5660/81

IC/81/233
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SYMMETRIC POSITIVE DIFFERENTIAL EQUATIONS AND FIRST ORDER HYPERBOLIC SYSTEMS *

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MIRAMARE - TRIESTE
December 1981

* To be submitted for publication.

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Abstract

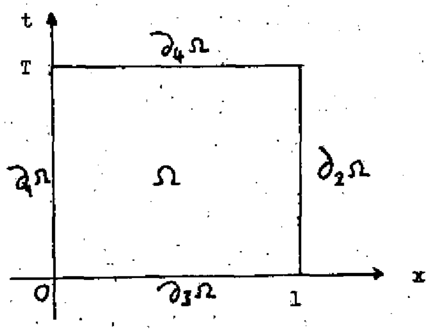
We prove that under some conditions the first order hyperbolic system and its associated mixed initial boundary conditions considered, for example, in Kreiss [2], Kreiss and Gustafsson [3] can be transformed into a symmetric positive system of P.D.E.'s with admissible boundary conditions of Friedrich's type [1].

Introduction

Let $\Omega =]0,1[\times]0,T[$, $T > 0$ and $\partial\Omega$ be the boundary of Ω

where $\partial_1\Omega = \{0\} \times [0,T]$, $\partial_2\Omega = \{1\} \times [0,T]$,

$\partial_3\Omega = [0,1] \times \{0\}$ and , $\partial_4\Omega = [0,1] \times \{T\}$



Let A_1 be a diagonal matrix of order n of the form

$$A_1 = \begin{pmatrix} A^I & 0 \\ 0 & -A^{II} \end{pmatrix}$$

$$A^I = \begin{pmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_r \end{pmatrix}$$

$$A^{II} = \begin{pmatrix} a_{r+1} & & 0 \\ & \ddots & \\ 0 & & a_n \end{pmatrix}$$

where $0 < a_i \leq a_{i+1}$, $i = 1,2,\ldots\ldots,r-1$

$0 < a_{j+1} \leq a_j$ $\quad j = r+1,\ldots\ldots,n-1$, $r < n$

Let A_2 be an $n \times n$ matrix .

Let f and v be vector functions : $\Omega \longrightarrow R^n$,

$$v = \begin{pmatrix} v_I \\ v^{II} \end{pmatrix} , \quad f = \begin{pmatrix} f^I \\ f^{II} \end{pmatrix}$$

$v^I = (v^1, v^2, \ldots\ldots v^r)^T$, $\quad f^I = (f^1, f^2, \ldots\ldots f^r)^T$

$v^{II} = (v^{r+1}, v^{r+2}, \ldots v^n)^T$, $\quad f^{II} = (f^{r+1}, f^{r+2}, \ldots\ldots f^n)^T$.

A model problem for hyperbolic systems can be written as (see Kreiss [2].)

$$(1) \qquad \frac{\partial v}{\partial t} + A_1 \frac{\partial v}{\partial x} + A_2 v = f$$

with the initial and boundary conditions

$$(2) \qquad \begin{aligned} v(x,0) &= 0 && \text{on } \partial_3\Omega \\ v^I(0,t) &= \alpha v^{II}(0,t) && \text{on } \partial_1\Omega \\ v^{II}(1,t) &= \beta v^I(1,t) && \text{on } \partial_2\Omega \end{aligned}$$

where α and β are rectangular matrices. The definition of a symmetric positive P.D.E.'s and its associated admissible boundary conditions was given by K.O. Friedrichs [1], Lesaint [4] as follows.

Let Ω be a bounded open subset of R^m with piecewise continuously differentiable boundary $\partial\Omega$. A lineary differential operator $A \in \mathcal{L}(R^n)$ defined by

$$(3) \qquad A = \sum_{i=1}^{m} A_i(x) \frac{\partial}{\partial x_i} + A_0(x) , \quad x \in \Omega$$

is said to be symmetric positive if for $i = 1,2,\ldots\ldots,m$, $A_i(x)$ is a symmetric matrix Lipschitz continuous in x and there exists a constant

$C_o > 0$ such that

(4) $$A_o(x) + A_o^*(x) - \sum_{i=1}^{m} \frac{\partial}{\partial x_i} A_i(x) - C_o I \geq 0$$

where I denotes the identity matrix of order n and $A_o^*(x)$ is the adjoint matrix of $A_o(x)$. Let $u : \Omega \longrightarrow R^n$.

A system of differential equations

(5) $$Au = f \quad \text{in } \Omega$$

is said to be a symmetric positive system of O.D.E. if the operator A is symmetric positive.

Let a matrix B be defined on $\partial\Omega$ by

(6) $$B(x) = \sum_{i=1}^{m} \nu_i A_i(x)$$

where ν_i, $i = 1,2,\ldots\ldots m$ are the components of the outer normal on $\partial\Omega$. The matrix $M(x) \in \mathcal{L}(R^n)$ be defined and continuous on $\partial\Omega$ and assume that

(7) $$\begin{cases} \text{(i)} \ M + M^* \geq 0 \\ \text{(ii)} \ \mathrm{Ker}\,(B - M) + \mathrm{Ker}\,(B + M) = R^n . \end{cases}$$

Then the admissible boundary conditions associated with the system (5) are given in the form

(8) $$(B - M)\, u = 0 \qquad \text{on } \partial\Omega .$$

<u>Transformations and theorems</u>

We assume that there exist constants $\lambda_o > 0$ and $C_o > 0$ satisfying

(9) $$A_o + A_o^* \geq C_o I$$

where $A_o = A_2 + \lambda_o I$.

Let $u(x,t) = e^{-\lambda_o t} v(x,t)$ then the system (1) is transformed into a symmetric positive system

(10) $$Au = u_t + A_1 u_x + A_o u = F$$

where $F = e^{-\lambda_o t} f$.

Note that if u is a solution of (10) then $v = e^{\lambda_o t} u$ is a solution of (1).

Let the matrices D_1 and D_2 be defined by

(11) $$\begin{aligned} D_1 &= (A^I \alpha)^* (A^I \alpha) - a_1 a_n I_{n-r} \\ D_2 &= (A^{II} \beta)^* (A^{II} \beta) - a_1 a_n I_r \end{aligned}$$

where I_r is the identity matrix of order r. We have the following theorems.

<u>Theorem 1</u>

A sufficient condition for the existence of an M satisfying (7) and that (2) can be written in the form (8) is the negative definiteness of D_1 and D_2.

This M is given by

$$M = \begin{pmatrix} A^I, & -2A^I\alpha \\ 0, & A^{II} \end{pmatrix} \qquad \text{on } \partial_1\Omega$$

$$M = \begin{pmatrix} A^I, & 0 \\ -2A^{II}\beta, & A^{II} \end{pmatrix} \qquad \text{on } \partial_2\Omega$$

$$M = I \qquad \text{on } \partial_3\Omega \cup \partial_4\Omega$$

<u>Proof</u>

Let $\{e_i\}$, $i = 1,2 \ldots\ldots r$ and $\{e_j\}$, $j = 1,2,\ldots\ldots n-r$, be bases for the column vectors in R^r and R^{n-r} respectively. Then the set

$$\left\{ \begin{pmatrix} e_i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ e_j \end{pmatrix} \right\}, \quad i = 1,2,\ldots\ldots r, \ j = 1,2,\ldots\ldots n - r$$

forms a basis for the column vectors in R^n.

Let

$$M = \begin{pmatrix} M_{11} & , & M_{12} \\ M_{21} & , & M_{22} \end{pmatrix}$$

where M_{11}, M_{12}, M_{21} and M_{22} are matrices, M_{11} is of order r , M_{22} is of order (n-r) , M_{12} is of order r x (n-r) and M_{21} is of order (n-r) x r.

Consider first the matrix B on the boundary $\partial_1\Omega$,

we have

$$B = \begin{pmatrix} -A^I & , & 0 \\ 0 & , & A^{II} \end{pmatrix}.$$

In order to put the condition $u^I = \alpha u^{II}$ on $\partial_1\Omega$ in the form of an admissible boundary condition we must choose M such that

$\text{Ker}\,(B - M) = \text{Ker} \begin{pmatrix} I_r & , & -\alpha \\ 0 & , & 0 \end{pmatrix}$. But since we have

$$(12)\quad \text{Ker} \begin{pmatrix} I_r & , & -\alpha \\ 0 & , & 0 \end{pmatrix} = \left\{ u : u = \begin{pmatrix} \alpha u^{II} \\ u^{II} \end{pmatrix} \right\}$$

$$= \text{Span} \left\{ \begin{pmatrix} \alpha e'_j \\ e'_j \end{pmatrix} \right\},$$

we see that $\begin{pmatrix} \alpha e'_j \\ e'_j \end{pmatrix} \in \text{Ker}\,(B - M)$

for all j = 1,2,........n-r . This implies that

$$(13)\quad -(A^I + M_{11})\,\alpha e'_j - M_{12} e'_j = 0$$

$$-M_{21}\,\alpha e'_j + (A^{II} - M_{22})\, e'_j = 0 \quad , \quad \forall j$$

From the above equations we have

$$M_{12} = (-A^I - M_{11})\,\alpha$$

$$M_{22} = (A^{II} - M_{21}\alpha),$$

that is

$$M = \begin{pmatrix} M_{11} & , & -(A^I + M_{11})\alpha \\ M_{21} & , & (A^{II} - M_{21}\alpha) \end{pmatrix}.$$

Furthermore, from (12), we have

$$(14)\quad \dim \text{Ker}\,(B - M) = n - r .$$

So, if we can show that

$$(15)\quad \begin{aligned} &\dim \text{Ker}\,(B + M) = r \\ &\text{Ker}\,(B - M) \cap \text{Ker}\,(B + M) = \underline{0} \end{aligned}$$

then the condition (7) (ii) holds.

In order to get (15) we choose $M_{11} = A^I$ and $M_{21} = 0$.

Thus we have

$$(16)\quad M = \begin{pmatrix} A^I & , & -2A^I\alpha \\ 0 & , & A^{II} \end{pmatrix}$$

$$(B + M) = \begin{pmatrix} 0 & , & -2A^I\alpha \\ 0 & , & 2A^{II} \end{pmatrix}$$

$$(17)\quad \text{Ker}\,(B + M) = \left\{ u : u = \begin{pmatrix} u^I \\ 0 \end{pmatrix} \right\}$$

$$= \text{span} \left\{ \begin{pmatrix} e_i \\ 0 \end{pmatrix} \right\}, \ i = 1,2 ,........r ,$$

This implies that $\dim \text{Ker}\,(B + M) = r$.

Now take any $u \in \text{Ker}\,(B - M)$, i.e $u = \begin{pmatrix} \alpha u^{II} \\ u^{II} \end{pmatrix}$,

then

$$(B + M)\,u = \underline{0} \text{ if and only if}$$

$$A^{II} u^{II} = \underline{0} \quad \text{and} \quad A^I \alpha u^{II} = \underline{0}$$

This implies $u^{II} = \underline{0}$ since A^{II} is non singular. Hence u = 0 and $\text{Ker}\,(B - M) \cap \text{Ker}\,(B + M) = \underline{0}$. Let us check the condition (7) (i) for such an M given in (16). We have

$$(18)\quad M + M^* = \begin{pmatrix} 2A^I & , & -2A^I\alpha \\ -2(A^I\alpha)^* & , & 2A^{II} \end{pmatrix}$$

For $u \in R^n$,

$$\langle (M+M^*)u,u\rangle = (u^I, u^{II})^T \begin{pmatrix} 2A^I, & -2A^I\alpha \\ -2(A^I\alpha)^*, & 2A^{II}\end{pmatrix}\begin{pmatrix}u^I\\ u^{II}\end{pmatrix}$$

$$= 2\left\{\langle A^I u^I, u^I\rangle + \langle A^{II}u^{II},u^{II}\rangle - 2\langle (A^I\alpha u^{II}, u^I\rangle\right\}$$

$$\geqslant 2\left\{a_1\|u^I\|^2 + a_n\|u^{II}\|^2 - 2\langle A^I\alpha u^{II},u^I\rangle\right\}$$

$$\geqslant 2\left\{2\sqrt{a_1a_n}\,\|u^I\|\,\|u^{II}\| - 2\langle A^I\alpha u^{II}, u^I\rangle\right\}$$

$$\geqslant 4\left\{\sqrt{a_1a_n}\,\|u^I\|\,\|u^{II}\| - \|A^I\alpha\|\,\|u^{II}\|\,\|u^I\|\right\} =$$

$$4\left\{(\sqrt{a_1a_n} - \|A^I\alpha\|)\,\|u^I\|\,\|u^{II}\|\right\}.$$

Hence if

(19) $(\sqrt{a_1a_n} - \|A^I\alpha\|) \geqslant 0$ then the condition (7) (i) holds.

Now we shall show that the condition (19) holds if and only if $D_1 \leqslant 0$.

(i) If $D_1 \leq 0$, then for all $v \in R^{n-r}$ we have

$$\langle v, D_1 v\rangle \leqslant 0$$

$$\langle v, ((A^I\alpha)^*(A^I\alpha) - a_1a_nI_{n-r})v\rangle \leqslant 0$$

$$\langle A^I\alpha v, A^I\alpha v\rangle \leqslant \langle v, a_1a_n v\rangle = a_1a_n\langle v,v\rangle .$$

Therefore from the above inequality we have

(20) $$(\|A^I\alpha\|)^2 = \sup_{\substack{v\in R^{n-r}\\ v\neq 0}} \frac{\langle A^I\alpha v, A^I\alpha v\rangle}{\langle v,v\rangle} \leqslant a_1a_n .$$

The inequality (20) implies the inequality (19).

(ii) If $\sqrt{a_1a_n} - (\|A^I\alpha\|) \geqslant 0$ one can easily show that this implies $D_1 \leqslant 0$.

Similar statements to those in the above proof can be used to verify the expression for M on $\partial_2\Omega$. For M on $\partial_3\Omega$ and $\partial_4\Omega$, since we have $B = -I < 0$ and $B = I > 0$ respectively, the results follow immediately.

Thus the theorem is proved. □

Theorem 2

In the case of $n = 2$, $r = 1$, a necessary condition for the existence of an M satisfying (7) and that (2) can be written in the form (8) is the non positive definiteness of D_1 and D_2.

Proof

Suppose that D_1 and D_2 are positive definite, that is,

(21) $$|\alpha| > \sqrt{a_2/a_1}$$

$$|\beta| > \sqrt{a_1/a_2}$$

We shall show that under these conditions we cannot find an M satisfying the condition (7) (i).

Consider first the boundary $\partial_1\Omega$. We have

$$B = \begin{pmatrix} -a_1, & 0 \\ 0, & a_2 \end{pmatrix}$$

Let $M = \begin{pmatrix} m_{11}, & m_{12} \\ m_{21}, & m_{22} \end{pmatrix}$ and suppose that

$u_1 = \alpha u_2$ belongs to Ker $(B - M)$.

Then we have

$$m_{12} = -(a_1 + m_{11})\alpha$$

$$m_{22} = (a_2 - m_{21}\alpha) .$$

Since B is neither positive nor negative semidefinite we can show that if the conditions (7) hold we have

$\det(B - M) = 0$ and

$\det(B + M) = 0$.

Hence we have

$$a_1\alpha m_{21} = a_2(a_1 - m_{11})$$

(i) If $m_{21} = 0$ then

$$m_{11} = a_1$$

Hence

$$M + M^* = \begin{pmatrix} 2a_1 , & -2a_1\alpha \\ -2a_1\alpha , & 2a_2 \end{pmatrix} \text{ and}$$

$M + M^* \geqslant 0$ if and only if $|\alpha| \leqslant \sqrt{a_2/a_1}$.

This is a contradiction.

(ii) If $m_{21} \neq 0$ then we have

$$m_{21} = \frac{a_2(a_1 - m_{11})}{a_1\alpha}$$

This gives

$$M + M^* = \begin{pmatrix} 2m_{11} , & -(a_1 + m_{11})\alpha + \frac{a_2}{a_1\alpha}(a_1 - m_{11}) \\ -(a_1 + m_{11})\alpha + \frac{a_2(a_1 - m_{11})}{a_1\alpha} , & \frac{2a_2 m_{11}}{a_1} \end{pmatrix}$$

We can show that $M + M^* \geqslant 0$ if and only if

$$|\alpha| \leq \sqrt{a_2/a_1}$$

This is also a contradiction. Similar statements to the above proof can be used to verify that if $D_2 > 0$ on $\partial_2\Omega$ then we cannot find an M satisfying the condition (7) (i). The theorem is proved. □

<u>Examples</u>

In this section we will show that many of the usual ordinary and partial differential equations may be expressed in the form (1), (2) and hence in symmetric positive form. The standard initial and boundary conditions may also be expressed as an admissible boundary condition..

<u>Example 1</u> Let $\Omega =]0, 1[\times]0, 1[$ and consider the wave equation

(22) $$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x^2} = f \quad \text{in } \Omega$$

(23)
$$v(x,0) = 0 , \quad v_t(x,0) = 0 \qquad (23.1)$$
$$v(0,t) = 0 , \quad v(1,t) = 0 \qquad (23.2)$$

Let $v_1 = \frac{\partial v}{\partial t}$, $v_2 = \frac{\partial v}{\partial x}$

$$V = (v_1 \quad v_2)^T , \quad S = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \text{ and}$$

$U = e^{-\lambda t} SV$ for constant $\lambda > 0$, then the equation (22) is transformed into a symmetric positive system:

(24) $$\frac{\partial U}{\partial t} + A_1 \frac{\partial U}{\partial x} + \lambda U = F \quad \text{in } \Omega$$

where $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $U = \begin{pmatrix} u^I \\ u^{II} \end{pmatrix}$

$u^I = e^{-\lambda t}(-v_1 + v_2)$, $u^{II} = e^{-\lambda t}(v_1 + v_2)$

and $F = e^{-\lambda t}\begin{pmatrix} -f \\ f \end{pmatrix}$. The condition (23) is transformed into

(25)
$$U(x,0) = 0$$
$$u^I(0,t) = u^{II}(0,t)$$
$$u^{II}(1,t) = u^I(1,t) .$$

The matrix B associated with the system (24), is

$$B(x,0) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B(0,t) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B(1,t) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B(x,1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The expression for the matrix M can be found, using Theorem 2 with $\alpha = 1$ and $\beta = 1$,

$$M(x,0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M(0,t) = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

$$M(1,t) = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \quad \text{and } M(x,1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The condition (25) can then be written as an admissible boundary condition

(26) $(B - M)\ U = 0$ on $\partial\Omega$.

Example 2 Let $\Omega =]a, b[$ and consider the following equation

(27) $$- \frac{d^2}{dx^2} u(x) + a_1(x) \frac{d}{dx} u(x) + a_o(x) u(x) = f(x)$$

for $x \in \Omega$ with one of the following pairs of conditions

(28)

$u(a) = 0$, $u(b) = 0$ (28.1)

$\frac{d}{dx} u(a) = 0$, $\frac{d}{dx} u(b) = 0$ (28.2)

$u(a) = 0$, $\frac{d}{dx} u(b) = 0$ (28.3)

$\frac{d}{dx} u(a) = 0$, $u(b) = 0$ (28.4)

Let $U = (u, \frac{du}{dx})^T$, $S = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

and $W = e^{-\lambda x} SU$, then the equation (27) is transformed into

(29) $$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{d}{dx} W + \begin{pmatrix} P + \lambda & R \\ C & Q - \lambda \end{pmatrix} W = e^{-\lambda x} \begin{pmatrix} -f \\ f \end{pmatrix}$$

where

$$P = \tfrac{1}{2}(a_o - a_1 + 1), \qquad R = \tfrac{1}{2}(a_o - a_1 + 1)$$

$$C = \tfrac{1}{2}(-a_o + a_1 + 1), \qquad Q = \tfrac{1}{2}(a_o + a_1 + 1)$$

The boundary condition (28) can be written as

(30)
$$w^I(a) = \alpha\ w^{II}(a)$$
$$w^{II}(b) = \beta\ w^I(b)$$

$W = (w^I \quad w^{II})^T$, $w^I = e^{-\lambda x}(-u + \frac{d}{dx} u)$ and $w^{II} = e^{-\lambda x}(u + \frac{d}{dx} u)$.

When $\alpha = 1$, $\beta = 1$ the condition (30) is equivalent to (28.1) and it is also equivalent to (28.2), (28.3) and (28.4) when $\alpha = -1$, $\beta = 1$, $\alpha = 1$ $\beta = -1$, and $\alpha = -1$ $\beta = 1$ respectively.

Choosing λ so that the matrix

$$\begin{pmatrix} 2(P + \lambda) & R + C \\ R + C & 2(Q - \lambda) \end{pmatrix}$$ is positive semidefinite, the system

(29) is then a symmetric positive system. From the system (29) and Theorem 2 we can find the expression for B and M, we get

$$B(a) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B(b) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$M(a) = \begin{pmatrix} 1 & -2\alpha \\ 0 & 1 \end{pmatrix}, \quad M(b) = \begin{pmatrix} 1 & 0 \\ -2\beta & 1 \end{pmatrix}$$

for $|\alpha| \leq 1$, and $|\beta| \leq 1$. Thus the condition (30) is the admissible boundary condition :

(31) $(B - M)\ W\big|_a = 0$ and

$(B - M)\ W\big|_b = 0$.

Conclusion

In a forthcoming paper we will deal with a numerical solution of this type of problem , using finite element techniques.

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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