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MERONS IN A GENERALLY COVARIANT MODEL WITH GÜRSEY TERM *

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ABSTRACT

We study meron solutions of the generally covariant and Weyl invariant fermionic model with Gürsey term. We found that, due to the presence of this term, merons can exist even without the cosmological constant. This is a new feature compared to previously studied models.

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Recently there was a revival of interest in the Gürsey model [1] proposed in 1956 as the only possible conformally invariant pure spinor model in four-dimensions containing only first order derivatives. Exact solutions of the model found by Kortel [2] long ago, with Heisenberg [3] ansatze, are instanton and meron-type solutions corresponding to spontaneous breaking of the conformal invariance [4,5]. The Poisson bracket structure of this model has been proposed by the introduction of auxiliary scalar fields and using the Dirac method for constraint systems as a possible passage to a quantum level [6].

A formulation of a generally covariant and Weyl invariant spinor model was given [7], but without pure fermionic interactions in its Weyl invariant formulation. Previous study of generally covariant models [7,8] led to a different treatment of instanton and meron solutions. Namely, the presence of instanton solutions was possible even without a cosmological constant. While in the meron case, solutions could not exist without a cosmological constant [8]. There were also attempts to formulate conformally invariant coupling of matter to gravity through rescaling of the gravitational field [9]. But the matter fields have non canonical dimension, and, especially in the case of the fermion field, such a dimension is zero and we could, in principle, have any power of self-interaction [7]. However, the meaning of such models is unclear as they possess no flat space limit.

In this paper we shall study a fermionic model coupled to gravity in its Weyl invariant formulation. We prefer Weyl formulation to the previously mentioned one because it keeps canonical dimensions of fields, and thus is more physical.

In order to work with fermions one has to introduce vierbien fields [10], as

$$g_{\mu\nu} = e_\mu^a e_{a\nu}\,; \quad \delta_{ab} = e_{a\mu} e^\mu{}_b \quad . \tag{1}$$

Latin indices refer to Lorentz degrees of freedom and Greek indices refer to covarient degrees of freedom. This leads to the definition of the generally covariant γ-matrices as

$$\gamma^\mu_{(x)} = e^\mu_a \gamma^a\,; \quad \{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu} \; . \tag{2}$$

Then, following the usual approach, one can build up a generally covariant Lagrangian. We can safely introduce Gürsey term in the Lagrangian because its flat space limit is equivalent to the Gürsey model.

Weyl transformations are defined as usual

$$\begin{aligned} g_{\mu\nu} &\mapsto e^{2\lambda} g_{\mu\nu} \\ g^{\mu\nu} &\mapsto e^{-2\lambda} g^{\mu\nu} \\ \psi &\mapsto e^{-d\lambda} \psi \end{aligned} \tag{3}$$

where d is canonical dimension of the matter field ψ. The conformally and Weyl invariant action consists in coupling the Einstein scalar curvature R, the spinor field ψ with an auxiliary scalar field φ and Gürsey term

$$\begin{aligned} A = \int d^4x \sqrt{g}\, \Big\{ & \frac{i}{2} \bar\psi \gamma^\mu \overleftrightarrow{\nabla}_\mu \psi + f \bar\psi \psi \varphi + \frac{3}{4} k (\bar\psi\psi)^{4/3} + \\ & + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{\varphi^2}{12} R - \frac{\lambda}{4} \varphi^4 \Big\} \; , \end{aligned} \tag{4}$$

where λ is a cosmological constant and

$$\nabla_\mu = \partial_\mu + \frac{1}{2} \omega_\mu{}^{ab} \sigma^{ab} \tag{5}$$

and,

$$\begin{aligned} \omega_\mu{}^{ab} = & \frac{1}{2} (\partial_\mu e_\nu{}^b - \partial_\nu e_\mu{}^b) e^{a\nu} + \\ & + \frac{1}{4} e^{a\rho} e^{b\omega} (\partial_\omega e_\rho{}^c - \partial_\rho e_\omega{}^c) e_\mu^c - (a \leftrightarrow b) \end{aligned} \tag{6}$$

is the spin connection and

$$\sigma^{ab} = \frac{1}{4} [\gamma^a, \gamma^b] \; . \tag{7}$$

Equations of motion are obtained by varying (4) with respect to vierbein and matter fields

$$\begin{aligned} & \frac{\varphi^2}{6} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = T_{\mu\nu} \\ & i \not\nabla \psi + f \varphi \psi + k (\bar\psi\psi)^{1/3} \psi = 0 \\ & \Box \varphi + \lambda \varphi^3 + \frac{\varphi}{6} R - f (\bar\psi\psi) = 0 \end{aligned} \tag{8}$$

with

$$\begin{aligned} T_{\mu\nu} = & \frac{i}{4} \left[\bar\psi \gamma_\mu \overleftrightarrow{\nabla}_\nu \psi + (\mu \leftrightarrow \nu) \right] + \partial_\mu \varphi \partial_\nu \varphi + \\ & + \frac{1}{6} \left[g_{\mu\nu} \Box - \partial_\mu \partial_\nu \right] \varphi^2 - g_{\mu\nu} \mathcal{L} \end{aligned} \tag{9}$$

matter energy-momentum tensor.

We are interested in solutions in a conformally invariant metric and we can use the advantage of Weyl invariance to transform (9) into a flat limit

$$
\begin{aligned}
&T_{\mu\nu} = 0 \\
&\not\partial\psi + f\psi\varphi + k(\bar\psi\psi)^{1/3} = 0 \qquad (10)\\
&\Box\varphi + \lambda\varphi^3 - f(\bar\psi\psi) = 0 \,.
\end{aligned}
$$

These equations will possess instanton and meron solutions. Instanton solutions are rather trival, due to their symmetry which always imposes $T_{\mu\nu}=0$, and they are essentially of the form given in [4], with a constraint on f, k and λ.

However, the condition $T_{\mu\nu}=0$, which is equivalent to solving the Einstein part, requires some restrictions on coupling constants for merons. To find the meron solutions we take the usual ansatze

$$
\varphi = \frac{C_\varphi}{\sqrt{x^2}} \qquad \psi = \frac{1}{(x^2)^{3/4}}\left(1 + \frac{i\gamma x}{\sqrt{x^2}}\right)C_\psi
$$

$$
g_{\mu\nu} = \delta_{\mu\nu} \quad , \qquad (11)
$$

where C_ψ is a constant spinor.

Then, the equations of motion (10) are

$$
T_{\mu\nu} = \frac{1}{x^4}\left[A\delta_{\mu\nu} - B\frac{x_\mu x_\nu}{x^2}\right] = 0 \quad , \qquad (12)
$$

$$
\lambda C_\varphi^3 - C_\varphi - 2f\bar C_\psi C_\psi = 0 \quad , \qquad (13)
$$

$$
(2\bar C_\psi C_\psi)^{1/3} = \frac{1}{k}\left(\frac{3}{2} - fC_\varphi\right) \,. \qquad (14)
$$

where

$$
\begin{aligned}
&A = \frac{C_\varphi^2}{4}\left(\lambda C_\varphi^2 - \frac{2}{3}\right) - \bar C_\psi C_\psi\left[2fC_\varphi - 2 + \frac{3}{2}k(2\bar C_\psi C_\psi)^{1/3}\right] \\
&B = \frac{C_\varphi^2 - 3\bar C_\psi C_\psi}{3} \,. \qquad (15)
\end{aligned}
$$

From (14) we find that

$$
T_{\mu\nu} = \frac{C_\varphi^2 - 3\bar C_\psi C_\psi}{12x^4}\left(\delta_{\mu\nu} - \frac{4x_\mu x_\nu}{x^2}\right) \,. \qquad (16)
$$

To solve the Einstein part we require

$$
C_\varphi^2 = 3\bar C_\psi C_\psi \,. \qquad (17)
$$

Although the real solutions exist for $\lambda \neq 0$, we are interested in the case $\lambda = 0$; from (13) and (14) we find the meron solutions *)

$$
\begin{aligned}
&\varphi = -\frac{3}{2f}\frac{1}{\sqrt{x^2}} \qquad \psi = \frac{1}{(x^2)^{3/4}}\left(1 + \frac{i\gamma x}{\sqrt{x^2}}\right)C_\psi \qquad (18)\\
&g_{\mu\nu} = \delta_{\mu\nu} \qquad\qquad \text{with} \quad \bar C_\psi C_\psi = \frac{3}{4f^2}
\end{aligned}
$$

---

*) We should mention that $\bar C_\psi C_\psi \neq 0$ actually means $\langle 0|\bar\psi\psi|0\rangle \neq 0$ at the quantum level, which is possible even if $\langle 0|\psi|0\rangle = 0$.

with constraint on coupling constants as

$$f^2 = \frac{k^2}{18} \quad . \tag{19}$$

In this paper we have presented a generally covariant and Weyl invariant model including a Gürsey term. The model has instanton solutions with a similar constraint on f, k and λ given in [4]. Furthermore, we found the meron solutions (11) even without cosmological constant with relation (19). Here one may also conclude that, compared to [7], the Gürsey term plays the role of the $\lambda\varphi^4$ term as long as meron solutions are concerned in this model.

It is known that merons are unstable in pure Yang-Mills theory [11] even with fermions present [12]. On the other hand, it was shown that merons are stable in a pure spinor model [13]. Then it is natural to suppose that even in this model the stability will be preserved.

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