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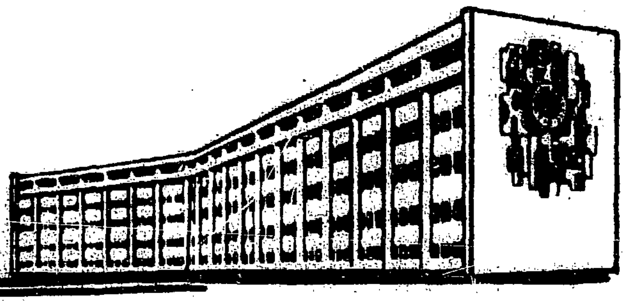
ENERGY SPECTRUM STRUCTURE AND "TRAP" EFFECTS IN A THREE-PARTICLE SYSTEM



КИЕВ

И.В.Сименог, А.И.Ситниченко ITP-82-17E

Структура энергетического спектра и эффект "ловушек" в системе трех частиц

Исследована структура порогового энергетического спектра системы трех бесспиновых частиц в зависимости от вида двухчастичного взаимодействия и показана корреляционная зависимость спектра и низкоэнергетических параметров рассеяния. Установлено новое явление "ловушек" для спектра в системе трех частиц с взаимодействием, имеющим компоненты с сильно различающимися радиусами действия.

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Energy Spectrum Structure and "Trap" Effects in a Three-Particle System

I.V.Simenog, A.I.Sitnichenko

1. Many-particle character of a quantum system of three particles manifests itself in the Efimov effect [1] ,i.e.in the appearance of effective long-range action in systems with a short-range interaction (consequently, in the presence of a developed threshold energy spectrum and a specific structure of the low-energy scattering amplitude (see survey [2])). The phenomenon of effective long-range action is universal for all pair interactions which are attractive at large distances if the binding energy of two particles $\mathcal{E}_2 = \alpha^2/m$ tends to zero. At the same time,rigorous conditions for the coincidence or nearness of two-particle thresholds in different states are required if the interaction depends on spin [2] . Beyond the region of the effective long-range action (in the intermediate range the constants of the interaction g between $g = g_{crit.}$ and the strong coupling range $g \gg 1$) for a spinless problem with simplest attractive interaction potentials, a characteristic near-threshold excited level is obtained in numerous calculations.The particle by pair scattering amplitude at low energies (the scattering length, the effective radius) correlates in a certain way with the spectrum structure [3,5] .

In the present paper using the Faddeev equations study is made how the threshold energy spectrum of three perticles in the intermediate range depends on the interaction specificity and a new phenomenon of "traps" for the energy spectrum is found when three-particle levels appear and disappear on a two-particle threshold in a certain narrow interval of two-particle parameters and all three-particle levels are essentially rearranged. The "trap" effect occurs under moderate coupling of two particles and is due to the neighbourhood of a new threshold-excited state of two particles. This is why the levels in a three-particle system in the case of interactions with strongly different radii of action are rearranged. The phenomenon of the level rearrangement for two oppositly charged particles, taking into account a short-range attraction is well-known and simply explained (it has recently been discussed in detail in [6,7]).

2. Efimov's levels are due to the effective long-range action or to the singularity of the Faggeev equations at small momenta $(p r_0) \to 0$ of the third particle relative to a coupled pair (r_0 is the characteristic interaction radius) [8-10] , and the ground state of three particles is determined mainly by the momentum region $(p r_0) \sim 1$. The first excited state (a unique excited state of a three spinless particle system for commonly studied simple interaction potentials in the intermediate range of two-particle parameters) near the two-particle breakdown threshold is also determined mainly by small momenta $(p r_0) \ll 1$ though it occupies a special position.

If the role of small momenta in the interaction is minimized, the region of the effective long-range action (when $g \to g_{crit}$, $\alpha \to 0$, $a \simeq 1/\alpha \to \infty$, a is the two-particle scattering length) is narrowed and may be, in principle, eliminated.

With increasing role of small momenta in the Faddeev equation the threshold spectrum of three particles becomes richer.

As a simple example we consider the influence of small momenta in the potential on the threshold spectrum structure in the case of the model interaction of separable form (the first-range separable potential) with the formfactor

$$u(pr_0)=\frac{(pr_0)^2+C}{[(pr_0)^2+1]^2}\ ,\ C\geq 0. \tag{1}$$

When $C\cong 1$ ($C=1$ - the so-called Yamaguchi potential) at all the values of two-particle energy $-\alpha^2/m$ besides the ground state there exists at least one more excited level in a three-particle system (about $\sim 10^2$ times near to the two-particle threshold than the ground state). With C decreasing to zero the threshold spectrum is essentially deformed. The dependence of the scattering length A (more exactly, dimensionless relation of the scattering lengths A/a) of the third particle by a coupled pair when $C=1$ in (1) in the intermediate range on $\delta=\ln(\alpha r_0)$ (For convenience, we shall further consider the dependence of the spectrum and scattering length on the parameter δ) has a specific behaviour [4] , it increases from $-\infty$ (where all the levels caused by the effective long-range action disappear) and $A>0$ at $\delta\approx-3+0$ and then the scattering length A decreases again to $-\infty$ when a subsequent excited level of three particles appears.

In (1) with parameter C decreasing from 1 to 0 the role of small momenta in an equation of motion is decreased, the structure of threshold levels and the dependence of the scattering length A on the value of the interaction constant (or on δ) are changed. When $C=1/3$ in the region $\delta\simeq-1$ the first excited level touches the two-body threshold and is broken and, consequently, the region of the parameter $\delta\approx-1$ appears where there is no excited levels in a three-particle system. In this case the branch of three-body scattering length A in the intermediate region relative to the parameter δ breaks at infinitely large positive values ($A\alpha$). With a further decreasing of parameter C in (1) to zero in the region $\delta < 0$ only one ground level of three particles is left, the role of small momenta is suppressed and the singularity of the Faddeev equation is eliminated and, consequently, an infinite set of levels with condensation point $\alpha = 0$ disappears. At $C \to 0$ a characteristic dependence $A/a = b_1 + b_2 tg\{s_0 \ln(d\alpha r_0)\}$ [3,4] is not valid for the scattering length in the limit $\alpha \to 0$ (for $C \to 0$ the two-body scattering length $a \to 0$, and the effective radius r_{eff} cannot be determined in this case). In the limit $C \to 0$ at $\alpha = 0$ for the ground state energy of three particles ($\mathcal{E}_{(3)} = -\varkappa_0^2/m$) we have ($\varkappa_0 r_0$)=1,2 , and the scattering length A is positive and is equal to A/r_0 =1,82. With increasing two-particle attraction (the increase in αr_0 from zero) for the case C =0, the length A slightly increases and then tends to $-\infty$ at δ = 0,7, when the first excited level of three particles appears. With a further increasing of αr_0 a typical situation is observed when the scattering length A changes from $+\infty$ to $-\infty$ in the interval where the next three-particle level appears from the two-body threshold.

The threshold infinite series of the levels at $C \to 0$ in (1) disappears since the interaction (1) at large distances (in the coordinate representation) becomes "repulsive" and abolishes the effective long-range action. In order to increase the number of threshold excited levels of a three-particle system in the intermediate range($\delta \sim -5 \div 1$), it is necessary to increase the role of small momenta in the interaction and to increase , thus, the Faddeev equation singularity at $(pr_0) \to 0$. We consider a pair separable interaction (of the first rank) with $u(p)$ in the form of a superposition of the Yukawa formfactors

$$u(p) = \sum_i \frac{g_i}{[(pR_i)^2 + 1]} \qquad (2)$$

with two components of largely different radii $R_2 \gg R_1$

$$u(p) = \frac{g_1}{(pR_1)^2+1} + \frac{g_2}{(pR_2)^2+1} \qquad (3)$$

where $0 \leqslant g_1 \leqslant 1$, $g_2 = 1 - g_1$.

In the limiting cases $g_1 \to 0$ and $g_1 \to 1$ in (3) we deal with one and the same potential and, for example, the plots of the dependence of the scattering length A/a on $\delta_1 \equiv \ln(\alpha R_1)$ [4] will be only displaced with respect to each other by $\ln(R_2/R_1)$ along the abscissa axis (at R_2/R_1 =100, $\ln(R_2/R_1)$ =4,6. However, when g_1 varies from 1 to 0 the spectrum structure of three particles varies. For example, for R_2/R_1 =100 (the most interesting region of parameter $R_2/R_1 \sim 10 \div 10^2$) and when g_1 decreases from 1 to 0,5 the intermediate region of the parameter δ_1, where only one excited level of three particles exists, narrows to the interval $\delta_1 = -3,3 \div 0,3$. In this region the scattering length A varies from $-\infty$, increasing and then decreasing again with increasing δ_1 , remaining negative all the time. With a further decreasing of g_1 in the region $g_1 \approx 0,3$ depending on δ_1 one more excited level splitts off from the two-body threshold, then at all two-particle energies there exist at least two excited three-particle levels.

For R_2/R_1 =100, g_1 =0,3 and δ_1 =-1 three bound states of three particles are characterized by the following values $\ln(\varkappa - \alpha)R_1$: -0,5; -5,0; -12,5. In this case two branches for A adjacent to the considered intermediate region in α and having opposite inclinations are closed at $A \to +\infty$ forming a more wide intermediate region $-9,0 \lessapprox \delta_1 \lessapprox +0,7$. In this intermediate region the three-body scattering lengths A/a and the effective radius R_{eff}/a weakly depend on δ_1 except this region boundary where $A \to 0$. When g_1 further tends to zero, the second excited level approaches gradually the threshold and breaks, at g_1 =0 we come again to the case g_1 =1 but the intermediate range is replaced in the parameter δ_1 to the left by $\ln(R_2/R_1)$.

The above characteristic regularities for the potential (3) take place in a wide region of differing radii in the interaction $R_2/R_1 \gtrsim 10$ as well as for the cases of a more general interaction form than that chosen to simplify the calculations of the superposition of the Yukawa formfactors (2).

It should be emphasized that from mentioned above it followes that the particle by coupled pair scattering length A for interaction with fixed low-energy two-particle parameters may be equaled to any forehand setting values both positive and negative and it essentially depends on an interaction form (unlike three-body binding energy)if the interaction is varied in a wide region.

Notice that low-energy two-nucleon phases up to~20÷30Mev both in triplet and singlet states can be described correctly by the parameter R_1 and the general intensity g of the potential with the formfactor (3) at fixed R_2/R_1 and g_1 (for example: R_2/R_1 =100, g_1 =0,3). At the energies 100Mev and higher the two-body scattering S-phase with such a potential is considerably larger than for the simplest form g_1 =1 in (3).

At fixed values of the two-body binding energy and the scattering length a(for example,in a triplet state)when for two particles the approach of the effective radius is justified (and $a \simeq 1/\alpha$), the family of potentials (2) as well as the potentials of more general forms at low energies will be characterized mainly by an independent parameter of the effective radius $r_{eff.}$. When the parameter of the "form" P for the potentials of sufficiently general form is small, it is not independent and is determined by the linear relation

$$f = (\alpha r_{eff})^3 P \simeq \left(1-\frac{1}{\alpha a}\right)-\frac{1}{2}(\alpha r_{eff.}) \qquad (4)$$

through a dimensionless parameter of the effective radius ($\alpha r_{eff.}$).Then different interaction potentials with a small parameter of the "form" f (typical values in the triplet states are the following: $r_t \approx$ 1,7 Fm, $\alpha r_t \approx$ 0,4 , $|f| \lesssim$ 0,005) are characterized only by one independent parameter (αr_{eff}). Far from critical values there holds a simple linear correlation dependence of three-particle values (generally speaking, these corollaries are applicable to more complex systems) such as the binding energy $\mathcal{E}_{(3)}$ and the scattering length A on a single dimensionless parameter(αr_{eff}) (at fixed binding energy and scattering length in a two-particle system) which changes in the small interval of the order of $|f| \lesssim 0{,}005$. Thus, a linear correlation dependence of three-body characteristics appears at low energies.

3. As a more realistic model of a three-particle system with the interaction having components strongly different in radii we consider the Faddeev equation with a second-rank separable interaction

$$V(p,p') = -\frac{g}{m\pi^2}\{g_1 u_1(pR_1)u_1(p'R_1) + g_2 u_2(pR_2)u_2(p'R_2)\};\ g_1+g_2=1. \qquad (5)$$

For a concrete definition we consider the Yukawa formfactors

$$u_i(pR_i) = 1/((pR_i)^2+1) \qquad (6)$$

in the case $R_2/R_1 \gg 1$. Here we represent the results of the analysis of the three-particle energy spectrum and the dependence of the particle by pair scattering length A on the two-particle parameters in the case $R_2/R_1 = 100$ and $g_1 = 0{,}001$. The parameters are chosen in a way that we can emphasize strikingly the level rearrangement , though the main conclusions are also valid for $R_2/R_1 \simeq 10 \div 10^2$ and the corresponding values g_1 as well as for a wide class of the assumed formfactors $u_i(pR_i)$ in the interaction (5) and more general models of the pair short-range interaction.

A two-particle system with strongly different-range interaction (5) has two states d_0 and d_1 (Fig.1;the values $\ln(R_2\sqrt{-mE})$ are given along the ordinate axis for two and three particles). This indicates the level rearrangement such as in the cases of the Coulomb and the nuclear interactions in a proton-antiproton "atom" considered in [6,7].

Excepting the vicinity of the critical value $g/R_2 = (g/R_2)_{crit.,2} \cong 10,46$ of the interaction constant for the two-particle excited state where the two-body levels of the same symmetry repulse, we can consider two states as independent since they belong to two potentials strongly different in the radius value. For each level taken separately the simple dependence (in the chosen interaction form)

$$g\cdot g_i = R_i(1+\alpha R_i)^2, \quad i=1,2$$

would be valid and the levels would intersect in the region $g/R_1 \approx 10^3 \sim 1/g_1$ since $\alpha \sim \frac{1}{R} f(g/R)$. Owing to the mixing of these two states the situation shown in Fig.1 is realized. Analogous considerations are also valid for a three-particle spectrum (the ground (0) and the first excited state(1)), excepting the vicinity of the point $g = g_{crit.,2}$ the three-particle spectrum is near to the spectrum calculated using each of the terms of the potential (5) taken separately.

In this case in the vicinity of the critical point $(g/R_2)_{crit.,2}$, two pairs of these levels, each of which taken separately corresponds to certain potential term, would intersect in four points. When one considers the potential in separate components each of the pair of levels intersects the two-particle levels being a threshold for the other pair of levels. When two terms equally important in the vicinity of the point $g = g_{crit.,2}$ are included then the converging levels are rearranged in the vicinities of the points of their possible intersection.

As a result two additional levels arise (except the ground and the first excited levels which exist at any values of two-particle binding energy) intersecting twice the two-particle threshold at some points in g , i.e. they exist in certain region in the coupling constant.

The number of levels in the vicinity of the point $(g/R_2)_{crit.,2}$ obtained due to simple summation of levels corresponding to the first and the second term of the potential (5), taken separately, is the largest possible number of levels under exact calculation using this potential (in the given case - four levels).Below we shall describe the case when only three of the above levels are realized. Thus,the rearrangement of two-particle levels in the region of parameters $\delta_2 = \ln(\alpha R_2) = 1{,}1$ and $g/R_2 = 10{,}0$ causes significant changes in three-particle levels as compared with the case $g_1 = 1$ ($g_1 = 0$) when there are only two states in the intermediate region. In addition to the excited level (1) in the above region two more excited near-threshold levels (2 and 3 in Fig.1) appear subsequently at the values $\delta_2 = 0{,}7$ and $1{,}0$ ($g/R_2 = 8{,}1$ and $9{,}7$).

These levels are shown in Fig.1 with a distance to the two-particle threshold increased by 10 and 10^3 times.

With increasing coupling constant g these two threshold levels (2 and 3) go again in the region above the threshold in the points $\delta_2 = 1{,}5$ and $2{,}6$, respectively. Thus, there takes place a peculiar phenomenon of "traps" for energy levels in a three-particle system.

The effect of "traps" for the three-particle energy spectrum consists in that the threshold three-particle levels appear and disappear on the two-particle threshold in a certain narrow interval of two-particle interaction parameters and all three-particle levels are essentially rearranged. The effect manifests itself with a moderate coupling of two particles near the threshold of the appearance of two-particle excited state and is due to the presence of the strongly different-range components in the interaction. In the vicinity of the critical point $g/R_2 = (g/R_2)_{crit.,1} = 1{,}01$ ($\delta_2 \to -\infty$) there is an infinite series of the Efimov levels condenced near the threshold (it is not shown in Fig.1).

The analogous series of the levels in the vicinity of the point $g=g_{crit.,2}$,corresponding to the potential (5) with the second term omitted, proves to be above the break-dawn threshold. The particle by a coupled pair scattering length A in the intermediate region of the "trap" effect is shown in Fig.2. As δ_2 decreases the value $A\alpha$ (the left part of Fig.2 shows partially the branch) decreases and goes to $-\infty$ and with a consequent decrease in $\delta_2 \to -\infty$ a common situation (see [2,4]) takes place, such as for simple forms of interaction. With increasing δ_2 (the branch is partially shown to the right in Fig. 2) the value $A\alpha$ decreases also and tends to $-\infty$ when a new level appears. The branches of the scattering length shown in Fig.2 are significantly different from the ones in the cases of simple interactions (g_1 =1) when in the intermediate region the scattering length $A\alpha$ (as the effective radius R_{eff}) weakly depends on δ_2 .

The above rearrangement of the three-particle spectrum and the phenomenon of"trap" may occur also with less differences in radii of two components in the interaction (5) which is due to the fact how strongly the level rearrangements of two particles are expressed. For the parameters R_2/R_1 =30 and g_1 =0,011 in the vicinity of $g/R_2=(g/R_2)_{crit.,2}$ = 3,7 a similar rearrangement of the levels takes place with the difference that in the intermediate region($-0,1 \leqslant \delta_2 \leqslant 1,4$) there exists only one excited state of three particles for which the phenomenon o "trap" manifests itself rather than the two ones for the parameters $R_2/R_1 = 100$, $g_1 =$ 0,001.

It should be emphasized that the considered phenomena of level rearrangement are realized at the given relation $R_2/R_1 \gg$ 1 in the narrow interval of the parameter $g_1 \approx$ 0 characterizing the weight of every interaction component. However, the variation of the formfactors $u_i(pR_i)$ in (5) can provide more favourable conditions for the manifestation of the above phenomena. The considered regularities are supposed to be also retained for the interactions represented by local potentials with components varying in their action ranges.

Notice that since the ground state of three particles ((0) in Fig.1) is subjected to a considerable rearrangement at $\rho/R_2 \sim$ 8,0 we should expect a considerable rearrangement of four-particle ground and excited levels. Under proper conditions (for the parameters corresponding to Fig.2) the "traps" for the threshold levels manifest themselves also in a four-particle system.

Finally we should note that for a three-nucleon problem the interactions of triplet and singlet states are different, but this difference, however, is not significant in comparison with that considered in the above variants of interactions. Moreover, beacause of the difference in two-particle thresholds in different states (channels) unfavourable conditions for the threshold phenomena considered above appear. The threshold phenomena can manifest themselves in a certain degree only quantitatively rather than on a qualitative level.

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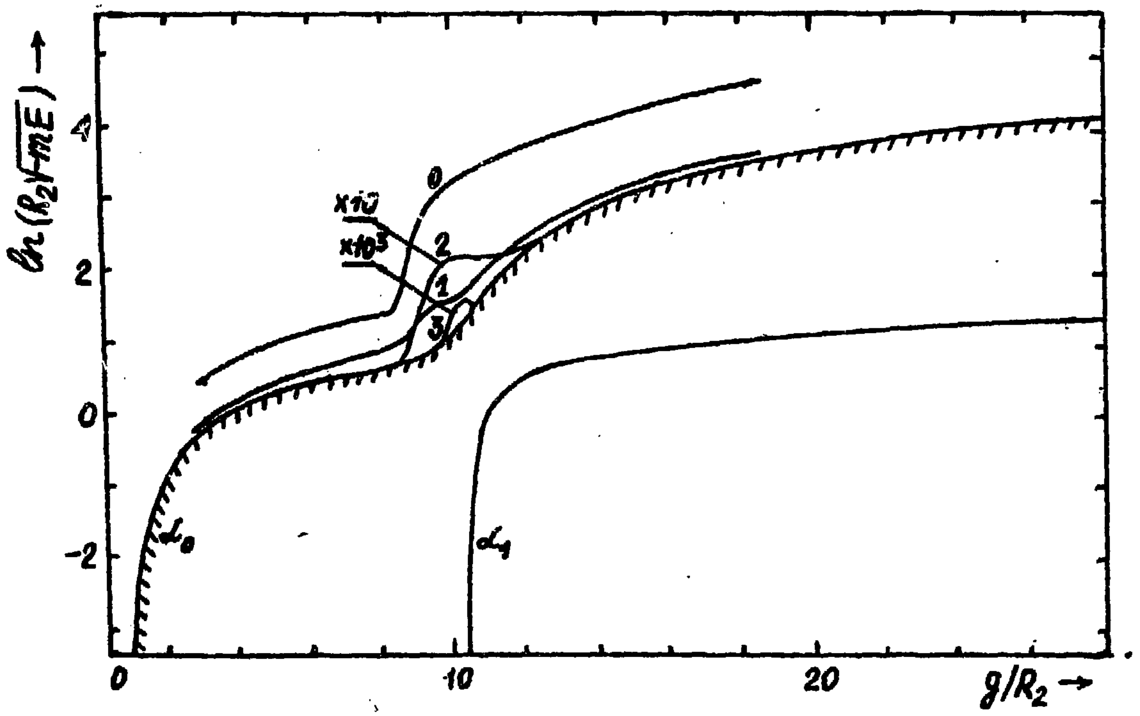

Fig.1. The two and three s⁻inless particle spectrum depending on the interaction intensity (g/R_2)(g_1 =0,001; R_2/R_1 =100). α_0, α_1 - are the ground (threshold) and excited states of two particles; 0 is the ground state of a three-particle system, 1,2,3 are the excited states of three particles.

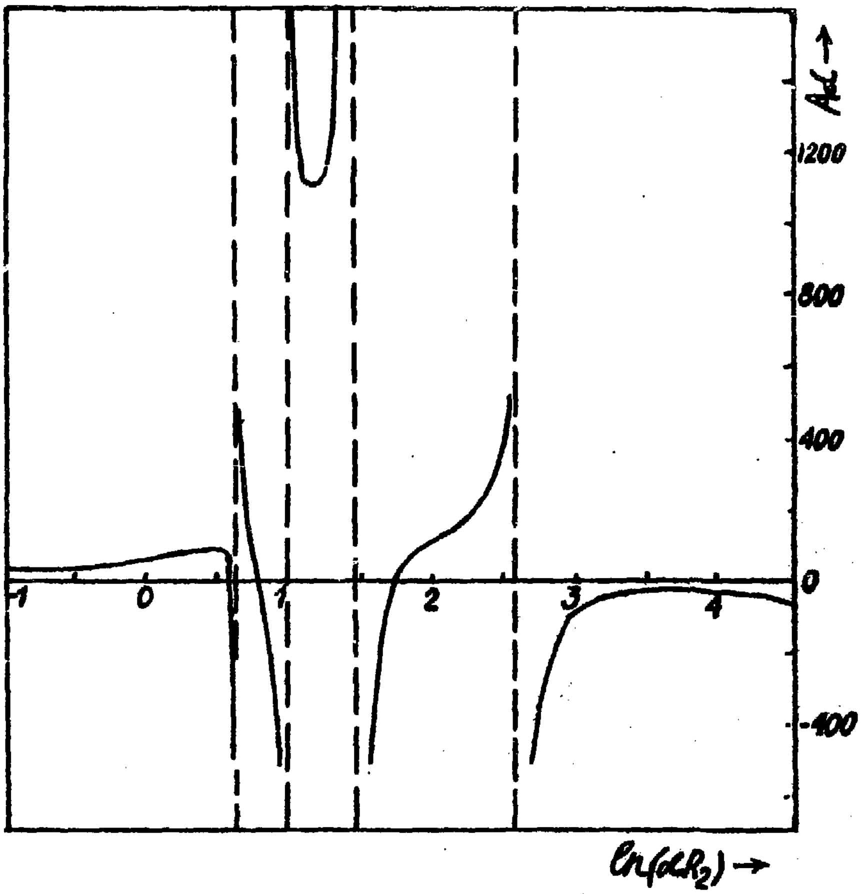

Fig.2. The particle by pair scattering length dependence in the region of the two- and three-particle spectrum rearrangement.

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