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THE INTERACTION OF AN ELECTROMAGNETIC WAVE WITH AN INHOMOGENEOUS PLASMA SLAB

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IPPCZ 242

July 1982

RESEARCH REPORT

POD VODÁRENSKOU VĚŽÍ 4, 180 69 PRAGUE 8
CZECHOSLOVAKIA

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In the connection with the problem of the plasma heating by high-frequency waves we have developed the numerical code which makes possible to study the incidence of an electromagnetic wave on an inhomogeneous plasma slab. In our one dimensional model we describe non-magnetized plasma by means of the two-fluid equations with a finite electron pressure and with the adiabatic condition for all processes.

It is shown, that at the normal incedence of a wave on a cold plasma, the wave is reflected from the region of the plasma resonance. A standing wave is arising which generates an electrostatic standing wave with twofold frequency. At the same time density gradient is sharply steepening in this region. In a warm plasma the incident wave again creates a standing wave, but the nonlinear perturbations propagate from the region of plasma resonance with the ion sound velocity to the whole volume of plasma. In this case the density gradient does not change very much. In the region of plasma resonance the ion sound waves are also generated.

1. INTRODUCTION

In recent years a great attention is paid to the study of the interaction of electromagnetic waves with a plasma. The understanding of physical processes which take place in plasma, and their quantitative evaluation are important both for the explanation of some effects in astrophysics and for the study of controlled fusion. In the last case the interaction of laser beams with dense plasma target and the rf heating of a rarefied magnetically confined plasma are subjects of considerable interest. In all these cases the interaction of intensive electromagnetic waves with a plasma takes place and the nonlinear effects play a dominant role. Thus, from the theoretical point of view it is possible to investigate these effects only by means of numerical methods.

The aim of our paper is to investigate the interaction of an electromagnetic wave with an inhomogeneous plasma using a simple numerical code which describes the most important nonlinear effects. The particle simulation codes give the most complete but very complicated description of these processes (see, e.g. [1], in relativistic case see [2]). However, the demands on the storage and speed of a computer are enormous; 10^5 particles for the two-dimensional model, 10^4 - 10^5 time steps. Another posibility is offered by the numerical solution of simple phenomenological equations - e.g. of the nonlinear Schrödinger equation for the averaged wave amplitude (see [3]). Because, these are usually derived for the electrostatic waves and the linear density profile, their applications are limited. In the small amplitude limit the starting equations (e.g. the kinetic equations or the fluid equations etc.) may be expanded with respect to the wave amplitude, eventually time averaged. From such equations we obtain expressions for ponderomotive forces (see [4]), the equations for second harmonic generation (see e.g. [5]), or equations for parametric decay of waves (see [6]). In mathematical formulation this problem is usually reduces to the solution of a set of ordinary differential equations or even to the set of algebraic equations (see [7]). There are, however, difficulties to describe the self-consistent interaction of electromagnetic waves with a plasma in these models.

In our paper we have chosen the mean course between the complicated particle code and between rather simplified theories using model or averaged equations. We have started from the Maxwell equations and from the equations of two-fluid hydrodynamics. Using this description we lost some nonlinear kinetic effects (e.g. the particle trapping in a strong wave), but as it was already shown in the first wave-particle simulations, the macroscopic quantities have approximately the same values as those obtained from mhd simulations [1]. There is a failure of this model, namely, the absence of the collosionless damping. This can be overcome by the introduction of a phenomenological damping to the momentum transport equations. The great advantage of this model is the fact that it deals with the electromagnetic wave directly and the initial and boundary conditions of our problem can be easily determined. Thus, we are able to investigate the time development of the system from the initial transient state till the state when the plasma parameters are strongly changed by nonlinear effects.

In this paper we use this model to study the interesting case of normal incidence of an electromagnetic wave on the inhomogeneous plasma slab. The main attention is paid to the effect of the electron temperature upon this process.

2. STARTING EQUATIONS

Investigating the interaction of an electromagnetic wave with an inhomogeneous plasma we shall start from the two-fluid equations and from the Maxwell equations. We shall study only the normal incidence of an electromagnetic wave on an inhomogeneous plasma slab. We shall supose that the slab is parallel to the (Y, Z) - plane and the electric field of incident wave is directed along the Z - axis. When the electron pressure p is different from zero (ions are always treated as cold) we shall assume that the plasma is confined in the direction of X - axis (i.e. in the direction of the inhomogeneity) by the gravitational field. The potential $U(x)$ of this field have been chosen in the form of a well. This gravitational field supports the stationary state of warm plasma. The relation between density, pressure and velocity is given by the adiabatic state equation.

We shall suppose that the plasma is stationary, prior to the incidence of an electromagnetic wave. In this stationary state we shall put electron and ion velocities equal to zero. Thus, no magnetic field is present in the slab at the initial state. We introduce

(1)
$$n_\alpha = N_\alpha + n_{1\alpha} \ , \ p = P + p_1 \ , \ E_x = E_{0x} + E_{1x} \ ; \ \alpha = i, e \ ,$$

where n_e, n_i are the electron and ion density, respectively, E_x is the X - component of electric field. The stationary quantites N_α, P, E_{0x} fulfil the following equations

$$\frac{dP}{dx} + N_e \left[e E_{0x} + m_e \frac{dU}{dx} \right] = 0 \ , \quad P = n_e T$$

(2)
$$e E_{0x} + m_i \frac{dU}{dx} = 0 \ , \quad \frac{dE_{0x}}{dx} = \frac{e}{\epsilon_0} \cdot \left(N_i - N_e \right) .$$

Here m_e m_i are the electron and ion mass respectively, e is the elementary charge $(e > 0)$, T is the electron temperature. If we introduce the particle flows $\vec{\Pi}_\alpha = n_\alpha \vec{V}_\alpha$ we can write the starting set of dynamic equations in the form

(3)
$$\frac{\partial n_{1\alpha}}{\partial t} - \frac{\partial}{\partial x} \Pi_{\alpha x} = 0 \ , \qquad \alpha = i, e$$

$$\frac{\partial \Pi_{ix}}{\partial t} + \frac{\partial}{\partial x} \left(\Pi_{ix} \ v_{ix} \right) = \frac{e}{m_i} \left(n_i \ E_{1x} - \Pi_{iz} \ B_y \right) ,$$

$$\frac{\partial \Pi_{ex}}{\partial t} + \frac{\partial}{\partial x} \left(\Pi_{ex} \ v_{ex} + \frac{p_1}{m_e} \right) = - \frac{e}{m_e} \left(n_e \ E_{1x} - \Pi_{ez} \ B_y \right)$$

$$- n_{1e} \frac{dU}{dx} \cdot \left(1 + \frac{m_e}{m_i} \right) ,$$

$$\frac{\partial \Pi_{\alpha z}}{\partial t} + \frac{\partial}{\partial x} \left(\Pi_{\alpha z} \ v_{\alpha x} \right) = s_\alpha \frac{e}{m_\alpha} \left(n_\alpha \ E_z - \Pi_{\alpha x} \ B_y \right) , \ s_e = -1 , \ s_i = 1 ,$$

$$\frac{\partial p_1}{\partial t} - \frac{\partial}{\partial x} \left(p \ v_{ex} \right) = \left(1 - \gamma \right) p \ \frac{\partial v_{ex}}{\partial x} \ ,$$

$$\frac{\partial B_y}{\partial t} - \frac{\partial E_z}{\partial x} = 0 \ , \ \frac{\partial B_y}{\partial x} - \epsilon_0 \mu_0 \frac{\partial E_z}{\partial t} = e \mu_0 \left(\Pi_{iz} - \Pi_{ez} \right) ,$$

$$\frac{\partial E_{1x}}{\partial t} = \frac{e}{\epsilon_0} \left(\Pi_{ex} - \Pi_{ix} \right) .$$

Here E_z, B_y represent the electromagnetic fields of a wave in plasma, μ_0 and ε_0 are permeability and permitivity of free space, respectively, γ is a ratio of the specific heats for the electron fluid, U is the potential of an artifical gravitational field having the shape of a well. This form of starting equation is suitable for the numerical solution by means of a finite-diference scheme.

3. Initial and boundary conditions

The stationary state is described by the quantities P, E_{0x}, U, N_α and possibly by T. Two of these quantities we may choose arbitrarily and the other can be determined from the set (2). In our case we have supposed $N_e(x)$ and $P(x)$ in the form

$$N_e = N_0 \cdot \left\{ \frac{1}{2} \left[1 - \cos(2\pi x/L) \right] \right\}^B , \qquad x \in \langle 0, L \rangle$$

(4)

$$P = N_e T_0 \cdot \left\{ \frac{1}{2} \left[1 - \cos(2\pi x/L) \right] \right\}^A .$$

Here N_0, T_0 are the density and the temperature of electrons in the middle of the slab, respectively. L is the thickness of the slab, A and B are the relative grandient of the temperature and the density of electrons in the point $x = L/4$, respectively. Our staionary state is fully determined by means of five constants N_0, T_0, A, B, L. . The profiles of the density, of the pressure and of the gravitional field are depicted in Fig. 1 for one special choise of these parameters. The difference between the electron and ion density is so small, that curves N_i and N_e merge. Let us note that all physical quantities and their derivative with respect to the x are equal to zero on the boundaries of the interval $\langle 0, L \rangle$. It is also important that the force $\frac{1}{N_e} \frac{dP}{dx}$ acting on electrons is equal zero at $x = 0, L$ and changes here continuously.

The initial conditions of dynamical quantities have been determined in such a way that in starting moment the plasma is in the stationary state, i.e. the quantities $n_{1\alpha}, \vec{\Pi}_\alpha, p_1, E_{1x}, E_z, B_y$ have been set to zero at $t = 0$ for all $x \in \langle 0, L \rangle$. The boundary condition splits into boundary conditions for the plasma quantities including the electric field E_{1x} and into boundary conditions for the electromagnetic wave. We have supposed that the quantities $n_{1\kappa}$, Π_{α}, ρ_1, E_{1x} are equal to zero for all $t \geqslant 0$ on the plasma boundaries, i.e. at the points $x = 0$ and $x = L$.

The boundary conditions for the electromagnetic wave call for a more detailed discussion. Our task consists in the formulation of the boundary conditions at the point $x = 0$ and $x = L$ which can be easily transformed into a finite difference scheme and which correspond to the free propagation of outgoing and incident waves. Actually we suppose that the plane wave with given amplitude and phase is incident on our slab from the point $x = -\infty$. Further, we suppose, that the front of incident wave reaches the point $x = 0$ at $t = 0$. Because there are no surface charges and no surface currents at $x = 0$ and $x = L$ the waves reflected from the plasma and the waves outgoing behind the plasma leave the slab freely.

The boundary conditions can be easily formulated only in free space. We introduce the auxiliary quantities

(5) $$E_{INC} = \frac{1}{2}\left(E_z - cB_y\right), \quad E_{DEC} = \frac{1}{2}\left(E_z + cB_y\right).$$

E_{INC} and E_{DEC} are in fact the intensities of electric field of the waves propagating in the direction of the increasing x and decreasing x, respectively. The boundary condition can be thus written as follows: on the left-hand side of plasma, i.e. at $x = 0$ we put for all $t \geqslant 0$

(6a) $$E_{INC} = S(t), \qquad \frac{\partial E_{DEC}}{\partial t} - c\,\frac{\partial E_{DEC}}{\partial x} = 0,$$

on the right-hand side of plasma, i.e. at $x = L$ we put for all $t \geqslant 0$

(6b) $$E_{DEC} = 0, \qquad \frac{\partial E_{INC}}{\partial t} + c\,\frac{\partial E_{INC}}{\partial x} = 0.$$

The source S has been chosen in the form

$$S(t) = -S_0(\omega t)\sin(\omega t)$$

(7) where $$S_0(\omega t) = E_0\left(1 - \cos(a\omega t)\right)^4/16 \quad \text{for} \quad t \in \left\langle 0, \frac{\pi}{a\omega}\right\rangle$$

$$= E_0 \quad \text{for} \quad t \geqslant \frac{\pi}{a\omega}$$

The number a is positive and a^{-1} gives the number of time periods τ ($\tau = 2\pi/\omega$) during which the amplitude of wave attains its maximum E_0

The amplitude of wave must grow slowly in numerical simulations and thus we have put $a = 0.05$.

4. The physical parameters of our problem

Seven parameters, namely $A, B, T_0, N_0, L, E_0, \omega$ occur in our model of the interaction of electromagnetic wave with the plasma slab. From the discussion of the physical similarity it follows that our problem is fully determined only by six dimensionless parameters. We have chosen these parameters

$$
(8) \qquad A = \left[\frac{L}{T}\cdot\frac{dT}{dx}\right]_{x=L/4}, \quad B = \left[\frac{L}{N_e}\,\frac{dN_e}{dx}\right]_{x=L/4}, \quad L' = L/\lambda,
$$

$$
E_0' = \frac{eE_0}{m_e c\omega}, \quad N_0' = \frac{e^2 N_0}{\epsilon_0 m_e \omega^2}, \quad T_0' = \frac{T_0}{m_e c^2},
$$

where

$$
\lambda = \omega/(2\pi c), \quad c = (\epsilon_0 \mu_0)^{-\frac{1}{2}} .
$$

The numerical values of the parameters (8) determine the character of the solved problem. If $N_0' > 1$ the plasma is dense and the waves are reflected from the slab. For $N_0' \ll 1$ the waves penetrate the plasma. The parameter L' gives the ratio of the wave length to the length of inhomogeneity. If $T_0'/E_0'^{2} \ll 1$ the plasma behaves as cold. At $T_0'/E_0'^{2} \sim 1$ we are studying processes in warm plasma. Because our equations (3) are nonrelativistic we can study only cases for which $E_0' \ll 1$ and $T_0' \ll 1$. Usually, we have chosen $10^{-3} \leqslant E_0' \leqslant 2\times 10^{-2}$ and $0 \leqslant T_0' \leqslant 5\times 10^{-3}$. In cases which are depicted on Figs. 1 - 11 we have put $A = 1$, $B = 1$, $L' = 2$ $N_0' = 2$. In addition to the parameters (8) we have two other constants in our problem, namely, the ratio of the ion mass to the electron mass m_i/m_e and the parameter μ . All results which are shown in Figs. 1 - 9, 11 were obtained for $m_i/m_e = 100$ and $\mu = 3$.

Before the numerical solution our set of equations have been transformed into the new dimensionless variables

$$
(9) \qquad t' = \omega t, \quad \vec{v}_\alpha' = \vec{v}_\alpha/c, \quad x' = \frac{\omega}{c}x,
$$

$$\vec{E}' = \frac{e}{m_e c \omega} \vec{E} , \quad \vec{B}' = \frac{e}{m_e \omega} \vec{B} , \quad U' = \frac{U}{c^2} \tag{9}$$

$$n'_\alpha = \frac{e^2}{\epsilon_0 m_e \omega^2} n_\alpha , \quad p' = \frac{e^2}{\epsilon_0 m_e^2 c^2 \omega^2} p .$$

Let us note that an arbitrary equation or an expression in the dimensionless variables can be obtained if we put $\epsilon_0 = \mu_0 = e = m_e = \omega = 1$ and substitute m_i / m_e for m_i in the dimension expressions.

3. The numerical code

The set of equations (3) is hyperbolic and its equations have a general form

$$\frac{\partial U(x,t)}{\partial t} + \frac{\partial}{\partial x} G(u,x,t) = R(u,x,t). \tag{10}$$

From many well known finite difference schemes which are applicable to the solution of (10) we have usod the leapfrog method described in (8). We have modified this method to be able to solve the nonconservative equations (10). This numerical technique consists of discretizing the space and time variables in steps Δx and Δt respectively. Introducing the usual notation

$$x_j = j \Delta x , \quad t_n = n \Delta t , \quad u_j^n = u(x_j \; t_n),$$

where $j = 0, 1 \dots M$ and $n = 0, 1, 2 \dots$ we can express the solution of (10) in a form

$$u_j^{n+1} = u_j^{n-1} + \left(R_j^n + \frac{G_{j-1}^n - G_{j+1}^n}{2 \Delta x} \right) 2 \Delta t . \tag{11}$$

The solution is numerically stable if $\Delta t < \Delta x / c$. The numerically acceptable values of $\Delta t' / \Delta x'$ and M are 0,9 and 100–200, respectively. The numerical accuracy of our solution has been checked on the fulfilment of the physical conservation laws: conservation of particles, of total energy and of total momentum. Using the prescription (11) we are able to calculate all values u_j^{n+1} from u_j^n and u_j^{n-1} . The exception forms the boundary values u_0^{n+1}, u_M^{n+1} which must be determined from the boundary conditions.

The boundary conditions (6a,b) are valid only in free space. Thus, we have shifted the plasma boundary on both sides of the slab about one lattice point inward. For $n_{\alpha}, \vec{\Pi}_{\alpha}, p_{1}$ and E_{1x} the boundary conditions must be written

(12) $$u_0^n = u_1^n = u_{M-1}^n = u_M = 0$$

for all n. Using the boundary conditions (6a,b) we can determine the boundary values of E_z, B_y. To be able to express these conditions sufficiently precisely in a difference form we have introduced the new variables

(13) $$\rho x - t = \xi \quad , \quad \rho x + t = \eta \quad , \quad \rho \equiv \Delta t / \Delta x$$

From equations (6a, b) and (13) we obtain

(14) $$\frac{\partial E_{DEC}}{\partial \eta} \cdot \left(1 - \rho c\right) - \frac{\partial E_{DEC}}{\partial \xi} \cdot \left(1 + \rho c\right) = 0 ,$$

$$\frac{\partial E_{INC}}{\partial \eta} \cdot \left(1 + \rho c\right) - \frac{\partial E_{INC}}{\partial \xi} \cdot \left(1 - \rho c\right) = 0 .$$

In these equations we can express the derivatives with respect to η and ξ by means of the central diferences

$$\frac{\partial U}{\partial \xi} = \frac{u_{j+1}^{n} - u_j^{n+1}}{\Delta} \quad , \qquad \frac{\partial U}{\partial \eta} = \frac{u_{j+1}^{n+1} - u_j^{n}}{\Delta} \quad ,$$

where $\Delta = \left((\Delta x)^2 + (\Delta t)^2 \right)^{\frac{1}{2}}$. For the values $E_{DEC,0}^{n+1}$ and $E_{INC,M}^{n+1}$ we then obtain from the equations (14)

(15) $$E_{DEC,0}^{n+1} = E_{DEC,1}^{n} + \left[E_{DEC,0}^{n} - E_{DEC,1}^{n+1} \right] \cdot q ,$$

$$E_{INC,M}^{n+1} = E_{INC,M-1}^{n} - \left[E_{INC,M}^{n} - E_{INC,M-1}^{n+1} \right] \cdot q ,$$

where $q = (1 - c\rho) / (1 + c\rho)$.

From (5), (15) and from two remaining conditions in (6a,,b) we get the boundary values for E_z, B_y.

$$(16)\qquad E_{z,0}^{n+1} = E_{DEC,0}^{n+1} + S^{n+1}, \quad B_{y,0}^{n+1} = E_{DEC,0}^{n+1} - S^{n+1}$$

$$E_{z,M}^{n+1} = E_{INC,M}^{n+1}, \quad B_{y,M}^{n+1} = -E_{INC,M}^{n+1}.$$

If $c\rho < 1$, the equations (15), (16) give a numerically stable generation of the electromagnetic wave in the interval $\langle 0, L \rangle$.

Our boundary conditions are more precisely than the conditions which are usually used , see e.g. [9]. Their main agvantage consists in half number of the difference conditions which must be solved and in the direct introduction of the source function S . This was attained by our separation of the electric field of wave in E_{DEC} and E_{INC} .

6. Discussion of our results

To be able to discuss our numerical results in detail it is necessary to mention an approximate anlytic solution of our problem. As far as the amplitude of the incident wave is small it is possible to develop the solution of the set of equations (3) to the series in powers of E_0' . At the same time we confine ourselves to the quadratic terms. We assume that the plasma slab is opaque (i.e. $N_0' > 1$) and we study the solutions only for $t > \pi/(a\omega)$ .At this time the standing wave is fully developed from the incident wave. As well, we confine ourselves to the case when $T_0' = 0$.

In linear approximation we obtain

$$E_z = E_1(x) \sin\varphi, \qquad B_y = -\frac{1}{\omega}\frac{dE_1}{dx}\cdot\cos\varphi,$$

$$(17)\qquad V_{ez} = \frac{eE_1}{m_e\omega}\cos\varphi, \qquad V_{iz} = -\frac{eE_1}{m_i\omega}\cos\varphi,$$

where $\varphi = \omega t + \alpha$ and α is an initial phase. The other quantities are equal to the stationary values. The profile of $E_1(x)$ depends on the density profile and can be determined as the solution of the equation

$$(18)\qquad \frac{d^2E_1(x)}{dx^2} - \epsilon_0\mu_0(\omega^2 - \omega_0^2)E_1(x) = 0,$$

where ω_0 is the local plasma frequency given by

$$\omega_0^2 = \frac{e^2 N_e(x)}{\epsilon_0 m_e} \cdot \left(1 + \frac{m_e}{m_i}\right) .$$

E_1 must **fulfil the boundary conditions mentined in paragraph** 3. **For the** quadratic quantities we obtain easily from (3)

$$E_{1x} = R(x) \cdot \left(1 - \frac{m_e}{m_i}\right) + Q(x) \cos 2\varphi + V_1(x) \sin \omega_0 t + V_2(x) \cos \omega_0 t ,$$

$$v_{ex} = \frac{e}{m_i} R \cdot t + \frac{e(R-Q)}{2 \omega m_e} \sin 2\varphi + \frac{e V_1}{m_e \omega_0} \cos \omega_0 t -$$

$$- \frac{e V_2}{m_e \omega_0} \sin \omega_0 t + V_3(x) ,$$

$$v_{ix} = \frac{e}{m_i} R \cdot t + \frac{e(R \cdot \frac{m_e}{m_i} + Q)}{2 \omega m_i} \sin 2\varphi - \frac{e V_1}{m_i \omega_0} \cos \omega_0 t +$$

$$+ \frac{e V_2}{m_i \omega_0} \sin \omega_0 t + V_3(x) ,$$

$$n_{1e} = - \frac{d}{dx} \left\{ N_e(x) \left[\frac{eR}{2m_i} \cdot t^2 - \frac{e(R-Q)}{4\omega^2 m} \cos 2\varphi + \frac{e V_1}{m_e \omega_0^2} \sin \omega_0 t + \right.\right.$$

$$\left.\left. + \frac{e V_2}{m_e \omega_0^2} \cos \omega_0 t + V_3(x) t \right]\right\} + V_4(x)$$

$$n_{1i} = - \frac{d}{dx} \left\{ N_e(x) \left[\frac{eR}{2m_i} \cdot t^2 - \frac{e(R \cdot \frac{m_e}{m_i} + Q)}{4 \omega^2 m_i} \cos 2\varphi - \right.\right.$$

$$\left.\left. - \frac{e V_1}{m_i \omega_0^2} \sin \omega_0 t - \frac{e V_2}{m_i \omega_0^2} \cos \omega_0 t + V_3(x) t \right]\right\} + V_5(x) .$$

Where $R(x) = - \frac{e}{4 m_e \omega^2} \frac{dE_1^2}{dx}$, $Q(x) = \frac{\omega_0^2 R}{\omega_0^2 - 4\omega^2} \cdot \left(1 - \frac{m_e}{m_i}\right)$,

and V_1, V_2, V_3, V_4, V_5 can be determined from the initial conditions. The quadratic terms in E_z, B_y, v_{ez}, v_{iz} are equal to zero. The formulas (17), (19) are valid only for a limited time as long as the changes of the stationary values of plasma parameters caused by the secular terms of (19) are small. This time t_c can be determined from the expressions for n_{1e} or n_{1i}. We can see that $n_{1e} \sim n_e$ if $t \sim t_c$, where $t_c / \tau \sim (m_i / m_e)^{\frac{1}{2}} / E_0'$.

The solution (17) desrcibes a standing electromagnetic wave in a plasma. The expressions (19) describe three processes. The first one is the acceleration of plasma caused by the radiation pressure. The second and the third processes are the generation of the forced oscillations on the second harmonic and the generation of the electrostatic oscillations of the cold plasma with the local plasma frequency ω_0 , respectively. The stationary term in E_{1x} is caused by the difference in the electron and ion velocities (v_{ez} and v_{iz}).

Our results of the numerical investigations of wave incidence on a cold plasma slab can be easily compared with the expressions (17), (19). If the amplitude of the incident wave is small ($E_0' << 10^{-2}$) the numerical results are in a good agreement with our theory for whole time of the simulation ($t \leqslant 200\tau$). A standing wave is created from the incident wave. The position of nodes and antinodes does not change with time. The same is valid about the oscillations of the electrons and ions in the field of this wave. Figure 2 shows a typical profile of E_z, B_y v_{ez}, v_{iz} . The electric field E_z is sketched at a time which is approximately equal to an even multiple of τ and in the antinode reaches its maximum value. The quantities B_y, v_{ez}, v_{iz} are sketched in time which is about $\tau/4$ later. Figure 3 shows the instantaneous profiles of the ion density perturbation n_{1i} , of the x - component of ion velocity v_{ix} and of the perturbation of the electrostatic field E_{1x} . The electrostatic field E_{1x} has two nodes, the first one at the plasma boundary, the second one at the antinode of the field E_z . The irregular shape of E_{1x} near the left plasma boundary is caused by the presence of cold plasma oscillations. The perturbation of ion density n_{1i} has three modes. The first one at the plasma boundary, the next one is in the region before the antinode of E_z and the third one is in the point of inflection of E_z . The ion velocity v_{ix} has a similar profile as E_{1x} Both these quantities show small time oscillations around the sketchend profiles. For the perturbation of electron density it holds $n_{1e} \propto n_{1i} \cdot v_{ex}$ has yet irregular shape at $t \leqslant 200\tau$. This is caused by the strong oscillations of this quantity both at the second harmonic and at the plasma frequency. Fig. 4 shows the time dependence of the ion density perturbation n_{1i} in the point oorresponding to the second antinode. In the same picture we can also see the time dependence of P_{ix} , where $P_{ix} = \int_0^L m_i\, n_i v_{ix}\, dx$. From this picture it is clear that n_{1i} grows quadratically with time and that the plasma as a whole gains a constant acceleration in the direction of X -axis. The excellent agreement between the results of our simulations and the expreesions (17), (19) follows from the fact that $t_c \gg 200\tau$ for $E_0' = 10^{-3}$.

If the wave amplitude $E_0' \geqslant 10^{-2}$, strong nonlinear effects are developed and the stationary parameters of plasma are changed substantially as time t grows (see Fig. 5). However, from the results of our simulations it follows that the equations (16) - (19) are valid if the stationary density N_α is replaced by the instantaneous density n_α . From the figure 5 we can also see that the density gradient is sharply increased in the region of plasma resonance and a shock wave is developed here. The bulk on density profile at the left boundary of slab is caused by our boundary conditions for plasma quantities. Both these effects prevent the succesful application of our model for long times if $E_0' > 5 \times 10^{-2}$.

When the plasma temperature is different from zero we confine ourselves only to the discussion of the numerical solution of eqs. (3). It follows from this solution that the electromagnetic wave is again reflected from the plasma (for $N_0' > 1$) and forms a standing wave. The profile of this wave is practically the same as that of cold plasma (see Fig. 2). The finite plasma temperature changes substantiously the space-time development of the quadratic quantities E_{1x} , v_{ix} , v_{ex} , n_{1i} , n_{1e} . These quantities and also the pressure perturbation p_1 are never more localized in the region where the amplitude of the standing electromagnetic wave is large. These perturbations spread out in the whole volume of plasma slab (see figs. 6 - 8). If we compare Fig. 3 with Fig. 6 we see that at the beginning of this process there is not a great difference between the cold and warm plasma in this point. For longer times the thermal motion of electrons transports these perturbations from the region of plasma resonance to the both plasma boundaries (see Figs. 6 - 8). Due to the requirement of quasineutrality these perturbation propagate with the ion sound velocity $V_s \sim ((\gamma T_e)/m_i)^{\frac{1}{2}}$. From the discussion of our numerical results it follows that the perturbations n_{1e} , n_{1i} , V_{ex} , V_{ix} and p_1 are composed from two components. The main component is a wave of compression which propagates with the ion sound velocity from the region of plasma resonance. This wave originates from the initial compression of plasma produced by the ponderomotive force corresponding to the reflected electromagnetic wave.

In cold plasma this compression is described by secular terms in (19). Because it holds that $n_{1i} \approx n_{1e}$ and $V_{ix} \approx V_{ex}$ (for $t > \tau/a$) no electric field is coupled to this wave. The perturbation of electric field E_{1x} corresponds to the ion sound wave which modulates the main perturbation of densities, velocities and pressure. To determine the group velocity V_G of this wave we have plotted the position of a local maximum of E_{1x} in dependence on time. This is done for several values of $T_0', \gamma,$ m_i/m_e in Fig 9. From this figure it follows that the local group velocity coincide very well with the local ion sound velocity V_S ; where $V_S = \sqrt{(\gamma T(x))(m_i)}$. For the group velocity in the middle of the slab we obtain $V_G' = 1.25 \times 10^{-2}$ for upper curve in Fig. 9 (the theoretical value of $V_S' = 1.22 \times 10^{-2}$) or $V_G' = 8.34 \times 10^{-3}$ for lower curve ($V_S' = 8.66 \times 10^{-3}$).

The time development of global parameters characterising the interaction of an electromagnetic wave with a warm plasma is given in Fig. 10. Here the perturbation of total thermal energy is given by

$$\Delta E_{TH} = \int_0^L \left(\frac{p_1}{\gamma - 1} \right) dx$$

and the perturbation of total potential energy of a plasma in the gravitional field is given by

$$\Delta E_{POT} = \int_0^L U(x) \cdot (m_e \, n_{1e} + m_i \, n_{1i}) \, dx .$$

At the beginning of the process the plasma is accelerated - the X - component of total ion momentum P_{iX} grows. At the same time the total thermal energy grows and the total potential energy decreases. After some time the gravitational force and the force given by the pressure gradient overcome the ponderomotive force of electromagnetic wave. Now, the plasma is slown down, the total thermal energy decreases and the total potential energy grows. From the results of our numerical simulations it follows that the increase in one type of these energies is approximately compensated by the decrease in the other type so that

$$\Delta E_{TH} + \Delta E_{POT} \approx 0.$$

When the incident wave has a large amplitude $\left(E_0' > 10^{-2} \right)$ the profiles of the density and the pressure change gradually as time grows (see Fig. 11). In warm plasma the density gradient does not grow in the region of plasma resonance to such extent as it does in cold plasma. This may be explained by the fact, that the density and pressure perturbations are transported by the thermal motion from the plasma resonance region to the whole volume of a plasma (compare Fig. 5 and Fig. 11).

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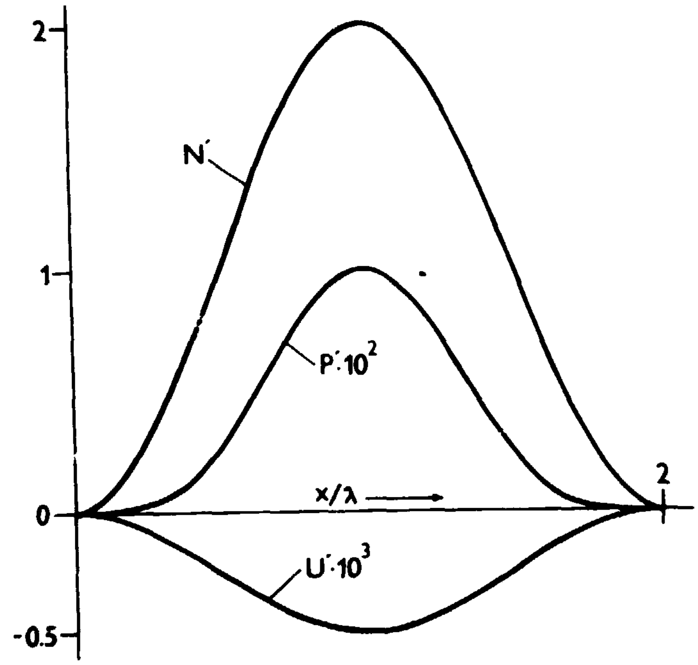

Fig. 1: Stationary state of plasma slab for $T_0' = 5 \times 10^{-3}$. The other parameters are given in paragraph 4.

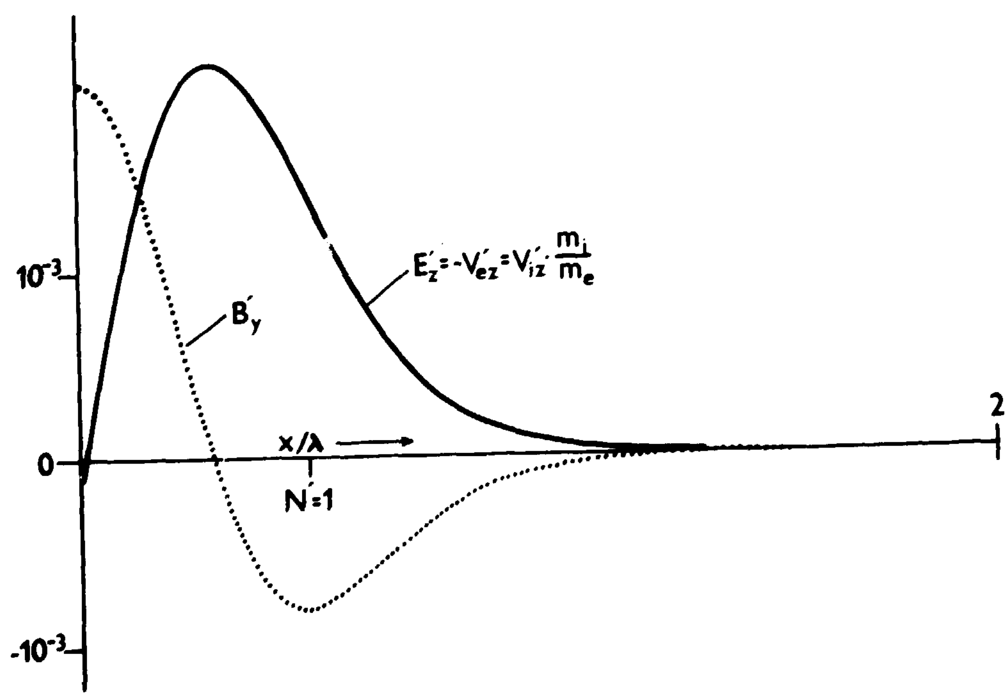

Fig. 2: Typical profiles of E_z', B_y', V_{ez}', V_{iz}' for $E_0' = 10^{-3}$ and $T_0' = 0$.

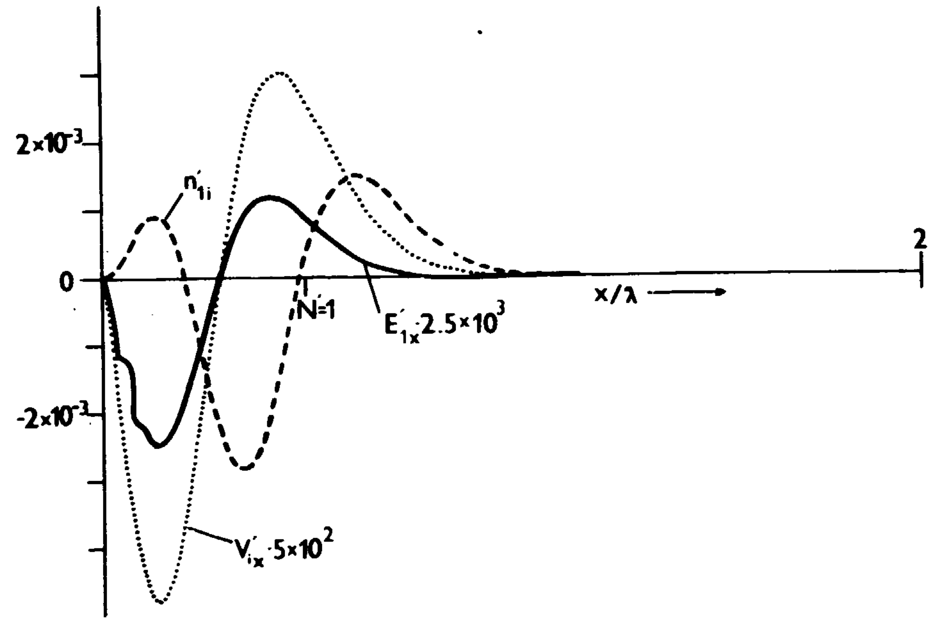

Fig. 3: Profiles of E'_{1x}, n'_{1i} and V'_{ix} for $E'_0 = 10^{-3}$, $T'_e = 0$ at $t = 165.8\,\tau$.

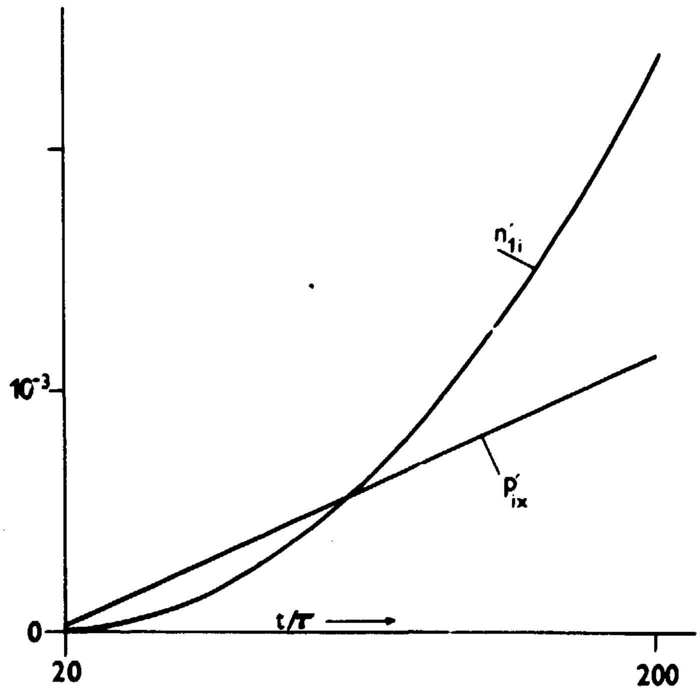

Fig. 4: Time dependence of the ion density perturbation and of the x-component of the total ion momentum for $E'_0 = 10^{-3}$, $T'_0 = 0$.

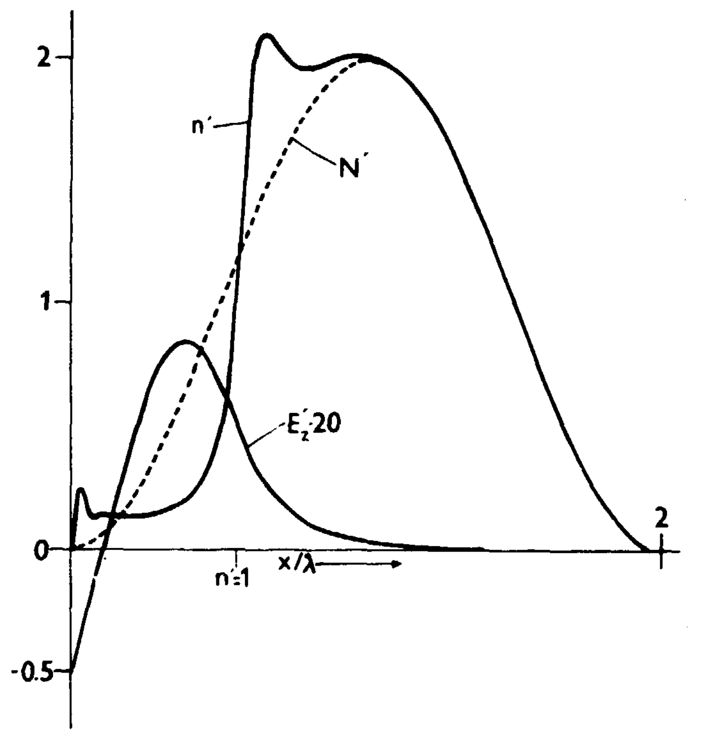

Fig. 5: Distortion of the plasma density profile and the self-consistent profile of the standing electromagnetic wave for

$E_0' = 2 \times 10^{-2}$, $T_0' = 0$, $t = 130\,\tau$.

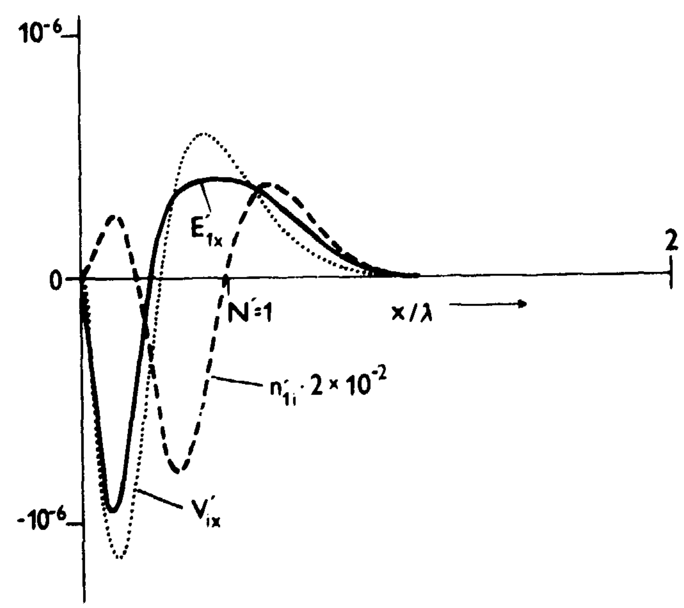

Fig. 6: Profiles of E_{1x}', n_{1i}' and V_{ix}' for $E_0' = 10^{-3}$, $T_0' = 5 \times 10^{-3}$ at $t = 36\,\tau$.

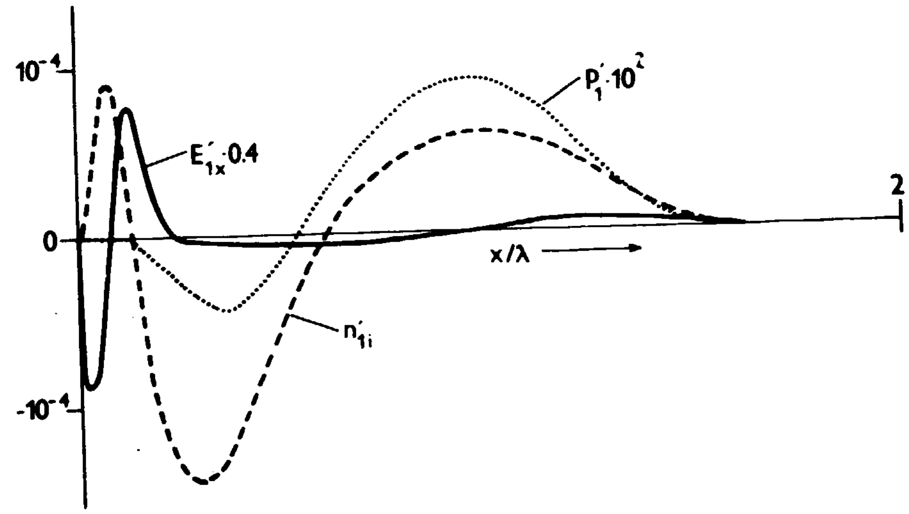

Fig. 7: Profiles of E_{1x}', n_{1i}' and p_1' for $E_0' = 10^{-3}$, $T_0' = 5 \times 10^{-3}$ at $t = 86.4\tau$.

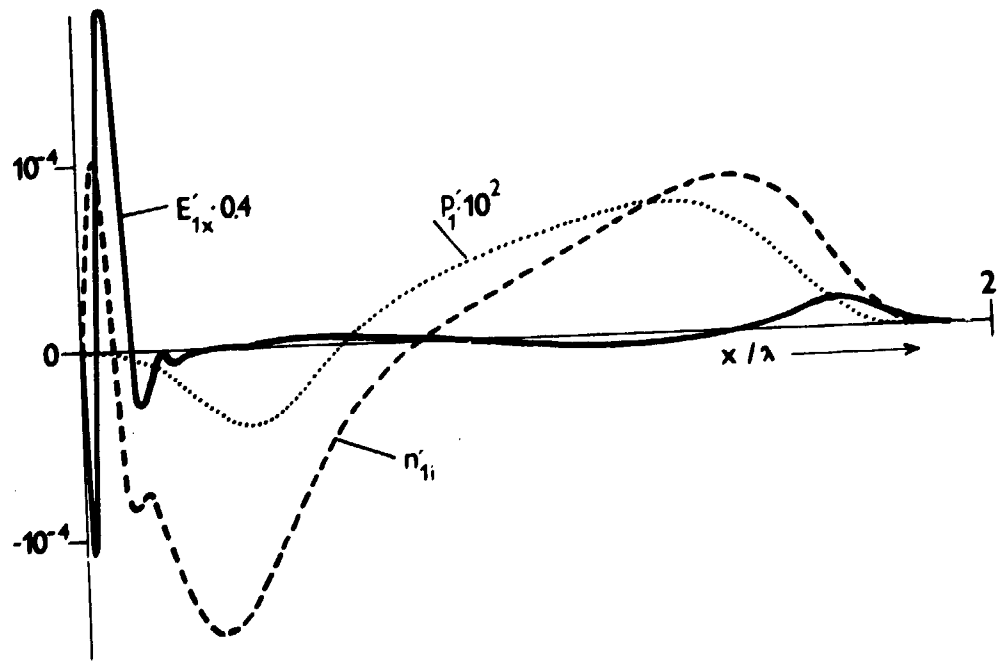

Fig. 8: Profiles of E_{1x}', n_{1i}' and p_1' for $E_0' = 10^{-3}$, $T_0' = 5 \times 10^{-3}$ at $t = 129.6\tau$.

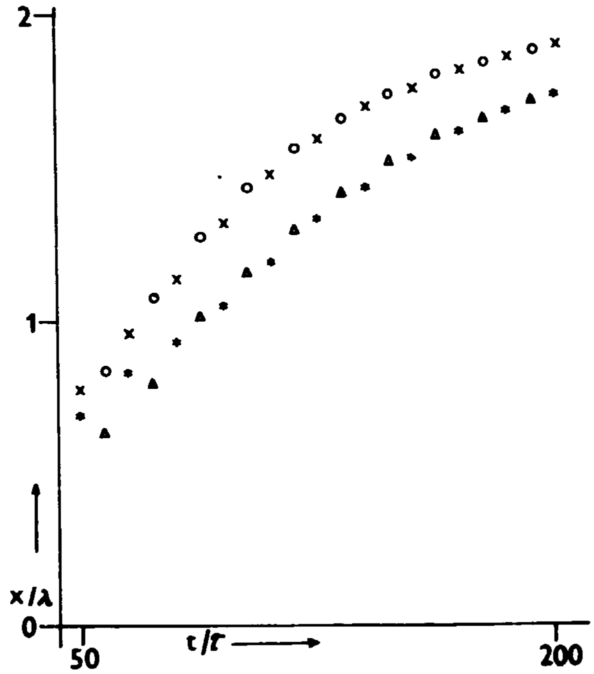

Fig. 9: Trajectories of a local maximum of the electrostatic field E_{ix} of the ion sound wave for the following parameters: $E_0' = 10^{-3}$ (all curves), ✱ corresponds to $T_0' = 2.5 \times 10^{-3}$, $\gamma = 3$, $m_i/m_e = 100$,
Δ corresponds to $T_0' = 5 \times 10^{-3}$, $\gamma = 1.5$, $m_i/m_e = 100$,
X corresponds to $T_0' = 10^{-3}$, $\gamma = 3$, $m_i/m_e = 20$,
O corresponds to $T_0' = 5 \times 10^{-3}$, $\gamma = 3$, $m_i/m_e = 100$.

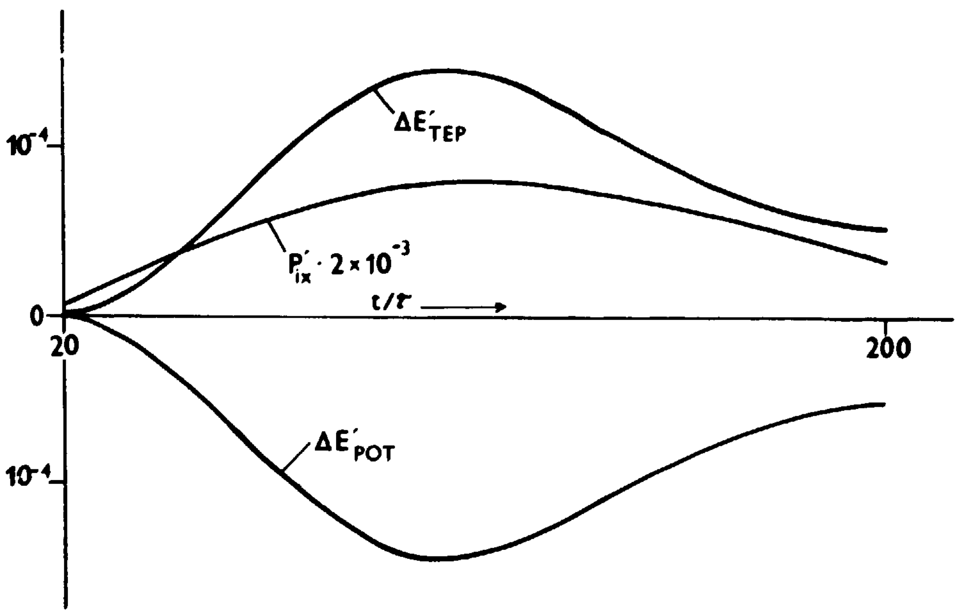

Fig. 10: Time dependence of $\Delta E'_{TH}$, $\Delta E'_{POT}$ and P'_{ix} for $E_0' = 10^{-2}$, $T_0' = 5 \times 10^{-3}$.

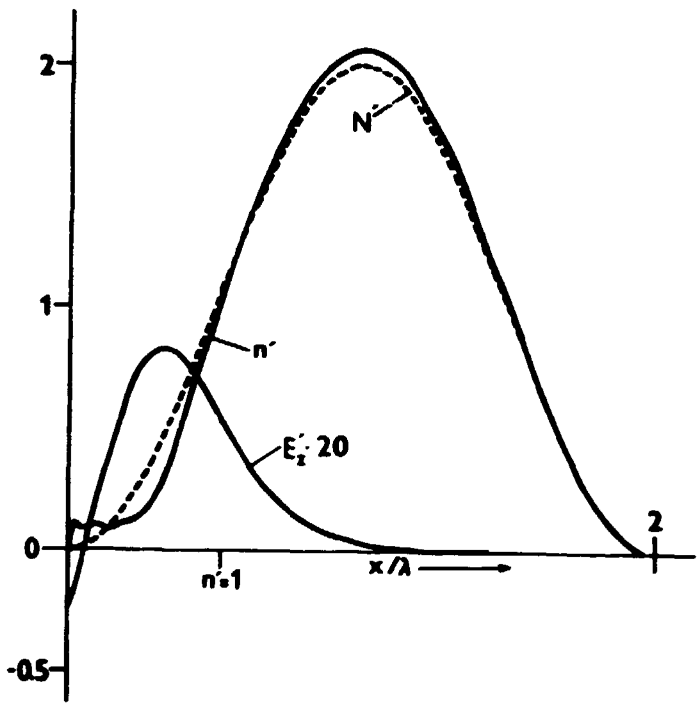

Fig. 11: Distortion of the plasma density profile and the self-consistent profile of the standing electromagnetic wave for $E_0' = 2 \times 10^{-2}$, $T_e' = 2 \times 10^{-3}$, $t = 130\tau$.