ITP-82-51E
April, 1982

G.F.Filippov, L.L.Chopovsky,
V.S.Vasilevsky

ON THE STATES WITH POSITIVE ENERGY WHICH RESULT FROM THE HAMILTONIAN DIAGONALIZATION ON THE OSCILLATOR BASIS

Academy of Sciences of the Ukrainian SSR
Institute for Theoretical Physics

Preprint
ITP-82-51E

G.F.Filippov, L.L.Chopovsky,
V.S.Vasilevsky

ON THE STATES WITH POSITIVE ENERGY WHICH RESULT FROM THE HAMILTONIAN DIAGONALIZATION ON THE OSCILLATOR BASIS

Kiev - 1982

УДК 539.I4

Г.Ф.Филиппов, Л.Л.Чоповский, В.С.Василевский

О состояниях с положительной энергией, появляющихся при диагонализации гамильтониана на осцилляторном базисе

Изучены состояния непрерывного спектра системы двух взаимодействующих кластеров. Показано, что диагонализация гамильтониана этой системы на осцилляторном базисе выделяет те состояния непрерывного спектра, амплитуды которых имеют узел при определенном числе осцилляторных квантов.

The states of continuous spectrum in a system of two interacting clusters are studied. It is shown that the Hamiltonian diagonalization on the oscillator basis isolates those states in a continuous spectrum whose amplitudes have a node at a certain number of oscillator quanta.

On the States with Positive Energy which Result from the Hamiltonian Diagonalization on the Oscillator Basis

G.F.Filippov, L.L.Chopovsky, V.S.Vasilevsky

1. As it is known the resonating group method(RGM) is used to study not only the states of continuous spectra in systems of interacting nucleon clusters but also the states of discrete spectra of the same systems [1-4]. In the latter case an algebraic version of RGM [5-7] is reduced to the diagonalization of the many-particle microscopic Hamiltonian of the system of clusters on the oscillator basis functions generated by the function of the resonating group method [7,8]. Having restricted ourselves by the matrix of the n-th - order Hamiltonian where n is the number of the employed basis functions and having solved the problem on the eigenvalues of this matrix,we obtain n different eigenvalues and the same number of the corresponding eigenvectors. Among the eigenvalues only the first ones (K,K < n) will be, generally speaking, negative. The rest eigenvalues are positive and, as a rule, they are not considered (see, however, Refs. [9,10]) since it is hardly conceived that the corresponding eigenvectors found in the representation of the limited oscillator basis would be associated with the wave functions of a

continuous spectrum. Nevertheless, this first impression of the eigenvectors with positive eigenvalues rejecting the validity of their study is erroneous. As a matter of fact, the eigenvectors with positive energies as well as the eigenvectors with negative energies, as the oscillator basis expands, tend to the exact solution and even at not very large values of n they give information about the scattering S-matrix. It should be noted that Hazi and Taylor showed in [11] the possibility to obtain the data on scattering S-matrix (the reaction cross-section, the resonance energies, the resonance widths) using positive eigenvalues of the Hamiltonian matrix in the cutoff oscillator basis. However, they considered only the simplest model system in which the interaction potential has a barrier and a core at small distances. We have considered the case of a real cluster model of two interacting subsystems for which one can find the scattering phases and eigenvectors corresponding to the states of continuous spectrum.

2. Let us remind that in the oscillator representation the wave function Ψ of a nucleon system is determined by an infinite number of coefficients $\{C_\nu\}$, $\nu = 1,2,\ldots,\infty$, resulting from the expansion of Ψ in a complete set of orthonormalized oscillator functions $\{\varphi_\nu\}$:

$$\Psi = \sum_{\nu=1}^{\infty} C_\nu \varphi_\nu \,. \qquad (1)$$

The coefficients C_ν satisfy the infinite system of homogeneous algebraic equations:

$$\sum_{\nu=1}^{\infty} \left\{ \langle \mu | \hat{H} | \nu \rangle - E \delta_{\mu\nu} \right\} C_\nu = 0 \,, \qquad (2)$$

$$\hat{H}=\hat{T}+\hat{V}\ ;\quad \mu=1,2,\ldots,\infty\ ,$$

where $\hat{H}$ is the Hamiltonian, $\hat{T}$ is the kinetic energy operator, $\hat{V}$ is the potential energy operator, and $\langle\mu|\hat{H}|\nu\rangle$ are the matrix elements of the Hamiltonian on the basis functions.

When the basis of the states φ_ν is generated by the wave function of a two-cluster variant of the resonating group method (in this case ν is, with accuracy to the additive constant, the number of oscillator excitation quanta of the wave functions of the two-cluster relative motion), the Hamiltonian matrix $\langle\mu|\hat{H}|\nu\rangle$ (see the Appendix) at large values μ and ν is essentially simplified and this enables one to understand the meaning of arbitrary solutions which result from diagonalizing the Hamiltonian on the cutoff oscillator basis $\{\varphi_\nu\}_n$, $\nu=1,2,\ldots,n$.

The matrix $\langle\mu|\hat{T}|\nu\rangle$ is three-diagonal and its matrix elements are distinct from zero if $\nu=\mu$ or $\nu=\mu\pm1$. The matrix elements of the operator $\hat{V}$ have the following asymptotics at large values μ and ν [6]:

$$\langle\mu|\hat{V}|\nu\rangle=\sum_{i=1}^{i_0}\frac{(-1)^{\mu+\nu}\alpha_i}{\sqrt{\nu+\mu}}\exp\left[-\beta_i\frac{(\nu-\mu)^2}{(\nu+\mu)}\right]+V_0\delta_{\mu\nu}\qquad(3)$$

where α_i and β_i are the coefficients depending on the parameters of the Gaussian nucleon-nucleon potential and V_0 is the threshold potential energy of the cluster channel equal to the potential energy of two non-interacting clusters. Therefore, if $\mu\gg1$, the equations of the system (2) take the form:

$$\langle\mu|\hat{T}|\mu-1\rangle C_{\mu-1}+(\langle\mu|\hat{T}|\mu\rangle+V_0-E)C_\mu+\langle\mu|\hat{T}|\mu+1\rangle C_{\mu+1}+$$

$$+\sum_{i=1}^{i_0}\sum_{\nu=1}^{\infty}\frac{(-1)^{\mu+\nu}\alpha_i}{\sqrt{\mu+\nu}}\exp\left[-\beta_i\frac{(\nu-\mu)^2}{\nu+\mu}\right]C_\nu=0\,. \tag{4}$$

Since the matrix elements of the potential energy operator are sign-alternating and the coefficients C_ν and the multiplier

$$A^i_{\mu\nu}=\frac{\alpha_i}{\sqrt{\mu+\nu}}\exp\left[-\beta_i\frac{(\nu-\mu)^2}{(\nu+\mu)}\right] \tag{5}$$

at large values μ and ν do not change practically if ν increases or decreases by unity, the sum over ν in (4) vanishes at $\mu \gg 1$. If $C_{n+1}=0$, then in virtue of the same reasons two sums vanish

$$\sum_{\nu=1}^{n}(-1)^{\mu+\nu}A^i_{\mu\nu}C_\nu\,,\qquad \sum_{\nu=n+1}^{\infty}(-1)^{\mu+\nu}A^i_{\mu\nu}C_\nu\,.$$

The second of these sums equals zero for all μ, if only $n \gg 1$.

We now assume that the set of the coefficients $\{C_\nu\}$ is determined by the vector matrix $\|\langle\mu|\hat{H}|\nu\rangle\|$ with the additional requirement $C_{n+1}=0$. Then since

$$\langle n|\hat{T}|n+1\rangle C_{n+1}=0\,,\quad \sum_{\nu=n+1}^{\infty}\langle\mu|\hat{V}|\nu\rangle C_\nu=0\,,$$

the first n of the equations (2) have only such C_ν in which $\nu \leq$ n, i.e. form a closed system of n linear

homogeneous algebraic equations for n unknown coefficients

$$\{C_\nu\}, \quad \nu = 1,2,\ldots, n . \qquad (6)$$

But the decidability condition of a set of homogeneous equations is its determinant vanishing. Therefore, the diagonalization of the Hamiltonian $\hat{H}$ on the basis of n functions φ_ν singles out in the Hamiltonian continuous spectrum the eigenvectors $\{C_\nu^\alpha\}$ with the zeroth (n+1)-th coefficient and the positive energy E_α counted from two-cluster decay threshold.

From the condition $C_{n+1}(E,L)=0$, where L is the angular momentum of the state with the energy E and the asymptotic expression for C_{n+1} (see [6,12]) it follows that

$$j_L(x_{n+1}^\alpha) - tg\delta_L^\alpha \, n_L(x_{n+1}^\alpha) = 0 ;$$

$$x_{n+1}^\alpha = K_\alpha \sqrt{4n + 2L + 4n_0 + 7} ; \quad K_\alpha = \sqrt{2E_\alpha} , \qquad (7)$$

where j_L and n_L are the spherical Bessel and Neiman functions, respectively, n_0 is the minimal permissible number of oscillator excitation quanta of the wave function of relative motion of two clusters, δ_L^α is the scattering phase in the state with the angular momentum L and the energy E_α . The units of measurement in (7) are chosen so that $\hbar = m_p = r_0 = 1$ where $\hbar$ is the Plank constant, m_p is the nucleon mass, r_0 is the oscillator radius. If the energy E_α of the state with the angular momentum L is known, then the formula (7) enables one to obtain the phase $\delta_L^\alpha(E_\alpha)$.

For the eigenvector $\{C_\nu^\alpha(E_\alpha, L)\}$ of the Hamiltonian in the resonating group method to be described

thoroughly it is necessary to know the value of coefficients C_ν^α with the numbers $\nu > n+1$. We can do it easily since for C_ν^α at $\nu > n+1 \gg 1$, the asymptotic expression

$$C_\nu^\alpha(E_\alpha, L) \sim B\sqrt{x_{\nu+1}^\alpha}\left[j_L(x_{\nu+1}^\alpha) - tg\delta_L^\alpha n_L(x_{\nu+1}^\alpha)\right] \qquad (8)$$

is valid, where the multiplier B is determined from the condition that the coefficient $C_\nu^\alpha(E_\alpha, L)$ calculated by the formula (8) coincides with the coefficient $C_n^\alpha(E_\alpha, L)$ of the eigenvector (6) resulting from the Hamiltonian diagonalization on n basis functions.

3. In order to illustrate general statements formulated above we use example of two interacting nuclei - ^{4}He and ^{3}H. These nuclei can form a coupled system - ^{7}Li in the states $J^\pi = 3/2^-$ and $1/2^-$. The wave functions of the bound states ^{7}Li and the states J^π belonging to the continuous spectrum can be written in the form

$$\Psi_{J^\pi}^\alpha = \sum_{\nu=1}^{\infty} C_\nu^{J^\pi\alpha} |\nu; J^\pi\rangle, \qquad (9)$$

where $\{|\nu; J^\pi\rangle\}$ is the set of orthonormalized configurations of the isotropic harmonic oscillator which is generated by the wave function of the resonating group method for the channel ^{4}He + ^{3}H (see the Appendix). The index α numerates the states with the same quantum numbers J^π in the order of increasing energy. The coefficients $C_\nu^{J^\pi\alpha}$ belonging to different eigenvectors ($\alpha = 1, 2, \ldots$) are calculated by the diagonalization of the seven-nucleon system Hamiltonian on n basis functions $|\nu; J^\pi\rangle$. They reveal a very simple dependence on ν represented in Fig.1. The eigenvectors

shown in the figure are obtained in the calculation with 50 basis functions. The nucleon-nucleon interaction was simulated by the Brink-Boeker potential [13] and the spin-orbital interaction GPT [14].

The amplitude coefficients $C_{\nu}^{3/2^-1}$ of the ground state of a system decreasing monotonically with increasing ν tend exponentially to zero. The coefficients $C_{\nu}^{3/2^-\alpha}$ of the states of continuous spectrum, as was mentioned above, vanish at $\nu = n+1$, and change the sign $\alpha-1$ times when ν varies from 1 to $n+1$. With increasing n the energies of all eigenvectors of a continuous spectrum vanish and the nodes of the coefficients $C_{\nu}^{3/2^-\alpha}$ move to the right along the axis ν. However, the first node of coefficients $C_{\nu}^{3/2^-2}$ of the state of a continuous spectrum with a minimal energy in the limit when $E \to 0$ ($n \to \infty$) tends to its limiting value $\nu_0 \sim \beta$. Since the energy of this eigenvector is near zero ($E^2_{3/2^-} = 7{,}05\ 10^{-2}$ if $n=50$), then the position of the first node of its coefficients can be associated with the coefficient a of the asymptotic expression for the state amplitude with the zeroth energy

$$\lim_{\nu \to \infty} C_{\nu}^{3/2^-2} = const\left[(4\nu_0+9)^{3/2} - 3a^3\right]. \qquad (10)$$

This connection is determined by the following relation

$$a = 3^{-1/3}\sqrt{4\nu_0+9}, \qquad (11)$$

where ν_0 is the limiting position of the first node at $n \to \infty$. Fig.2 represents the dependence of ν_0 on the energy $E = E^{\alpha}_{J\pi} - E_{thr}$ (E_{thr} is the threshold

energy). The behaviour of the function (11), whose limiting value at $\nu \to \nu_0$ yields the length of the elastic $t+\alpha$ scattering, is shown in this figure. Assuming $\nu_0 = 8$ we obtain that $a = 4{,}43$.

Note that

$$a^3 = \lim_{k \to 0} \frac{\delta_{3/2^-}(k)}{k^3} . \qquad (12)$$

Fig.3 shows the phases $\delta_{3/2^-}$ and $\delta_{1/2^-}$ of the elastic scattering of ^3He by ^4He in the states $3/2^-$ and $1/2^-$, respectively, depending on energy. In order to calculate the phases we used the relation (7) and the energies $E^{\alpha}_{J^\pi}$ of eigenstates that result from diagonalization of Hamiltonian with a different number n of basis functions.

We now can calculate the coefficient a using the formula (12) and the known phase $\delta_{3/2^-}$ in the point with the minimal energy:

$$E_{3/2^-} = 4{,}948 \cdot 10^{-2} ; \quad \delta_{3/2^-} = 2{,}78 \cdot 10^{-2}$$

The formula (12) yields the following estimate

$$a = 4{,}28$$

which is in good agreement with the above value.

The qualitative trend of the scattering phases obtained from theoretical calculations and the values of the phase splitting is in agreement with the experimental data of the phase analysis [15-17] .

The dependence of the elastic scattering phase in the states $J^\pi = 7/2^-$ and $J^\pi = 5/2^-$ is shown in Fig.4. The experimental values of the phase at the energy (in the center of mass system of particles involved in nuclear reactions) less than 3MeV are taken from the paper of Boykin et. al. [16] and at the energies more than 3MeV from

the paper of Hardy et. al. [18] . The theoretical phase-energy dependence is determined using the relation (7). The energy is calculated in MeV units and the phase in degrees.

Both theoretical and experimental phases at the energy 2÷5 MeV rapidly increase, pass through the point $\delta = \pi/2$. In this case the theoretical phases increase more slowly than experimental ones. Note, however, that in resonance region the phase of $J^{\pi}=7/2^{-}$ state passes through the point $\delta = \pi/2$ with a larger derivative than the phase shift of the state $J^{\pi}=5/2^{-}$ which is in good agreement with the experimental dependence of the phase.

According to the calculated phases we have obtained the partial cross-sections $\sigma_{J^{\pi}}(E)$ of the elastic scattering $t + \alpha$ which are also represented in Fig.4. The maximum of the cross-section $\sigma_{7/2^{-}}(E)$ is at $E_{res} \cong 4.63$ MeV (experimental value of the resonance energy $E_{res} = 4.63$ MeV) and in the case of the resonance $J^{\pi}=5/2^{-}$ the maximum of the cross-section is at $E_{res} \cong 6,3$ MeV (in experiment E=6,68 MeV).

The position $7/2^{-}$ and $5/2^{-}$ of the resonances ^{7}Li is consistent with that of collective quadrupole excitations $7/2^{-}$ and $5/2^{-}$ determined in [19] as a result of diagonalisation of the Hamiltonian on the basis of oscillator functions of longitudinal oscillations of the inertia ellipsoid (Sp(2,R) basis). This coincidence is not arbitrary but is determined by the fact that, firstly, the overlap integral of oscillator functions of the "cluster" basis and oscillator functions of the basis of longitudinal oscillations of the inertia ellipsoid is near to unity until the number ν of their oscillator excitation quanta is small and, secondly, that the lowest collective excitations are formed mainly on

the basis functions of longitudinal oscillations with small γ ,i.e. on the functions represented, to a certain degree, in the cluster basis too.

Having restricted ourselves to the basis of longitudinal oscillations we can calculate only the energy of quadrupole resonances but not their width since longitudinal oscillations cannot disintegrate a nucleus even at relatively large excitation energies. In its turn, the "cluster" basis reveals the channel of decay of ^{7}Li in α and t and, therefore, enables us to calculate not only the energy of the lowest quadrupole resonances but also their width though with a certain overestimate due to inadequate reproduction of the collective quadrupole mode stabilizing the resonance. The theoretical estimates calculated relative to the cross-sections $\sigma_{7/2}(E)$ and $\sigma_{5/2}(E)$, equal, respectively, to $\Gamma_{7/2^-}$=2,6 MeV, $\Gamma_{5/2^-}$=3,1 MeV and the experimental values are $\Gamma_{7/2^-} \approx$ 0,1 MeV, $\Gamma_{5/2^-} \approx$ 0,9 MeV.

Consequently, the calculation of the resonance states taking into account both bases under consideration is of special interest. Such a calculation, beyond our theme,could enable one to define more exactly the width of quadrupole resonances.

So the method under consideration gives a very simple possibility to calculate the phases of the elastic scattering and the wave functions of a continuous spectrum at the energies which are determined by the given position of the coefficient node of the wave function expansion of continuous spectrum in the oscillator basis.

The generalization of the considered approach for the case of multichannel processes will be made in the following paper.

APPENDIX

In Ref. [8] the generating matrix elements (g.m.e.) of the nuclear Hamiltonian of p-shell with the nucleon number $A=4+K$, $K \leq 4$ are constructed. They are obtained using the generating function $\Psi(R,\beta)$ which generates independently two different bases of oscillator functions. The parameter β of the generating function at $R \to 0$ generates the excitations of the dominant collective branch of the total collective basis in the minimal approximation of the generalized hyperspherical function method. This branch is given by the definite irreducible representation σ of $Sp(2,R)$ group. If $\beta=0$, then the parameter R generates excitations of oscillation degrees of freedom of two clusters, the α-particle cluster and the K-nucleon cluster. In this case g.m.e. of Ref. [8] are essentially simplified. We represent their explicit form.

The generating matrix elements of the unit operator

$$F(\xi) = \langle \Psi(R,0) | \tilde{\Psi}(\tilde{R},0) \rangle = sh^K \xi \, e^{d\xi};$$

$$\xi = R\tilde{R}t \; ; \quad t = \cos\theta \; ; \quad d = \frac{K(4-K)}{A} . \tag{A.1}$$

In order to project onto the states of normal and anomalous ($\pi_1 = (-1)^{\bar{f}}$, $\bar{f} = A-4$; $\pi_2 = (-1)^{\bar{f}}$, $\bar{f} = \bar{f}+1$, respectively) parity (see [19]) we represent the overlap integral $F(\xi)$ in the form

$$F(\xi) = \sum_{r=0}^{K} D_r^K \left[sh\,\varepsilon_{Kr}\xi + ch\,\varepsilon_{Kr}\xi \right], \tag{A.2}$$

where

$$D_r^K = \frac{(-1)^r K!}{r!(K-r)!} \; ; \quad \varepsilon_{Kr} = K - 2r + d = \frac{8K}{A} - 2r .$$

Then the contribution to m.e. on the states of positive parity is made only by $ch\xi$ and for negative parity by $sh\xi$. Thus,

$$F(\xi) = \sum_{r=0}^{K} D_r^K M(\varepsilon_{Kr}\xi), \tag{A.3}$$

where

$$M(x) = \begin{cases} shx, & \pi = -1; \\ chx, & \pi = +1. \end{cases}$$

As a result, after projecting onto the states with a definite value of the orbital momentum L the partial m.e. corresponding to m excitation quanta of the wave function of relative motion of cluster takes the form

$$\langle mL|mL\rangle = \sum_{r=0}^{K} \frac{D_r^K \varepsilon_{Kr}^{2m+n}}{(2m+n)!} J_L^{2m+n}, \tag{A.4}$$

where

$$n = \begin{cases} K, & \pi = (-1)^{\bar{L}}; \\ K+1, & \pi = (-1)^{\bar{\bar{L}}}, \end{cases}$$

and the integral J_L^{2m+n} is determined in [19]. The generating matrix element of the kinetic energy operator

$$T(\xi)=\frac{\hbar^2}{2m_0}\left\{\frac{3}{2}(A-1)-\frac{4K}{A^2}(R^2+\tilde{R}^2)+\frac{\partial}{\partial z}\right\}F(z\xi)\Big|_{z=1}$$

Hence, taking into account (A.3),(A.4) we obtain the diagonal m.e. :

$$\langle mL|\hat{T}|mL\rangle=\frac{\hbar^2}{2m_0}\left[\frac{3}{2}(A-1)+2m+n\right]$$

and nondiagonal

$$\langle mL|\hat{T}|m+1\,L\rangle=\langle m+1\,L|\hat{T}|mL\rangle=-\frac{\hbar^2}{m_0}\cdot\frac{2K}{A}\sqrt{\frac{B_m}{B_{m+1}}(2m+n+2-L)(2m+n+3+L)}$$

where

$$B_m=\sum_{r=0}^{K}D_r^K\,\varepsilon_{Kr}^{2m+n}.$$

The overlap integral with the operator of central NN-interaction can be easily calculated by the formulas in Ref. [8] at $\beta=\tilde{\beta}=0$

$$V_c(\xi)=\langle\Psi(R,0)|\sum_{i>j=1}^{A}\hat{V}_c(r_{ij})|\tilde{\Psi}(\tilde{R},0)\rangle=$$

$$=V_{ss}f_{ss}^R+V_{sp}^{(+)}f_{sp}^{(+)R}-V_{sp}^{(-)}f_{sp}^{(-)R}+V_{pp}f_{pp}^R.$$

In this case the constants V_{ss}, $V_{sp}^{(\pm)}$ and V_{pp} are determined so as in [8] and the functions f_{ss}^R, $f_{sp}^{(\pm)R}$ and f_{pp}^R have the form :

$$f_{ss}^R(\xi)=\gamma^{3/2}\,F(\xi);$$

$$f_{sp}^{(\pm)R} = \gamma^{3/2} \sum_{r=0}^{K-1} D_r^{K-1} \left\{ Q_{R\tilde{R}} M\left[\xi(\varepsilon_{Kr} - 1 \pm \gamma)\right] - Q_{R0} M\left[\xi(\varepsilon_{Kr} - 2)\right] \right\};$$

$$f_{pp}^{R} = f_{ss}^{R} + 2\gamma^{3/2} \sum_{r=0}^{K-2} D_r^{K-2} \left\{ Q_{R\tilde{R}} M\left[\xi(\varepsilon_{Kr} - \gamma - 3)\right] + Q_{R\tilde{R}} \left[\xi(\varepsilon_{Kr} + \gamma - 3)\right] - \right.$$

$$\left. - Q_{R0} M\left[\xi(\varepsilon_{Kr} - 2)\right] - Q_{R0} M\left[\xi(\varepsilon_{Kr} - 4)\right] \right\},$$

where

$$Q_{R\tilde{R}} = \exp\left[-\frac{1-\gamma}{2} (R^2 + \tilde{R}^2) \right].$$

It is easy to see that the above g.m.e. possess only standard blocks

$$\Delta(c_1, c_2, c_3) = e^{-c_1 R^2 - c_2 \tilde{R}^2} M(c_3 \xi)$$

with different sets of constants c_1, c_2 and c_3. The partial m.e. corresponding to them has the form

$$\langle m_1 L | \Delta | m_2 L \rangle = (-1)^{m_1+m_2} \sum_{\rho=0}^{\bar{m}} \frac{c_3^{2\rho+\alpha} c_1^{m_1+n-\rho} c_2^{m_2+n-\rho} J_L^{2\rho+\alpha}}{(2\rho+\alpha)!(m_1+n-\rho)!(m_2+n-\rho)!} \sqrt{\frac{(2m_1+\eta)!(2m_2+\eta)!}{J_L^{2m_1+\eta} J_L^{2m_2+\eta}}} \delta_{m_1 m_2}$$

where

$$\alpha = \begin{cases} 0, & \pi = +1; \\ 1, & \pi = -1; \end{cases}$$

$$\bar{m} = \min\left\{ m_1 + n,\ m_2 + n \right\}.$$

As to g.m.e. and the corresponding m.e. of the spin-orbital interaction operator, they can be easily constructed by analogy with that made for the central interaction.

REFERENCES

1. Microscopic Method for the Interactions between Complex Nuclei. Progr. Theor. Phys.Suppl., 1977, N62, p. 1-294.
2. Comprehensive Study of Structure of Light Nuclei. Progr. Theor. Phys., Suppl., 1980, N68, p.1-320.
3. К.Вильдермут,Я.Тан.Единая теория ядра.М.,Мир,1980,502с.
4. Y.C.Tang,M.Le Mere, D.R.Thompson. Resonating-Group Method for Nuclear Many-Body Problems. Phys.Rep.,1978, 47, p. 167-223.
5. Г.Ф.Филиппов,И.П.Охрименко.О возможности использования осцилляторного базиса для решения задач непрерывного спектра.Ядерная физика,1980,32, вып.4,с.932-939.
6. Г.Ф.Филиппов.Об учете правильной асимптотики в разложениях по осцилляторному базису.Ядерная физика,33,вып. 4,1981m с. 928-931.
7. G.F.Filippov, V.S.Vasilevsky, T.P.Kovalenko.Algebraic Version of Resonating Group Method. Preprint ITP-81-107E,Kiev, 1981, 21 p.
8. V.S.Vasilevsky, G.F.Filippov, L.L.Chopovsky. Generating Matrix Elements of the Central and Spin-Orbital Interaction Operator in the Lightest Nuclei of p-Shell.Preprint ITP-81-13E,Kiev,17p.
9. L.Mailing,J.Rizek, Z.Pluhar, Yu.F.Smirnov. On Some Peculierities of the Variational Calculations in the Harmonic Oscillator Basis. J.Phys.G.: Nucl.Phys., N2, N6, 1976.

10. Л.Майлинг, И.Ржизек, З.Плугарж. Волновая функция тритона в осцилляторном базисе.Известия АН СССР,сер. физ., 1974,т.38, №10, с.2141-2147.
11. A.U.Hazi, H.S.Taylor, Stabilization Method of Calculating Resonance Energies: Model Problem. Phys. Rev., A1, 1970, p. 1109-1120.
12. Ю.И.Нечаев,Ю.Ф.Смирнов.Некоторые особенности задачи рассеяния в осцилляторном представлении.В кн.:Тезисы докладов XXXI.Совещания по ядерной спектроскопии и структуре атомного ядра.(Самарканд,1981), Наука,485с.
13. D.M.Brink, E.Boeker. Effective Interactions for Hartree-Fock Calculations.Nucl.Phys., A91, N1,1967, p. 1-27.
14. D.Gogny, P.Pires, R. de Tourreil. A Smooth Realistic Local Nucleon-Nucleon Force Suitable for Nuclear Hartree-Fock Calculations. Phys.Lett., 32B, N7,1970, p. 591-595.
15. C.D.Jacobs, Jr., R.E.Brown. Elastic Scattering of ^{3}He by ^{4}He from 17,8 to 30,0 MeV. Phys.Rev., 1C, N5, 1970, p. 1615-1621.
16. W.R.Boykin, S.D.Baker, D.M.Hardy. Scattering of ^{3}He and ^{4}He from Polarized ^{3}He between 4 and 10 MeV. Nucl. Phys., A195, 1972, p. 241-249.
17. R.J. Spriger, T.A.Tombrello. Scattering of ^{3}He by ^{4}He and of ^{4}He by Tritium. Phys. Rev.,vol.163, N4, 1967, p. 964-984.
18. D.M.Hardy, R.J.Spriger,S.D.Baker, Y.S.Chen,T.A.Tombrello. Scattering of ^{3}He and ^{4}He from Polarized ^{3}He between 7 and 18 MeV. Nucl.Phys., A195, 1972,p.250-256.
19. G.F.Filippov, L.L.Chopovsky, V.S.Vasilevsky.On the Connection between Collective and Spin Degrees of Freedom in the Sp(2,R) Basis. The Lightest Nuclei of p-Shell. Preprint ITP-81-72E,Kiev, 1981, 50 p.

Received March 1, 1982

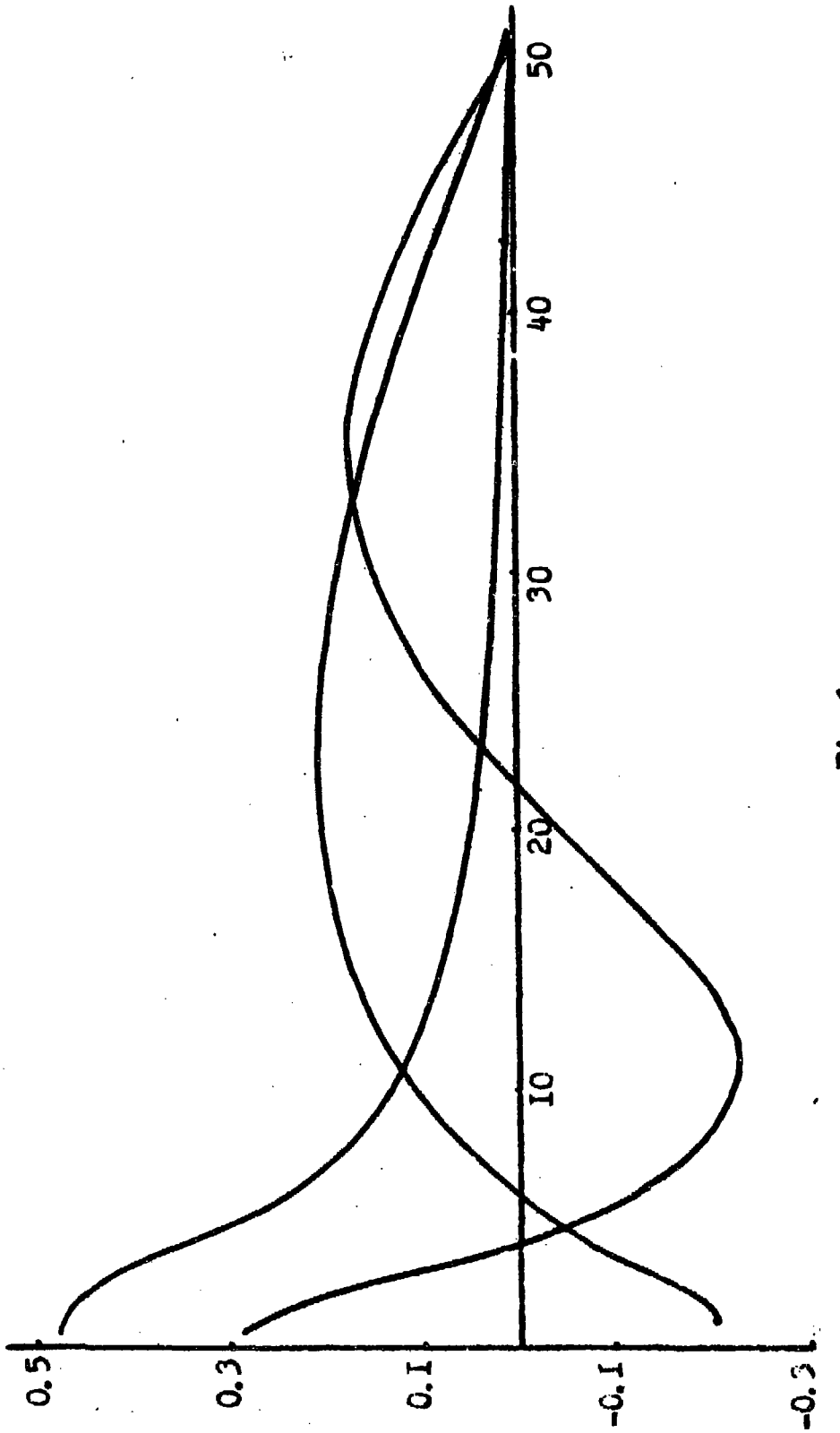

Fig. 1

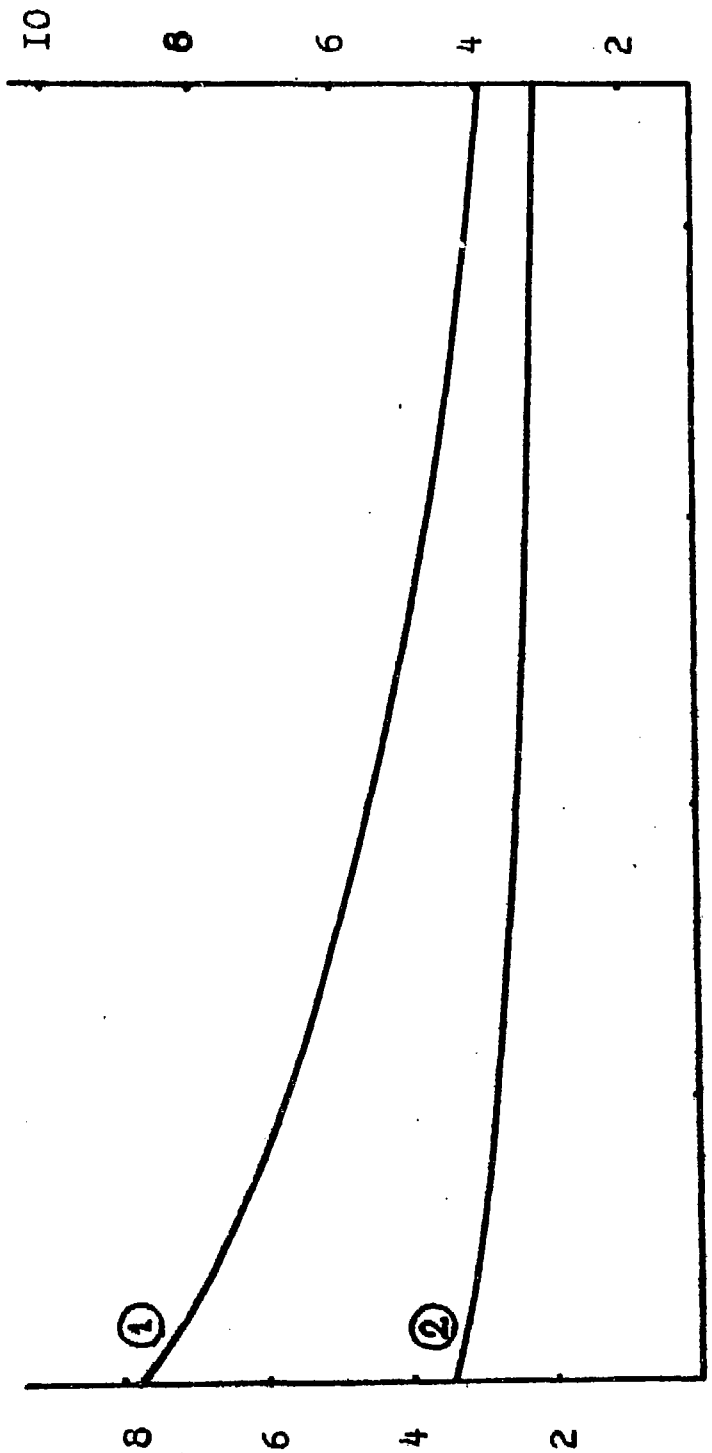

Fig. 2

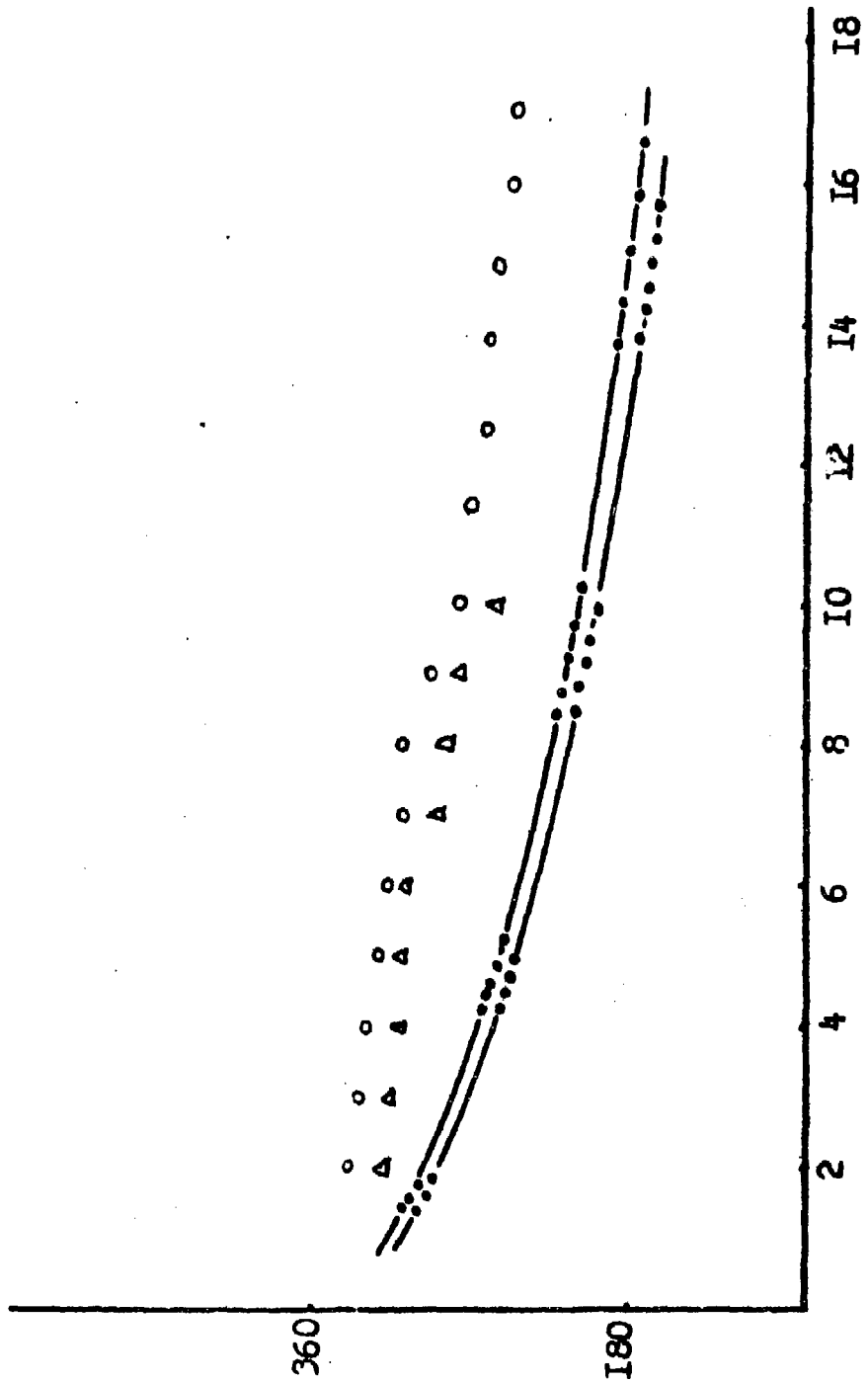

Fig. 3

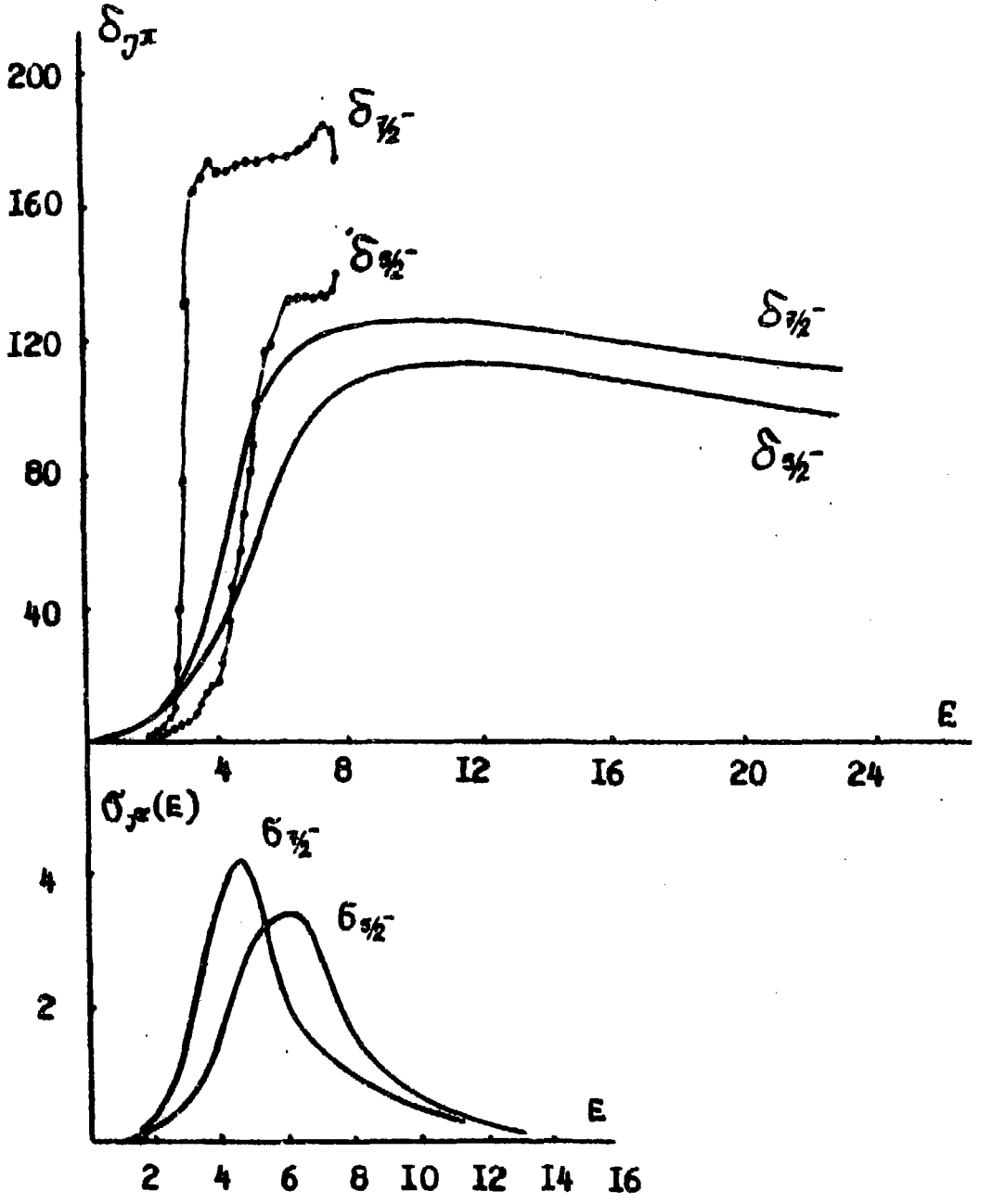
$\delta_{\gamma\pi}$
200
160
120
80
40
$\delta_{1/2^-}$
$\delta_{3/2^-}$
$\delta_{3/2^-}$
$\delta_{5/2^-}$
E
4
8
12
16
20
24
$\sigma_{\gamma\pi}(E)$
$\sigma_{1/2^-}$
$\sigma_{3/2^-}$
4
2
E
2
4
6
8
10
12
14
16

Fig.4

FIGURE CAPTIONS

Fig.1. The dependence of the coefficients $C_\nu^{J^\pi \alpha}$ of the expansion (9) for the three lowest states $J^\pi = 3/2^-$ ($\alpha = 1,2,3$) on energy.

Fig.2. The dependence of the position of the first node ν_0 of the function $C_\nu^{J^\pi \alpha}$ (the curve I) and the function a (the curve 2, see (12)) on energy.

Fig.3. The behaviour of the phase shifts $\delta_{3/2^-}$ and $\delta_{1/2^-}$ ($L^\pi = 1^-$). The signs o and Δ denote the experimental phases $\delta_{3/2^-}$ and $\delta_{1/2^-}$. The points connected by smooth curves represent the calculated dependence of the same phases on energy.

Fig.4. The dependence of the phase shifts $\delta_{7/2^-}$ and $\delta_{5/2^-}$ ($L^\pi = 3$) on energy. The isolated points reproduce experimental values of the phase and the smooth curves reproduce the calculated ones. The cross-sections of the elastic $t + \alpha$ scattering are given in the bottom part of the figure.

Геннадий Федорович Филиппов
Леонид Леонидович Чоповский
Виктор Семенович Василевский

О состояниях с положительной энергией, появляющихся при диагонализации гамильтониана на осцилляторном базисе

Редактор А.А.Храброва Техн.редактор Л.П.Давиденко

Зак. 229 Формат 60x90/16.Уч.-изд.л. 1,2
Подписано к печати 20.04.82 г. Тираж 295. Цена 9 коп.

Офсетная лаборатория Института теоретической физики АН УССР

Препринты Института теоретической физики АН УССР
рассылаются научным организациям и отдельным ученым
на основе взаимного обмена.

Наш адрес: 252130, Киев-130
ИТФ АН УССР
Информационный отдел

The preprints of the Institute for Theoretical Physics
are distributed to scientific institutions and individual
scientists on the mutual exchange basis.

Our address:

Information Department
Institute for Theoretical Physics
252130, Kiev-130, USSR