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ON INTEGRATION OVER FERMI FIELDS  
IN CHIRAL AND SUPERSYMMETRIC THEORIES

M O S C O W

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**Abstract**

Chiral and supersymmetric theories are considered which cannot be formulated directly in Euclidean space or regularized by means of massive fields in a manifestly gauge invariant fashion. In case of so called real representations a simple recipe is proposed which allows for unambiguous evaluation of the fermionic determinant circumventing the difficulties mentioned. As application of the general technique we calculate the effective fermionic interactions induced by instantons of small size within simplest chiral and supersymmetric theories ( $SU(2)$  as the gauge group and one doublet of Weyl spinors or a triplet of Majorana spinors, respectively). In the latter case the effective Lagrangian violates explicitly invariance under supersymmetric transformations on the fermionic and vector fields defined in standard way.

As is known, theories of interacting two-component (Weyl) spinors run into serious difficulties. Consider the simplest case: SU(2) as gauge group and a doublet of right-handed fermions interacting with the gauge bosons. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} (G_{\mu\nu}^a)^2 + \psi^\dagger i \sigma_\mu^R \mathcal{D}_\mu \psi \quad (1)$$

where  $G_{\mu\nu}^a$  ( $a = 1, 2, 3$ ) is field strength tensor constructed from fields  $A_\mu^a$ .  $\psi_\alpha^k$  ( $k, \alpha = 1, 2$ ) are Weyl spinors,  $k$  and  $\alpha$  being the colour and Lorentz indices, respectively;  $\sigma_\mu^R$  are 2x2 matrices ( $\sigma_\mu^R = (1, \vec{\sigma})$ ) and the covariant derivative  $\mathcal{D}_\mu$  is defined as  $\mathcal{D}_\mu = \partial_\mu + ig T^a A_\mu^a$  (for a colour doublet  $T^a = \tau^a/2$ ).

Although nothing is seemingly wrong with Lagrangian (1) in Minkowski space (see, however, for a discussion below) no Lagrangian containing spinor field  $\psi$  alone is feasible in Euclidean space. Indeed,  $\psi$  transforms according to (0, 1/2) representation of the Lorentz group while its Hermitian conjugate,  $\psi^\dagger$ , transforms as (1/2, 0). On the other hand, in Euclidean space complex conjugation does not change the representation for the field and left-handed fields are to be added as independent. In other words, in Euclidean space it turns impossible to define the involution operation and integration over Fermi fields in standard way [1]. As a manifestation of all this, the operator  $\sigma_\mu^- \mathcal{D}_\mu$  ( $\sigma_\mu^- = (1, \vec{\sigma})$ ) arising from continuation of  $\sigma_\mu^R \mathcal{D}_\mu$  into Euclidean space is not Hermitian. The problem has been realized by many authors and nowadays is exposed, say, in a modern textbook [2]. The overall conclusion sometimes is that there exists no theory of a single Weyl doublet.

The necessity to consider Euclidean space and all the problems involved becomes apparent ones one turns to evaluate nonper-

turbative effects induced, say, by instantons. Within perturbation theory the problems do not look so imminent. However, they are echoed by the difficulties of constructing a manifestly gauge invariant regularization procedure. Indeed, introduction of independent left-handed fields (along with the right-handed ones originally present) is called for now by regulator fields which are massive. Moreover, mass term for these regulators violates gauge invariance. Note also that regularization of chiral theories on a lattice is thoroughly discussed in a recent paper [3], with the conclusion that chiral theories cannot be put on a lattice.

In this paper we would like to propose and exploit a simple recipe to perform integration over Fermi fields which is free of the difficulties described above. The scope of applicability of the procedure is not wide, however, and is limited to the cases when fermions belong to so called real representations of colour group (for the definition see eq.(10)). Still it covers some interesting cases. Note that for real representations there are no triangle anomalies [4].

Concentrate first on the example (1). The functional generating fermionic Green's functions in this theory takes the form

$$Z(\eta, \eta^+) = \int \mathcal{D}A e^{-\frac{i}{4} \int d^4x (G_{\mu\nu}^a)^2} \Phi(A, \eta, \eta^+) \quad (2)$$

where

$$\Phi(A, \eta, \eta^+) = \int \mathcal{D}\psi \mathcal{D}\psi^+ \cdot \exp i \int d^4x \{ \psi^+ \sigma_\mu^R \mathcal{D}_\mu \psi + \eta^+ \psi + \psi^+ \eta \}$$

Here  $\eta$  is an external source and  $\psi, \psi^+$  are Grassmann variables.

Perform now a linear change of variables :

$$\varphi_\alpha^{\kappa} = (\varepsilon_2)^{\kappa m} (\sigma_2)_{\alpha\beta} X_\beta^{*m} \quad (3)$$

and the corresponding change for  $\varphi^\dagger$ . In the spinor space the substitution implies that a left-handed particle is called now a right-handed antiparticle. Under substitution (3) the kinetic term goes into

$$\varphi^\dagger \sigma_\mu^R \partial_\mu \varphi = - X^\dagger \sigma_\mu^L \partial_\mu X \quad (4)$$

where  $\sigma_\mu^L = (-1, \vec{\sigma})$ ,  $\sigma_\mu^R = -(\sigma_2 \sigma_\mu^L \sigma_2)^T$ . When deriving eq.(4) we have accounted for the anticommuting nature of  $X$  and added a total derivative. The interaction term is rewritten as

$$\begin{aligned} g A_\mu^a \varphi^\dagger \sigma_\mu^R \frac{\Sigma^a}{2} \varphi &= -g A_\mu^a X^\dagger \sigma_\mu^L (-\tau_2 \frac{\tau^a}{2} \tau_2)^T X = \\ &= -g A_\mu^a X^\dagger \sigma_\mu^L \frac{\Sigma^a}{2} X \end{aligned} \quad (5)$$

Here we have used the fact that the  $SU(2)$  doublet is a self conjugated representation, i.e. generator matrix acting on the representation and on its conjugate are the same. Finally, the integral (2) is rewritten as

$$\begin{aligned} \Phi(A, \eta, \eta^\dagger) &= \int \mathcal{D}X \mathcal{D}X^\dagger \cdot \exp i \int d^4x \cdot \\ &\cdot \{-X^\dagger \sigma_\mu^L i \mathcal{D}_\mu X - X^\dagger (\sigma_2 \tau_2 \eta^*) - (\eta^\dagger \sigma_2 \tau_2) X\} \end{aligned} \quad (6)$$

Neither (2) nor (6) allow for a Euclidean space formulation. Now, the trick is to represent  $\Phi$  as a root square of the product of (2) and (6). Then continuation into Euclidean space poses no problems. Indeed,

$$\begin{aligned} \Phi^2(A, \eta, \eta^\dagger) &= \int \mathcal{D}\varphi \mathcal{D}\varphi^\dagger \mathcal{D}X \mathcal{D}X^\dagger \cdot \\ &\cdot \exp i \int d^4x \{ \varphi^\dagger \sigma_\mu^R i \mathcal{D}_\mu \varphi - X^\dagger \sigma_\mu^R i \mathcal{D}_\mu X + \\ &+ \eta^\dagger \varphi - (\eta^\dagger \sigma_2 \tau_2) X + \varphi^\dagger \eta - X^\dagger (\sigma_2 \tau_2 \eta^*) \} \end{aligned} \quad (7)$$

and this equation can be readily rewritten in terms of four-component spinors  $\Psi, \bar{\Psi}$  :  $\bar{\Psi} = (\chi^+, \varphi^+)$ ,  $\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$

$$\Phi^2(A, \eta, \eta^+) = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \cdot \exp i \int d^4x \cdot \{ \bar{\Psi} \gamma_\mu i \mathcal{D}_\mu \Psi + \bar{j} \Psi + \bar{\Psi} j \} \quad (8)$$

where we assume the spinor representation for the  $\gamma$  matrices ( $\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ ) and the four-component source  $j$  is expressed in terms of  $\eta, \eta^+$  introduced originally:

$$j = \begin{pmatrix} -\epsilon_2 \sigma_2 \eta^* \\ \eta \end{pmatrix} \quad (9)$$

So far we performed identical transformations. Now we fix (8) to be the definition of  $\Phi$ . Such definition leaves unspecified the sign of  $\Phi$  but this is not a setback<sup>3</sup>. The advantages of using definition (8) need not be explained in length: both Euclidean space formulation and explicitly gauge invariant regularization by Pauli-Villars fields are no longer hampered by the Weyl nature of the spinors.

We hope that the numerous definitions and redefinitions have not veiled the simplicity of the recipe proposed. Essentially the situation is the same as with evaluation of the integral  $\int e^{-\frac{x^2}{2}} dx$  by squaring it and introducing polar coordinates. Note also that squaring of the partition function of the Ising model has been previously used to rewrite it as a functional in-

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<sup>3</sup> For topologically trivial sector of  $A_\mu$  the sign of  $\Phi$  is fixed by the conventions of perturbation theory. Perturbation theory does not fix the sign for topologically nontrivial  $A_\mu$  but the corresponding amplitudes do not interfere with the perturbative ones, and the sign is arbitrary.

tegral over Fermi fields ( see Ref. [5] ).

We will use eq.(8) for explicit calculations. But let us discuss first how general the result is. First, any fermion representation can be written in terms of right-handed Weyl spinors. Then the fields form some ,generally speaking reducible, multiplet of colour group. To apply the machinery developed we need to have the complex conjugated multiplet transforming as the same representation of the colour group. In more detail, let the generator matrices acting on the fields  $\psi$  be  $T^a$ . Then for the complex conjugated multiplet the generators are  $(-T^a)^T$ . Existence of an equivalence transformation provided by a unitary matrix  $U$  :

$$T^a = U(-T^a)^T U^\dagger \quad (10)$$

suffices for our purposes.

The meaning of condition (10) is that the theory is actually vectorlike. That is why eq.(8) which avoids any reference to an axial-vector type interaction stays valid. Intuitively one would expect that the meaning of the condition of cancellation of triangle anomalies is the same. In the case considered this turns to be true and eq.(10) guarantees both cancellation of the anomalies [4] and validity of eq.(8). However, this is not generally true that any anomaly-free theory can be put to the form (8). In particular, within the standard electroweak theory the anomalies cancel on the sum of a quintet and of a decuplet of  $SU(5)$  (or ,what is the same, on the fundamental spinor representation of  $SO(10)$ ). But neither  $SU(5)$  nor  $SO(10)$  is vectorlike and we fail to evaluate the fermionic integral in this important case along the lines outlined above. We believe that this failure might indicate that something is profoundly wrong with this type

of theories (outside the scope of perturbation theory) <sup>κ</sup>.

The above consideration applies to supersymmetric theories as well. Usually these are formulated in terms of Majorana spinors but since the latter can be rewritten in terms of Weyl spinors, the problems are just the same.

In particular, SU(2) supersymmetric gluodynamics with triplets of vector and Majorana fields cannot be realized in straightforward way in Euclidean space [6]. However this theory is apparently vectorlike and, therefore, we can apply eq.(8) to evaluate any amplitude in this theory.

After these general remarks turn to some specific examples. First, let us calculate one-instanton contribution within the theory described by Lagrangian (1). The example is interesting since one-instanton transition is to be accompanied by production of a single fermion so that the fermion number does not conserve [7,9].

Having established eq.(8) we can proceed further exactly in the same way as 't Hooft did [8] in case of Dirac spinors. The result is

$$\begin{aligned} \Phi^2 (A = A_{\text{instanton}}, \eta, \eta^+) = \\ = d_f(\rho) \int d^4x \bar{j}(x) u_0(x) \int d^4y u_0^+(y) j(y) \end{aligned} \quad (11)$$

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<sup>κ</sup> The right-left symmetry of weak interactions could restore at some higher mass scale if there exist right-handed fermions interacting with the same intermediate bosons. We note in passing that these fermions cannot be too heavy. The typical mass scale is about 100 GeV. Otherwise the observed fine tuning of the  $W$ - and  $Z_0$ -boson masses ( $\rho \approx 1$ ) would be destroyed by higher order effects.

where  $d_f$  is the factor in the instanton density associated with non-zero fermionic modes,  $\rho$  is the instanton size and  $u_0$  is the zero mode in the field of a given instanton; the external source  $j(x)$  is defined in eq.(9). Explicitly,

$$u_0(x) = \frac{1}{\pi} \frac{\rho}{(x^2 + \rho^2)^{3/2}} \begin{pmatrix} \psi_0 \\ 0 \end{pmatrix} ; (\psi_0)_a = \epsilon_{\kappa a} \quad (12)$$

$$d_f(\rho) = e^{0.292} \cdot \exp\left(\frac{2}{3} \ln M \rho\right)$$

where  $M$  is the mass of Pauli-Villars regulator field and the dependence of  $d_f(\rho)$  on  $\rho$  is related to the fermion contribution to renormalization of the coupling constant  $g$ .

Substituting eq.(9) into eq.(11) we get the answer in terms of the two-component sources  $\eta$  :

$$\Phi = \pm [d_f(\rho)]^{1/2} \frac{1}{\pi} \int d^4x (\eta^+(x))_a \epsilon_{\kappa a} \frac{\rho}{(x^2 + \rho^2)^{3/2}} \quad (13)$$

Eq.(13) describes production of a single fermion as is expected according to the selection rule based on the triangle anomaly (in the external current counting the number of fermions,  $\psi_\mu \sim \varphi^\dagger \sigma_\mu^R \varphi$ ). Note that eq.(13) gives the answer explicitly in terms of two-component quantities and still there is no uncertainty involved.

For an instanton of a small size the result can be represented in a form of effective Lagrangian:

$$[\Delta \mathcal{L}(x)]_{\text{instanton}} = \pi [\varphi^+(x)]_a^i R_{ik}(\vec{n}) \epsilon_{\kappa a} d(\rho) \frac{d\rho}{\rho^5} \frac{d^3 n}{(8\pi e)} \quad (14)$$

$$d(\rho) = d_0(\rho) [d_f(\rho)]^{1/2}$$

where  $d_0(\rho)$  is the ordinary instanton density for  $SU(2)$  gauge group,  $d^3 n$  is the volume element of the  $SU(2)$  group and the matrix  $R_{ik}$  specifies the instanton group orientation. Antiinstantons provide the Hermitian conjugate of Lagrangian (14). It goes

without saying that averaging over instanton orientation would nullify the effective Lagrangian (14).

It is not for the first time that evaluation of the effect of instantons in theory (1) has been attempted and a few remarks on literature are now in order. In Ref. [9] some extension of the rules of integration over Fermi fields has been proposed to cover the case when right and left diagonal bases do not coincide with each other (because of non Hermiticity of Lagrangian (1) in Euclidean space). The recipe does not fix the phase, however. This point, among other things, was emphasized in Ref. [10] which starts the calculation from the basis of perturbation theory. The effect of rotation of the right and left diagonal bases with respect to the perturbative one is then traced and is argued to produce some extra factor. We believe this extra factor to be unity so that our answer is close to that of Ref. [10] (the reason for vanishing of the rotation effect is just vectorlike nature of theory (1) which is explained in length above). Moreover, there is no ambiguity in phase. In particular, once  $\theta$ -term is introduced into Lagrangian (1) the extra factor  $\exp(i\theta)$  enters the effective Lagrangian.

Evaluation of the instanton induced effects in supersymmetric theories is of more immediate interest (in connection with the problem of spontaneous breaking of supersymmetry instantons were discussed recently in Ref. [11]). As was mentioned above explicit evaluation is possible here and we proceed to this now.

In terms of Majorana spinors  $\lambda_a^\alpha$  ( $a=1,2,3, \alpha=1,2,3,4$ ) the Lagrangian is

$$\mathcal{L} = -\frac{1}{4} (G_{\mu\nu}^a)^2 + \frac{1}{2} \lambda^T \gamma_0 \gamma_\mu i \mathcal{D}_\mu \lambda \quad (15)$$

and the generator matrices  $T^a$  entering the covariant derivative are understood to be in triplet representation,  $(T^a)_{bc} = -i \epsilon_{abc}$ . The Lagrangian reduces to form (1) upon expressing the Majorana spinors in terms of Weyl spinors,  $\varphi^a \sim (1-\gamma_5)\lambda^a$  and substituting  $T^a$  in triplet representation. Therefore, the result for the fermion determinant can be read off eq.(8) with the following trivial changes. First, eq.(3) looks now as

$$\varphi_a^a = (\sigma_2)_{a\beta} \chi_\beta^a, \quad (a = 1, 2, 3)$$

and the external source in a four-component form reduces to

$$j^a = \begin{pmatrix} -\sigma_2 \eta^{*a} \\ \eta^a \end{pmatrix}, \quad \bar{j}^a = (\eta_a^+, -\eta_a^T \sigma_2) \quad (16)$$

To proceed with explicit computation we need knowledge of the zero modes. There are four fermionic zero modes and these can be readily constructed. Let us represent the result in the form

$$u_{(i)}^a = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi_{(i)}^a, \quad (i) = 1, 2, 3, 4.$$

where  $\varphi_{(i)}^a$  is a two-component spinor, (i) is the mode number and

$$\varphi_{(1,2)}^a = \frac{\sqrt{2}}{\pi} \frac{\rho^2}{(x^2 + \rho^2)^2} \sigma^a v_{1,2}$$

$$\varphi_{(3,4)}^a = \frac{1}{\pi} \frac{\rho}{(x^2 + \rho^2)^2} \sigma^a \sigma_\mu^+ \chi_\mu w_{3,4} \quad (17)$$

Here  $\sigma_\mu^+ = (-i, \vec{\sigma})$  and  $v_{1,2}, w_{3,4}$  are some constant spinors; for example,  $v_1 = w_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v_2 = w_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  and matrices  $\sigma^a$  act on these spinors mixing in this way the space and colour indices. It is worth noting that the group theoretical meaning of the zero modes has been elucidated in Ref. [6].

In terms of the zero modes  $\Phi$  is given by

$$\begin{aligned} \Phi (A = A_{\text{instanton}}, \eta, \eta^+) &= \\ &= \pm \sqrt{d_{\text{nonzero}}^{(\varphi)} (M\rho)^4} \prod_{(i)=1}^4 \int d^4x \eta^{a+}(x) \varphi_{(i)}^a(x) \end{aligned} \quad (18)$$

where  $M$  is the regulator mass, as before;  $d_{\text{nonzero}}$  accounts for all nonzero modes in  $d(\rho)$  while the factor  $(M\rho)^4$  is due to the four fermionic zero modes. Such a distinction between contributions of the zero and nonzero modes acquires its full significance in supersymmetric models since the effect of all nonzero modes is cancelled out once both fermionic and bosonic degrees of freedom are accounted for. The coupling constant renormalization is also entirely now due to the zero modes (4 fermionic and 8 bosonic modes) :

$$\frac{2\pi}{d_s(\rho)} = \frac{2\pi}{d_s(M)} + \left(\frac{1}{2} \cdot 4 - 8\right) \ln M\rho$$

(For a discussion of the role of zero and nonzero modes in the standard calculations of  $d(\rho)$  see Ref. [12]).

Thus the instanton contribution into the partition function takes the form

$$Z_{\text{instanton}} = \left(\frac{2\pi}{d_s}\right)^4 \exp\left(-\frac{2\pi}{d_s(\rho)}\right) \frac{d\rho}{\rho^5} \prod_{(i)=1}^4 \int d^4x \gamma^{a+}(x) \varphi_{(i)}^a(x) \quad (19)$$

The result can be put again into form of effective fermion interaction induced by an instanton of small size:

$$\begin{aligned} (\mathcal{L}_{\text{eff}})_{\text{instanton}} &= 2\pi^4 \left(\frac{2\pi}{d_s}\right)^4 \exp\left(-\frac{2\pi}{d_s(\rho)}\right) \frac{d\rho}{\rho^5} \\ &\varphi^{a+}(x) \sigma^a \sigma^b \sigma_2 \varphi^{b*}(x) \partial_\mu \varphi^{a+}(x) \sigma^a \sigma_\mu^- \sigma_\nu^+ \sigma^b \sigma_2 \partial_\nu \varphi^{b*}(x) \end{aligned} \quad (20)$$

Antiinstantons are responsible for the Hermitian conjugate.

Averaging over the instanton group orientation we get

$$\begin{aligned} \overline{(\mathcal{L}_{\text{eff}})_{\text{instanton}}} &= \frac{8\pi^4}{3} \left(\frac{2\pi}{d_s}\right)^4 \exp\left(-\frac{2\pi}{d_s(\rho)}\right) \frac{d\rho}{\rho^5} \\ &\cdot \left\{ \varphi^{+a} \sigma_2 \varphi^{a*} \partial_\mu \varphi^{+b} \sigma_2 \partial_\nu \varphi^{b*} + \right. \\ &\quad \left. + \varphi^{+a} \sigma_c \sigma_2 \varphi^{b*} \partial_\mu \varphi^{+c} \sigma_c \sigma_2 \partial_\nu \varphi^{a*} \right\} \end{aligned} \quad (21)$$

Adding up the Hermitian conjugate ( due to anti instantons) and going back to Majorana notations we get

$$\begin{aligned}
 (\mathcal{L}_{\text{eff}})_{\text{instanton} + \text{antiinstanton}} &= \frac{4\pi^4}{3} \left( \frac{2\pi}{\alpha_5} \right)^4 \exp\left(-\frac{2\pi}{\alpha_5(\rho)}\right) \frac{d\rho}{\rho^5} \\
 &\left\{ \lambda^{qT} \delta_0 \lambda^a \partial_\mu \lambda^{6T} \delta_0 \partial_\mu \lambda^6 - \frac{1}{2} \lambda^{qT} \delta_0 \sigma_{\alpha\beta} \lambda^6 \partial_\mu \lambda^{6T} \delta_0 \sigma_{\alpha\beta} \partial_\mu \lambda^q \right. \\
 &\quad \left. + \lambda^{qT} \delta_0 \delta_5 \lambda^a \partial_\mu \lambda^{6T} \delta_0 \delta_5 \partial_\mu \lambda^6 \right\} \quad (22)
 \end{aligned}$$

Eq.(22) is our final result for the instanton induced interaction and as far as applications are concerned it is the central point of present paper.

A few remarks concerning eq.(22) are now in order. Effective Lagrangian (22) keeps only lowest power of the fields. This is the reason why it contains ordinary, not covariant, derivatives and, superficially, is not gauge invariant. Since the whole procedure is gauge invariant at each step we are guaranteed that in next order in expansion in the fields (this time, in the bosonic fields) the ordinary derivatives will be build up to the covariant ones. However, extra terms constructed from  $G_{\mu\nu}^a$  would also arise and we are not interested in them here.

In the approximation considered the effective Lagrangian contains fermionic but no bosonic fields. The reason for this asymmetry is obvious : only fermionic zero modes make the determinant vanish. But the same trivial remark implies that Lagrangian (22) cannot be invariant under supersymmetry transformations which substitute bosonic fields for the fermionic ones. The only way, therefore, for (22) to be compatible with supersymmetry is to reduce to a total derivative. But this is not so and the four-fermion interaction (22) gives nonvanishing scattering amplitude. An analogy with the famous U(1) problem immediately comes to mind. In that case the effective Lagrangian is also noninvariant

under symmetry (  $U(1)$  ) transformations. In that case noninvariance of the effective Lagrangian does imply explicit (not spontaneous ) breaking of the symmetry. We think that the same is true with supersymmetry. This point deserves detailed discussion which , however, goes beyond the scope of the present paper.

The authors are grateful to V.A.Novikov and M.A.Shifman for thorough and productive discussions.

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Об интегрировании по фермионным полям в киральных и суперсимметричных теориях

Работа поступила в ОНТИ 6.01.82

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|                             |                  |                    |
|-----------------------------|------------------|--------------------|
| Подписано к печати 13.01.82 | Т00239           | Формат 60x90 1/16. |
| Офсетн.печ.                 | Усл.-печ.л.0,75. | Уч.-изд.л.0,5.     |
| Заказ 6                     | Индекс 3624      | Тираж 290 экз.     |
|                             |                  | Цена 8 коп.        |

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Отпечатано в ИТЭФ, П17259, Москва, Б.Черемушкинская, 25

ИНДЕКС 3624