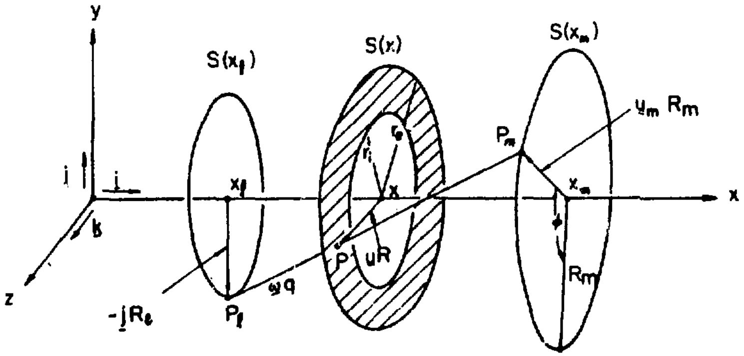

Fig. 2

Obscuration of the line of sight $p_\ell p_m$ produced by the ring S(x).

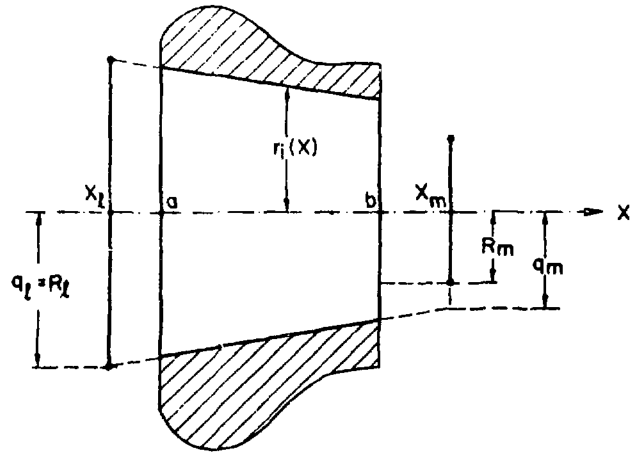

Fig. 3

Numerical instability occurring in an envelope for the case A=0.

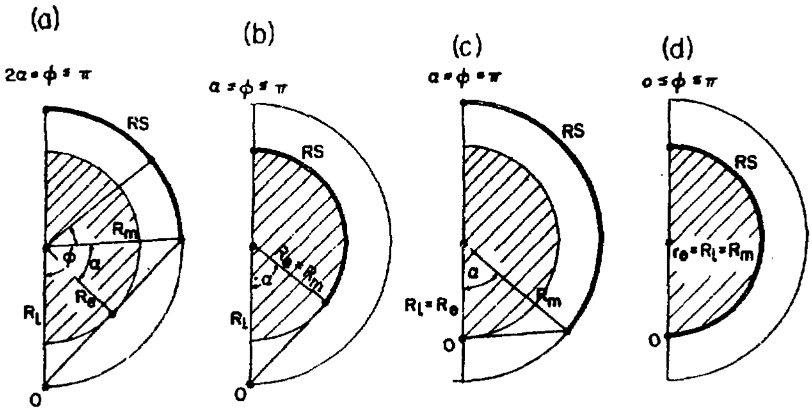

Fig. 4

Obscurations at $x_\ell = x_m$ $(q_\ell = q_m = r_e)$

O - point of observation

φ - central angles at which obscurations occur

RS - radiating surface

═══ - obscured surface

(hatched area) - obscuring body

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Analysis of radiative heat transfer in the presence of obscurations.

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Numerical simulation of radiative heat transfer problems in general axisymmetric geometry in the presence of an active gas is considered. Such simulation requires subdivision of the radiating surfaces into discrete

elements, which are in the present case radiating rings. While the effect of a participating medium is easily taken into account by integration along the lines of vision between the surface elements, the calculation of the different obscurations poses the main difficulty. We have written a closed expression which formulates the problem exactly, and then developed a systematic and compact computational approach to the obscuration problem in complex configurations. The present procedure is particularly suited to computer calculations associated with engineering applications in the aircraft and furnace industries.