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PERIODIC INSPECTION FOR SAFETY OF
CANDU HEAT TRANSPORT PIPING
SYSTEMS - A PROBABILISTIC APPROACH

PHASE I-A

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SUMMARY

Heat transport and emergency core cooling piping systems are vital components of a nuclear power plant, and their reliability and performance during the design life are of a paramount importance to the owner as well as regulatory agencies.

Periodic inspection is intended to maintain an adequate level of safety throughout the life of a plant, and to protect plant personnel and the public from the consequences of a failure and release of fission products.

This report outlines a rational approach to the periodic inspection based on a fully probabilistic model, and describes the work done on the Phase I(a) of the proposed study. It demonstrates a methodology based on sound theoretical treatment and a large body of experimental data, whereby the strength of a pressurized pipe (or vessel) containing a defect could be evaluated. It also shows how the extension of the defect at various lifetimes could be predicted. These relationships are pre-requisite for the probabilistic formulation and analysis for the periodic inspection of piping systems.

1. INTRODUCTION

Periodic inspection serves two purposes. It is a very important aspect of preventive maintenance to protect the productivity of the plant, and it can, in some cases, make an important contribution to the safety of plant personnel and the public. Periodic inspection for safety has for a long time been an integral part of the licensing process for conventional boilers and pressure vessels, but in a rather crude way looking for gross effects such as massive corrosion or tube blockage and requiring a complete shut down of the plant for what has been mostly a visual inspection.

The monetary penalties resulting from the total shut down of process plants for inspection purposes becomes less and less tolerable as the unit size increases. To relieve this situation, procedures to monitor identified causes of deterioration, and the basing of the inspection interval on a running analysis of measured critical criteria are increasing. These procedures require that a close connection be achieved between the design of the plant and its performance. In aviation, periodic inspection was at first considered in the context of a preset safe life. This was to be carried out at intervals determined by a programme of testing and of major and minor overhauls which could be modified in time by evidence coming from the inspection itself. With increasing fleet, unit size, and service frequency, periodic inspection is in many cases carried out at intervals determined by analysis of data from a line performance monitoring.

For nuclear reactors, the hostile radiation environment presents serious problems for re-inspection, requiring specially developed equipment as well as the lengthening of time required to carry out the inspection. To date the light water type has set the pattern for periodic inspection of reactors. The special problems of the large thick walled pressure vessel has caused great emphasis to be placed on the danger of catastrophic failure of the pressure envelope.

It is important to remember that the cyclic refuelling of light water reactors provides an opportunity for periodic inspection which does not exist with the continuous on power Candu type refuelling. For Candu reactors it is very important then to give careful attention to the *requirement* for periodic inspection because an overspecification will have a serious impact on production as well as on accumulated radiation doses.

The purpose of this study is to provide a rational approach to periodic inspection based on a probabilistic model.

1.1 Periodic Inspection for Safety

Periodic inspection for safety is intended to maintain an adequate level of safety throughout the life of the plant. In the case of nuclear reactors a defence in depth is supposedly used against the potential hazard of the release of the fission product inventory of the core. This defence in depth takes the form of multiple barriers with the additional protection of "safety" systems which are intended to mitigate against the effect of the failure of an individual barrier;

for example, emergency core cooling mitigates against the effect of the loss of normal coolant from the core which would result if a rupture of the pressurized reactor coolant system envelope occurred. There has been a requirement associated with the use of nuclear reactors that the residual risk must be reduced to the greatest extent possible. This is the situation for both routine (non-accidental) and accident conditions. This has resulted in the adoption of a postulatory approach to satisfy in which an upper limit accidental event is described for which the effects must be minimized within prescribed regulatory limits.

The problem of establishing the amount and frequency of re-inspection in practical terms has been a concern of the aviation industry since its inception. However, to survive in aviation it is necessary to learn to operate on the brink of a catastrophe, and consequently the safety margins are therefore small and very dependent on re-inspection procedures. In nuclear matters in general the margins should be much greater and there need be less reliance on re-inspection to maintain a satisfactory safety level throughout the life of the plant.

The ASME Section XI Code presently deals only with the reactor coolant pressure boundary. The code then requires partial re-inspection at each refuelling outage (12-18 months) to achieve 100% re-inspection in each 10 year interval.

Proposals so far made for a Candu Reactor Code also deal with the reactor pressure coolant boundary and certain interfaces with the secondary side (for example to steam generators), but classifies re-inspection requirements according to the effect a component failure would have on the safety of people and the likelihood of the event.

For example re-inspection for safety might be limited to components the failure of which would initiate the emergency cooling system, since other events would not seriously tax the overall containment system. The Canadian plan is in the development stage and this study, in part, is intended to help in this process.

Explicit in the Canadian approach is the postulate of a component failure. One may then ask what is the process of such a failure and what is the probability of its occurrence? In the following the process of failure is discussed and a methodology for predicting it is proposed.

2. FAILURE PROCESS

All structural components contain defects or cracks of varying degrees even before their installation. They may vary from atomic scale to those detectable by the present day techniques. When these structural components are subjected to stresses or strains, the contained energy may force the cracks to propagate.

In the first stage, the cracks are normally of a microscope scale so that their existence does not alter the mechanical characteristics of the material. However, as the component is subjected to further load applications, the cracks propagate, and the ensuing damage may cause a reduction of the material strength. The cracks or defects in this stage of the component life are generally of a macroscopic scale. Subsequent applications of load would lead to a critical crack length which causes the failure of the component.

The process of failure described above varies in time for a population of similar components. Figure 1 shows a schematic diagram of the crack propagation in a population and various stages of it.

The philosophy of the fail-safe concept could be described by this figure. It relies on leaving the component or structure in service until the cracks (or defects) are detected by a planned inspection before they reach the dangerous level. A serious problem with this method is the assurance of sufficient residual strength to provide safety until the defects are detected. Several models have been proposed for predicting the safe life of a component in aviation as summarized by Payne [1].

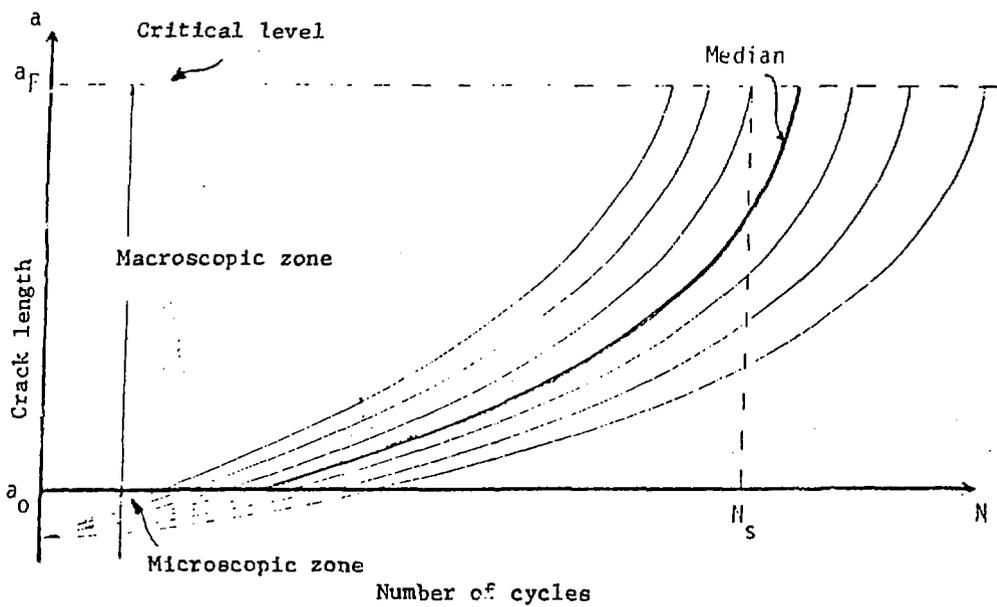


Fig. 1. Evolution of a defect during service-life of a population.

One of the major problems to deal with is the variability of strength, loading, and component behaviour. For example, at a given lifetime, similar components will exhibit different residual strengths, and will contain different defects or crack sizes. Figure 2a shows such a behaviour where a probability density function could be assigned for various crack lengths at a specified lifetime. Similarly, to achieve a pre-determined crack size, there would be a variation in the required number of cycles. Hence, one could define a conditional probability density function for the fatigue life. Figure 2b shows such a distribution.

To calculate the risk of failure under any prescribed load spectrum, one would require a description of the variability in residual strength and the variability in crack propagation. In this report, we will attempt to calculate first the mean distribution for these variables. The distribution about the mean and calculation of risk of failure probabilities will be the subject of another report.

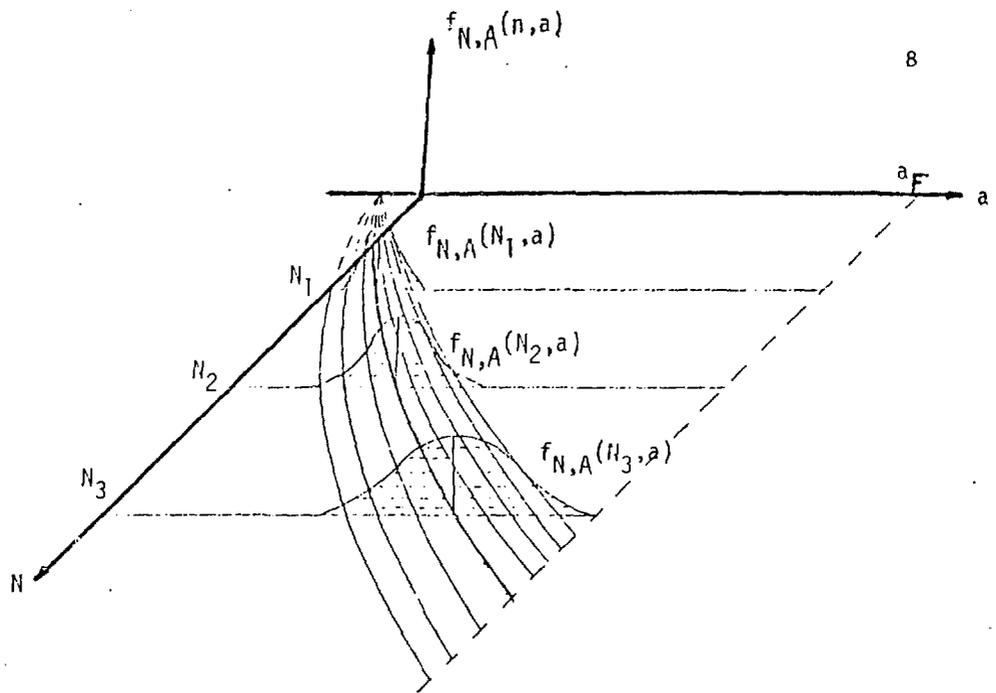


Fig. 2(a). Probability density function of the crack length at a specified lifetime.

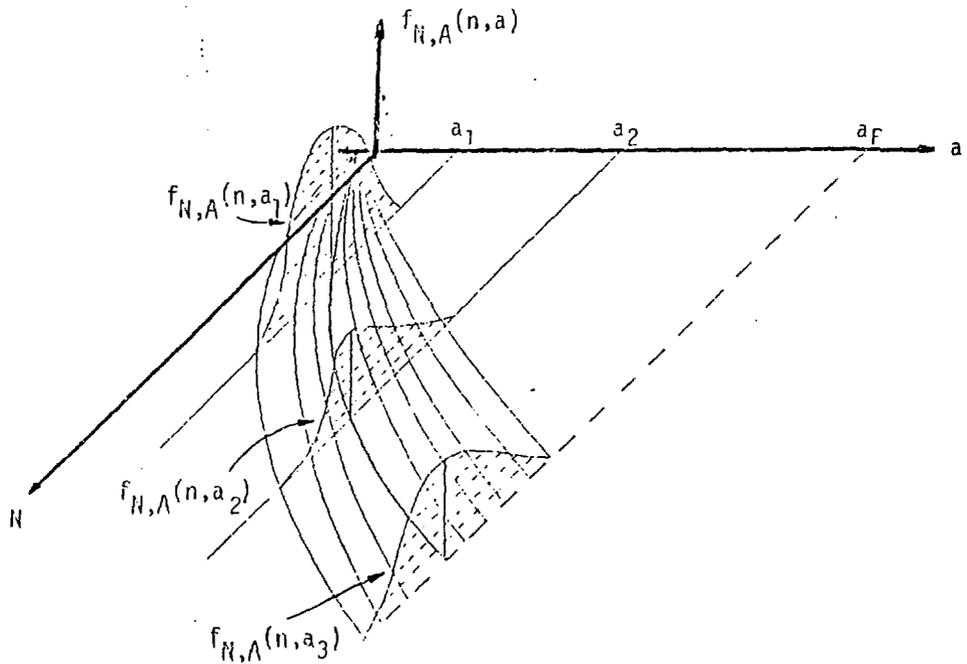


Fig. 2(b). Probability density function of the fatigue-life for a specified crack length.

3. CRACK PROPAGATION

There exists numerous crack propagation laws, the majority of which claim to be verified by the experimental data (albeit restrictive) analyzed in their perspective papers. These different propagation rules may be divided into three categories. In general, they treat cracks in an infinite plate subject to a uniform stress perpendicular to the crack, or can be applied to that configuration.

There are theoretical studies based either on the dimensional analysis, or on the crack tip displacement. The dimensional analyses depend essentially on the list of variables introduced in the formulation. They could thus lead to complicated rules which may not correspond to the observed behaviour, e.g.,

$$\frac{da}{dN} = f(\Delta\sigma, a, \epsilon_y, \sigma_y, \epsilon_f, c) \quad (1)$$

where $\Delta\sigma$ is the stress range, a = half the crack length, ϵ_y and σ_y are the yield strain and stress, respectively, ϵ_f is a characteristic of the fracture strain, and c is some other dimensionless variable such as strain hardening exponent. A useful result of this type of analysis is that the rate of crack propagation ($\frac{da}{dN}$) is proportional to the crack length (a). Examples of this type of approach may be found in Refs. [2-4]. The failure of laws based on limited parameters of relation (1) to fit test data in a broad range has been amply demonstrated (see for example Refs. [4 & 5]).

The theories based on the crack tip displacement have indicated that the crack propagation rate is a function of either second or third power of stress range $\Delta\sigma$. References [6, 7] are typical examples of this class of theories.

Finally, the semi-empirical laws tend to correlate the rate of crack propagation to the stress intensity variation factor (a function of applied stress and current length) $\frac{da}{dN} = f(\Delta K)$. The well-known law of this group is the "Paris type" relation,

$$\frac{da}{dN} = C_0 \Delta K^m \quad (2)$$

where C_0 and m are the material constants. This type of law has been experimentally verified by several researchers, e.g., Refs. [5, 8-11] for pressure vessel and piping materials.

Our study of experimental data has indicated that within nominal data scatter, a simple law relationship exists between crack growth rate $\left(\frac{da}{dN}\right)$ and the range of stress intensity factor (ΔK) at room temperature for different piping steels. The effect of test temperature, however, varies with the type of steel.

The experiments of interest are those conducted by the General Electric Company for the former U.S. Atomic Energy Commission (AEC), Ref. [12]. These experiments are of particular interest since they reflect the material condition and temperature at the operating range of the heat transport piping systems. The experimental results show that there is a strong influence of test temperature on the crack growth rate in *carbon steel* at all levels (3 to 5 times greater than

in room temperature). This effect is negligible for Cr-Mo alloy and stainless steels up to 600°F. The heat transport piping system in Candu reactors is constructed of carbon steel, thus the strong influence of operating temperature on the crack propagation has to be taken into consideration.

Another observation is the influence of mean stress. In piping systems, the alternating stress is super-imposed on a mean stress. The test results show that the crack growth rate increases with the applied mean stress. Again this influence is negligible for 304 stainless steel.

An attempt has been made to correlate the experimental results of Brothers [12] on A-212B, piping steel at 600°F with the power law of (2). Given the experimental data, and the expression for stress intensity factor for test specimen, we have developed a procedure and corresponding computer programme to correlate the test results by means of the least square method. For the A-212B, carbon steel, the crack propagation law becomes:

$$\frac{da}{dN} = 6.6181467 \times 10^{-10} (\Delta K)^{2.9535877} \quad (3)$$

where ΔK is in units of $\text{ksi}\sqrt{\text{in.}}$ and $\frac{da}{dN}$ in units of in. per cycle. For a more detailed procedure, see Ref. [13]. Figure 3 shows the relation (3) in a log-log scale along with the experimental data.

It is to be noted that the ΔK is proportional to stress range $\Delta\sigma$, and if the value of stress range is inaccurate by a given factor, α , the rate of propagation would be inaccurate by the cubic power of

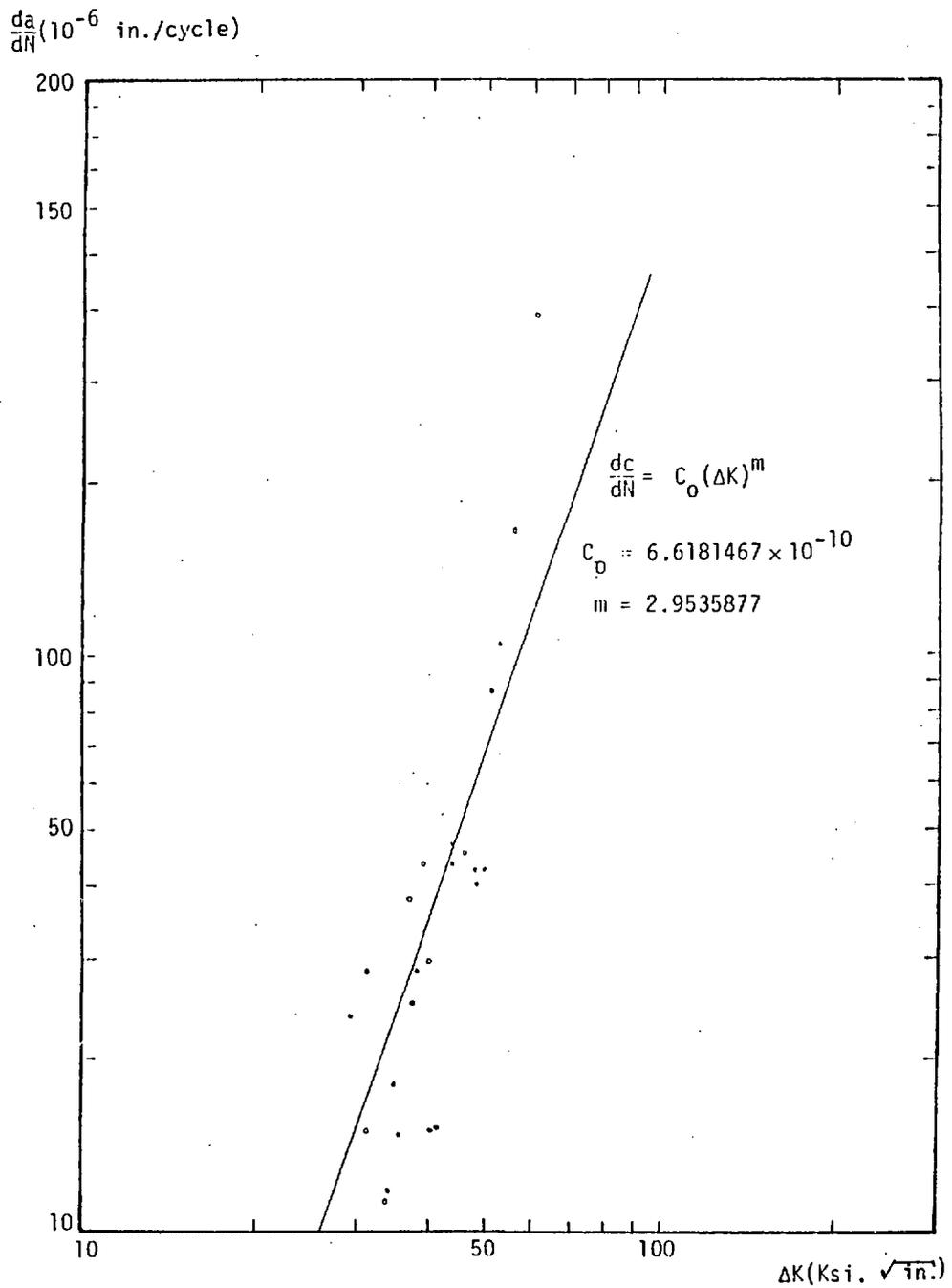


Fig. 3. Rate of crack extension versus stress intensity factor and experimental data.

that factor, i.e., a^3 . This then points out the importance of knowing the stresses in the vicinity of the defects for making an evaluation of the critical pressure.

Once the crack growth rate is determined empirically, the relation (3) could be used to estimate crack growth rates in more complex situations encountered in design. In principle, the procedure would be limited only by the availability of formulation for stress intensity factors for crack geometry and loading conditions of complex piping situations. Expressions for the stress intensity factors for various defect geometries may be found in Ref. [14].

In order to determine the number of cycles to failure for a plate containing a defect of length $2a$ (Brothers experimental configuration), one must first define the corresponding stress intensity factor. For the plate shown in Figure 4, the stress intensity factor is,

$$(\Delta K)^2 = (\Delta\sigma)^2 W \tan \left[\frac{\pi a}{W} \right] \quad (4)$$

where $\Delta\sigma$ is the applied stress range and W the width of the specimen.

The substitution of (4) into (3) then yields the relationship between crack propagation rate and the applied stress range for the experiments of Ref. [12]:

$$\frac{da}{dN} = 6.6181467 \times 10^{-10} \left\{ \Delta\sigma \left[W \tan \left(\frac{\pi a}{W} \right) \right]^{\frac{1}{2}} \right\}^{2.9535877} \quad (5)$$

To obtain the number of cycles for a defect to propagate from a given length to another one, it will suffice to integrate Eqn. (5). For the method of integration consult Ref. [13]. Therefore, we have established the methodology to obtain the mean curve of Figure 1.

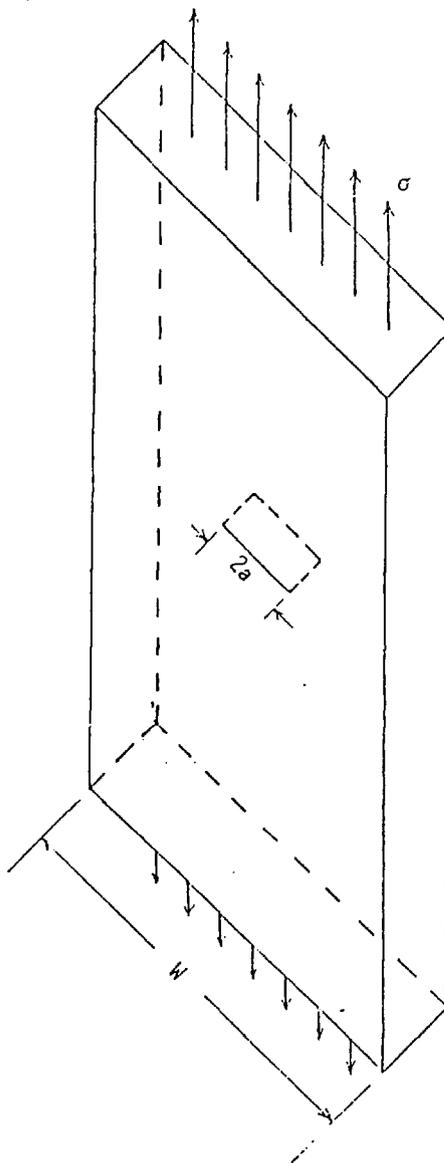


Fig. 4. A plate with a defect in its centre subjected to a uniform tensile stress.

4. RESIDUAL STRENGTH OF A PRESSURIZED PIPE OR VESSEL CONTAINING A DEFECT

Most theoretical and experimental works in fracture and crack propagation, as mentioned earlier, have been concerned with a flat plate configuration. The reason for this constraint is that both theoretical and experimental extensions to other configurations have proven difficult to solve and/or complex for practical applications. In a pressure vessel or piping system, the driving force for crack extension has at least two components. One is associated with the hoop stress, and the other is a consequence of the radial pressure which tends to bulge the wall adjacent to a defect or crack. The radial-pressure component has no counterpart in the flat plate test, but in a pressurized cylinder it can make an even larger contribution to the crack extension than the hoop stress.

Earlier tests carried out by NACA, Ref. [15], indicated the strong influence of the curvature and crack length. These tests also clearly demonstrated that the strength of a pressure vessel is substantially less than that of a corresponding flat plate in tension. Consequently, Peter and Kuhn [15] proposed an empirical relationship which contained a curvature correction term. This relation has since been further developed and given theoretical backing by several researchers. The basic premise is that a cylindrical pressure vessel or pipe can be treated as a flat plate of the same material, thickness and length, and containing the same crack (see Fig. 5) provided that the applied stress σ is taken to be:

$$\sigma = M \sigma_H \quad (6)$$

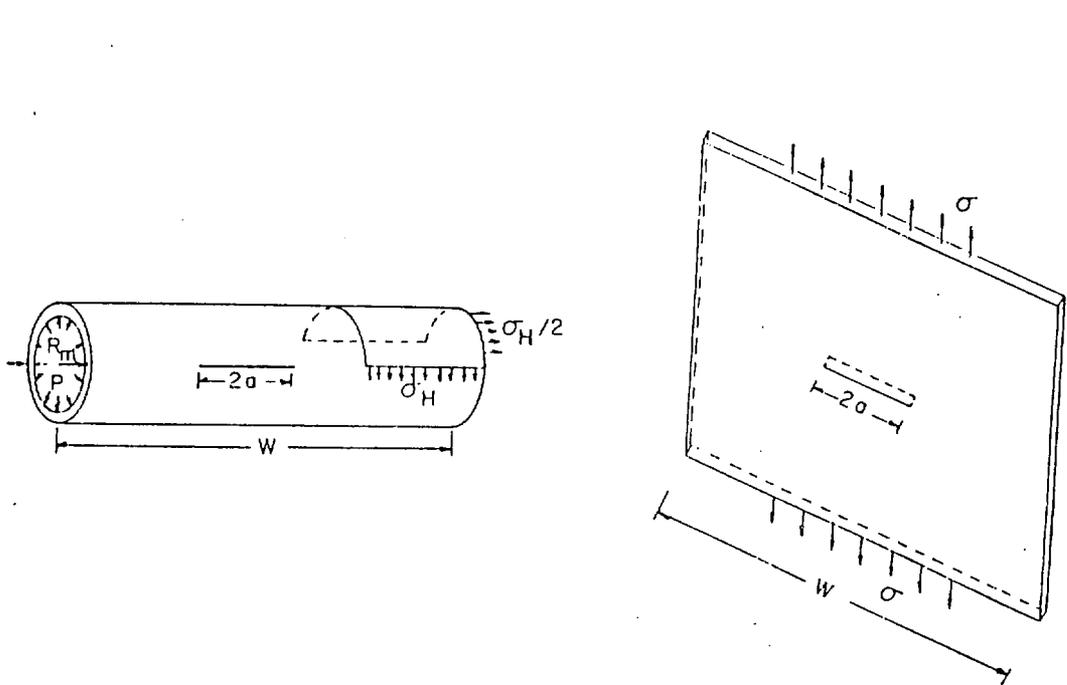


Fig. 5. A pressurized pipe or vessel containing an axial flaw and corresponding flat plate subjected to a stress $\sigma = M\sigma_H$.

The multiplier M is a function of crack length $2a$, the mean pipe radius R_m , pipe thickness t . The critical hoop stress (or pressure) σ_H^* for crack extension can be therefore expressed in terms of the nominal stress in the flat plate σ^* (Fig. 5), i.e.,

$$\sigma_H^* = \sigma^* M^{-1} \quad (7)$$

The above relation is attractive since it permits the utilization of a large body of fracture toughness data existing for flat plates. Relation (7) is further extended to include the effects of plastic flow at the crack region. Another advantage of this type of formulation is that it lends itself to making predictions.

Various expressions for the multiplier M are proposed, where the major ones are summarized in Table 1. Folias' expression [16] is based on analytical derivation of stress fields and linear elastic fracture mechanics (LEFM).

Anderson and Sullivan, Ref. [17], have enhanced the versatility of the approach by making it possible to draw on the larger body of fracture toughness data which exists for flat plates. They have coupled relation (7) with the linear elastic fracture mechanics by replacing σ^* (which depends on crack length) with the fracture toughness parameter K_c (which is independent of crack length). The factor $\phi = \phi(\sigma^*/\bar{\sigma})$ is a plasticity correction to the LEFM which becomes significant when $\sigma^*/\bar{\sigma} \geq 0.6$, where $\bar{\sigma}$ is the average plastic flow stress for the material. It is to be noted that K_c varies with the plate thickness, and that the K_c value inserted in the expression C of Table 1 should be consistent with the wall thickness of the pipe or vessel.

TABLE 1
 CRITERIA FOR CRACK EXTENSION IN PRESSURIZED PIPES
 OR CYLINDRICAL VESSELS

	FAILURE CRITERION	M	ϕ	REFERENCES
A	$\sigma_H^* = \sigma^* M^{-1}$	$(1 + 9.2 a/R_m)$	----	PETERS and KUHN [15]
B	$\sigma_H^* = \sigma^* M^{-1}$	$[1 + 1.61(a^2/R_m^2)R_m/t]^{1/2}$	----	FOLIAS [16]
C	$\sigma_H^* = \frac{K_c M^{-1}}{(\pi a \phi)^{1/2}}$	$(1 + \beta_a/R_m)$	$[1 + \frac{(M\sigma_H^*)^2}{2\sigma_y^2}]$	ANDERSON and SULLIVAN [17]
D	$\sigma_H^* = \sigma^* M^{-1}$	$[1 + 0.81 (\frac{a}{R_m t})^{1/2}]^2$	----	CRICHLow and WELLS [18]
E	$\sigma_H^* = \frac{K_c M^{-1}}{(\pi a \phi)^{1/2}}$	$[1 + 1.61 a^2/R_m t]^{1/2}$	$\sec \frac{\pi \sigma_H^*}{\sigma_y + \sigma_u}$	DUFFEY, McCLURE, EIBER, MAXEY [19,20]

Expression D was obtained from fitting experimental data to the available forms of analytical formulae to arrive at an empirical relation [18].

Duffey, McClure, Eiber, and Mixey [19, 20] have obtained a useful expression by combining Folias' equation for M , fracture mechanics approach and an estimate of ϕ derived from the Dugdale's crack model [21]. The expression given in line E of Table 1 contains two material properties K_c and $\bar{\sigma}$ which could be measured independently. The expression derived by Duffey et al. is shown to be in good agreement with various experimental data.

It has been pointed out that the Duffey et al. criterion may have certain limitations when applied to very tough materials at ambient temperature. In this case, determination of fracture toughness parameter K_c may require large test pieces.

Hahn, Sarrate, and Rosenfield [22,23] suggested improvement to Duffey et al. criterion by incorporating a more rigorous plasticity correction based on the crack-tip displacement. When this correction is employed it becomes apparent that the fracture toughness plays a minor role in the extension of short cracks in tough materials.

Hahn et al. [22,23] in essence proposed three closely related criteria for the extension of axial cracks in pressurized cylindrical vessels or pipes: (i) a fracture-toughness criterion mainly for low- and medium-tough materials, (ii) a flow-stress criterion for short cracks in tough materials, and (iii) a modification of (i) for very thin vessels. These three criteria are described in Table 2. The predictions of these criteria have been compared with a large body of experimental results, and have been shown to be in fairly good agreement.

TABLE 2

CRITERIA FOR THE EXTENSION OF AN UNSTABLE CRACK IN A PRESSURIZED VESSEL OR PIPE [22,23]

CATEGORY	SPECIFICATION		CRITERION	M
	R_m/t	$\frac{K_c^2}{\left(\frac{\bar{\sigma}}{\sigma_y}\right)^2} a^{-1}$	$\sigma_H^* =$	
A) Intermediate wall thickness with low- to medium- toughness.	> 5 < 50			$\left[1 + 1.61 \frac{a^2}{R_m t}\right]^{\frac{1}{2}}$
1. Short cracks		> 7	$\frac{\bar{\sigma}}{K_c} M^{-1}$	
2. Medium cracks		< 7 > 1.2	$\frac{M^{-1}}{(\pi a \phi_3)^{\frac{1}{2}}}$	
3. Long cracks		< 1.2	$K_c (\pi a M^2)^{-\frac{1}{2}}$	
B) Intermediate wall thickness with high toughness.	> 5 < 50			$\left[1 + 1.61 \frac{a^2}{R_m t}\right]^{\frac{1}{2}}$
1. Short cracks		> 7	$\frac{\bar{\sigma}}{K_c} M^{-1}$	
2. Medium cracks		< 7 > 1.2	$\frac{M^{-1}}{(\pi a \phi_3)^{\frac{1}{2}}}$	
3. Long cracks		< 1.2	$K_c (\pi a M^2)^{-\frac{1}{2}}$	
C) Very thin wall with low- to medium- toughness.	> 50			$\left[1 + 1.61 \frac{a^2}{R^2} (50 \tanh R_m/50t)\right]^{\frac{1}{2}}$
1. Short cracks		> 7	$\frac{M^{-1}}{K_c} M^{-1}$	
2. Medium cracks		< 7 > 1.2	$\frac{M^{-1}}{(\pi a \phi_3)^{\frac{1}{2}}}$	
3. Long cracks		< 1.2	$K_c (\pi a M^2)^{-\frac{1}{2}}$	

$\bar{\sigma}$ = Average flow stress, which varies between σ_y and σ_u ; $\sigma_y < \bar{\sigma} < \sigma_u$. For this study, we will assume an average value, i.e., $\bar{\sigma} = (\sigma_u + \sigma_y)/2$.

$$\phi_3 = \text{Plasticity correction factor} = \left(\frac{\pi \sigma_H^* M}{2 \bar{\sigma}}\right)^{-2} \ln \left[\sec \frac{\pi \sigma_H^* M}{2 \bar{\sigma}}\right]^2.$$

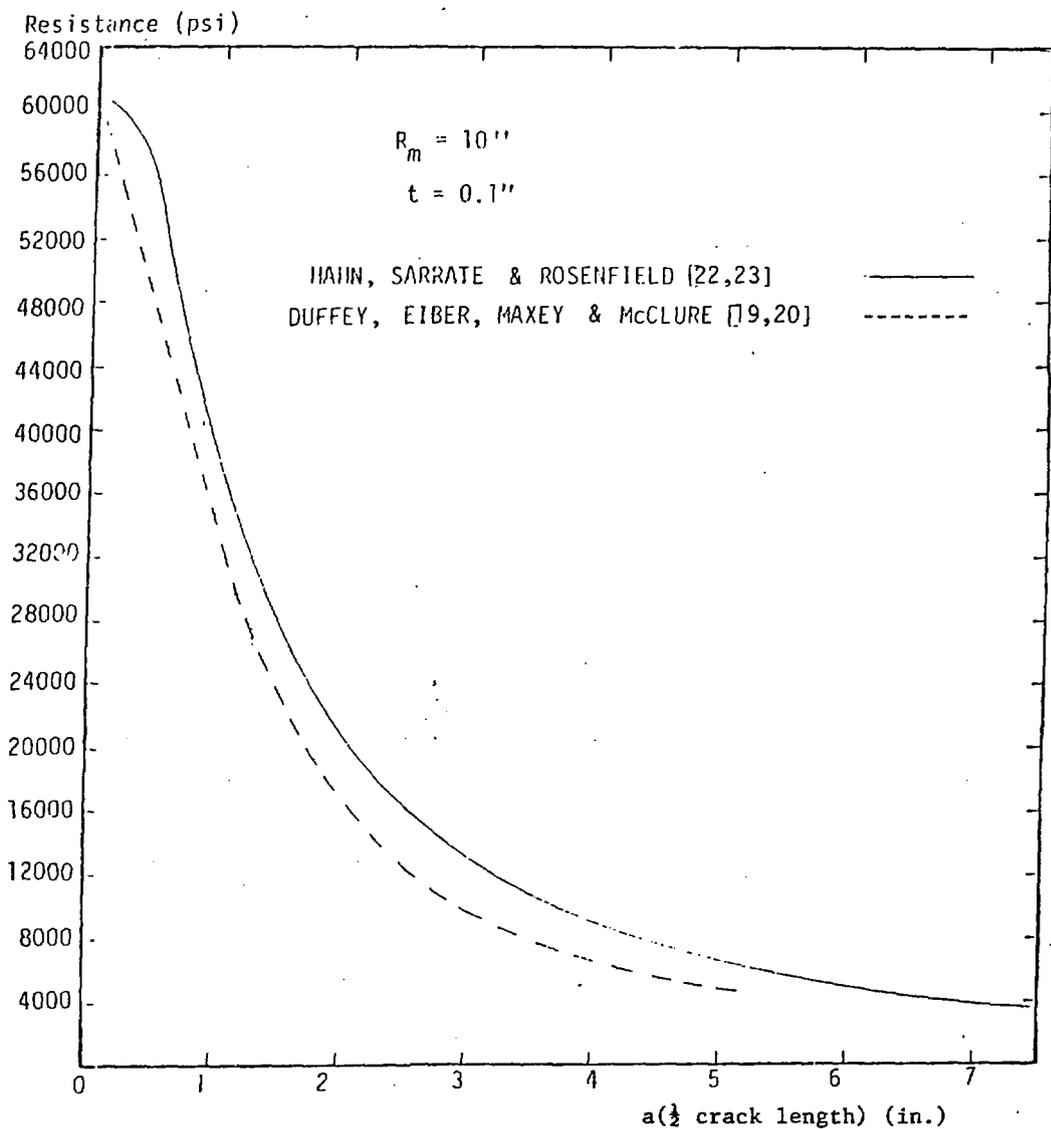


Fig. 6. Residual strength versus crack length for a thin-walled pipe.

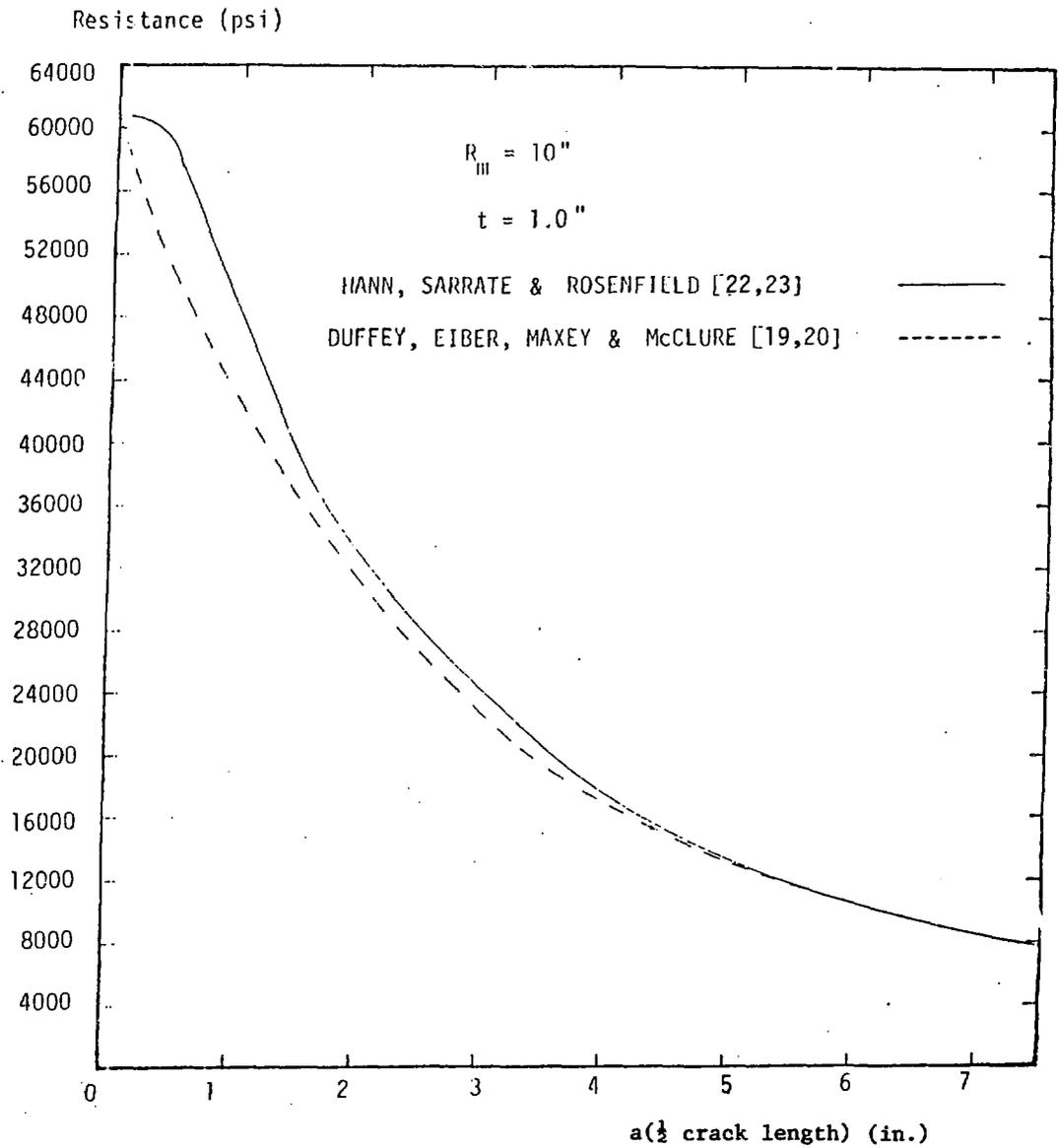


Fig. 7. Residual strength versus crack length for a thick-walled pipe.

5. NUMERICAL RESULTS AND COMPARISON

To predict the residual strength of a pipe (or vessel) containing a defect which is propagating in time, we have chosen the models due to Duffey et al. [19,20] and Hahn et al. [22,23].

Data for the pipe material was taken from Brothers' tests [12] on A-212B piping steel at 600°F. Two additional material constants have to be defined, Poisson's ratio ν and critical stress-intensity value, K_{Ic} . They are also taken from the available experimental data and are 0.3 and 118,000 psi $\sqrt{\text{in.}}$, respectively. With the material properties fully defined, two extreme cases were studied for a 20 in. diameter pipe: a thin walled one ($t = 0.1$ in.), $R/t = 100$ and a thick one ($t = 1$ in.), $R/t = 10$. These two limits define boundaries of theoretical transition from a plane stress condition to the plane strain one.

Due to the nature of governing equations, different methods of solution were required to obtain the residual strength predicted by each theory. Computer programmes were prepared to solve the functional relationships of Hahn, Sarrate and Rosenfield [22,23] and Duffey et al. [19,20]. The results are given in Figs. 6 and 7. One notes the pronounced effect of thickness t on the residual strength. The other observation is that for a given stress level, the critical crack length is smaller for a thicker pipe. The two theories give about the same value, and they indicate the same tendency with respect to the crack length. Either theory could therefore be used for the subsequent study. However, it is to be noted that Duffey et al.'s functional form is easier to calculate than that of Refs. [22,23].

6. CONCLUDING REMARKS

The stated objective of this Phase I(a) for the proposed work (see Ref. [24]) was to find relationships describing the crack propagation rate, and the residual strength of a pressurized pipe containing a crack or defect. These relationships are to be used in the subsequent probabilistic formulation and analysis for the periodic inspection of piping systems.

It is to be noted that the assurance of sufficient residual strength is central to providing safety until defects are detected.

We have been able to demonstrate a methodology for evaluating the strength of a pressurized pipe (or vessel) containing a crack (or defect). Inversely for a given crack length or defect size, we have been able to find the critical stress level at which the crack extension would commence.

The relationships found in this study which are based on sound theoretical treatment as well as experimental verification, could now be used as a median curve for a fully probabilistic formulation. It is to be noted that variations in the stress range and/or thickness to diameter ratio produce curves of a similar shape for either crack length vs. number of applied cycles, or the residual strength vs. crack length. This behaviour then confirms the probabilistic modelling which was described in Figure 2 based on experimental observations and heuristic arguments.

7. REFERENCES

- [1] Payne, A.O., "A Reliability Approach to the Fatigue of Structures", *Probabilistic Aspects of Fatigue*, ASTM, STP 511, American Society for Testing and Materials, 1972, pp. 106-155.
- [2] Frost, N.E., and Dugdale, D.S., "The Propagation of Fatigue Cracks in Sheet Specimens", *Journal of Mechanics and Physics of Solids*, Vol. 6, No. 2, 1958, pp. 92-110.
- [3] Liu, H.W., "Fatigue Crack Propagation and Applied Stress Range - An Energy Approach", *Journal of Basic Engineering*, Trans. ASME, Vol. 85, No. 1, 1963, pp. 116-122.
- [4] Yang, C.T., "A Study of the Law of Crack Propagation", *Journal of Basic Engineering*, Trans. ASME, Vol. 89, No. 3, 1967, pp. 487-495.
- [5] Paris, P., and Erdogan, F., "A Critical Analysis of Crack Propagation Laws", *Journal of Basic Engineering*, Trans. ASME, Vol. 85, No. 4, 1963, pp. 528-534.
- [6] Frost, N.E., and Dixon, J.R., "A Theory of Fatigue Crack Growth", *International Journal of Fracture Mechanics*, Vol. 3, No. 4, 1967, pp. 301-316.
- [7] McClintock, F.A., "Fatigue Crack Propagation", in *Influence of Metallurgical Structures*, ASTM, STP-415, 1967, pp. 169-174.
- [8] Kiss, E., Heald, J.D., and Hale, D.A., "Low-Cycle Fatigue of Prototype Piping", AEC R & D Report, GEAP 10135, General Electric Co., Calif., (Jan. 1970), 93 pages.
- [9] Cook, T., and Pontch, D.J., "Further Work on a Fracture Mechanics Endurance Determination for a Welding Institute Test Vessel", *Central Electricity Generating Board Research Dept.*, RD/B/N2793, Berkely Nuclear Laboratories, (Oct. 1973), 19 pages.
- [10] Cook, T., and Meads, J.M., "Pressure Vessel Fatigue: Crack Initiation and Propagation From a Notch Corner", *Ibid*, RD/B/N2812, (Jan. 1974), 26 pages.
- [11] Priddle, E.K., "Fatigue Crack Propagation Due to Random Amplitude Vibrations: Prediction Using Constant Amplitude Data", *Ibid*, RD/B/N2784, 15 pages.
- [12] Brothers, A.J., "Fatigue Crack Growth in Nuclear Reactor Piping Steels", AEC Research and Development Report, GEAP-5607, General Electric Co., California, (March 1968), 52 pages.

- [13] Ellyin, F., "Inspection Périodique pour sécurité de tuyauteries du système de caloportage des réacteurs Candu - Une approche probabilistique - Phase I-A", *Ellyin & Associates Consultants, Ltd.*, AECB Contract No. 1SQ78-00127, March 1979.
- [14] Rooke, D.P., and Cartwright, D.J., *Compendium of Stress Intensity Factors*, Her Majesty's Stationery Office, London, U.K., 1976.
- [15] Peters, R.W., and Kuhn, P., "Bursting Strength of Unstiffened Pressure Cylinders With Slits", *National Advisory Committee for Aeronautics, (NACA)*, Technical Note 3993, Washington, (April 1957), 20 pages.
- [16] Folias, E.S., "An Axial Crack in a Pressurized Cylindrical Shell", *International Journal of Fracture Mechanics*, Vol. 1, No. 2, 1965, pp. 104-114.
- [17] Anderson, R.B., and Sullivan, T.L., "Fracture Mechanics of Through-Cracked Cylindrical Pressure Vessels", *National Aeronautics and Space Administration (NASA)*, Report No. TND-3252, Washington, 1966.
- [18] Crichlow, W.J. and Wells, R.H., "Crack Propagation and Residual Static Strength of Fatigue-Cracked Titanium and Steel Cylinders", *ASTM, STP-415*, American Society for Testing and Materials, 1967, pp. 25-70.
- [19] Duffey, A.R., McClure, G.M., Eiber, R.J., and Maxey, W.A., "Fracture Design Practices for Pressure Piping", in *Fracture*, by H. Liebowitz (ed.), Vol. 5, Chapter 3, 1969, pp. 159-232.
- [20] Duffey, A.R., Eiber, R.J., and Maxey, W.A., "Recent Work on Flaw Behaviour in Pressure Vessels", *Proceedings of the Symposium on Fracture Toughness Concepts for Weldable Structural Steel*, Risley, U.K., (April 1969), pp. M1-M34.
- [21] Drydale, D.S., "Yielding of Steel Sheets Containing Slits", *Journal of the Mechanics and Physics of Solids*, Vol. 8, No. 2, 1960, pp. 100-104.
- [22] Hahn, G.T., Sarrate, M., and Rosenfield, A.R., "Criteria for Crack Extension in Cylindrical Pressure Vessels", *International Journal of Fracture Mechanics*, Vol. 5, No. 3, 1969, pp. 187-209.
- [23] Hahn, G.T., and Sarrate, M., "Failure Criteria for Through-Cracked Vessels", *Proceedings of the Symposium on Fracture Toughness Concepts for Weldable Structural Steel*, Risley, U.K., (April 1969), pp. P1-P15.
- [24] *Ellyin & Associates Consultants, Ltd.*, "Periodic Inspection for Safety of Candu Heat Transport Piping Systems - A Probabilistic Approach", EAEL-14-78, DSS-AECB-1, (UP-E-68), June 1978.