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QUARK MASS EFFECTS IN QCD

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In the renormalization group (RG) treatment of QCD the parametrization by scale parameter Λ is widely accepted and used in the analysis of data. It seems to be quite natural for quantum field models with asymptotic freedom in the region of energies much larger than all particle masses.

The most popular, at present, 2-loop approximation for invariant coupling (IC) is of the form

$$\bar{g}(Q^2, f) = \frac{1}{\beta_f L} - \frac{b_f \ln L}{(\beta_f L)^2} \quad , \quad g = \frac{\alpha_s}{4\pi} \tag{1}$$

$$L = \ln \frac{Q^2}{\Lambda^2} \quad , \quad \beta_f = 11 - \frac{2}{3} f \ , \ b_f \beta_f = 102 - \frac{38}{3} f \ .$$

where f is the number of flavours.

However, the domain of nowaday physical applications contains thresholds of the heavy-quark-pair creation. Due to this the notion of "operative" quarks was introduced, i.e., the dependence of f on Q^2 that can be discrete:

$$f = 3 \qquad Q^2 < M_4^2 \ ,$$
$$f = 4 \qquad M_4^2 < Q^2 < M_5^2$$

or continious[1]

$$f(Q^2) = \sum_n \left\{ 1 + \frac{5}{4} \frac{M_n^2}{Q^2} \right\}^{-1} , \tag{2}$$

where M_n is a threshold of the n-th quark-pair $q\bar{q}$ creation. As was noticed in paper[2] the Q^2 dependence of number f yields the analogous dependence of scale parameter $\Lambda \rightarrow \Lambda(Q^2)$.

In recent papers of the author[3,4] a quantitative analysis of the $\Lambda(Q^2)$ was performed at the 2-loop level. This report contains the resume of these publications. Main results are exposed in three subsequent Sections. First (Section 2) we perform a generalization of the 1-loop expression for the jump Λ_{f-1}/Λ_f of the scale parameter from paper[2] to the 2-loop case and find that the account of the second loop effect leads to an increase of the jump. Second (Section 3) we give approximate solution of the 2-loop RG differential equation for $\bar{g}$ with account of heavy quark masses. The resulting expression (see below Eq. (10)) is of a high accuracy and correctly reflects the threshold singularities. In the last Section 4 we give results of the numerical comparison of our new Eq. (10) with popular Eq. (1) inside which we use for the flavour number f the continious expression (2) by Georgi-Politzer. The results of comparison are exposed in the Figure.

2. The qualitative effect of variation of the scale parameter with changing Q^2 at the points of transition from one number of flauvours to another follows from the condition of continuity of the invariant coupling $\bar{g}$ at these points

$$\bar{g}(M_f^2, f-1) = \bar{g}(M_f^2, f). \tag{3}$$

Using, for a qualitative estimate the 1-loop approximation to Eq. (1) and expanding in the small parameter $\delta_f = (\beta_{f-1} - \beta_f)/\beta_f \sim 10^{-1}$ one can get[2]

$$\ln \frac{\Lambda_{f-1}^2}{\Lambda_f^2} = \frac{2}{33-2f} \ln \frac{M_f^2}{\Lambda_f^2}. \tag{4}$$

It follows from this expression that Λ_f decreases with growing f, i.e., with energy and that the relative jump increases in magnitude with growing the threshold number f and threshold mass M_f. The inclusion of the 2-loop term in r.h.s. of Eq. (1) slightly enlarges the jump value

$$\ln \frac{\Lambda_{f-1}^2}{\Lambda_f^2} = \frac{\delta_f + C[\xi_f - \delta_f]}{1 + C\,\dfrac{\ln L_f - 1}{\ln L_f}}\, L_f \quad , \quad L_f = \ln \frac{M_f^2}{\Lambda_f^2} > 1 \,,$$

$$\xi_f = (b_{f-1} - b_f)/b_f > \delta_f \quad , \quad C = \left[\frac{\beta_f L_f}{b_f \ln L_f} - 1\right]^{-1} > 0 \,. \tag{5}$$

3. For a more accurate description of threshold effects one has to analyse the 2-loop RG equations written with account of masses. We perform this analysis in the standard subtraction scheme which in the modern slang is called the "MOM-scheme". Notice here that due to mass dependences the MOM, and, e.g., MS schemes, turn out to be essentially nonequivalent even at the 2-loop level. In other words the transition from the given below expression (10) to the MS case is not reduced to the change of the momentum scale. However, the corresponding generalization is rather straightforward, and qualitatively, our conclusions are scheme-independent.

Starting with the standard perturbation theory in the MOM scheme

$$\bar{g}_{p.th}(Q^2, m^2, \mu^2, g) = g - g^2[U(Q^2/m^2) - U(\mu^2/m^2)] + \\ + g^3[U(Q^2/m^2) - U(\mu^2/m^2)]^2 - g^3[V(Q^2/m^2) - V(\mu^2/m^2)] + \dots \tag{6}$$

where U is the sum of one-loop contributions

$$U(z) = 9 \ln z - \frac{2}{3} I(\xi_4 z) - \frac{2}{3} I(\xi_5 z) \dots \quad , \xi_i = \frac{m^2}{M_i^2} \quad , \\ I(z) = 6 \int_0^1 dx\,(1-x)x \ln[1 + x(1-x)z] \to \ln z \quad , \tag{7}$$

and V - the analogous sum of 2-loop contributions

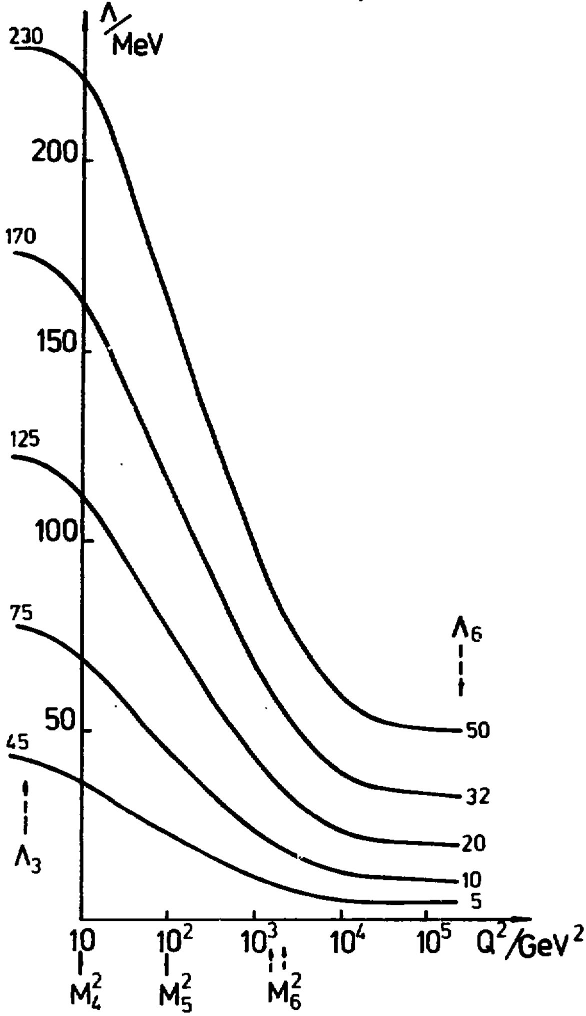
$$V(z) = 64 \ln z - \frac{38}{3} I_2(\xi_4 z) - \frac{38}{3} I_2(\xi_5 z) - \dots \tag{8}$$

and by solving approximately the RG differential group equation with mass dependences (first introduced in papers[5,6])

$$\frac{\partial \bar{g}_{RG}(Q^2)}{\partial \ln Q^2} = \beta\left(\frac{Q^2}{m^2}, \bar{g}_{RG}\right) \\ \beta\left(\frac{\mu^2}{m^2}, g\right) = \left[\frac{\partial}{\partial \ln Q^2} \bar{g}_{p.th}(Q^2, m^2, \mu^2, g)\right]_{Q^2=\mu^2} \tag{9}$$

one can obtain[3]

$$\bar{g}_{RG}(Q^2, m^2, \mu^2, g) = \tag{10}$$
$$= \frac{1}{1 + g\,U(Q^2, m^2, \mu^2) + g\,\dfrac{V(Q^2, m^2, \mu^2)}{U(Q^2, m^2, \mu^2)}\,\ln\left[1 + g\,U(Q^2, m^2, \mu^2)\right]} \,.$$



Here functions $U(Q^2,\dots)$ and $V(Q^2,\dots)$ are coefficients of the perturbation expansion (6), and g - the coupling constant defined by

$$g = \bar{g}(\mu^2, m^2, \mu^2, g) . \tag{11}$$

Note that the obtained formula (10) under the expansion over g powers corresponds to the initial perturbation expression (6). On the other hand, in the "pure logarithmic" regions at $Q^2 \ll M_i^2$ and $Q^2 \gg M_i^2$ when

$$U \simeq \beta_1 \ln Q^2/\mu^2 \quad , V \simeq \beta_2 \ln \frac{Q^2}{\mu^2}$$

Eq. (10) goes to the well known 2-loop logarithmic expression./7/

4. To establish the correspondence between Eq. (10) and the popular Eq. (1) using the scale parameter Λ , it is possible to employ the procedure of numerical comparison. This was done in/4/. As the basis of comparison several exact solutions (10) were used. They were fixed by g values at the point $\mu^2 = 10$ GeV^2:

$100g = 1.05$; 1.20; 1.35; 1.50 and 1.65.

The values of the invariant QCD coupling in the usual normalization are as follows

$\bar{\alpha}_s(10\ \mathrm{GeV}^2) = 0.13$; 0.15; 0.17; 0.19; 0.21 .

To these solutions there correspond scale parameter Λ values in the 3-quark region at $Q^2 \lesssim 1$ GeV^2

$\Lambda_3 = 45$; 75; 125; 170 and 230 MeV .

The functions $\Lambda(Q^2)$ were obtained by the numerical comparison of Eq.(10) at given above values of g with Eq. (1) in which the f dependence was given by Eq. (2). The resulting curves monotonically decreasing with growing Q^2 argument are given in the Figure. Two astonishing features can be seen from this Figure:

(a) The curves $\Lambda(Q^2)$ between the heavy-quark-pairs thresholds at $M_4^2 \simeq 10$ GeV^2, $M_5^2 \simeq 100$ GeV^2 and $M_6^2 \gtrsim 1000$ GeV^2 do not cease their rather rapid decreasing and as a whole look like monotonic smooth curves. This is due to the mutual proximity of thresholds M_4^2, M_5^2 and, possibly, M_6^2 in the logarithmic scale.

(b) The total variation of scale parameter values between the 3-quark region (Λ_3 values given in the Figure) and the 6-quark region (Λ_6 values) turns out to be rather large. The ratio Λ_3/Λ_6 grows with diminishing Λ_3 and the absolute α_s values and varies for the considered solutions between 4.5 and 9. Qualitatively these effects are seen from the simple formula (4). Quantitatively they can be estimated from the more complicated Eq. (5).

Hence the parametrization of the QCD coupling $\bar{\alpha}_s(Q^2)$ in the region of modern experiments (for the Q^2 values of an order of several dosen and hundreds of GeV^2) with the help of the popular 2-loop expression turns out to be complicated by the dependence $\Lambda \to \Lambda(Q^2)$. This dependence in contradistinction to the $f(Q^2)$ dependence according to Eq. (2) varies with changing α_s , and hence, has no universal nature. Due to this the popular Eq.(1) can be used for rather narrow intervals of the energy (momentum) variable Q^2 with the "local" scale parameter Λ value. Relations between different local Λ values can be established on the basis of Eq.(10) or curves given in the Figure. The use of the information obtained in the region of modern experiments in the high-energy region, especially in the region of grand unification must be made with high precautions.

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Ширков Д.В. Учет масс кварков в КХД E2-82-465

В работе подытожены результаты исследований энергетической зависимости инвариантного заряда $\bar{\alpha}_s(Q)$ в КХД с учетом влияния 2-петлевых поправок, а также масс тяжелых кварков. Основные результаты:

/1/ Вывод формулы для $\bar{\alpha}_s$ в 2-петлевом приближении с учетом эффектов "включения" тяжелых кварков.

/2/ Количественное определение на основе этой формулы энергетической зависимости параметра шкалы Λ и вывод о его неадекватности в современной области энергий.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1982

Shirkov D.V. Quark Mass Effects in QCD E2-82-465

In this paper recent studies of invariant QCD coupling $\bar{\alpha}_s(Q^2)$ in the 2-loop approximation with account of fermionic mass effects are summarized. The main results are:

(1) An explicit expression for $\bar{\alpha}_s(Q^2)$ in the 2-loop approximation with accurate account of heavy quark masses.

(2) A quantitative analysis on the basis of the above-mentioned expression for $\bar{\alpha}_s(Q^2)$ of the energy dependence of the scale QCD parameter Λ and the conclusion about its inadequacy in the modern energy region.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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