IC/82/133

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MASSIVE NEUTRINOS IN ASTROPHYSICS

Asghar Qadir

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

1982 MIRAMARE-TRIESTE

IC/82/133

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

MASSIVE NEUTRINOS IN ASTROPHYSICS *

Asghar Qadir **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

Massive neutrinos are among the big hopes of cosmologists. If they happen to have the right mass they can close the Universe, explain the motion of galaxies in clusters, provide galactic halos and even, possibly, explain galaxy formation. Tremaine and Gunn have argued that massive neutrinos cannot do all these things. I will explain, here, what some of us believe is wrong with their arguments.

MIRAMARE - TRIESTE
August 1982

---

* To appear in the Proceedings of the "Enrico Fermi" School on Gamow Cosmology, Varenna, Italy, July 13th to July 23rd, 1982.

** Permanent address:: Mathematical Department, Quaid-i-Azam University, Islamabad, Pakistan and Centre of Basic Sciences (UGC), Islamabad, Pakistan.

There have been many talks, here, expounding on the importance of neutrinos in cosmological considerations. The history of the subject is not long - only about ten years as a serious idea - but it is taking more and more of the attention of cosmologists, and making up an ever larger part of the litterature. This flood of work started with two papers by Cowsik and Mc Clelland 1) which pointed out that if neutrinos have masses, even as small as ~ 1 eV/c^2, they could play an important role in cosmology and astrophysics. They could supply: (i) enough mass to close the Universe (i.e. keep it finite in volume); (ii) the "dark matter" postulated to explain the motion of galaxies in clusters and (iii) even the postulated galactic halos, which (as we have heard from Professors Rubin and Ruffini) are required to explain the flattening of the velocity curves and to keep the galaxy dynamically stable. This very attractive suggestion of Cowsik and Mc Clelland was not really followed up till much later, when a paper was published on the subject by Tremaine and Gunn 2) (TG). This paper claimed to demonstrate, on the basis of some thermodynamic considerations, that the suggestion of Cowsik and Mc Clelland was untenable. This debate assumed much greater importance with the independent announcements of two teams of experimentalists that neutrino mass had been observed 3). Despite later doubts about the validity of these announcements 4), the initial excitement did not die down. It was argued that even if neutrinos could not have the right mass to do all that was suggested by Cowsik and Mc Clelland, they may still play an important role in cosmology, and perhaps even in astrophysics 5). Since then there have been any number of papers on the various aspects of neutrinos in cosmology that have been discussed here. In this talk I shall try to explain why some of us believe that the arguments of TG are wrong, and should, in fact, actually, be interpreted as evidence in favour of believing that there are large diffuse halos around galaxies, such as might be expected if these haloes were composed of neutrinos having masses ~ 10 eV/c^2.

Let us, very briefly, review the argument of TG. They argue that the maximum mass of matter inside galaxies is known, and hence, if the number of neutrinos in the galaxy is known, we have as upper bound on the mass of the neutrinos. The number of neutrinos is estimated by direct analogy with the number of photons which must arise (as was pointed out by George Gamow) from the hot big bang. They, then, obtain a lower bound for the neutrino mass if it is to be consistant with the observed motion of: (1) galaxies in a cluster; (2) binary galaxies about each other; or (3) the stars in an individual galaxy. To be able to tie their thermodynamic considerations to observation they needed to make certain assumptions: (A) the maximum coarse-grained phase-space density must decrease with time; (B) the density of matter in galaxies is not more than one twentieth of the critical density (which is the density required to have a flat Friedmann universe); and (C) the bound system of neutrinos resembles an isothermal gas sphere. Their whole argument is based on the implicit assumption: (D) the "core of the galaxy" used in their calculations is the "core of the visible component of the galaxy".

Using assumption (C), TG take a Maxwell-Boltzmann distribution for the neutrinos. Then assumption (A) yields the requirement that

$$m_\nu^4 \gtrsim (\rho_c/2g_\nu)(h^2/2\pi\sigma^2)^{3/2} \tag{1}$$

where σ is the one-dimensional velocity dispersion ρ_c, the core density, is given by

$$\rho_c = 9\sigma^2/4\pi G r_c^2 \tag{2}$$

r_c is the core radius and g_ν is the number of helicity states of neutrinos of the same mass. For a given neutrino species g_ν would be two (one each for neutrino and anti-neutrino) if there are no right-handed neutrinos which are relevant. (This will be the case in most GUT theories 6), where the right-handed neutrinos are superheavy.) If there is only one species with a mass in the relevant range, we would have $g_\nu = 2$ for that species, and zero otherwise. If there are two species with the same mass $g_\nu = 4$, and if there are three species $g_\nu = 6$. If there is only one species and the right-handed neutrino has the same mass we would have $g_\nu = 4$. If there were only one species of massive neutrinos and enough lepton number asymmetry (incidentally, giving a finite chemical potential) we could even take $g_\nu = 1$.

Using assumption (B), TG obtain the constraint for the sum of neutrino masses

$$\Sigma g_\nu m_\nu \lesssim 5 h_o^{-1} \ eV/c^2 \tag{3}$$

where $h_o = H \times 10^{17.5}$sec, H being the Hubble parameter. From observation h_o lies between 1 and 2. On the other hand, Eq.(1) leads to the constraint

$$m_\nu \gtrsim (101\, eV/c^2)(100\, km.sec^{-1}/\sigma)^{1/4}(1 kpc/r_c)^{1/2} \tag{4}$$

Using assumption (d), observed values of σ and r_c can be put in, giving

$$m_\nu \gtrsim h_o^{-1/2} g_\nu^{-1/4} (k_1\, eV/c^2) \tag{5}$$

where k_1 is 2.5 for galactic clusters, 7 for binary galaxies and 15 for individual galaxies. It is apparent that only in the case of galactic halos is it at all possible to consider the two equations to be consistent, giving a value ~ 3 eV/c^2 for the neutrino mass. TG allow that some further uncertainties may possibly permit smaller values of σ and r_c, and thus even binary galaxies may be argued to be consistent , but there is no way to reconcile Eqs.(3) and (5) for individual galaxies. By Occam's razor, they rule out the suggestion of Cowsik and Mc Clelland entirely. Schramm and Steigman 5), going purely by the final statement of the conclusion by TG, suggest that neutrinos with mass $\sim$ 3-10 eV/c^2 may provide much of the missing matter and even close the Universe, despite their being unable to provide galactic halos.

There are various problems with the arguments of TG. The most crucial one 7) is concerned with their assumption (D), that the entire matter content in a galaxy can be deduced from the motion of its visible component. From the motion of the stars, in principle one can deduce the matter distribution interior to the "orbits" of the stars, but not the matter distribution exterior to their orbits. Since the upper bound on neutrino masses comes from the assumption (modified by including some estimates based on the dynamics of galactic systems) the upper bound should be relaxed. With sufficiently large and diffuse halos, arbitrarily large amounts of matter can be contained in the halo. The more important aspect of relaxing assumption (D) is that the thermodynamic arguments are drastically affected. They depend crucially on confining the matter within a given region and using only the velocity dispersions in that region. If there is enough neutrino matter, in a large halo, to dominate galactic dynamics it will become necessary to consider the region containing most of the galactic matter (i.e. the neutrinos) and the velocity dispersions of that matter. Thus our "core" has to be changed from the "visible core" to the "neutrino core". The "neutrino core" must yield the observed motion of the visible component of the galaxy. The situation may be visualized as depicted in Fig.1. Notice that matter in the small bump, which represents the "visible core", will have a very different dynamical behaviour from the "neutrino core" matter. Clearly, assumption (B) has to be modified in accordance with the modification of assumption (D).

Assumption (C) needs to be considered very carefully. The bound system of neutrinos can be regarded as a collisionless gas because the neutrinos interact weakly. As such they have no means of thermalizing, and the initial velocity distribution can be regarded as being "frozen in" even when the gas cools. At first sight, the neutrinos may seem to have a scaled down form of their original velocity distribution, which was isothermal. Due to the absence of collisions, however, the distribution will have no thermal form as the gas cools. Thus it would be wrong to assume a Maxwell-Boltzmann distribution for the neutrino gas. It has been argued that the neutrinos may be able to interact through the mean field (which is the gravitational field), and may, then, be able to thermalize. There would still remain a worry as to the final thermal distribution. Since neutrinos are fermions, the isothermal distribution will only apply if its temperature, $T_{iso}^{(\nu)}$, is greater than the Fermi temperature $T_F^{(\nu)}$. Now, if we take matter, of density ρ_{vis}, to be uniformly distributed in a sphere, and the neutrinos to have a density ρ_ν, the Fermi temperature will be

$$T_F^{(\nu)} = (\hbar^2/m_\nu)(N/V)^{2/3} \simeq 10(\rho_\nu/\rho_{vis})^{2/3}(1\,eV/c^2/m_\nu)\ {}^\circ K \quad (6)$$

while the inferred temperature of the isothermal gas sphere will be

$$T_{iso}^{(\nu)} = m_\nu \sigma^2/3k \quad (7)$$

Taking the neutrino matter and the visible matter confined to the same core radius, if neutrinos are to dominate the galaxy, the TG argument would only hold for $m_\nu \gtrsim 100\ eV/c^2$ (having used $\sigma \sim 100$ km/sec. as done by TG). Otherwise the Fermi temperature will be greater (orders of magnitude greater for $m_\nu \sim 10\ eV/c^2$) than the inferred isothermal gas sphere temperature. Thus the isothermal gas sphere approximation, as used by TG, is untenable. If, instead, we take the neutrino halo to extend much further out, the approximation can, possibly, improve considerably as ρ_ν/ρ_{vis} will decrease.

Let us, now, consider the consequences of approximating the bound system of neutrinos by a degenerate Fermi gas. Calculations of a degenerate, self-gravitating system of Fermi particles [8] and similar calculations for galactic size objects [9] show that there is no inconsistancy in this case (such as was claimed by TG) for a galaxy approximated by a sphere of visible matter

$\sim$ 10 kpc radius, $\sim 2 \times 10^{-24}$ gm/cm^3 density and of neutrinos $\sim$ 100 kpc radius and $\sim 10^{-24}$ gm/cm^3, if the neutrino mass $\sim 10\ eV/c^2$ and the neutrino velocity distribution $\sim$ 700 km/sec.. It is easily verified that this configuration gives $T_{iso}^{(\nu)} \quad T_F^{(\nu)}$, with little difference between the two temperatures. One would expect, then, that the degenerate configuration could be approximated by an isothermal configuration. Putting the appropriate numbers into Eq.(1) no contradiction appears. i.e. the isothermal approximation implemented in this way is quite consistent with the degenerate Fermi gas approximation, and does not provide any argument to say that galactic halos cannot be composed of neutrinos with mass $\sim 10\ eV/c^2$.

I have explained why the argument of TG is invalid and how to correct it [10]. It became clear that the corrected argument does not disallow massive neutrino halos, but not that it provides evidence for their existance. Let me now explain why I claim that the TG arguments provide evidence of large diffuse halos such as would be expected if they were composed of neutrinos having masses $\sim 10\ eV/c^2$. Having recognized that assumption (D) is wrong, one can ask oneself how significant the error is in particular cases. From Fig.2 it is clear that the error is least for rich galactic clusters, more for binary galaxies and most for individual galaxies, as more of the invisible halo matter is effectively "seen" (by the motion of visible matter) in larger collections of galaxies. In fact, putting in the numbers mentioned above gives the same sort of discrepancy noticed by TG! I would like to mention one more "piece of evidence" in favour of the "case for very large diffuse halos". The typical distribution of galaxies in a rich cluster, according to the simplified picture depicted in Fig.1, would be much like that shown in Fig.3, which is the actual distribution of galaxies in the Coma cluster.

ACKNOWLEDGEMENTS

I would like to thank the organizers of the Enrico Fermi School on Gamow Cosmology for arranging for my participation at such short notice. I would also like to express my gratitude to Professor Abdus Salam, the International Atomic Energy Agecny and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. I would also like to thank Professors R. Ruffini and D.W. Sciama for discussions on the subject.

REFERENCES AND FOOTNOTES

1) R. Cowsik and J.M. Mc Clelland, Phys. Rev. Lett. 29, 669 (1972); Ap. J. 180, 7 (1973).

2) S. Tremaine and J.E. Gunn, Phys. Rev. Lett. 42, 407 (1979).

3) F. Reines, H.W. Sobel and E. Pasierb, Phys. Rev. Lett. 45, 1307 (1980); V.A. Lyubimov, E.G. Nozik, E.Z. Tretyakov and V.S. Kosik, ITEP preprint No.62, Moscow 1980.

4) R.P. Feynman and Vogel, MIT preprint 1980.

5) D.N. Schramm and G. Steigman, Ap. J. 243, 1 (1981); Gen. Rel. and Grav. 13, 101 (1981).

6) R.E. Marshak, R.N. Mohapatra and Riazuddin, Proc. of Muon Worshop, TRIUMF, Vancouver 1980; G. Lazarides and Q. Shafi, Phys. Letters 99B, 113 (1981).

7) A. Qadir and R, Ruffini, ICTP preprint IC/80/122, Trieste.

8) R.Ruffini and S. Bonazzola, Phys. Rev. 187, 1767 (1969).

9) J.G. Gao and R. Ruffini, Phys. Letters 97B, 388 (1980); 100B, 47 (1981); A. Crollalanza, J.G. Gao and R. Ruffini, Nuovo Cim. Lett. 32, 411 (1981).

10) It should be pointed out that when D.W. Sciama presented the same sort of objections (Astrophysical Cosmology, edited by H.A. Bruck, G.V. Coyne and M.S. Longair, Pontificiae Academie Scientarium Scripta Varia 48, 1982) Gunn recorded disagreement on that point, on the basis that there are observational limits on the total of matter permissible in the Universe. It should be stressed that for a sufficiently diffuse halo the only observational limits are that the density is less than twice the critical density.

FIGURE CAPTIONS

Fig.1 Explanation of the large diffuse halo by a simple analogy. (a) Imagine a rubber sheet with water poured into it. (b) If we now pour iron filings into the water we get a "bump" at the centre. (c) On stirring up the water the iron filings will spread out the bump. The dynamics of the filings in the bump will be very different from that of the main body of water. For our purposes the water is replaced by neutrinos, the iron filings by visible matter and the rubber sheet, of course, by space-time. Stirring corresponds to introducing a finite temperature. The analogy breaks down at the thermodynamic level as the two types of matter do not tend to the same temperature in galaxies.

Fig.2 (a) For a single galaxy most of the halo matter is not apparent in the motion of the visible component of the galaxy as it is exterior to it. (b) Much more of the halo becomes apparent in the case of binary galaxies. (c) Still more becomes apparent in the case of galactic clusters. The richer the clusters the greater the portion which becomes apparent.

Fig.3 The distribution of galaxies in the Coma cluster of galaxies. Qualitatively this is exactly what would be expected from Fig.1(c). Though the models are, by no means reliable enough to be used for a detailed fitting with the data, it is nice to know that they are in good agreement with observation.

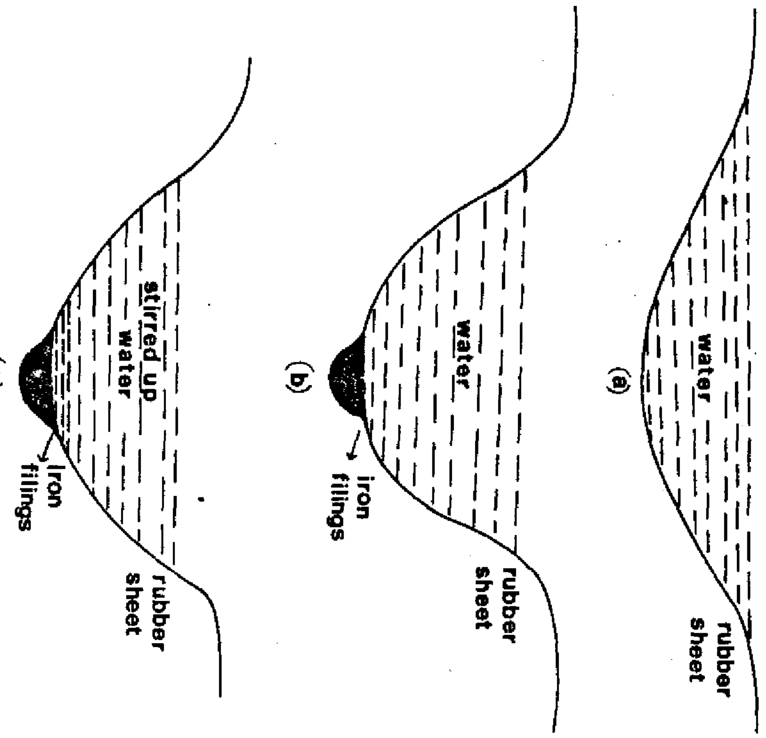
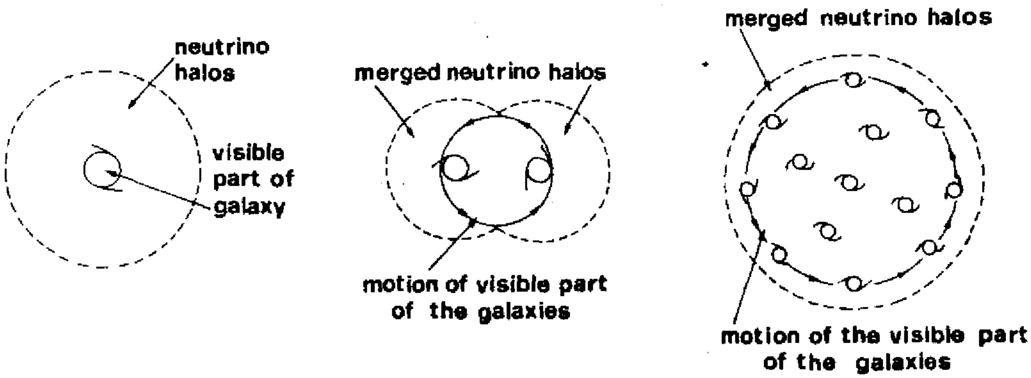


Fig.1



(a) (b) (c)

Fig.2

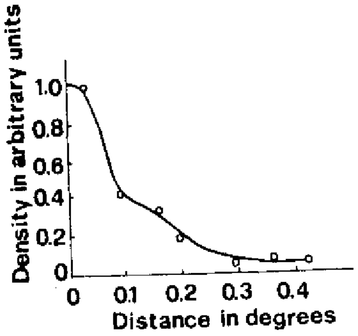


Fig. 3

CURRENT ICTP PUBLICATIONS AND INTERNAL REPORTS

IC/82/23 INT.REP.*	SUN KUN OH - Mass splitting between B^+ and B^0 mesons.
IC/82/24 INT.REP.*	A. BREZINI - Self-consistent study of localization near band edges.
IC/82/25 INT.REP.*	C. PANAGIOTAKOPOULOS - Dirac monopoles and non-Abelian gauge theories.
IC/82/26	CAO CHANG-qi and DING XING-fu - Intermediate symmetry $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ and the SU(N) unification series.
IC/82/27	H.B. GHASSIB and S. CHATTERJEE - ^{4}He-impurity effects on normal liquid ^{3}He at low temperatures - I: Preliminary ideas and calculations.
IC/82/28	C. PANAGIOTAKOPOULOS, ABDUS SALAM and J. STRATHDEE - Supersymmetric local field theory of monopoles.
IC/82/29 INT.REP.*	G.A. CHRISTOS - Concerning the proofs of spontaneous chiral symmetry breaking in QCD from the effective Lagrangian point of view.
IC/82/30 INT.REP.*	M.I. YOUSEF - Diffraction model analyses of polarized ^{6}Li elastic scattering.
IC/82/31	L. MIZRACHI - Electric-magnetic duality in non-Abelian gauge theories.
IC/82/32 INT.REP.*	W. MECKLENBURG - Hierarchial spontaneous compactification.
IC/82/33	C. PANAGIOTAKOPOULOS - Infinity subtraction in a quantum field theory of charges and monopoles.
IC/82/34 INT.REP.*	M.W. KALINOWSKI, M. SEWERYNSKI and L. SZYMANOWSKI - On the F equation.
IC/82/35 INT.REP.*	H.C. LEE, LI BING-AN, SHEN QI-XING, ZHANG MEI-MAN and YU HONG - Electroweak interference effects in the high energy $e^+ + e^- \rightarrow e^+ + e^-$ + hadrons process.
IC/82/36	G.A. CHRISTOS - Some aspects of the U(1) problem and the pseudoscalar mass spectrum.
IC/82/37	C. MUKKU - Gauge theories in hot environments: Fermion contributions to one-loop.
IC/82/38 INT.REP.*	W. KOTARSKI and A. KOWALEWSKI - Optimal control of distributed parameter system with incomplete information about the initial condition.
IC/82/39 INT.REP.*	M.I. YOUSEF - Diffraction model analysis of polarized triton and ^{3}He elastic scattering.
IC/82/40 INT.REP.*	S. SELZER and N. MAJLIS - Effects of surface exchange anisotropy in Heisenberg ferromagnetic insulators.
IC/82/41 INT.REP.*	H.R. HAROON - Subcritical assemblies, use and their feasibility assessment.
IC/82/42	W. ANDREONI and M.P. TOSI - Why is AgBr not a superionic conductor?

THESE PREPRINTS ARE AVAILABLE FROM THE PUBLICATIONS OFFICE, ICTP, P.O. BOX 586,
I-34100 TRIESTE, ITALY.

IC/82/43	N.S. CRAIGIE and J. STERN - What can we learn from sum rules for vertex functions in QCD?
IC/82/44 INT.REP.*	ERNEST C. NJAU - Distortions in power spectra of digitised signals - I: General formulations.
IC/82/45 INT.REP.*	ERNEST C. NJAU - Distortions in power spectra of digitised signals - II: Suggested solution.
IC/82/46 INT.REP.*	H.R. DALAFI - A critique of nuclear behaviour at high angular momentum.
IC/82/47	N.S. CRAIGIE and P. RATCLIFFE - Higher power QCD mechanisms for large p_T strange or charmed baryon production in polarized proton-proton collisions.
IC/82/48 INT.REP.*	B.R. BULKA - Electron density of states in a one-dimensional distorted system with impurities: Coherent potential approximation.
IC/82/49 INT.REP.*	J. GORECKI - On the resistivity of metal-tellurium alloys for low concentrations of tellurium.
IC/82/50	S. RANDJBAR-DAEMI and R. PERCACCI - Spontaneous compactification of a (4+d)-dimensional Kaluza-Klein theory into $M_4 \times G/H$ for arbitrary G and H.
IC/82/51 INT.REP.*	P.S. CHEE - On the extension of H^P-functions in polydiscs.
IC/82/53 INT.REP.*	S. YOKSAN - Critical temperature of two-band superconductors containing Kondo impurities.
IC/82/54	M. ÖZER - More on generalized gauge hierarchies.
IC/82/55	N.S. CRAIGIE and P. RATCLIFFE - A simple method for isolating order α_S and α_S^2 corrections from polarized deep-inelastic scattering data.
IC/82/56	C. PANAGIOTAKOPOULOS - Renormalization of the QEMD of a dyon field.
IC/82/57 INT.REP.*	J.A. MAGPANTAY - Towards an effective bilocal theory from QCD in a background.
IC/82/58	S. CHAKRABARTI - On stability domain of stationary solitons in a many-charge field model with non-Abelian internal symmetry.
IC/82/59 INT.REP.*	J.K. ABOAYE and D.S. PAYIDA - High temperature internal friction in pure aluminium.
IC/82/60	Y. FUJIMOTO and ZHAO ZHIYONG - N-$\bar{N}$ oscillation in SO(10) and SU(6) supersymmetric grand unified models.
IC/82/61	J. GORECKI and J. POPIELAWSKI - On the application of the long mean free path approximation to the theory of electron transport properties in liquid noble metals.
IC/82/62	I.M. REDA, J. HAFNER, P. PONGRATZ, A. WAGENDRISTEL, H. BANGERT and P.K. BHAT - Amorphous Cu-Ag films with high stability.
IC/PD/82/1 INT.REP.	PHYSICS AND DEVELOPMENT (Winter College on Nuclear Physics and Reactors, 25 January - 19 March 1982).