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THE QUESTION OF AN UPPER BOUND ON ENTROPY *

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ABSTRACT

We discuss the possibility, and significance, of an upper bound on entropy in the light of the arguments of Bekenstein and Unruh and Wald. We obtain a stricter bound than Bekenstein does, and point out some limitations with regard to its significance.

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Bekenstein has recently proposed [1)-3)] that there exists a universal upper bound for the entropy of a system of energy E and maximum length scale R,

$$S_{>} = 2\pi RE \qquad (1)$$

(in the natural units $c = \hbar = k = 1$) with the maximum entropy being achieved by a black hole. His original argument was based on a contradiction with the second law of black hole thermodynamics if the bound did not apply. Unruh and Wald [4)] have argued that no such contradiction arises if the argument is considered in full detail. Thorne [5)] has derived an internal contradiction within Bekenstein's argument by considering a non-cubic rectangular box being lowered into a black hole, and showing that the smaller, rather than the larger, length scale is required by Bekenstein's argument in that case. Since the smaller length scale could not apply in any case (it is violated by observation) he argues that Bekenstein's bound is not valid.

We contend that the essence of Bekenstein's argument is not limited by black hole considerations. It is merely a counting of the maximum number of states in a finite system having a finite energy. Let the volume of the system be V and the energy E. Then the phase space volume is certainly bounded above by E^3V since the momentum is bounded above by energy. For definiteness consider a cubic box of length L. By the uncertainty principle, applied to a rigid box, the smallest momentum interval that can be used meaningfully in a statistical analysis is $1/2L$. Similarly, since the momentum has an upper bound E, the minimum spatial interval which we can use is $1/2E$. Thus the minimum volume of an element in phase space is $1/(4EL)^3$. Thus the maximum number of distinct states that can be meaningfully discussed is

$$N_{>} = (8E^3V)^2 \qquad (2)$$

which gives an upper bound of entropy. It is easily verified that the same result is obtained with a rectangular box or a spherical box. Taking the spherical shape for convenience, we get

$$S_{>} = 6\, \ell n\, [ER(16\pi/3)^{1/3}] \quad . \qquad (3)$$

It is interesting to note that the bound obtained here is different from that obtained by Bekenstein, being logarithmic instead of linear. Clearly, if ER is large the bound given by Eq.(3) is much stricter than that given by Eq.(1). Also, it is not meaningful to take $ER < 1$. It is not clear how seriously the bound should be taken at $ER \approx 1$. However, if we take the numerical coefficients given above seriously we get the same order of magnitude here as given by Bekenstein's bound. (In fact, if we take the values obtained in the two cases exactly, Eq.(3) gives 5.6 while Eq.(1) gives 6.3. However, the actual value obtained by Eq.(3) may not be so reliable). Thus we see that we can obtain a bound for the maximum entropy without reference to black holes, and it is stronger than the bound discussed by Bekenstein.

Bekenstein has discussed the application of his bound to black holes and to the number of possible families of fermions. For a black hole, using Planck units (i.e. G also taken to be unity)

$$N_{>}^{bh} = (2\pi/3)^2 R^{12} \ , \quad S_{>}^{bh} = 2 \, \ln \, (2\pi R^6/3) \quad . \qquad (4)$$

The entropy usually assigned for a black hole is greater than the maximum entropy permissable by one bound! This is not really very surprising. Implicit in our bound is the assumption that the black hole is like a "rigid box". However, the usual assignment of the entropy of a black hole comes from a consideration of Hawking radiation, i.e. regarding it as a "leaky box". Thus there is no contradiction between the usual assignment of entropy and our bound. It must be pointed out, here, that our argument does not "prove that black holes radiate", but merely that the usual assignment of entropy requires that they radiate. It is not clear to us how seriously the application of any such considerations to black holes should be taken.

Even more problematic is the significance of any thermodynamic considerations to individual elementary particles and applying thermodynamics to counting states for internal symmetries [3]. The argument for doing so depends on an identification of thermal entropy with "internal symmetry entropy" which comes from Hawking radiation considerations. Despite the doubts about the applicability of our bound to elementary particles we present the results for the sake of comparison with Bekenstein's results. If we have a truly elementary particle it can be completely contained in a sphere of a radius equal to its Compton wavelength. Thus, for the elementary particle we would get

$$N_{>}^{ep} = (2\pi/3)^2, \ S_{>}^{ep} = 2 \, \ln(2\pi/3) \qquad (5)$$

since the number of states is an integer not greater than N^{ep}, we find that our bound gives a limit of four internal states for a truly elementary particle.

It has been argued [6] that we can have negative energy systems (like cavities) which violate Bekenstein's bound. Clearly such systems would also violate our bounds if taken literally. In fact, if we look at the derivation of our bound it is the magnitude of the energy of the system which is relevant and not its sign. There is anyhow a fallacy in the argument [6]. The negative energy cavity is an open system. The bound can only apply to a closed thermodynamic system. Thus, the walls of the cavity would have to be included as well. The complete system is of positive energy and so the argument [6] does not apply. The fallacy here is like the fallacy of arguing that the usual second law of thermodynamics is violated by a refrigerator. We, therefore, contend that our bound can, and does, apply in a regime where quantum statistics in its usual form applies.

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