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RELATIVISTIC GENERALIZATION OF THE NEWTONIAN FORCE *

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ABSTRACT

Whereas there is no denying the essential contribution of geometrodynamics, it must be admitted that our physical intuition is still firmly based in the Newtonian concept of force. Here we extend some earlier work re-introducing the Newtonian force concept into relativity theory. Some fundamentally new insights into the relativistic effects due to charge and rotation are presented.

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The essence of Einstein's general theory of relativity is to replace Newtonian dynamics by geometrodynamics. Thus the usual concept of "force" is given up and a new concept of the curvature of space-time, introduced. However, most of our physical intuition still relies on the concept of "force". Without in any way denying the fundamental significance of geometrodynamics it has been argued that the re-introduction of the Newtonian concept of "force" would yield new insights into relativity theory and its physical implications [1]. This would apply particularly where detailed calculations are not practicable, but one wants to see the relativistic effects "at a glance" as it were.

It is usually taken for granted that no forces are felt in a freely falling frame (FFF). What is often forgotten is that this statement holds true only for point-like observers. As is very well known, any extended object experiences tidal forces, even in an FFF. In fact the basic identification of "gravitational forces" with space-time curvature relies on the expression of the tidal acceleration vector in terms of the curvature tensor. Further, a genuinely point-like observer would never be able to observe anything anyhow. To be able to perform an experiment, or observe anything, spatial extension is essential. Thus, what we really mean by saying that no forces are experienced in an FFF is, that if the spatial extension of our measuring device is negligible compared to the distance from the gravitational source, then there is no force noticable in the FFF. However, for any given size, a sufficiently accurate (ideal) measuring device (such as is indicated in Fig.1) could always detect the tidal force. We are not, here, wanting to look at the higher order "corrections" due to tidal forces, but merely to use them to make a point of principle clear. The point of principle is that the Newtonian type "gravitational" forces are still meaningful in FFFs. To be able to demonstrate our point we need to be able to relate the Newtonian type force with the observable, tidal, force. In Newtonian mechanics the tidal force is simply the gradient of the magnitude of the central force. The generalization of this result will lead to a relativistic analogue of the Newtonian force, which we shall call the pseudo-Newtonian (ψN) force. All space-times admitting N-forces could, then, be appropriately called N-space-times. These are the space-times which are sufficiently similar to the Minkowski space-time for usual, Newtonian, concepts to be used, with some suitable modification. It is expected that seeing relativistic effects as "corrections" of Newtonian results will lead to some new insights into the significance of the relativistic results. This point will be amplified later.

As will be shown later, space-times admitting two isometries, one along a time-like vector and the other along a space-like vector, are ψN-space-times. (This is not to say that more general space-times, without these symmetries, will necessarily not be ψN.) It turns out that in these space-times the tidal force can be related to the gradient of a quantity which is naturally identifiable as the ψN force. The role of the ψN-force in relativity may be compared, in some sense, with the role of the potential of a conservative field in Newtonian physics. In both cases the physically observable effect is related to the gradient of the quantity concerned. Like the potential leads to a deeper understanding of Newtonian physics, it is not unreasonable to suppose that the ψN-force will provide a further understanding of Einsteinian physics.

It is traditionally supposed that a conformal transformation "switches off" the gravitational field and the FFF physically realizes this "switching off". Now the electromagnetic field can be "switched off" by a complex conformal transformation. However, it is argued, there is no physical equivalent of the FFF in this case to realize that "switching off". This argument leads to the conclusion that there remains a fundamental difference between gravitation and electromagnetism. Essentially, the Einsteinian attempt was a <u>geometric</u> unification of gravitation and electromagntism, but it appeared that the unification was incomplete because of the absence of a physical mechanism to "switch off" electromagnetism. The problem was that the electromagnetic effects were observable in an FFF but the gravitational effects were supposed to be absent. However, since gravitational effects will also be observable by an extended obsever and electromagnetic effects would not be observable by a point-like observer, there appears to be no difference of principle between the two. The only difference apparent seems to be one of degree - the electromagnetic effects would be much easier to detect than the gravitational effects, in that the degree of accuracy required for the measuring device would be much less for electromagnetism than for gravitation. Thus it could be argued that the unification of gravitation and electromagnetism has already been achieved (by Reissner and Nordstrom [2] in fact) in that there will be "gravitational effects" of charges and "electromagnetic effects" of neutral matter (on charges) as has been shown [1].

It may seem at first sight, that with the Glashow-Salam-Weinberg [3] unification of weak and electromagnetic forces, it could be claimed that the three weaker forces in nature have been unified. This claim would not be valid for the following reason. The GSW unification is of a very different

nature (group-theoretic instead of geometrical) and couched in an entirely different language (of quantum field theory). Thus we can only claim the unification of the long-range forces by the Einsteinian approach. To be able to extend this unification using the GSW unification, we need to be able to write the Einstein-Maxwell field in quantum field-theoretic terms. There is reason to hope that the ψN-force will lead to a simpler (in principle) attempt at quantization of relativity. Our expectation is that this approach to quantization will lead to a modification of Hawking's results on radiation from black holes [4] and would be relevant for a possible "great grand unification" of all the four forces of nature. Another place where ψN-forces are expected to lead to some better understanding is in the usual theory of the formation of pulsars. According to the standard model the pulsar is a neutron star formed during a supernova explosion, being the very dense core of the star which explodes. One would, then expect to find a pulsar at the centre of a supernova remnant (unless it has become a black hole) and to find a supernova remnant around all pulsars (unless it has happened to get pulled off by some other star). Anyhow we should generally expect to find the pulsars and supernova remnants very closely related. This is not found to be the case [5]. Only two pulsars (the Crab and Vela) are found within supernova remnants and one more collapsed object (SS433 [5]) may be related to a supernova remnant. If only a few pulsars and a few supernova remnants had been found not related to each other one could try to find some explanation for these exceptional cases. However, when the vast majority are like this a general explanation is required. Preliminary calculations [6] seem to show that the ψN-force may provide a valuable insight into this problem. In any case, expressing relativity in terms of ψN-forces allows us to pose questions which could not have been posed in purely relativistic terms, and to provide answers to some of those questions.

The plan of the paper is as follows. In Sec.II we examine tidal forces in fairly general space-times. In the next section we obtain an expression for the ψN-force. Then, in Sec.IV, we take the Kerr-Newman (KN) geometry as an example for our previous, general, discussion. Finally we give a brief summary and discussion of our results in Sec.V.

II. TIDAL FORCES

It is not clear how a general space-time could be dealt with to yield the required extension of the classical force concept. For our purposes we consider a space-time having two Killing vectors which we denote by $\partial/\partial x^0$ (which is time-like) and $\partial/\partial x^3$ (which is space-like). The metric tensor can then be partitioned into blocks

$$(g_{ab}) = \left(\begin{array}{c|c} g_{ij} & 0 \\ \hline 0 & g_{rs} \end{array}\right) \qquad \left.\begin{array}{l} a,b = 0,1,2,3 \\ i,j = 1,2 \\ r,s = 0,3 \end{array}\right\} \quad . \tag{1}$$

The first block, g_{ij}, can clearly be diagonalized without affecting the block, g_{rs}, which carries the isometries. (It is useful to think of the i,j as two-dimensional polar co-ordinates for concreteness. However, our results do not at all depend on what those co-ordinates may be.)

It is easy to see that if we work in a suitably chosen local Lorentz frame (LLF) the tidal force is just a gradient of the usual Newtonian gravitational force in a Schwarzschild geometry. This result can be extended[1] to a Reissner-Nordstrom geometry. To deal with more general space-times we use Riemann normal co-ordinates[7], which are the co-ordinates for an LLF. These co-ordinates are defined by

$$x^{a'} = x^a - \bar{x}^a + \frac{1}{2}\ \bar{\Gamma}^a_{bc}(x^b-\bar{x}^b)\ (x^c-\bar{x}^c) \quad , \tag{2}$$

where the bar on top merely signifies that the relevant expression is to be evaluated at P, the pole of the Riemann normal co-ordinate system. Thus

$$\overline{(\partial x^{a'}/\partial x^b)} = \overline{(\partial x^a/\partial x^{b'})} = \delta^a_b \tag{3}$$

Obviously, every quantity has the same value in the primed and unprimed co-ordinates at P. In Riemann normal co-ordinates the Riemann tensor can be written as

$$\bar{R}^{a'}{}_{b'c'd'} = \bar{\Gamma}^{a'}{}_{b'c';,d'} - \bar{\Gamma}^{a'}{}_{b'd',c'} \quad . \tag{4}$$

The standard formula for the tidal force (per unit mass of the test particle) is[8]

$$F^{a'}_T = - R^{a'}{}_{b'c'd'}\ t^{b'}\ \ell^{c'}\ t^{d'} \quad , \tag{5}$$

where $t^{a'}$ is a unit time-like vector and $\ell^{a'}$ is a space-like separation vector. Taking $t^{a'}$ to be our Killing vector, Eqs.(3)-(5) lead to the result that

$$\bar{F}^{a'}_T = - \bar{\Gamma}^{a'}{}_{0'0',b'}\ \bar{\ell}^{b'} \quad . \tag{6}$$

Given the Killing vector $\partial/\partial x^0$ only, there appears to be no problem to continue with out formulation of the ψN - forces. However, the formulation at this level of generality leads to certain complications due to which it is not absolutely clear that no problems can arise. To simplify the calculations, and because the simpler physical cases do have an extra symmetry, we assume the existence of a second Killing vector $\partial/\partial x^3$. In this case the index b' could be replaced by an index i' (having the range 1',2'). Using Eq.(3) the primes can now be dropped. Taking the block g_{ij} to be diagonalized and evaluating the connection symbols, the surviving components of the tidal force are seen to be

$$\left.\begin{array}{l} \bar{F}^1_T = \frac{1}{2}\ \bar{g}^{11}\ \bar{g}_{00,1i}\ \ \bar{\ell}^i \\ \bar{F}^2_T = \frac{1}{2}\ g^{22}\ \bar{g}_{00,2i}\ \ \bar{\ell}^i \end{array}\right\} \quad . \tag{7}$$

Let us imagine an observer in his space-ship approaching a large black hole. Traditionally, he is supposed to be unaware of the proximity of the black hole because his ball does not drop when he releases it. However, our observer does notice the presence of the black hole because he is armed with his accelerometer (the measuring device depicted in Fig.1). He detects the tidal force exerted by the black hole due to the swing of the needle on a dial. He now swivels the accelerometer about till he gets a maximum (or minimum) reading. He has now found the principal axes of the tidal force. In terms of matrix algebra, he has placed his accelerometer along an eigenvector of the tensor $-\bar{\Gamma}^i{}_{00,j}$ (in the $x^{1'}x^{2'}$-hyperplane). Let the two eigenvalues be denoted by $\lambda_\pm$. Then Eqs.(6) and (7) yield the eigenvalues

$$\left.\begin{array}{l} \lambda_\pm = \frac{1}{4}\Big[(g^{11}g_{00,11} + g^{22}g_{00,22}) \pm \{(g^{11}g_{00,11} - g^{22}g_{00,22})^2 \\ \qquad\qquad + 4\ g^{11}g^{22}\ (g_{00,12})^2\}^{1/2}\Big] \end{array}\right\} \quad . \tag{8}$$

The corresponding eigenvectors are then given, up to a scale factor determined by the length of the accelerometer, by

$$\ell^1_\pm/\ell^2_\pm = g^{11}g_{00,12}/(2\lambda_\pm - g^{11}g_{00,11}) \quad , \qquad (9)$$

where we have dropped the bar above for convenience, as we shall henceforth always be dealing with quantities at P. It is easily verified that the discriminant in Eq.(8) is always positive outside the event horizon, if one exists, and if no event horizon exists is always positive. Thus $\lambda_+ \geq \lambda_-$, with the equality holding when the discriminant is zero. Taking only the maximum tidal force for our present purpose, we drop the "+" everywhere from now on. If the length of the accelerometer is L we see that

$$g_{11}(\ell^1)^2 + g_{22}(\ell^2)^2 = L^2 \quad . \qquad (10)$$

Eqs.(9) and (10) together determine ℓ^1 and ℓ^2

$$\left.\begin{aligned} \ell^1 &= Lg^{11}(g^{22})^{1/2}\, g_{00,12}[(2\lambda - g^{11}g_{00,11})^2 + g^{11}g^{22}(g_{00,12})^2]^{-1/2} \\ \ell^2 &= L(g^{22})^{1/2}\,(2\lambda - g^{11}g_{00,11})\,[(2 - g^{11}g_{00,11})^2 + g^{11}g^{22}(g_{00,12})^2]^{-1/2} \end{aligned}\right\} \qquad (11)$$

and the tidal force is completely determined by Eqs.(8) and (11) as $F^i_T = \lambda\ell^i$. Its magnitude is thus simply given by λ L. If we take the x^1, x^2 co-ordinates to be usual polar co-ordinates the direction of the accelerometer is given by ℓ^2, as **shown in Fig.2. The** angle the accelerometer makes with the "radial" direction is then, (see Fig.2)

$$\chi_T = \tan^{-1}[(g_{22}/g_{11})^{1/2}\,(\ell^2/\ell^1)] \quad . \qquad (12)$$

It is worth noting here that the problem introduced by removing the extra symmetry, provided by the Killing vector $\partial/\partial x^3$, is that we would have a three-dimensional eigenvalue problem to solve, and hence a cubic equation. The reality and maximality of the relevant roots, and hence the physical interpretation of the results is not so clear in that case.

III. THE ψN-FORCE

We have obtained an expression for the tidal force which can be computed in terms of some of the metric coefficients only. However, we have not so far made explicit the relationship between the tidal force and the ψN-force, whereby our observer in his space-ship could hook up his accelerometer to his ship-board computer and obtain a value for the ψN-force he is experiencing. This result is essentially contained in Eq.(6). If we make use of the geodesic equations we can write

$$F^i_T = \ddot{x}^i_{\ ,j}\, \ell^j \qquad (13)$$

(since the tidal force lies in the x^1x^2 plane, as does the separation vector). In Newtonian physics the tidal force was the gradient, along a specified direction of the Newtonian gravitational force. The Newtonian force (per unit mass) was just the second derivative of the position vector. Here again, the tidal force turns out to be just the gradient, along a particular direction (given by the separation vector) of the second derivative of the position vector. It would seem natural to define the ψN-force as the second derivative of the relevant position vector. However, we must be careful as only the directed gradient is observationally defined and not the ψN-force. There will, in general, be an "integration constant" to be accounted for. A similar problem arises when defining the potential in Newtonian physics. In that case the integration constant is determined by setting the potential equal to zero infinitely far away from the gravitational source, because "we know that there is no effect at an infinite distance". Similarly, when defining the ψN-force, we require that it be zero in Minkowski space "because we know that there are no forces in a Minkowski space". Thus we define the ψN-force, F^i, by

$$F^i = \ddot{x}^i - \ddot{x}^i_M \quad , \qquad (14)$$

where $\ddot{x}^i_M$ is $\ddot{x}^i$ evaluated in Minkowski space. Hence, using the geodesic eqautions, we obtain the expression for the ψN-force

$$F^i = (\Gamma^i_{ab} - \Gamma^i_{Mab})\, \dot{x}^a \dot{x}^b \quad , \qquad (15)$$

where Γ^i_{Mab} is the connection symbol for Minkowski space in the same co-ordinate system as is being used for the other calculations. In Cartesian co-ordinates this quantity is zero, but even in cylindrical polar co-ordinates this quantity is non-zero. This is the generalization of the expression for force which is found to give the usual Newtonian force for a Schwarzschild metric, and a $-Q^2/r^3$ correction to it (where Q is the charge of the gravitating source) in the Reissner-Nordstrom metric [1].

The ψN-force may be regarded as the "Newtonian fiction" which "explains" the same motion (geodesic) as the "Einsteinian reality" of the curved space-time does. We can, thus, translate back to Newtonian terms and concepts where our intuition may be able to lead us to ask, and answer, questions that may not have occurred to us in relativistic terms. Notice that, whereas the tidal force exists in a curved space-time (and hence picks up the local Lorentz factor mentioned in Ref.1), the ψN-force lives in a fictitous flat space-time (and hence does not pick up the local Lorentz factor). Thus the magnitude of the ψN-force will be given by (in polar type co-ordinates)

$$F = [(F^1)^2 + r^2(F^2)^2]^{1/2} \tag{16}$$

and not $[g_{11}(F^1)^2 + g_{22}(F^2)^2]$ as may have been expected.

The direction of the ψN-force, if polar co-ordinates were being used, would be given by F^2, in the same sense as ℓ^2 gave the direction of the separation vector in Fig.2. In terms of the angle made with the radial direction, again in the sense of Fig.2, we would have the angle

$$\chi = \tan^{-1}(rF^2/F^1) \quad . \tag{17}$$

This is the representation which is easier to visualize.

IV. APPLICATION TO THE KN GEOMETRY

We consider the KN geometry in Boyer-Lindquist co-ordinates. In these co-ordinates the non-zero metric coefficients are

$$\left.\begin{aligned} g_{00} &= -[1 - (2mr - Q^2)/\rho^2]; \quad g_{11} = \rho^2/\Delta \\ g_{22} &= \rho^2; \quad g_{33} = \Lambda \sin^2\theta/\rho^2 \; ; \\ g_{03} &= g_{30} = -a^2(1 + g_{00}) \sin^2\theta \end{aligned}\right\} \; , \tag{18}$$

where the terms ρ, Λ, Δ are defined by

$$\left.\begin{aligned} \rho^2 &= r^2 + a^2\cos^2\theta, \Delta = r^2 - 2mr + Q^2 + a^2 \\ \Lambda &= (a^2 + r^2)^2 - a^2\Delta\sin^2\theta \end{aligned}\right\} \; , \tag{19}$$

where m, Q and a are the mass, charge and intrinsic angular momentum per unit mass of the gravitating source, in geometrical (gravitational) units $G = c = 1$. Notice that for a black hole $m^2 \geqslant Q^2 + a^2$, and the outer event horizon is at $r_+ = m + \sqrt{m^2 - Q^2 - a^2}$. Thus $m \leqslant r_+ \leqslant 2m$.

Referring to Eqs.(11) and substituting for the inverses and derivatives of the metric coefficients (see Appx.A), we can obtain the vector ℓ^i. Notice that here ℓ^1 is the radial and ℓ^2 the angular component of the separation vector, as shown in Fig.2. Thus the maximum tidal force is in a direction given by χ_T as determined by Eq.(12). The magnitude of the tidal force is given by λL, where λ can be obtained by inserting Eqs.(A.1) and (A.3), of Appx.A, into Eq.(8), for λ_+. Since the resultant expressions are very complicated and do not give much insight into the problems we shall not present them here in the general form, but will content ourselves with giving the expressions in particular cases later.

For the ψN-force in the KN geometry we use Eq.(15), with the surviving connection coefficients of the KN geometry, given in Appx.A, and of the Minkowski metric, to obtain

$$\left.\begin{aligned} F^1 &= \Gamma^1_{00}\dot{t}^2 + 2\Gamma^1_{03}\dot{t}\dot{\varphi} + \Gamma^1_{11}\dot{r}^2 + 2\Gamma^1_{12}\dot{r}\dot{\theta} \\ &\quad + (\Gamma^1_{22} - \underset{M}{\Gamma}{}^1_{22})\dot{\theta}^2 + (\Gamma^1_{33} - \underset{M}{\Gamma}{}^1_{33})\dot{\varphi}^2 \\ F^2 &= \Gamma^2_{00}\dot{t}^2 + 2\Gamma^2_{03}\dot{t}\dot{\varphi} + \Gamma^2_{11}\dot{r}^2 + \Gamma^2_{22}\dot{\theta}^2 \\ &\quad + 2(\Gamma^2_{12} - \underset{M}{\Gamma}{}^2_{12})\dot{r}\dot{\theta} + 2(\Gamma^2_{33} - \underset{M}{\Gamma}{}^2_{33})\dot{\varphi}^2 \end{aligned}\right\} \; . \tag{20}$$

Inserting Eqs.(B.1)-(B.3), of Appx.B, into Eqs.(20), and simplifying, we finally obtain

$$F^1 = \rho^{-6}\Big[-a^2 VW \sin 2\theta + 2ar U(a\varepsilon \sin^2\theta - L) - \mu^2\{\rho^2(m-r)(na^2+r^2) - r\Delta(2na^2 - a^2\cos^2\theta + r^2)\} + r\rho^2\{W^2 + (aU/\Delta - a\varepsilon + L/\sin^2\theta)^2 \cdot \sin^2\theta\}\Big]$$

$$F^2 = \rho^{-6}\Big[2rVW - a^2\{\mu^2(na^2+r^2) - (a^2\varepsilon^2 + L^2/\sin^4\theta)\cdot \sin^2\theta + a\varepsilon L - \rho^2(\varepsilon^2 + \mu^2 - L^2/a^2\sin^4\theta)/2\}\sin 2\theta - \rho^2\{2VW/r - (aU/\Delta - a\varepsilon + L/\sin^2\theta)^2(\sin 2\theta)/2\}\Big] \quad (21)$$

where U, V, W are defined in Eq.(B) and ε , μ , L, n are the constants of integration of the geodesic equations. The first two are related to the energy and mass of the test particle (which would be the same if the particle is dropped and not thrown), L is related to the z-component (i.e. along the axis of rotation of the gravitating particle) of the angular momentum of the test particle, and n is the fourth constant of motion related to the total, relative, angular momentum of the test particle. For convenience, we shall take $L = 0$ generally and $\varepsilon = \mu = 1$. n must have a value greater than or equal to unity. In the case that it is equal, there is no relative angular momentum of the test particle. Using Eqs.(18) and (21) in conjunction with Eqs.(16) and (17), the magnitude and direction of the ψN-force can be obtained. There does not seem to be any point in displaying the resultant expressions.

At this stage one would like to check the results against those obtained earlier [1] for the RN geometry. This check is not so simple. If we were to directly set $a = 0$ our expressions for the tidal force would not be defined, principally because in that case

$$\lambda = g^{11} g_{00,11}/2 \quad (22)$$

and hence ℓ^1 and ℓ^2, in Eqs.(11), become indeterminate. Consequently, we must take the limiting procedure as $a \to 0$. Inserting Eqs.(A.1), (A.3) and (19) into Eq.(8) we can easily obtain

$$2\lambda - g^{11}g_{00,11} = (g^{22}/g_{00,11})(g_{00,12})^2 + (g_{00,22}/g^{11})(g^{22}g_{00,12}/g_{00,11})^2 + \cdots$$
$$= C a^4 + O(a^6) \quad . \quad (23)$$

where

$$C = \frac{2(3m/r - 2Q^2/r^2)^2 \sin^2 2\theta}{r^6(2m/r - 3Q^2/r^2)} \quad . \quad (24)$$

From Eqs.(A.1), (A.3), (19) and (23) we get

$$\lambda = A + B a^2 + O(a^4) \quad , \quad (25)$$

where

$$A = \frac{1}{r^2}\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)\cdot\left(\frac{2m}{r} - \frac{3Q^2}{r^2}\right)$$
$$B = \frac{1}{r^4}\left[\left(\frac{2m}{r} - \frac{3Q^2}{r^2}\right) - \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)\left(\frac{6m}{r} - \frac{Q^2}{r^2}\right)\cos^2\theta\right] \quad (26)$$

The above-mentioned equations, together with Eqs.(11), give

$$\frac{\ell^1}{L} = \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)^{1/2} + Da^2 + O(a^4)$$
$$D = \frac{\sin^2\theta + (2m/r - Q^2/r^2)\cos^2\theta}{r^2(1 - 2m/r + Q^2/r^2)^{1/2}}$$
$$\frac{\ell^2}{L} = \frac{(3m/r - 2Q^2/r^2)\sin 2\theta}{r^3(1 - 2m/r + Q^2/r^2)^{1/2}} + O(a^4) \quad \cdot (27)$$

Thus, taking the limit as $a \to 0$ we obtain $\lambda = A$, $\ell^2 = 0$ and $\ell^1 = (1-2m/r + Q^2/r^2)^{1/2}L$, in entire agreement with the previous results for tidal forces in an RN geometry. For the ψN-force the procedure is much simpler as we can directly put $a = 0$ in the relevant expressions. Again we get complete agreement with the previous calculations, the ψN-force being $(m/r^2 - Q^2/r^3)$ and lying along the radial direction. The small a corrections to the RN geometry for the tidal force are given by Eqs.(25)-(27). The direction in terms of χ_T can be obtained by inserting these values into Eq.(12). For the ψN-force corrections we choose the special case mentioned earlier ($L = 0$, $\varepsilon = \mu = 1$) in Eqs.(21) to obtain for F^1 and F^2, respectively

$$-(a^2/r^3)\,[(1-3m/r + 2Q^2/r^2) + (3m/r - 2Q^2/r^2)\cos^2\theta]\cdot$$
$$\cdot\frac{1}{2}(a^2/r^4)(2m/r - Q^2/r^2)^2 \sin 2\theta/(1 - 2m/r + Q^2/r^2)^2 \quad . \quad (28)$$

Notice that both the tidal force and the ψN-force pick up small angular components in general, and are hence directed off the radial direction. However, at $\theta = 0$, $\pi/2$ and π the extra term disappears and the forces, to order a^2, do become radial in direction.

The general expressions are too complicated to provide much wisdom. The small a "corrections" to the RN geometry certainly help in one way. However, it would still be very useful to look at certain other, special, cases. In particular, we shall present the formulae when the point P (at which our entire discussion is taking place) lies on the equatorial plane ($\theta = \pi/2$) and when it lies on the axis ($\theta = 0$ or π). In either case we are forced to take the limit, as either the ψN-force or the tidal force, goes indeterminate at these points. In the first case we get

$$\left.\begin{aligned}
&\lambda = \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}+\frac{a^2}{r^2}\right)\left(\frac{2m}{r}-\frac{3Q^2}{r^2}\right)\cdot\frac{1}{r^2}\ ;\\
&\ell^1 = \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}+\frac{a^2}{r^2}\right)^{1/2}L\ ;\ \ell^2 = 0\ ;\\
&F^1 = \Big[\left(\frac{m}{r}-\frac{Q^2}{r^2}\right)+\frac{a^2}{r^2}\cdot\Big\{\frac{3m}{r}-\frac{2Q^2}{r^2}-\frac{2a^2(n-1)}{r^2}\\
&\qquad +\left(\frac{2m}{r}-\frac{Q^2}{r^2}\right)\cdot\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}+\frac{a^2}{r^2}\right)^{-2}\Big\}\Big]\Big/\left(1+\frac{a^2}{r^2}\right)^3\ ;\\
&F^2 = 0\,.
\end{aligned}\right\}\quad(29)$$

and in the second case we get

$$\left.\begin{aligned}
&\lambda = \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}+\frac{a^2}{r^2}\right)\left[\left(\frac{2m}{r}-\frac{3Q^2}{r^2}\right)-\frac{a^2}{r^2}\left(\frac{6m}{r}-\frac{Q^2}{r^2}\right)\right]\Big/\left[r^2\left(1+\frac{a^2}{r^2}\right)^4\right];\\
&\ell^1 = \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}+\frac{a^2}{r^2}\right)^{1/2}L\Big/\left(1+\frac{a^2}{r^2}\right)^{1/2}\ ;\ \ell^2 = 0\ ;\\
&F^1 = \frac{\left[\left(\frac{m}{r}-\frac{Q^2}{r^2}\right)+\frac{a^2}{r^2}\left\{\frac{3m(n-1)}{r}-\frac{Q^2(2n-1)}{r^2}-\frac{a^2mn}{r^3}\right\}\right]}{r\left(1+\frac{a^2}{r^2}\right)^3}\ ;\\
&F^2 = \frac{2a^3(n-1)^{1/2}\left[\left(\frac{2m}{r}-\frac{Q^2}{r^2}\right)-\frac{a^2}{r^2}\left\{n-1-\frac{2mn}{r}+\frac{nQ^2}{r^2}+\frac{a^2(n-1)}{r^2}\right\}\right]}{r^5\left(1+\frac{a^2}{r^2}\right)^5}\,.
\end{aligned}\right\}\quad(30)$$

Here we see that Eqs.(29) do not agree with the values obtained previously [1]. The reason for this difference is that in that case θ was set as $\pi/2$ from the start and kept fixed. Thus various terms coming from the derivatives of θ were not included. In particular, the constant of integration of the

geodesic equations for θ did not enter into those calculations. Notice that ℓ^2 is zero in these special cases only, but is generally non-zero.

Another very interesting case is when we take $n = 1$. In this case Eq.(B.3) and the geodesic equations are consistently satisfied by a constant value of θ, say θ_0. Thus the geodesic of such a test particle would spiral inward on the surface of a cone of angle θ_0 as shown in Fig.3. The tidal force is not affected by the choice of n, but the ψN-force is. The general expression for the ψN-force now becomes

$$\left.\begin{aligned}
&F^1 = \frac{\Big[\left(1+\frac{a^2}{r^2}\right)\left(\frac{m}{r}-\frac{Q^2}{r^2}-\frac{ma^2\cos^2\theta_0}{r^3}\right)+\frac{a^2}{r^2}\Big\{\left(\frac{2m}{r}-\frac{Q^2}{r^2}\right)\sin^2\theta_0 +\left(1+\frac{a^2\cos^2\theta_0}{r^2}\right)\left(\frac{2m}{r}-\frac{Q^2}{r^2}\right)^2\sin^2\theta_0\cdot\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}+\frac{a^2}{r^2}\right)^{-2}\Big\}\Big]}{r\left(1+\frac{a^2\cos^2\theta_0}{r^2}\right)^3}\ ;\\
&F^2 = \frac{a^2\left(\frac{2m}{r}-\frac{Q^2}{r^2}\right)^2\sin 2\theta_0}{r^4\left(1+\frac{a^2\cos^2\theta_0}{r^2}\right)^2\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}+\frac{a^2}{r^2}\right)^2}\,.
\end{aligned}\right\}\quad(31)$$

In the case that $\theta = \pi/2$, Eqs.(31) give us

$$F^1 = \Big[(m/r - Q^2/r^2) + (a^2/r^2)\ \{3m/r - 2Q^2/r^2 + (2m/r - Q^2/r^2)^2/ (1 - 2m/r + Q^2/r^2 + a^2/r^2)^2\}\Big]\ /r \qquad (32)$$

and in the case that $\theta = 0$, we get

$$F^1 = (m/r - Q^2/r^2 - ma^2/r^3)/[r(1 + a^2/r^2)^2]\ , \qquad (33)$$

while F^2 is zero in both cases. Notice that for an extreme KN black hole F^1 will become zero at the horizon if the approach is along the axis.

To give a more general picture of the tidal and ψN-forces we have presented the results graphically in Figs.4–8. Notice, particularly, the variation of the angles χ_T and χ with changes of the angle θ. Also notice the difference between the behaviour of the forces with Q and with a. There is not much qualitative difference, or even quantitative difference, for variations of n.

V. SUMMARY AND DISCUSSION

We have seen that we can define a relativistic analogue of the Newtonian force, which we have called the ψN-force, in terms of observations made with an accelerometer (the device depicted in Fig.1). In essence, what we are doing is to take the curved space-time, given by general relativity, and "flatten it". In so doing we have to supply a "force" to "explain" the geodesic as the result of "bending a straight path". This procedure may appear to complicate a very simple and beautiful theory, unnecessarily. The reason for pursuing this convoluted procedure is that whereas relativity is very elegant, our basic physical intuition is completely Newtonian. For most purposes we will still ask what the force is which causes something to occur, and still try to unify the forces of nature, despite the fact that relativity uses neither of these concepts. It is useful, in this sense, to see the relativistic effects as "corrections" to the Newtonian forces. There is an additional reason to feel that it would be useful to re-educate our physical intuition in the understanding of the implications of geometry in terms of "forces". The problems of the quantization of relativity are too well known to need re-iteration. They stem from the use of a curved space-time background in which the quantization is done [4]. When gravity is quantized the metric itself has to be quantized, and this leads naturally to the requirement that the space-time be quantized [8]. An attempt to achieve this type of quantization is provided by twistors [9] and their associated complications [10]. While the approach seems to be very sound it cannot be denied that the complications of this method obscure any wisdom that could be obtained from it. Another approach is to try the quantization of relativity, great-grand-unification and the unification of fermions and bosons in one fell swoop of supergravity [11]. Doubtless this is another very promising approach, but it is undeniably complicated. Again there is no simple physical wisdom to be obtained from it. Nor is it clear what predictions would be forthcoming from this approach. To summarize, the various current attempts at quantum relativity are too complicated, in terms of basic concepts, to give much insight, and are unlikely to lead to observationally significant results. We feel that our approach of providing a "force" in a Minkowski background should be much easier to use for quantization purposes. Of course we have provided a force, whereas what is required is a "potential" analogous to the Newtonian potential - a ψN-potential, ξ . For the Schwarzschild geometry it is just the Newtonian potential, m/r, while for the RN geometry it is

$$\xi = (m/r - Q^2/2r^2) \qquad (34)$$

which is just $(1-g_{00})/2$ as may have been expected. However, for the KN geometry it cannot be so simple.

The insights into the workings of the relativity, obtained in terms of "forces", should ultimately educate us to be able to directly understand the significance of the geometry. However, it appears to us to be necessary to keep a link with the older, more familiar concepts, while trying to appreciate the more elegant concepts of geometrodynamics. This seems to be particularly important when trying to merge two different sets of new concepts derived from the older concepts. It would also be useful when it is not practicable to describe an entire situation in terms of the geodesics of all constituent particles, as in the case of a supernova explosion, for example. It will certainly help in framing physically relevant questions such as "when does the relativistic force exceed (for example) the Coulomb force?" which could not have been asked in purely relativistic terms.

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Using Eqs.(18) and (19), the required inverses of the metric coefficients and the relevant derivatives are

$$g^{00} = -\Lambda/(\rho^2\Delta)\;;\; g^{11} = \Delta/\rho^2\;;\; g^{22} = 1/\rho^2\;;$$
$$g^{03} = g^{30} = -a(2mr - Q^2)/(\rho^2\Delta)\;;$$
$$g^{33} = (\Delta - a^2\sin^2\theta)/(\rho^2\Delta\sin^2\theta). \qquad \text{(A.1)}$$

$$g_{00,1} = -(mr^2 - Q^2r - ma^2\cos^2\theta)/\rho^4\;;$$
$$g_{00,2} = a^2(2mr - Q^2r)(\sin 2\theta)/\rho^4\;;$$
$$g_{03,1} = 2a(mr^2 - Q^2r - ma^2\cos^2\theta)\sin^2\theta/\rho^4\;;$$
$$g_{03,2} = -a(2mr - Q^2)(a^2 + r^2)\cdot(\sin 2\theta)/\rho^4\;;$$
$$g_{11,1} = 2[\rho^2(m - r) + r\Delta]/\Delta^2\;;$$
$$g_{11,2} = -a^2(\sin 2\theta)/\Delta\;;$$
$$g_{22,1} = 2r\;;\; g_{22,2} = -a^2\sin 2\theta\;;$$
$$g_{33,1} = 2[r\rho^4 - a^2(mr^2 - Q^2r - ma^2\cos^2\theta)\sin^2\theta]\cdot(\sin^2\theta)/\rho^4\;;$$
$$g_{33,2} = [\rho^4(r^2 + a^2) + a^2(2mr - Q^2)(2\rho^2 + a^2\sin^2\theta)\cdot\sin^2\theta\cdot(\sin 2\theta)/\rho^4. \qquad \text{(A.2)}$$

$$g_{00,11} = 2[2mr^3 - 3Q^2r^2 - a^2(6mr - Q^2)\cos^2\theta]/\rho^6\;;$$
$$g_{00,12} = -2a^2(3mr^2 + 2Q^2r + ma^2\cos^2\theta)(\sin 2\theta)/\rho^6\;;$$
$$g_{00,22} = 2a^2(2mr - Q^2)(a^2\sin^2 2\theta + \rho^2\cos 2\theta)/\rho^6. \qquad \text{(A.3)}$$

The surviving connection coefficients required for our computations are

$$\Gamma^1_{00} = -\tfrac{1}{2}g^{11}g_{00,1}\;;\quad \Gamma^2_{00} = -\tfrac{1}{2}g^{22}g_{00,2}\;;$$
$$\Gamma^1_{03} = \Gamma^1_{30} = -\tfrac{1}{2}g^{11}g_{03,1}\;;\quad \Gamma^2_{03} = \Gamma^2_{30} = -\tfrac{1}{2}g^{22}g_{03,2}\;;$$
$$\Gamma^1_{11} = \tfrac{1}{2}g^{11}g_{11,1}\;;\quad \Gamma^2_{11} = -\tfrac{1}{2}g^{22}g_{11,2}\;;$$
$$\Gamma^1_{12} = \Gamma^1_{21} = \tfrac{1}{2}g^{11}g_{11,2}\;;\quad \Gamma^2_{12} = \Gamma^2_{21} = \tfrac{1}{2}g^{22}g_{22,1}\;,$$
$$\Gamma^1_{22} = -\tfrac{1}{2}g^{11}g_{22,1}\;;\quad \Gamma^2_{22} = \tfrac{1}{2}g^{22}g_{22,2}\;;$$
$$\Gamma^1_{33} = -\tfrac{1}{2}g^{11}g_{33,1}\;;\quad \Gamma^2_{33} = -\tfrac{1}{2}g^{22}g_{33,2}\;. \qquad \text{(B.1)}$$

The first integrals of motion for a particle moving in the gravitational and electromagnetic field outside the event horizon of a KN black hole are [7]

$$\dot t = [(a^2 + r^2)U/\Delta - a^2\varepsilon\sin^2\theta + aL]/\rho^2\;;$$
$$\dot r = V/\rho^2\;;\quad \dot\theta = W/\rho^2\;;$$
$$\dot\varphi = (aU/\Delta - a\varepsilon + L/\sin^2\theta)/\rho^2. \qquad \text{(B.2)}$$

where

$$U = \varepsilon(a^2 + r^2) - aL\;;$$
$$V^2 = U^2 - \mu^2(r^2 + na^2)\Delta\;;$$
$$W^2 = a^2[\mu^2 n - \mu^2\cos^2\theta - (\varepsilon\sin^2\theta - L/a)^2/\sin^2\theta]. \qquad \text{(B.3)}$$

where the significance of μ, ε, L and n is explained in the text. Notice that if $L = 0$, $\mu = \varepsilon$, we get

$$W = a\mu(n - 1)^{1/2} \qquad \text{(B.4)}$$

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FIGURE CAPTIONS

Fig.1 A spring of length L connects two masses. One end of the spring has a pointer which rotates about on a dial. This "accelerometer" measures forces by showing compression (-) for a repulsive force, extension (+) for an attractive force and showing no change (0) if there is no force.

Fig.2(a) An accelerometer, greatly exaggerated for clarity, is shown falling freely towards a gravitating source, O, in the Newtonian version of gravity using a Euclidean geometry. The polar components of the separation vector, defined by the accelerometer, are depicted by PQ and ℓ^2. The "Cartesian type" component corresponding to ℓ^2 is given by PR, and the angle of the accelerometer with the radial direction by χ_T. The component ℓ^2, and hence χ_T, would be zero if the accelerometer fell radially inward along PO. The maximum tidal force, F_T^i, would lie along ℓ^1.

Fig.2(b) The same situation is now depicted in a curved space-time. The straight lines of Fig.2(a) are now replaced by geodesics. The ℓ^2 is now a "polar-type" co-ordinate. Here the accelerometer would spiral in along a complicated path and so ℓ^2 and χ_T would be non-zero. The maximum tidal force would still lie along ℓ^1.

Fig.3 The geodesic of a test particle with $n = 1$ lies on the cone $\theta = \theta_0$, as shown here, with its axis along the axis of rotation of the gravitating source. The geodesic is simply a conical spiral. Nevertheless, the ψN-force experienced by the test particle is not simple.

Fig.4(a) The variation of the tidal force magnitude, $\lambda(r)$, with the distance of the dial end of the accelerometer, r, from a Kerr black hole is shown for $n = 1$, $a = 0.01\ m$, for different values of θ. $\lambda(r)$ is entirely insensitive to change of θ. Notice the fact that $\lambda(r)$ turns over at just short of $r = 3m$.

Fig.4(b) The variation of $\lambda(r)$ for an extreme Kerr black hole. Here the variation of $\lambda(r)$ with θ is noticeable. The turnover, just after $r = m$ could not be shown on the scale used.

Fig.5(a) The variation of the direction of the tidal force, $\chi_T(r)$, for an extreme Kerr black hole, for various values of θ. The variation of $\chi_T(r)$ with θ is noticeable due to the large value of a.

Fig.5(b) The variation of the direction of the tidal force, $\chi(r)$, for an extreme Kerr black hole. The variation of $\chi(r)$ with θ is striking. For $\theta = \pi/2$, $\chi(r) = 0$. Comparison with Fig.5(a) shows that $\chi(r) \neq \chi_T(r)$.

Fig.6(a) The variation of the ψN-force magnitude, F(r), for a Kerr black hole with a = 0.01 m. The variation of F(r) with θ has been made noticeable by choosing a sufficiently small window in which to look at the ψN-force.

Fig.6(b) The value of a is taken at 0.5 m here. The variation with θ shows up on a much larger scale.

Fig.6(c) For the extreme Kerr black hole the variation of the ψN-force with θ is dramatically apparent.

Fig.7(a) The effect of introducing a charge $Q = \sqrt{0.5}$ m for a = 0.01 m is shown. Comparison with Fig.6(a) shows how the presence of charge makes the curves sharper. (The shift from $r_+ \approx 2m$ to $r_+ \approx 1.71m$ is to be discounted in the comparison.)

Fig.7(b) A nearly extreme Kerr-Newman black hole with $Q = \sqrt{0.99}$ m, a = 0.01 m. The remarkable feature to notice is the dip appearing in the ψN-force around 1.5 m. This dip is particularly noticeable as we approach the axis of rotation, coming to a minimum near 1.1 m. The extreme KN black hole and the axis had to be avoided to avoid numerical instabilities.

Fig.7(c) A larger a(= 0.5 m) and a smaller Q (= $\sqrt{0.5}$ m) are shown for comparison with the previous diagrams. There is no striking feature in this case.

Fig.8(a) The variation of the ψN-force with n is depicted here for an extreme KN black hole. The difference becomes apparent only when n is changed by a factor of 10. Clearly the ψN-force is not very sensitive to changes of n. Here a = 0.01 m.

Fig.8(b) Changing a to 1 m reduces the sensitivity to n and greatly enhances the ψN-force. No dramatic feature is apparent here.

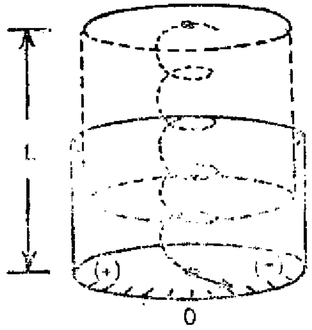


Fig.1

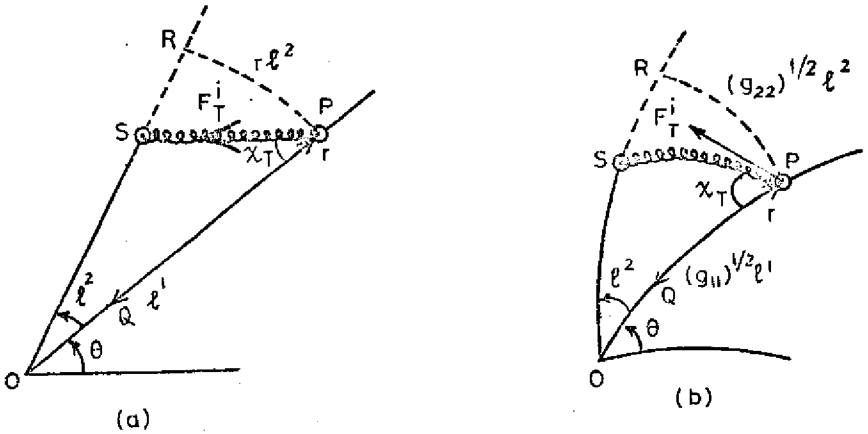

R
$r\ell^2$
F_T^i
S
P
χ_T
r
ℓ^2
Q
ℓ^1
θ
O
(a)
R
$(g_{22})^{1/2}\ell^2$
F_T^i
S
P
χ_T
r
ℓ^2
Q
$(g_{11})^{1/2}\ell^1$
θ
O
(b)

Fig.2

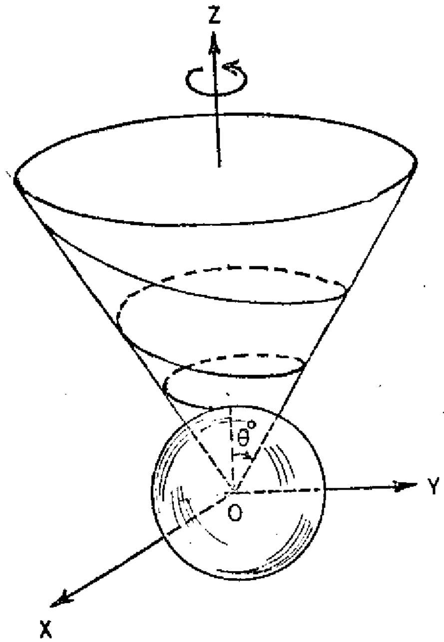

Z
θ^0
Y
O
X

Fig.3

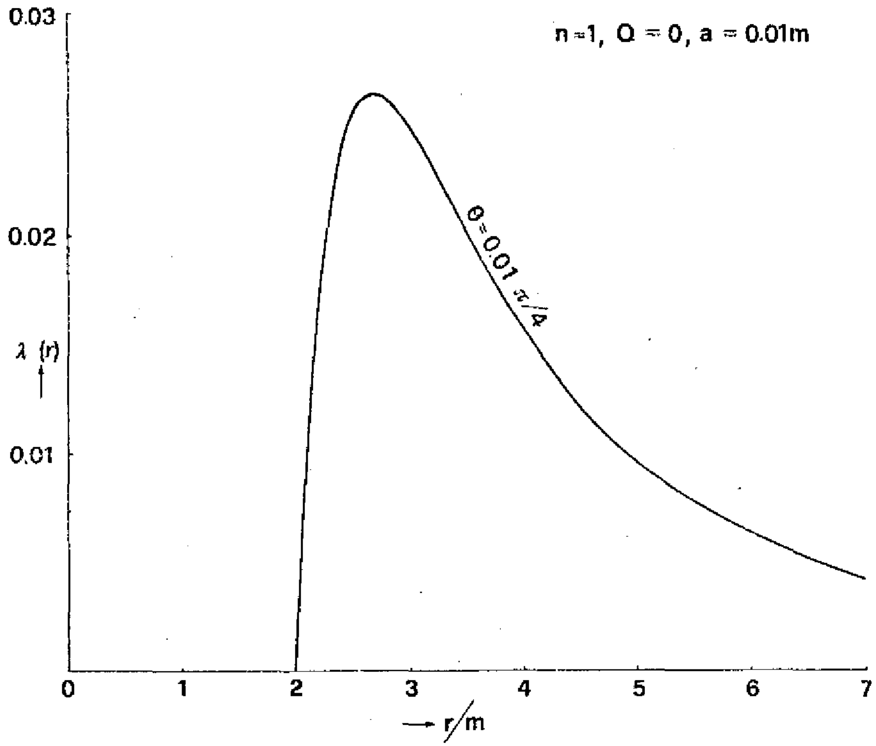

n = 1, Q = 0, a = 0.01m
0.03
0.02
0.01
λ (r)
θ = 0.01 π/4
0
1
2
3
4
5
6
7
r/m

Fig. 4a

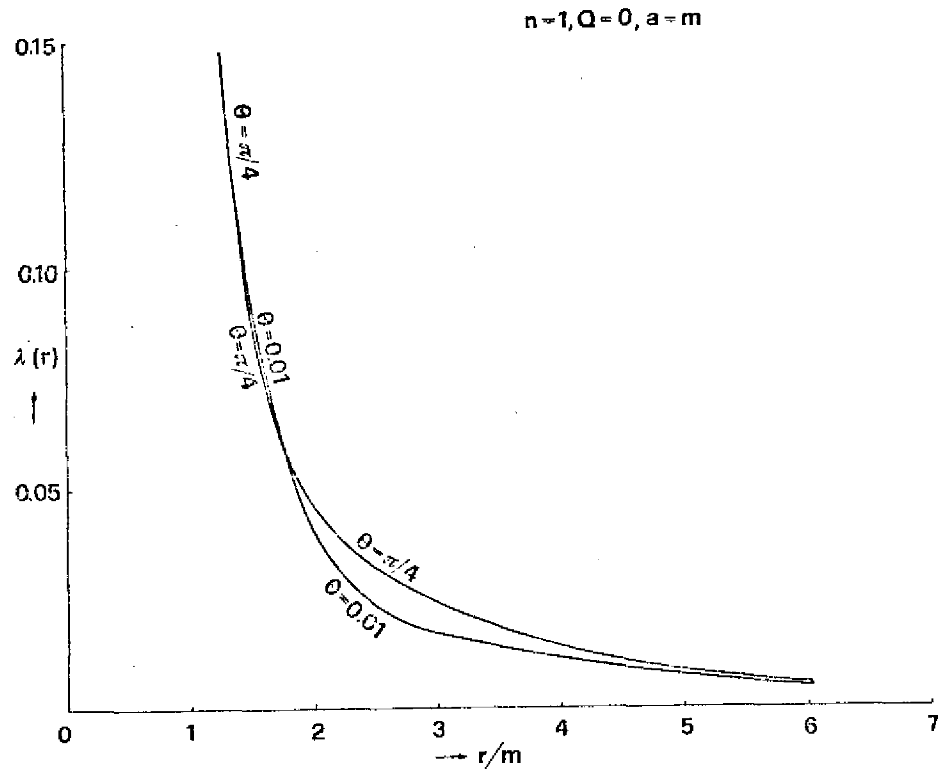
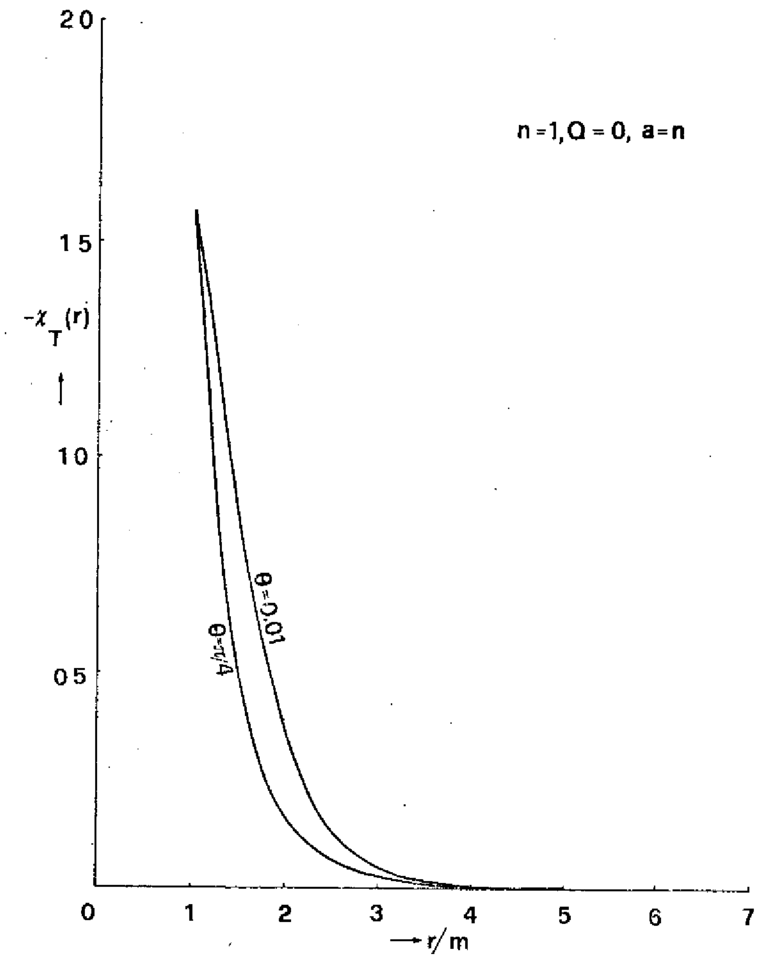
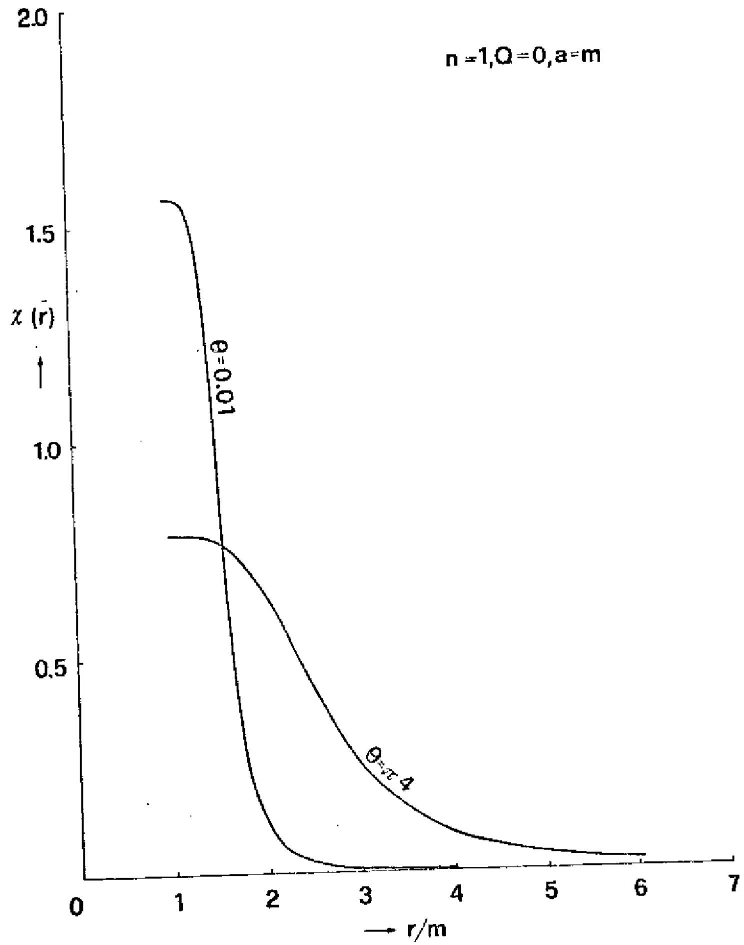

n = 1, Q = 0, a = m
0.15
0.10
0.05
λ (r)
θ = π/4
θ = 0.01
θ = π/4
θ = π/4
θ = 0.01
0
1
2
3
4
5
6
7
r/m

Fig. 4b



2.0
1.5
1.0
0.5
$-\chi_T(r)$
n = 1, Q = 0, a = n
θ = π/4
θ = 0.01
0
1
2
3
4
5
6
7
→ r/m

Fig. 5a



2.0
1.5
1.0
0.5
$\chi(r)$
n = 1, Q = 0, a = m
θ = 0.01
θ = π/4
0
1
2
3
4
5
6
7
→ r/m

Fig. 5b

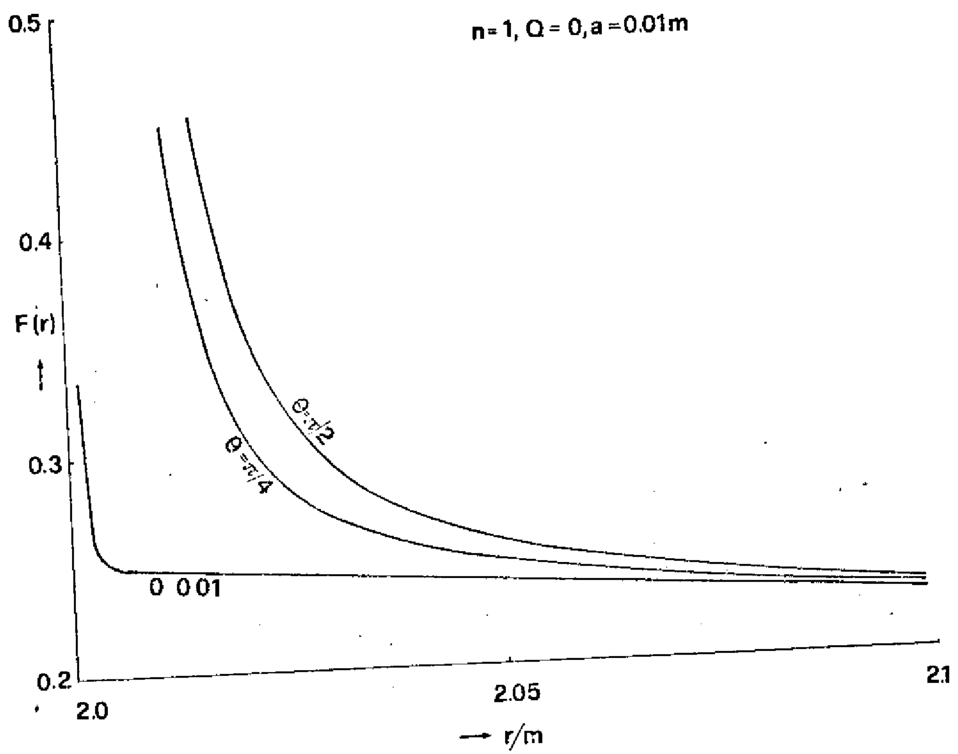

n= 1, Q = 0, a =0.01m
0.5
0.4
0.3
0.2
F (r)
θ=π/2
θ=π/4
0 0.01
2.0
2.05
2.1
r/m

Fig.6 a

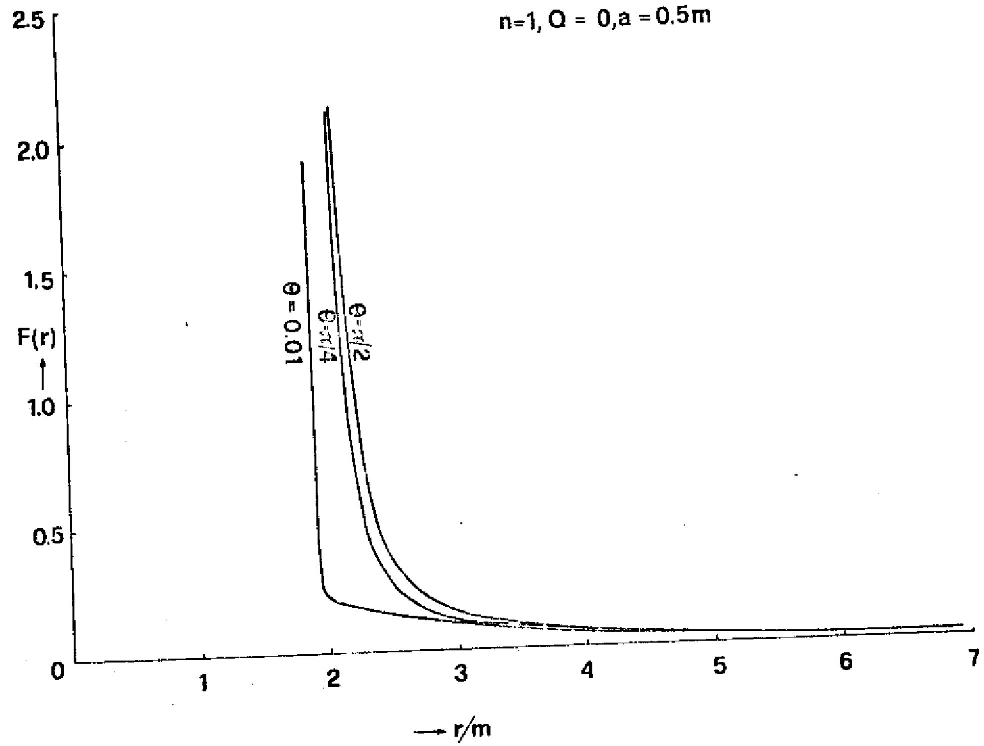

n=1, Q = 0, a = 0.5m
2.5
2.0
1.5
1.0
0.5
0
F(r)
θ = 0.01
θ=π/4
θ=π/2
1
2
3
4
5
6
7
r/m

Fig. 6b

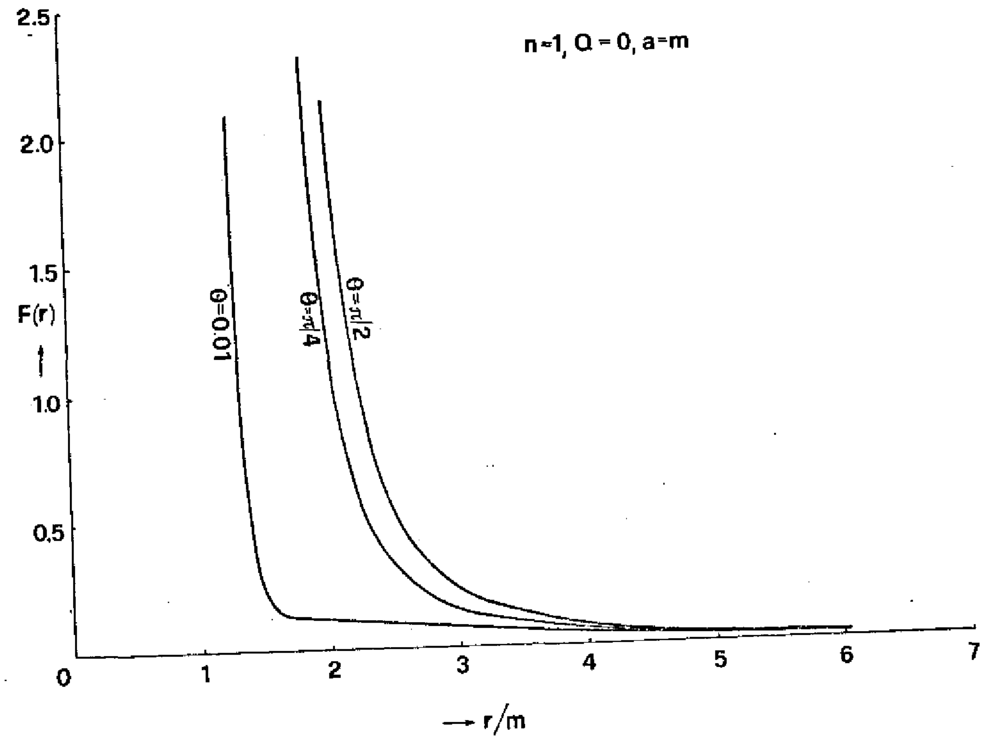

n=1, Q = 0, a=m
2.5
2.0
1.5
F(r)
1.0
0.5
0
1
2
3
4
5
6
7
θ=0.01
θ=π/4
θ=π/2
r/m

Fig.6c

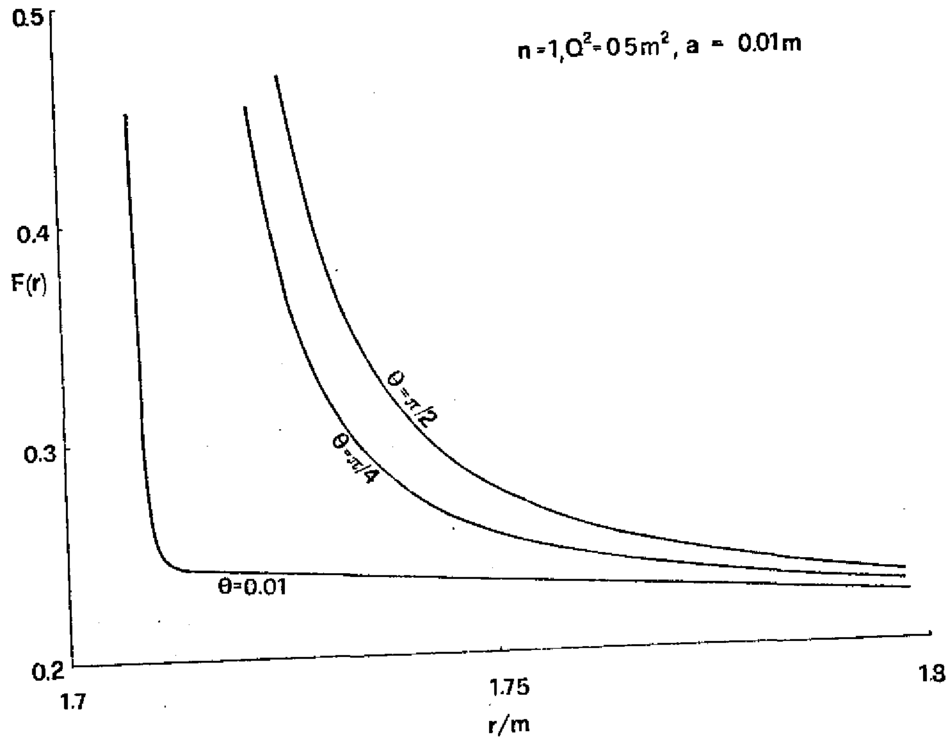
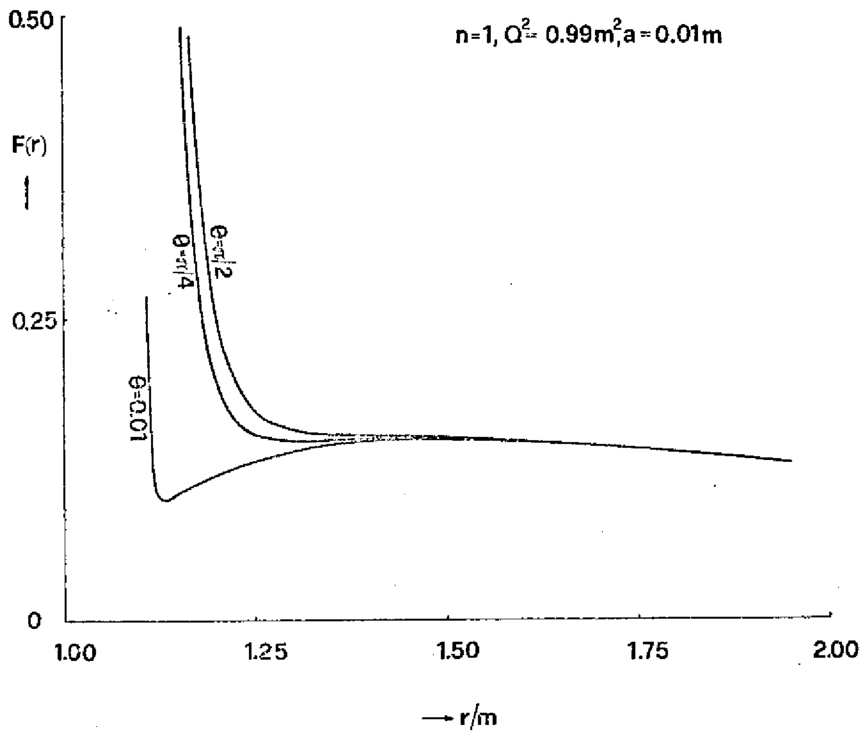
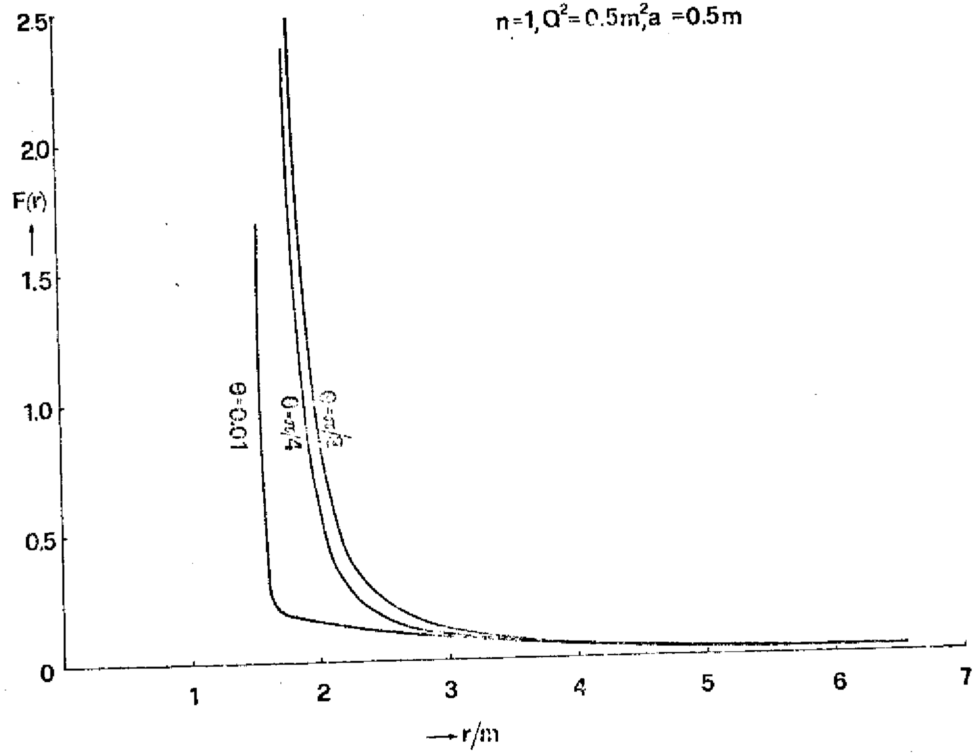

n=1, Q²=0.5m², a = 0.01m
0.5
0.4
F(r)
0.3
0.2
1.7
1.75
1.8
θ=π/2
θ=π/4
θ=0.01
r/m

Fig.7a



n=1, Q²= 0.99m², a=0.01m
0.50
0.25
0
F(r)
θ=π/2
θ=π/4
θ=0.01
1.00
1.25
1.50
1.75
2.00
r/m

Fig.7b



n=1, Q²=0.5m², a =0.5m
2.5
2.0
1.5
1.0
0.5
0
F(r)
θ=0.01
θ=π/4
θ=π/2
1
2
3
4
5
6
7
r/m

Fig.7c

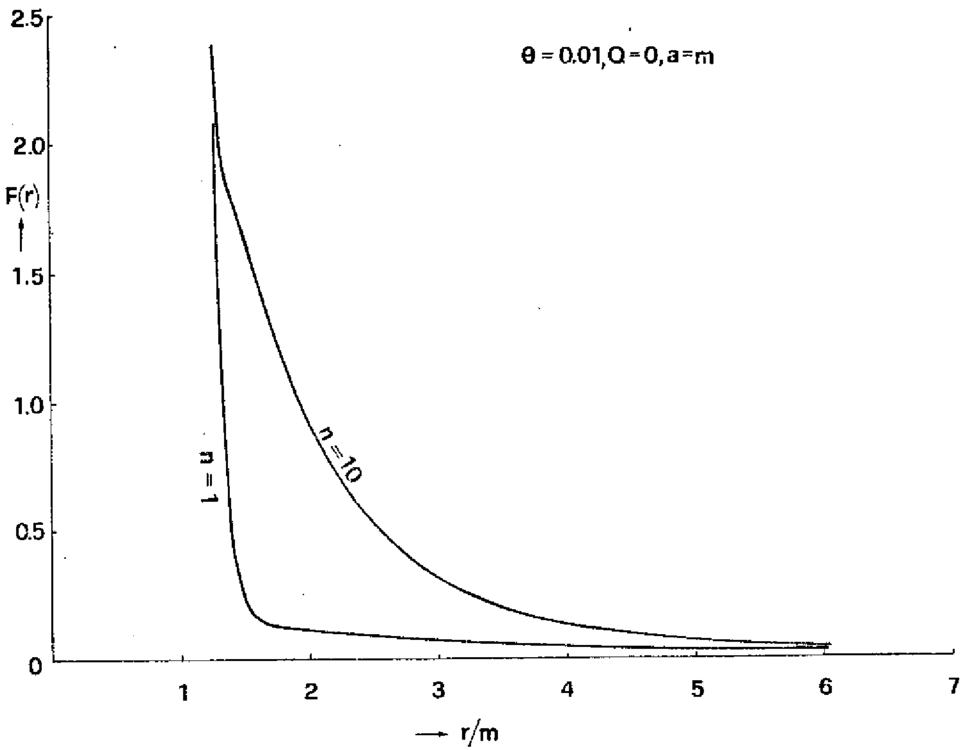


Fig.8a

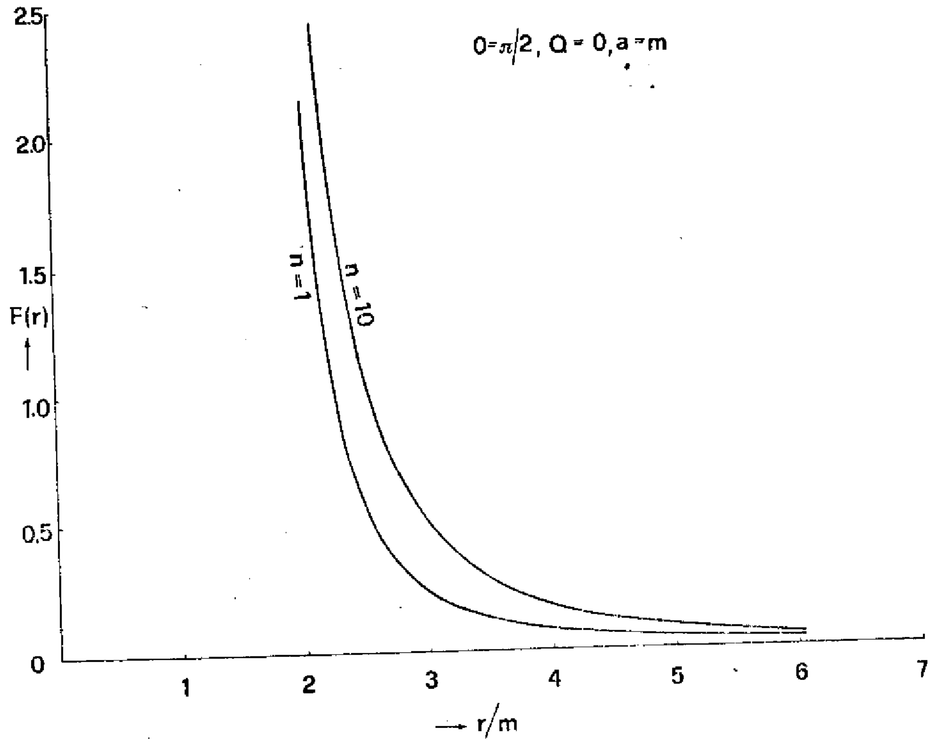


Fig.8b

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