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IN RELATIVISTIC PLASMA

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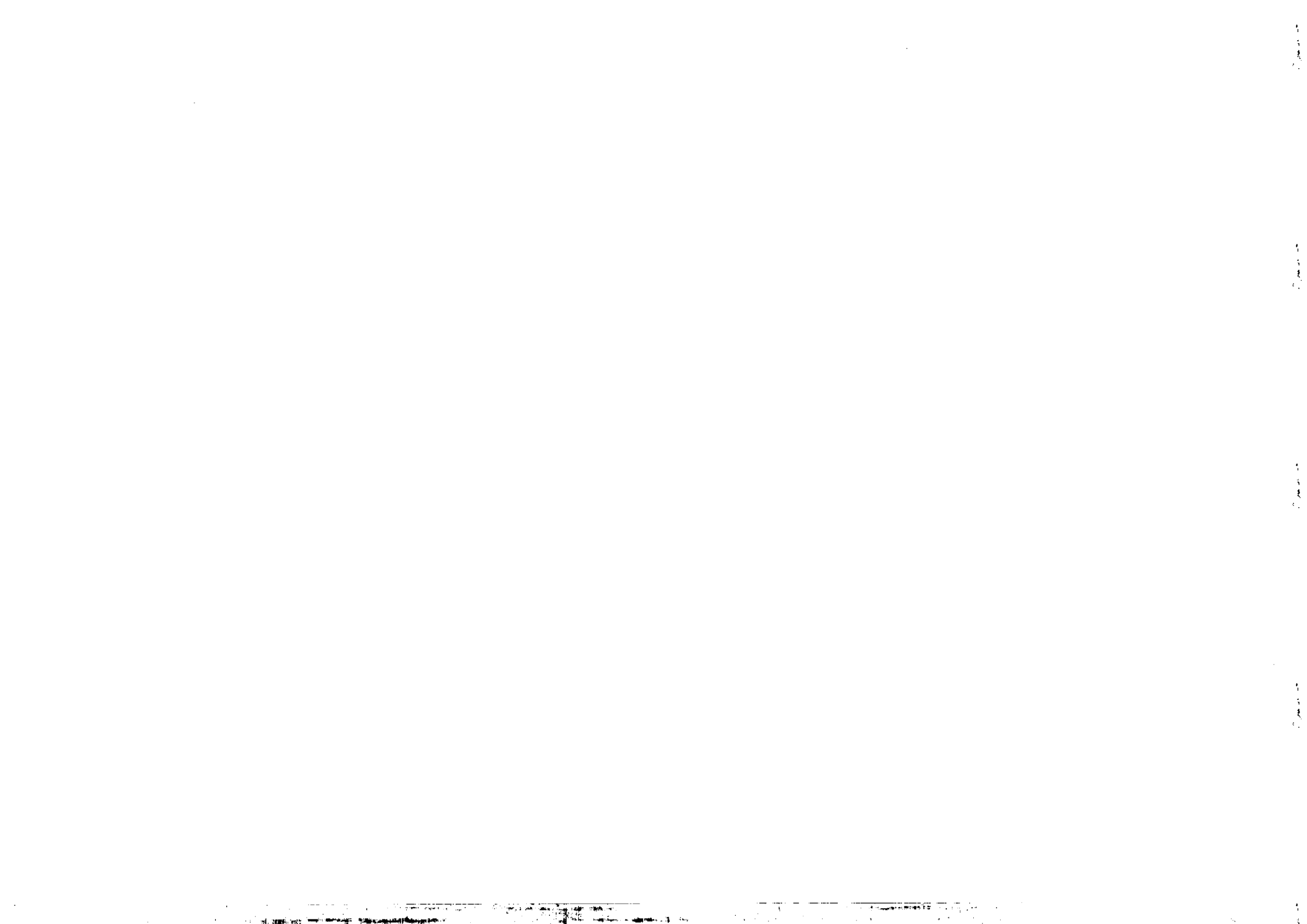


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ON THE KINETIC THEORY OF PARAMETRIC RESONANCE
IN RELATIVISTIC PLASMA *

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ABSTRACT

The instability of relativistic hot plasma located in high-frequency external electric field is studied. The dispersion relation, in the case when the plasma electrons have relativistic oscillatory motion, is obtained. It is shown that if the electron Debye's radius is less than the wave length of plasma oscillation and far from the resonance on the overtones of the external field frequency, the oscillation build-up is possible. It is also shown that taking into account the relativistic motion of electrons leads to a considerable decrease in the frequency at which the parametric resonance takes place.

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INTRODUCTION

When we shine on a plasma an electromagnetic wave with frequency close to the electron plasma frequency, a parametric coupling is induced between the electrostatic waves (the electron plasma wave and ion acoustic wave) and the electromagnetic wave. Nowadays the interaction between waves and strong energetic charged particles is one of the fundamental problems of the theory of collective processes in plasma. This phenomena has been receiving a growing amount of interest in recent years due to the different types of instabilities occurring in it as a result of the electromagnetic field. In particular, the investigation of plasma instabilities in presence of high-frequency electric field has a special attraction due to the possibility of occurring of parametric excitation, which plays a significant part in plasma heating.

Silin [1] was the first who made a systematic study of the phenomenon. Based on the hydrodynamic equation for a cold plasma, he obtained several interesting results for the growth rate in the case when the field intensity is well above threshold. DuBois and Goldman [2] considered the case of weak external field and estimate the threshold for excitation by making allowance to the Landau damping. Later on, Alliev and Silin [3] investigated the irrotational oscillations of fully ionized plasma located in a strong high-frequency electric field. It was shown there that the oscillations of the particles with respect to each other induced by the external field can lead to spatial dispersion. The presence of high-frequency field modifies branches of the dispersion relations and is also responsible for the appearance of a new branch. An elegant experiment was carried out by Stern and Tzoor [4] who observed the excitation of a couple of electron plasma wave and ion acoustic wave at a relatively low external field. Nishikawa discussed the case of a weakly ionized plasma and pointed a possible reduction of the

threshold power.

With the achievement of laser techniques, it becomes possible to obtain very strong electric field. The plasma electron velocity under the influence of such field may be considered to be comparable with the velocity of light (c) and accordingly the relativistic effects should not be ignored. The oscillation of the relativistic electron mass in the field of an intense electromagnetic wave lead to a substantial increase in the wave absorption in a plasma, as was shown in Ref. [7]. Tsintsadze, who advanced the investigation of relativistic electron plasma, showed that the oscillation of the electron mass in an external high-frequency field leads to parametric build up of both potential and non-potential high-frequency oscillations. In Ref. [8] it was studied the possibility of parametric instability in magnetoactive plasma. These earlier results point to the possibility of parametric build up of waves in a plasma due to oscillation of the relativistic electron mass at the frequency of the external field. This parametric effect is distinguished from that described in Ref. [9] by the circumstance that in the present case the parametric resonance can occur in a pure electron plasma, in which case ion oscillation can be neglected.

To observe these effects in laboratory high power levels are required, but modern rf power supplies can produce fields in a plasma strong enough for the electrons to acquire relativistic velocities. In astrophysical situation these strong fields may be produced by the electromagnetic radiation from various astrophysical objects (galactic nuclei, radio galaxies, and quasars, for example).

Galeev and Kranshchel'skikh have recently studied the propagation of circularly polarized electromagnetic wave along an external magnetic field (H_0) for the case in which the plasma electrons acquire ultrarelativistic velocities in the field of pump wave. As shown in Ref [10], the resonant

nature of the wave causes the electrons to become ultrarelativistic even in "weak" pump fields, i.e. even if the amplitude E of the electromagnetic pump wave satisfies the inequality:

$$\underline{E} < \underline{H}_0$$

The nonlinearity in the parametric resonance in a plasma due to oscillations in the relativistic electron mass is analyzed in [11], while the propagation of nonlinear, circularly polarized electromagnetic wave along an external magnetic field has been analyzed in [12].

A plasma wave is strong enough to derive ions relativistic if

$$\mu \nu \gg 1$$

where $\mu = (m_0 / m_1)$ and $\nu = e E_{\max} / m_0 c \omega$.

At present, a wave of such intensity is not expected to be encountered in laboratory conditions.

As it was mentioned before, Silin [1] has studied the condition of parametric excitation and resonance on the frequency of the external field or its overtones in cold, and later on he [14] studied the same problem in hot plasma, where the thermal properties were taken into consideration. Tsintsadze [7] advanced the parametric resonance in cold relativistic electron plasma.

Taking into account the relativistic oscillatory motion of plasma electron in presence of high-frequency electric field, and basing on the relativistic kinetic equation for hot plasma, the problem of plasma instability and the possibility of parametric resonance on the frequency of the external field, is the objective of the present paper.

BASIC EQUATIONS

Considering the wave length of proper plasma oscillation to be much smaller than the characteristic length of plasma medium, then the equilibrium state of plasma located in high-frequency electric field:

$$\underline{E}(t) = \underline{E}_0(0) \sin \omega_0 t$$

can be described by an equilibrium distribution function $f_{\alpha 0}$, satisfying the following kinetic equation:

$$\frac{\partial f_{\alpha 0}}{\partial t} + (e_{\alpha}/m_{\alpha}) \underline{E}(t) \frac{\partial f_{\alpha 0}}{\partial v_{\alpha}} = 0 \quad (1)$$

where e_{α} , m_{α} , v_{α} are the charge, mass, and velocity of particle α (α may be electron e or ion i). The solution of (1) may be written as

$$f_{\alpha 0}(\underline{u}_{\alpha}) \equiv f_{\alpha 0}(1 \underline{u}_{\alpha} - \underline{u}_{\alpha 0} \cos \omega_0 t) \quad (2)$$

where

$$\underline{u}_{\alpha} = \underline{v}_{\alpha} (1 - v_{\alpha}^2/c^2)^{-1/2}$$

and

$$\underline{u}_{\alpha 0} = (e_{\alpha} \underline{E}_0 / m_{\alpha} \omega_0)$$

To study the stability of such plasma equilibrium state, let us write the electric field and the distribution function as a sum of equilibrium (to be denoted by the suffix 0) and perturbed (denoted by the prime) parts, i.e.,

$$\underline{E} = \underline{E}_0 + \underline{E}', \quad f_{\alpha} = f_{\alpha 0} + f_{\alpha}'$$

The primed quantities represent the spatial fluctuation due to wave motion in plasma.

Hereafter, we limit ourselves to the following conditions:

(i) The ion and electron temperatures are considered to be non-relativistic.

This condition enables us to replace the equilibrium distribution function by, for example, Maxwellian distribution function.

(ii) The condition

$$\omega_0 \gg \omega_{Le} v_{Te} / c$$

(where ω_{Le} , and v_{Te} are the electron Langmuir frequency and electron

thermal velocity, and c is the light velocity), is considered to be satisfied. This enables us to neglect high-frequency magnetic field and to consider the dependence of the perturbed quantities on the coordinate to be of the form $[\exp(i \underline{k} \cdot \underline{r})]$.

Thus, taking into account the above conditions, one may obtain the following linearized kinetic equation 12

$$\frac{\partial \phi_{\alpha}}{\partial t} + i \phi_{\alpha} k_{\parallel} u_{\parallel \alpha} (1 + \beta_{\alpha}^2 \cos^2 \omega_0 t)^{-1/2} + k_{\perp} u_{\perp \alpha} (1 + \beta_{\alpha}^2 \cos^2 \omega_0 t)^{-1/2} - i k \left(\frac{\partial f_{\alpha 0}}{\partial u_{\alpha}} \right) \sum_b \frac{4\pi e_{\alpha} e_b}{m_{\alpha} k} \int d\underline{u}_b \phi_b \exp \left\{ i \int \underline{k} \cdot [\underline{v}_{\alpha 0}(t) - \underline{v}_{\alpha 0}(t')] dt' \right\} = 0 \quad (3)$$

where

$$\phi_{\alpha}(\underline{u}, r, t) = f_{\alpha}'(1 u_{\alpha} + u_{\alpha 0} \cos \omega_0 t) \exp \left\{ i \int \underline{k} \cdot \underline{v}_{\alpha 0}(t') dt' \right\}$$

$$v_{\alpha 0} = u_{\alpha 0} \cos \omega_0 t [1 + \beta_{\alpha}^2 \cos^2 \omega_0 t]^{-1/2}, \quad (4)$$

$$\beta_{\alpha} = u_{\alpha 0} / c$$

k_{\parallel} , $u_{\parallel \alpha}$, and k_{\perp} , $u_{\perp \alpha}$ are the components of the wave vector \underline{k} and the velocity \underline{u}_{α} parallel and perpendicular to the electric field respectively.

From the above integral equation (3) one may obtain for the functions U_n and W_n ; defined by

$$U_n = \left(\frac{e}{m_e} \right) \int \phi_{e,n}(u) du$$

$$W_n = \left(\frac{e_i}{m_i} \right) \int \phi_{i,n}(u) du$$

the following set of linear algebraic equations:

$$U_s [1 + \delta \epsilon_e (s\omega_0 + i\gamma, k)] + \sum_{l=-\infty}^{\infty} |P_{ei}|_{l+s} W_l \delta \epsilon_e (s\omega_0 + i\gamma, k) = 0 \quad (5)$$

$$W_s [1 + \delta \epsilon_i (-s\omega_0 + i\gamma, k)] + \sum_{l=-\infty}^{\infty} |P_{ei}|_{l+s} U_l \delta \epsilon_i (-s\omega_0 + i\gamma, k) = 0 \quad (6)$$

where

$$\delta \epsilon_e (\omega + i\gamma, k) = \frac{4\pi e^2}{m_e k^2} \int \frac{k \left(\frac{\partial f_{e0}}{\partial u_{\alpha}} \right)}{\omega - i\gamma - C_{||\alpha} k_{||} u_{||\alpha} - C_{\perp\alpha} k_{\perp} u_{\perp\alpha}} du \quad (7)$$

$$C_{||\alpha} = \left(\frac{\omega_0}{2\pi} \right) \int_0^{2\pi/\omega_0} (1 + \beta_{\alpha}^2 \cos^2 \omega t)^{-1/2} dt \quad (8)$$

$$C_{\perp\alpha} = \left(\frac{\omega_0}{2\pi} \right) \int_0^{2\pi/\omega_0} (1 + \beta_{\alpha}^2 \cos^2 \omega t)^{-3/2} dt \quad (9)$$

and

$$|P_{ei}| = \left(\frac{\omega_0}{2\pi} \right) \int_0^{2\pi/\omega_0} \exp \left\{ i \int_0^t \underline{k} \cdot [\underline{v}_{e0}(t') - \underline{v}_{i0}(t')] dt' \right\} dt \quad (10)$$

Taking into account the smallness of the parameter (u_0/m_i) , then one may approximately express all the functions U_s and W_s through three functions $W_0, U_{\pm n}$ [1]. The results are:

$$W_m = - \left\{ 1 - \frac{1}{1 + \delta \epsilon_i (-m\omega_0 + i\gamma, k)} \right\} \left\{ U_n |P_{ei}|_{m+n} + U_{-n} |P_{ei}|_{m-n} - W_0 \sum_{l \neq \pm n} |P_{ei}|_{l+m} |P_{ei}|_l \left[1 - \frac{1}{1 + \delta \epsilon_e (m\omega_0 + i\gamma, k)} \right] \right\}, \quad m \neq 0 \quad (11)$$

$$U_m = - \left\{ 1 - \frac{1}{1 + \delta \epsilon_e (m\omega_0 + i\gamma, k)} \right\} \left\{ W_0 |P_{ei}|_m - \sum_{l \neq 0} |P_{ei}|_{l+m} \left[1 - \frac{1}{1 + \delta \epsilon_e (m\omega_0 + i\gamma, k)} \right] \left[U_n |P_{ei}|_{l+m} - U_{-n} |P_{ei}|_{l-m} \right] \right\}, \quad m = \pm n \quad (12)$$

CONCLUSION

Restricting ourselves to the case of relativistic velocity of electrons, i.e., $\beta_e \gg 1$, we have

$$C_{||e} = 2 R(\beta_e / (1 + \beta_e^2)) / \pi (1 + \beta_e^2)^{1/2}, \quad C_{||i} = 1$$

$$C_{\perp e} = 2 G(\beta_e / (1 + \beta_e^2)) / \pi (1 + \beta_e^2)^{1/2}, \quad C_{\perp i} = 1$$

$$|P_{ei}|_n = J_n \left(\frac{k U_{i0}}{\omega_0} \right) \left\{ \frac{\sin \left\{ \left[\frac{k U_{e0} c}{\omega_0 U_{e0}} - n \right] \left(\frac{\pi}{2} \right) \right\}}{\left[\frac{k U_{e0} c}{\omega_0 U_{e0}} - n \right] \left(\frac{\pi}{2} \right)} \right\}$$

where J_n is Bessel function, and $G(\rho), R(\rho)$ are elliptic integral of the first and second type.

It is noted here that equations (9-12) are reducible to those obtained in Ref. [1] in the case of non-relativistic electron velocity.

It is easy to obtain the following dispersion relation from the above equations 14 :

$$1 + \frac{1}{\delta \epsilon_i (i\gamma, k)} = \sum_{l=-\infty}^{\infty} |P_{ei}|_l^2 \left\{ \frac{-1}{1 + \delta \epsilon_e (l\omega_0 + i\gamma, k)} + 1 \right\} \quad (13)$$

It is also clear that equation (13), in the case of non-relativistic electron motion, is reduced to equation (4.8) of Ref. [1].

If $\gamma \gg k U_{Te}$, (where $U_{Te} = \sqrt{T_e} S_e'$, $V_{Te} = (T_e/m_e)^{1/2}$ is the thermal electron velocity, and

$$S_e' = (k_{||}^2 C_{||e}^2 + k_{\perp}^2 C_{\perp}^2) / k^2$$

then equation (13) is reduced to equation (3.36) of Ref. [1].

If the electron Debye's radius (R_{De}) is less than the wave length of plasma oscillation, one may obtain the following expressions:

$$\gamma^2 = \omega_{Li}^2 \left\{ -1 + |P_{ei}|^2 \frac{1 - J_+(i\gamma/kU_{Te})}{kR_{De} + 1 - J_+(i\gamma/kU_{Te})} + \sum_{l \neq 0} \frac{\omega_{Le} S_e'' |P_{ei}|^2}{\omega_{Le} S_e'' - l^2 \omega_0^2} \right\} \quad (14)$$

where

$$S_e'' = (k_{||}^2 C_{||e}^2 + k_{\perp}^2 C_{\perp}^2) / k^2 \ll 1$$

and

$$J_+(x) = x e^{-x^2/2} \int_{+i\infty}^x dy e^{y^2/2}$$

It is concluded from equation (14) that far from the resonance on the overtones of the external field frequency, the oscillation build-up increment is possible. In other words, an oscillation growth can be possible if the external field frequency is less than the relativistic electron Langmuir frequency ω_e , where

$$\omega_e^2 = \omega_{Le}^2 S_e''$$

Since $S_e'' \ll 1$, therefore taking into account the relativistic motion of electron in presence of high-frequency electric field leads to a considerable decrease in the frequency at which the parametric resonance takes place.

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