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GLOBAL QUANTITY FOR DYONS WITH VARIOUS CHARGE DISTRIBUTIONS *

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ABSTRACT

The spatial volume integral of Tr *F F characterizes the dyons globally. This integral is investigated for the dyons with various electric and magnetic charge distributions, which can be probed by the scattering of the test particle in these dyon fields.

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A magnetic monopole solution was discovered in the SU(2) gauge theory with a Higgs triplet by 't Hooft [1] and Polyakov [2]. Julia and Zee [3] extended this 't Hooft-Polyakov monopole to the dyon, a magnetic monopole with electric charge. In the limit of a zero Higgs potential, Prasad and Sommerfield [4] explicitly obtain the dyon solution with a point magnetic charge surrounded by a cloud of electric charge.

For dyons, Marciano and Pagels [5] have found the spatial volume integral of Tr *F F to be non-zero, where F and *F are the field tensor and its dual respectively. Recently the relationships between this integral and the Pontryagin index are recently discussed by Christ and Jackiw [6]. This integral for the dyon has several interesting properties. This gauge invariant integral describes the "Yang-Mills" $E^a \cdot B^a$ globally, and is the product of total magnetic and electric charge apart from a parameter C that governs the fall-off of the fields. On the other hands, the mass of dyon is sum of the squares of the total magnetic and electric charge with the same parameter C [7]. The spatial volume integral of Tr *F F and the mass are the informative quantity describing the global property about the dyons.

In this paper, the spatial volume integral of Tr *F F is investigated for dyons with various charge distributions. These charge distributions are possible due to several different choice of abelian field tensor from the Non-abelian ones, and could be probed by the scattering of the test particle in these fields[8].

The explicit solutions by Prasad-Sommerfield [[illegible]] are of the form

$$A_i^a = \epsilon_{aij}\,\hat{r}_j\,\{1-K(r)\}/er, \tag{1}$$

$$A_0^a = \hat{r}_a\,J(r)/er, \tag{2}$$

$$\varphi^a = \hat{r}_a\,H(r)/er, \tag{3}$$

with

$$K = Cr/\sinh(Cr), \tag{4}$$

$$J = \sinh\gamma\,\{Cr\coth(Cr)-1\}, \tag{5}$$

$$H = \cosh\gamma\,\{Cr\coth(Cr)-1\}. \tag{6}$$

Here C is a parameter that governs the fall-off of the Yang-Mills fields, and γ is an arbitrary parameter.

The spatial volume integral of Tr *F F in terms of these field components is

$$q = -\frac{1}{16\pi}\int d^3x\,{}^*G^a_{\mu\nu}\,G^{a\mu\nu} \tag{7}$$

$$= \frac{1}{4\pi}\int d^3x\,E^{ai}B^{ai},$$

where ${}^*G^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\,G^{a\alpha\beta}$, and E^{ai}, B^{ai} are SU(2) "electric" and "magnetic" Yang-Mills fields. By inserting the exact solutions eqs. ([illegible]), ([illegible]), and (6), the above integral becomes

$$q = -\frac{C\sinh\gamma}{e^2} \tag{8}$$

$$\times\int_0^\infty d\rho\,\Big[1+\frac{\rho^4}{\sinh^4\rho}-\frac{4\rho^3\cosh\rho}{\sinh^3\rho}+\frac{2\rho^4\cosh^2\rho}{\sinh^4\rho}\Big]/\rho^2.$$

The integral in eq. (8) sums to unity and thus,

$$q = -\frac{C\sinh\gamma}{e^2}. \tag{9}$$

The Abelian electromagnetic field tensor can now be identified to pick up a long-range component in the above solutions. However the way to define this is not unique as observed by Actor [9]. The electromagnetic field tensor is chosen by 't Hooft as [1], [4], [10],

$$F^A_{\mu\nu} = \hat{\varphi}^a G^a_{\mu\nu} - \frac{1}{e}\epsilon^{abc}\,\hat{\varphi}^a D_\mu\hat{\varphi}^b D_\nu\hat{\varphi}^c, \tag{10}$$

where $\hat{\phi}^a = \phi^a/(\phi^b\phi^b)^{1/2}$,

and

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc} A^b_\mu A^c_\nu . \qquad (11)$$

The superscript A is used to distinguish this from other choices that will shortly follow. This identification describes a point monopole with

$$B^{iA} = \frac{1}{2}\epsilon^{ijk} F^A_{jk} = g\frac{\hat{r}_i}{r^2}, \qquad (12)$$

surrounded by a cloud of electric charges producing the electric field

$$E^{iA} = F^{0iA} = Q\frac{\hat{r}_i}{r^2}\left(1 - \frac{C^2 r^2}{\sinh^2(Cr)}\right). \qquad (13)$$

Here the total magnetic charge g and electric charge Q are [4]

$$g = -\frac{1}{e} \qquad (14)$$

$$Q = \frac{1}{e}\sinh\gamma \qquad (15)$$

For this 't Hooft Abelian electromagnetic field tensor eq. (10), the spatial volume integral of Tr *F F is

$$q = -\frac{C\sinh\gamma}{e^2}\int_0^\infty d\rho\,\left[1 - \frac{\rho^2}{\sinh^2\rho}\right]/\rho^2 \qquad (16)$$
$$= -\frac{C\sinh\gamma}{e^2}.$$

Although the integrands in eqs. (8) and (16) are different, this integral is the magnetic charge times the electric charge apart from C.

A different definition was proposed by Actor as [9]

$$F^B_{\mu\nu} = \hat{\phi}^a G^a_{\mu\nu} \qquad (17)$$

This definition leads to a magnetic field

$$B^{iB} = \frac{1}{2}\epsilon^{ijk} F^B_{jk} = g\frac{\hat{r}_i}{r^2}\left(1 - \frac{C^2 r^2}{\sinh^2(Cr)}\right), \qquad (18)$$

and the electric field

$$E^{iB} = F^{0iB} = \hat{r}_i \frac{Q}{r^2}\left(1 - \frac{C^2 r^2}{\sinh^2(Cr)}\right), \qquad (19)$$

corresponding to an extended distribution of both magnetic and electric charges concentrated around the origin. For this charge distribution, the spatial volume integral is calculated to be

$$q = CgQ\left(\frac{5}{3} - \frac{\pi^2}{9}\right) \qquad (20)$$

A new definition of the electromagnetic field tensor

$$F_{\mu\nu}^{C} = \hat{\varepsilon}^a G_{\mu\nu}^a + \frac{1}{2} Q \epsilon_{\mu\nu\alpha\beta} \epsilon_{abc} \hat{\varepsilon}^a D^\alpha \hat{\varepsilon}^b D^\beta \hat{\varepsilon}^c \qquad (21)$$

enables one to have a dyon with a point electric charge surrounded by a cloud of magnetic charge. Only the first term in eq. (21) contributes to the magnetic field as

$$B^{ic} = \frac{1}{2} \epsilon^{ijk} F_{jk}^{C} = g \hat{r}_i \left(1 - \frac{C^2 r^2}{\sinh^2(Cr)}\right) / r^2 \qquad (22)$$

Both terms in eq. (16) contributes to the electric field E as

$$E^{iC} = F^{0iC} = Q \frac{\hat{r}_i}{r^2} . \qquad (23)$$

With a new definition of the electromagnetic field tensor, the role of electric and magnetic charge distributions is interchanged with that of 't Hooft identification eq. (10). With the electric field eq. (22) and the magnetic field eq. (23), the spatial volume integral of Tr *F F is

$$q = CgQ \int_0^\infty d\rho \, \frac{1}{\rho^2} \left(1 - \frac{\rho^2}{\sinh^2 \rho}\right) = CgQ \qquad (24)$$

The global behaviour of E·B for this new identification $F_{\mu\nu}^a$ eq. (21) turns out to be same for the field tensor $G_{\mu\nu}^a$ eq. (11) and the electromagnetic field tensor $F_{\mu\nu}^A$ eq. (10).

Furthermore, another identification

$$F_{\mu\nu}^{D} = \hat{\varepsilon}^a G_{\mu\nu}^a + \left(g \delta_{\mu\nu\alpha\beta} + \frac{1}{2} Q \epsilon_{\mu\nu\alpha\beta}\right) \epsilon^{abc} \hat{\varepsilon}^a D^\alpha \hat{\varepsilon}^b D^\beta \hat{\varepsilon}^c \qquad (25)$$

gives fields of both point electric and magnetic charges. The spatial volume integral diverges for this charge distributions, since the integrand diverges at the origin.

In conclusion, the spatial volume integral of Tr *F F characterizing the dyons globally are investigated for the dyons with various charge distributions. This integral takes averages of $E^a \cdot B^a$ over the isotopic and spatial space, and is the product of total magnetic and electric charge with parameter C apart from the charge distribution dependent factors. These factors are dependent on the detailed charge distributions of dyons, and measure the spatial average of E B . These charge distributions can be probed physically by the scattering of the test particle in the dyon fields with these charge distributions.

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REFERENCES

1 G. 't Hooft, Nucl. Phys. B79 (1974) 276.

2 A. Polyakov, JETP Lett. 20 (1974) 194.

3 B. Julia and A. Zee, Phys. Rev. D 11 (1975) 2227.

4 M. K. Prasad and C. M. Sommerfield, Phys. Rev. Lett. 35 (1975) 760.

5 W. Marciano and H. Pagels, Phys. Rev. D 14 (1976) 531.

6 N. Christ and R. Jackiw, Phys. Lett. 91B (1980) 228.

7 C. Montonen and D. Olive, Phys. Lett. 72B (1977) 117.

8 Y. Kazama, C. N. Yang, and A. S. Goldhaber, Phys. Rev. D 15 (1977) 2287; D. G. Boulware, L. S. Brown, R. N. Cahn, S. D. Ellis and C. K. Lee, Phys. Rev. D 14 (1976) 2708; P. Rossi, Nucl. Phys. B127 (1977) 518.

9 A. Actor, Rev. Mod. Phys. 51 (1979) 461.

10 J. Arafune, P. G. O. Freund, and C. J. Goebel, J. Math. Phys. 16 (1975) 433.

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