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DOES NATURE LIKE NAMBU-GOLDSTONE BOSONS? *

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ABSTRACT

We argue here that many (up to around 30 species) so far undetected Goldstone bosons could exist in nature, for example, associated to the spontaneous breaking of a horizontal global symmetry, provided the breaking scale is $V \gtrsim 10^{10}$ GeV. Since Goldstone bosons do not generate r^{-1} but spin-dependent r^{-3} non-relativistic long-range potentials, the apparently most dramatic effect of massless bosons - new long-range forces competing with gravitation and electromagnetism - is easily avoidable (the Glashow-Weinberg-Salam breaking scale is enough). $\mu \to eG$ and $K \to \pi G$ provide the most restrictive bounds and probably the only possibility to look for Goldstone bosons in laboratory.

I. INTRODUCTION

Goldstone bosons were generally believed to be unacceptable in a realistic model for elementary particle interactions, the main concern being that such massless bosons would mediate new gravitational or Coulomb-like r^{-1} potentials. However, it has been recently found, in two recent models (with [1)] and without [2)] right-handed neutrinos) that the Goldstone boson associated with the spontaneous breaking of the global lepton number conservation ("Majoron") could easily escape detection. The reasons for its elusiveness are essentially the same as for the "invisibility" of another almost Goldstone boson, the invisible axion [3)], generated by the spontaneous breaking of the Peccei-Quinn [4)] symmetry [F1].

Here we would like to point out general properties of the Goldstone bosons and general features of the models with spontaneously broken global symmetries, by which inconsistencies with present experiments are avoided.

We start by noting that the exchange of Goldstone bosons cannot mediate $V(r) \sim r^{-1}$ potentials, but only spin-dependent $V(r) \sim [\vec{s}_1 \cdot \vec{s}_2 - 3(\vec{s}_1 \cdot \hat{r})(\vec{s}_2 \cdot \hat{r})] r^{-3}$ potentials as, $r \to \infty$. The bounds on the Goldstone bosons coupling constants imposed by the possible existence of new long-range forces are, hence, much weaker. These bounds are trivial in comparison with bounds coming from cross-sections or decay rates of reactions involving the Goldstone bosons. In the models considered so far this property shows up in the pseudoscalar couplings of Majorons [1),2)] and axions (in the limit in which they are true Goldstone bosons [6)]. This is a crucial fact for the order of magnitude of the new scales involved. Actually, at the tree level, as we will see, Goldstone bosons have only derivative couplings with fermions (equivalent, to first order, to pure pseudoscalar couplings with fermions of equal mass, i.e. for flavour diagonal couplings [F2]). Derivative couplings must be weighted by an inverse mass, as can be immediately seen from dimensional considerations. This mass is the energy scale V of the global symmetry breaking at which the Goldstone boson is generated. The larger is V the weaker is the coupling [F3].

The presence of a hierarchy of scales is a common problem which will be ignored in this paper. The questions we intend to answer are: how large should the scale V be in order to produce (so far) undetected Goldstone bosons? How many species of Goldstone bosons could exist? (this refers exclusively to cosmological bounds).

By considering the experimental bounds on $\mu \to e\,G$ or $K \to \pi\,G$, and the astrophysical considerations on $e \to e\,G$ (where G is a Goldstone boson) we find $V \gtrsim 10^{10}$ GeV. Since such weakly coupled particles decouple very early in the history of the universe, the cosmological bounds on the number N_G of species are rather weak $N_G \lesssim 30$.

In Sec.II we analyse the Goldstone boson emission vertex by fermions, using current algebra, as well as the coupling of Goldstone bosons with fermions in the Lagrangian. We see that Goldstone boson exchanges can never mediate long-range r^{-1} potentials, a crucial aspect for their elusiveness. In Sec.III a recipe for invisible Goldstone bosons, and its application to horizontal global symmetries are given.

In the following we estimate the experimental bound on the new energy scale through flavour diagonal (Sec.IV) and flavour changing (Sec.V) reactions. In Sec.VI the cosmological bound on the number of Goldstone bosons N_G is studied.

II. HOW GOLDSTONE BOSON EXCHANGE GENERATES ONLY TENSORIAL SPIN DEPENDENT LONG-RANGE POTENTIALS

Let us first indicate using the available experimental bounds why this issue is of crucial importance for the elusiveness of new long-range forces. Following Feinberg and Sucher [7] we parametrize the spin independent, S.I., and tensor, T, potentials among fermions as

$$V^{SI} \sim \frac{\lambda_N}{r}\left(\frac{r_0}{r}\right)^{N-1}\hbar c \quad , \tag{1}$$

$$V^{T} \sim \frac{\lambda^T}{r}\left(\frac{r_0}{r}\right)^{2} S\hbar c \quad , \tag{2}$$

where $r_0 = 1$ Fermi, and $S = \vec{s}_1\,\vec{s}_2 - 3\,\vec{s}_1\cdot\hat{r}\;\vec{s}_2\cdot\hat{r}$ with $\vec{s}_i$ the fermion spins. The most restrictive limits on the strengths λ obtained by comparing V with known gravitational and electromagnetic effects are [7] $\lambda_1 < 10^{-47}$ (from Eötvös type experiments); $\lambda_2 < 10^{-26}$, $\lambda_3 < 10^{-12}$ (from Cavendish type experiments); $\lambda^T < 10^{-6}$ (from the hyperfine splitting of molecular levels). As we will see the Goldstone boson-fermions vertex contains a γ_5 factor and yields only V^T potentials. With respect to a Coulomb r^{-1} potential we gain 40 orders of magnitude in the coupling constant, i.e. 20 orders of magnitude in the scale V of global symmetry breaking (because $\lambda \sim V^{-2}$).

The non-relativistic potential $V(r)$ between two fermions f is related to the Goldstone boson fermion vector $\mathcal{G}(q) \equiv \langle f|fG\rangle$ (where q is the momentum transfer) by

$$V_{ff}(r) \sim \int d^3\vec{q}\;\frac{|\mathcal{G}(q)|^2_{N.R.}\;e^{-i\vec{q}\cdot\vec{r}}}{|\vec{q}|^2} \quad , \tag{3}$$

where N.R. means "in the non-relativistic limit", i.e. $E \sim m_f \gg |\vec{p}_f|$, so $q^0 = 0$ (to zero order in m_f^{-1}). The dominant contributions as $r = |\vec{r}| \to \infty$ comes from $q^\mu \to 0$. Only if $\mathcal{G}(0) \neq 0$ we get a Coulomb-like potential $V(r) \sim \mathcal{G}(0)^2/r$. For Goldstone bosons this is not the case.

The vanishing of the Goldstone boson emission vertex from a fermion in the soft $q^\mu \to 0$ limit is a special case of a general result, originally derived in the context of PCAC and soft pion physics, the "Adler consistency condition" [8]. Let us consider the matrix element of the conserved current J_μ associated to the Goldstone boson between one-fermion initial and final states, f_1 and f_2. In particular, $f_1 \equiv f_2$ is the only relevant case for long-range interactions

$$\langle f_2|\,J_\mu\,|f_1\rangle \equiv e^{iqx}\,M_\mu \quad . \tag{4}$$

Here we neglect internal indices, $q = p_{f_2} - p_{f_1}$ is the momentum transfer. Furthermore, we know that

$$\langle 0|\,J_\mu\,|G\rangle \sim q^\mu\,F_G \quad , \tag{5}$$

with F_G the Goldstone boson form factor. M_μ can be separated into the zero mass Goldstone boson pole and the remaining amplitude N_μ which, by definition, does not contain a q^2 pole:

$$M_\mu \sim \frac{\mathcal{G}_{f\to Gf}(q^2=0)\;F_G\;q^\mu}{q^2} + N_\mu \quad . \tag{6}$$

Current conservation implies

$$q^\mu M_\mu = 0 \quad . \tag{7}$$

It follows that

$$g_{f\to Gf}^{(q^2=0)} \sim \frac{q^\mu N_\mu}{F_G} \quad . \qquad (8)$$

The only mechanism which could yield a non-vanishing vertex g when $q^\mu \to 0$ is the presence of singularities $(q\cdot p)^{-1}$ in N_μ. In the derivation of the Adler consistency condition the matrix element of the current is taken between multiparticle initial and final states. Then, singularities in N_μ of this kind arise from fermionic propagators when J^μ is attached to external legs without changing the mass at the vertex. This is not possible in our case, therefore the amplitude N_μ is regular when $q_\mu \to 0$.

We see, then, to zero order in q_μ that $g_{f\to Gf}(0) = 0$ so no r^{-1} long-range potential can be generated. At first order in q^μ the relation (8) implies that the Goldstone boson-fermion coupling is pseudoscalar. A Lorentz decomposition of N_μ contains the four-vertex γ_μ, $q_\mu = p_{f_2} - p_{f_1}$ and $p_\mu = p_{f_2} + p_{f_1}$, if the current is a vector or $\gamma_5\gamma_\mu$, $\gamma_5 q_\mu$, $\gamma_5 p_\mu$ if it is an axial vector. When $f_2 \equiv f_1$, $q_\mu p_\mu = 0$, and apart from terms of order q^2 or more, we get

$$g_{f\to Gf} \sim q^\mu N_\mu \sim \bar{u}\, q^\mu \gamma_\mu \gamma_5 u \sim 2 m_f \bar{u} \gamma_5 u \quad . \qquad (9)$$

In the non-relativistic limit $\bar{u}\gamma_5 u \sim \vec{\sigma}\cdot\vec{q}$, with $\vec{\sigma}$ the Pauli matrices. Thus, we get a potential like the one in Eq.(2). This is a general property of Goldstone boson vertices.

Let us present now a simple argument to show that at the tree level, the coupling of Goldstone bosons with fermions is only of a derivative type [F2]. Let us consider any global symmetry group with any number of Higgs particles and fermions. If the symmetry were local we could eliminate the Goldstone bosons in the Lagrangian by going to the unitary gauge. The required gauge transformation is [9] [F4]

$$U(x) = e^{-i\frac{G^a(x)}{V} T^a} \quad , \qquad (10)$$

where a is the index of the symmetry group, T^a are the broken generators, that is, $T^a v \neq 0$, v is the vacuum expectation value, V.E.V. of the Higgs

fields, $V = |v|$ and $G^a(x)$ are the Goldstone fields. All the Higgs fields can be written as a unique vector ϕ in a real eventually reducible representation of the group as [F4]

$$U(x) = e^{i\frac{G^a(x) T^a}{V}} (v + \eta(x)) \quad , \qquad (11)$$

where $\eta(x)$ represents as many independent fields as there are dimensions in the part of the Higgs representation space orthogonal to all the vectors $T^a v$.

In this gauge the Goldstone bosons being non-physical particles, disappear from the Lagrangian. That is, when the gauge vector bosons are present they transform under $U(x)$ in such a way that they cancel the terms containing derivatives of $U(x)$. Returning to our case, when the symmetry is global, one can still perform on the fields the local transformation $U(x)$, which now is not a symmetry of the Lagrangian. The Lagrangian changes (but still, the theory described is the same [F5]) by terms containing $\partial_\mu U$ (which would be cancelled by gauge bosons if present). When the global symmetry group is a U(1), then $(\partial_\mu U)\, U^{-1} \equiv \partial_\mu G/V$, and only the derivative of the Goldstone boson field remains. When the group is non-Abelian, however, a whole series is generated, but still the one Goldstone boson exchange graph depends on the fermion-fermion vertex with one boson in which only $\frac{1}{V}\partial_\mu G$ appears. This one gives the leading order in q^μ.

The terms from which the couplings of the Goldstone bosons with fermions originate with this procedure are not the Yukawa couplings but the kinetic term of fermions. Only an axial vector derivative coupling, $\frac{1}{V}\partial_\mu G\, \bar{f}\gamma^\mu\gamma_5 f$, yields a non-zero flavour diagonal (axial) coupling. Thus, if the theory is completely vector-like (in the sense that the global group makes no distinction between left and right) Goldstone bosons produce no long-range potential.

For the vertex $f \to f\, G^a$ in the case $q \to 0$ (non-relativistic limit, long-range tail of the G^a exchange) we are interested in the coupling of fermions to the zero momentum mode of the Goldstone bosons, $G^a(x) = G^a$, constant independent of x. For this mode the transformation in Eq.(10) is a global transformation, a symmetry of the Lagrangian. Thus, the $q = 0$ part of the Goldstone bosons can be rotated away. This is another way of seeing that the vertex $f \to f\, G^a$ is zero when $q = 0$.

As we have seen the coupling of Goldstone bosons with fermions is of the order

$$\frac{m_f}{V}\ \bar{f}\gamma_5 f \quad . \tag{12}$$

From the experimental bound on V^T in Eq.(2) we have

$$\lambda^T \sim \frac{m_f^2}{V^2} \sim \frac{(1-10^3)\,MeV^2}{V^2} \lesssim 10^6 \quad . \tag{13}$$

Hence $V \gtrsim 10$-100 GeV. Thus, we conclude that choosing V just at the scale of the Glashow-Weinberg-Salam model will suffice to ensure the non-observability of long-range forces.

III. INVISIBLE GOLDSTONE BOSONS AND HORIZONTAL GLOBAL SYMMETRIES

The recipe for making the Goldstone bosons sufficiently elusive is obvious: take an appropriately large scale V. As we will see in the next sections, V must be larger than the standard scale of 250 GeV. If this is so we need some other Higgs field, singlet of the W-S group, but not of the global group, whose V.E.V is at the scale V. The usual Higgs fields $SU(2)_L$ doublets may or may not break the global group, but if they do break it, it must happen that the same generators are also broken at the higher scale V.

Are there realistic candidates for global symmetries of nature? Besides the $U(1)_{B-L}$ one can only think of horizontal symmetries. The striking pattern of the generations and the remarkable lack of intergenerational transitions has not yet received any convincing explanation. Once Goldstone bosons are not considered as a trouble to be avoided, the use of horizontal global symmetries in trying to relate generations arises naturally as a possibility to be explored. Global symmetries are always present in the Lagrangian without Higgs fields and a part of them could remain when these bosons are added. In the last years local horizontal symmetries [10] have been introduced to obtain relations between the masses of the fermions. The game to be played with the V.E.V.'s of the Higgs fields to get mass matrices is the same either with local or global symmetries. There is, however, one essential difference in the assignment of particles to various representations of the group. It arises from the fact that in global symmetries one does not need

to worry about anomalies, with one exception, the U(1) group. An anomalous U(1) actually breaks into a Z_n group, and the cosmological problem of avoiding domains would appear (as for the $U(1)_{PQ}$ [11]). Just to give an example, in an SU(3) global horizontal symmetry the left- and right-handed fermions, f_L and f_R $((e_L,\mu_L,\tau_L)$ and (e_R,μ_R,τ_R), etc.) could be assigned to conjugate fundamental representations, 3 and $\bar{3}$, respectively. Allowed representations for Higgs fields coupled to $\bar{f}_L f_R$ would then be $\bar{3}$ and 6 which have not yet been explored. One should also take into account that radiative corrections, by exchange of Goldstone bosons, to the mass matrices induced in global horizontal symmetries are much smaller than the corrections due to vector bosons in horizontal local symmetries. The reason is basically the difference on the required breaking scales, for gauged interactions it is $M > 10^5$ GeV, while for global symmetries, as we have seen, it is $V > 10^{10}$ GeV. Every vector boson propagator contributes to the amplitude with a factor M^{-2}, every scalar boson propagator joined to two fermion vertices contributes to the amplitude with V^{-2}. Therefore, in the global case, the vacuum expectation value of Higgs fields should produce the desired features of the fermionic mass matrix without relying heavily on radiative corrections.

Let us take as the total symmetry $g_H \times g_V$ (a horizontal group g_H and a vertical one g_V), the horizontal symmetry is global and just consider the vertical local group as the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$. The usual Higgs doublets φ, must carry non-trivial quantum numbers of the global group g_H, since their V.E.V's produce the fermion masses and generation mixings. The new elements of the theory are Higgs singlets of G-W-S fields Φ (may be coupled to the right-handed neutrinos) which break g_H at the higher scale V at least parallely to the breaking induced by the doublets. After both breakings the Goldstone bosons are

$$G^a \sim \mathrm{Im}\left\{ \langle \Phi \rangle^+ T^a \Phi + \langle \varphi^+ \rangle T^a \varphi \right\} \quad . \tag{14}$$

Only the φ fields have direct Yukawa couplings with charged fermions f (and ν_L) with coupling constants $\alpha_{Yuk} \sim m_f/\langle\varphi\rangle$. Therefore the coupling constants of fermions f and G^a are

$$g_{Gff} \sim \alpha_{Yuk} \cdot \frac{\langle \varphi \rangle}{\langle \Phi \rangle} \sim \frac{m_f}{\langle \Phi \rangle} \sim \frac{m_f}{V} \quad . \tag{15}$$

as we had already obtained. We will investigate in the following sections the limits imposed on these couplings by experimental bounds on flavour changing and conserving processes.

If we pursue the idea that all Higgs fields couple to some fermion, then ϕ couples to right-handed neutrinos, and its V.E.V. induces large Majorana masses for them. Therefore, through the now usual mechanism [12] of diagonalizing a mass matrix of the type

$$\begin{pmatrix} M & m \\ m & 0 \end{pmatrix} ,$$

where M is a 3×3 Majorana mass matrix for the ν_R, and m a 3×3 Dirac mass matrix induced by the doublets, one hopes to get the usual neutrinos with masses of order m^2/M and their heavier partners at the scale of $M \sim V$. This idea has been introduced already [13] for gauged horizontal interactions, the only difference being the order of magnitude of the scale V.

The idea of having a global horizontal symmetry group SU(2) (or SU(3)) comes naturally when dilatational excitations of massless (or massive) extended objects along its internal axes are considered [14]. Four Lorentz invariant quantities composed of the internal variables can be defined which transform as the generators of a group SU(2) × U(1). The first can be taken as a horizontal symmetry, while the second could be taken as the Peccei-Quinn group.

IV. FLAVOUR DIAGONAL COUPLINGS OF GOLDSTONE BOSONS

The apparently most dramatic effect of diagonal couplings of Goldstone bosons are the long-range forces discussed above. The bounds imposed on the new scale V turned out to be just of the order of the Glashow-Weinberg-Salam scale. Other possible effects are Goldstone bosons bremsstrahlung in pp collisions testable in beam dump experiments and the energy loss by stars through Goldstone bosons emission by electrons, which turn out to provide the most sensitive bound, just like for axions.

Beam dump experiments can be used to establish bounds on the production of penetrating particles, provided one can estimate their own interaction cross-section [15]. In particular, an upper limit on the ratio of the number of axions (Hansl [15]) to the number of π^0 has been inferred, which apply

equally to the number of Goldstone bosons, $n_G/n_{\pi^0} \lesssim 10^{-8}$. The number of particles produced are inversely proportional to the respectively associated breaking scales. For the pion we take the pion form factor $F_\pi \sim 100$ MeV. Thus $n_G/n_{\pi^0} \sim F_\pi^2/V^2$ and $V > 10^3$ GeV, a rather weak bound. A much stronger one comes from the energy loss in stars [16]. Since the Goldstone bosons couple weakly to matter, this mean free path is much larger than a typical size of stars and they could remove stellar energy too fast unless their coupling strength to matter is limited. From the existence of red giant cores one gets a bound on the coupling with electrons $g_{Gee} < 10^{-11}$. Since $g_{Gee} \sim m_e/V$ we get $V > 10^8$ GeV. Not much can be gained by considering the process on a proton (or u,d quarks). It is amazing that this bound on V almost coincides with those comming from laboratory experiments on flavour changing neutral reactions.

V. GENERATION CHANGING COUPLINGS OF GOLDSTONE BOSONS

The Goldstone bosons associated with spontaneous breaking of the horizontal group could be emitted in transitions $f_i \to f_j + G^a$, with $f_i f_j$ fermions with equal charge and colour in different generations. The most restrictive limits come from the reactions $\mu \to eG$, $K \to \pi G$, $K^0 \leftrightarrow \bar{K}^0$.

The signature for $\mu \to eG$ would be a bump in the electron spectrum at an energy $m_\mu/2$ superposed to the Curie plot for $\mu \to e\nu\bar{\nu}$. We believe that a branching ratio $B \sim \frac{\Gamma(\mu \to eG)}{\Gamma(\mu \to e\nu\nu)} < 10^{-4}$ is enough to hide such eventual bump [F7] (that is $\Gamma(\mu \to eG) \lesssim 5.10^2\ \mathrm{sec}^{-1}$). Neglecting the electron mass we have

$$\Gamma^{th}(\mu \to eG) = g_{\mu eG}^2 \frac{m_\mu}{16\pi} . \tag{16}$$

Taking the coupling constant $g_{\mu eG} \sim m_\mu/V$; we find $V > 10^9$ to 10^{10} GeV. We get the same result comparing $K \to \pi G$ with $K \to \pi^0 e\nu$ and assuming that the strong interaction effects cancel in the ratio, therefore

$$\frac{\Gamma^{th}(K \to \pi G)}{\Gamma^{th}(K \to \pi e\nu)} = \frac{m_K^3}{16\pi} \left(\frac{G_F^2 m_K^5}{192\pi^3} \right)^{-1} . \tag{17}$$

The experimental values $B(K \to \pi G) \lesssim 4.10^{-8}$ [18] $(\Gamma(K \to \pi G) < 5.10^{1}\ \mathrm{sec}^{-1})$ and $B(K \to \pi e \nu) \sim 5.10^{-2}$ imply a bound $V > 10^{10}$ GeV. K and μ decays may be the best hunting ground for Goldstone bosons associated with spontaneous breaking of global horizontal symmetry. A weakly interacting missing low mass particle discovered in K decay would usually be interpreted as an axion. If the axion is very light it would be impossible to resolve it in this experiment from a massless Goldstone boson. We note however two ways in which this issue could be decided:

i) The axion emission in K decay proceeds via the $\sin^2\theta_c$ Cabibbo mixing of d quark inside s. On the other hand, μ-e admixture must be exceedingly small due to the very small bound on $\mu \to e\gamma$. Hence, if both decays, $K \to \pi G$ and $\mu \to eG$ were seen, the axion interpretation would be strongly excluded. In particular, we would expect both decay rates to be comparable.

ii) The axion being massive will most likely decay into two photons. Hence, in the case of axions a signal should also be seen in $K^+ \to \pi^+\gamma\gamma$. Actually, axions have been searched for using this decay mode [19]. Clearly no such signal should occur for the massless Goldstone bosons.

Another important two-body decay is $\nu_H \to \nu_L G$, where H = heavy, L = light neutrinos. Even with the fairly strong bounds on its strength these decays could proceed sufficiently fast so as to avoid cosmological constraints on the number of heavy neutrinos as well as $\nu_H \to \nu_L + \gamma$ decays into photons [20]. Notice that the mean lifetime due to $\nu_H \to \nu_L + \gamma$ decay is proportional to m_H^{-3}

$$\tau_{\nu_H} \sim \tau_{\mu \to eG} \left(\frac{m_\mu}{m_{\nu_H}}\right)^3 \sim \left(\frac{m_\mu}{m_{\nu_H}}\right)^3 10^{-2}\,\mathrm{sec} \qquad (18)$$

is smaller than the age of the universe for $m_{\nu_H} > 10^2$ eV and of the order of one year for $m_{\nu_H} \sim 100$ keV.

Evidence for decays in which the Goldstone bosons appear as free particles, would be the best way of differentiating eventual global from gauged horizontal interactions. $K^0 \longleftrightarrow \bar{K}^0$ transitions, instead, would proceed in both cases at first order. In evaluating the t-channel Goldstone boson exchange (assume the s-channel exchange is of the same order) we use vacuum insertion and PCAC (as in the original work of Gaillard and Lee [21])

$$\partial_\mu \langle K^0 | \bar{d}\, \gamma_\mu \gamma_5\, s | 0 \rangle \sim f_K\, m_K^2 \qquad (19)$$

and take $\bar{d}\, \partial_\mu \gamma_\mu s \sim m_k$. Therefore

$$\langle \bar{K}^0 | K^0 \rangle \sim \langle \bar{K}^0 | \bar{d}\, \gamma_5\, s | 0 \rangle \frac{1}{\langle q^2 \rangle} \langle 0 | \bar{d}\, \gamma_5\, s | K^0 \rangle \sim$$
$$\sim g_{KG}^2 f_K^2 \ . \qquad (20)$$

We have taken the average Goldstone boson momentum $\langle q^2 \rangle \sim m_k^2$, f_k is the kaon form factor and g is the coupling constant of the Goldstone boson with the quarks $g \sim m_k/V$. From the real part of this amplitude, using the limit on

$$\frac{m_{K_L} - m_{K_S}}{m_K} \sim \left(\frac{f_K}{V}\right)^2 < .7\ 10^{-14} \qquad (21)$$

one gets a relatively low bound $V > 10^6$ GeV. Even assuming maximal CP violation (that is taking the amplitude as imaginary), from $\varepsilon < 10^{-3}$ we get $V > 10^8$ GeV, a less restrictive bound.

VI. HOW MANY GOLDSTONE BOSON SPECIES COULD EXIST?

The number of massless Goldstone bosons expected in a specific model is just the number of broken generators. Considerations of helium abundance in the framework of the standard big bang models strongly bounds the number of allowed neutrino species to $N_\nu < 4$. Do similar restrictions apply to the number of Goldstone bosons? If it were so an horizontal group like SU(3), with eight Goldstone bosons should be excluded. As we will see, however, the restriction is rather weak. This is due to the extremely small cross-sections of Goldstone bosons. If it were because of their interactions with charged fermions the Goldstone bosons would always remain out of equilibrium after their appearance at $T \sim V$. The role of the right-handed neutrinos for these purposes depends strongly on the details of the model. However, the four bosons terms in the Higgs potential ensure that Goldstone bosons will be in equilibrium with the usual Higgs doublets of the G-W-S model, up to the temperature in which they decay, around the G-W-S scale. Therefore the decoupling temperature of the Goldstone bosons is $T_G \sim 250$ GeV. At such a high temperature we expect a large number of degrees of freedom in the radiation: photons, gluons, free quarks and all types of leptons, around 54, besides the number of Goldstone bosons N_G. At the decoupling temperature of left-handed neutrinos $T_\nu \sim 1$ MeV only the neutrinos and e^+e^- are in equilibrium with photons, the rest has decayed into these lighter species. Thus, the neutrino number density increased considerably from T_G to T_ν in such a way that the contribution of one additional neutrino species at the temperature of helium synthesis, 1 MeV, is equivalent to 30 Goldstone bosons species. Hence only a weak bound can be imposed $N_G \lesssim 30$.

VII. CONCLUSIONS

Goldstone bosons, if associated with a scale larger than 10^{10} GeV are harmless. The existence of global horizontal interactions could be the only possibility to discover them in laboratory experiments, through flavour changing reactions as $\mu \to eG$ and $K \to \pi G$. Flavour conserving reactions associated with the necessary scale would not provide any testable laboratory predictions.

While writing this paper we knew about the work of D.B. Reiss [22] on horizontal global symmetries in which some of the points raised here have been considered.

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FOOTNOTES

[F1] To complete the list of Goldstone bosons recently proposed in the literature, we mention a peculiar one considered by P. Fayet. In order to break supersymmetry spontaneously a new gauge group $\mathcal{U}(1)$ is added, whose gauge vector boson at low energies, in the limit of vanishing mass and gauge coupling constant, behaves as a spin zero Goldstone boson [5].

[F2] Using the Dirac equation for the interacting fermions fields f_1 and f_2, $(\gamma_\mu \partial_\mu - m_i)\, f_i = \ldots\ldots$, one gets the equalities

$$\partial_\mu G\, \bar{f}_1 \gamma_\mu f_2 = (m_1 - m_2)\, \bar{f}_1 f_2 G + \cdots$$
$$\partial_\mu G\, \bar{f}_1 \gamma_5 \gamma_\mu f_2 = (m_1 + m_2)\, \bar{f}_1 \gamma_5 f_2 G + \cdots$$

When $f_1 \equiv f_2$, only the pseudoscalar coupling remains. The points indicate fermion bosons interaction terms.

[F3] The case of the Majoron model without right-handed neutrinos [2] is different. This is the only case we know in which the new scale V is lower than the standard one of the Glashow-Weinberg-Salam model. The coupling of the Goldstone boson with all charged fermions occurs here through the mixing of Higgs fields, induced by the Higgs mechanism which gives mass to the Z^0. This is a small number, the ratio of the new over the standard scale.

[F4] Let us see how V^{-1} does appear in the exponent. Assume we parametrize the Higgs field ϕ with non-dimensional fields ξ^a (a,b run over the broken generators T) similarly to Eq.(11)

$$\phi_i = e^{i \xi^a T^a_{ij}} (v_j + \eta_j)$$

The notation is defined in the text. The contribution of ϕ to the conserved currents is

$$J_\mu{}^b = -i\, \partial_\mu \phi_i\, T^b_{ij}\, \phi_j + \cdots = \partial_\mu \xi^a v_i\, T^a_{ij}\, T^b_{jk} v_k + \cdots$$

These are the only terms linear in a field, so they must contain the Goldstone bosons G^a, whose dimension is M^1. Let us define an a_ dimensional vector $\hat{v}$ such that $v_i = V \hat{v}_i$ and $V = |v|$ thus $J_\mu{}^a = V(\partial_\mu G^a) A^{ab}$ where $G^a = V \xi^a$ are a basis for the Goldstone bosons since the matrix $A^{ab} = \hat{v}_i T^a_{ij} T^b_{jk} \hat{v}_k$ is orthogonal positive definite. [9]

[F5] When a local transformation is made on the fields the Lagrangian can change but still it describes the same theory. This can be easily seen in a path integral formulation in which a transformation on the fields amounts to a change of variables.

[F6] In the case of Majorons even if the lepton number corresponds to a vector group U(1), the Yukawa couplings with neutrinos violate parity explicitly in the Lagrangian. For the axion the group U(1) is axial.

[F7] Notice that the limit for $\mu \to eG$ is not to be taken as the bound on $\mu \to e\gamma$, because the experiments searching for the last reaction [17] look for $e-\gamma$ coincidences. Since the decay branching ratio $\frac{\Gamma(\ell \to eG)}{\Gamma(\ell \to e\nu\bar{\nu})} \sim \frac{1}{m_\ell^2}$ the eventual signal in τ decay would be smaller. We thank L. Maiani for this remark.

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