IN8300144

B.A.R.C.-1148

B.A.R.C.-1148

सत्यमेव जयते

भारत सरकार

GOVERNMENT OF INDIA

परमाणु ऊर्जा आयोग

ATOMIC ENERGY COMMISSION

FINITE ELEMENT COMPUTATION OF NATURAL CONVECTION IN ENCLOSURES

by

H. S. Kushwaha
Reactor Engineering Division

भाभा परमाणु अनुसंधान केन्द्र
BHABHA ATOMIC RESEARCH CENTRE
बंबई, भारत
BOMBAY, INDIA
1982

B.A.R.C.-1148

GOVERNMENT OF INDIA
ATOMIC ENERGY COMMISSION

B.A.R.C.-1148

FINITE ELEMENT COMPUTATION OF NATURAL CONVECTION IN ENCLOSURES

by

H.S. Kushwaha

Reactor Engineering Division

BHABHA ATOMIC RESEARCH CENTRE
BOMBAY, INDIA
1982

INIS Subject Category : E11

Descriptors

NATURAL CONVECTION

ADVECTION

FINITE ELEMENT METHOD

BOUNDARY CONDITIONS

NONLINEAR PROBLEMS

TWO-DIMENSIONAL CALCULATIONS

INCOMPRESSIBLE FLOW

NAVIER-STOKES EQUATION

ISOTHERMAL PROCESSES

FINITE ELEMENT COMPUTATION OF NATURAL CONVECTION IN ENCLOSURES

by

H.S. Kushwaha

1. INTRODUCTION:

Free or Natural convection problems are distinguished by the fluid motion being produced by beuyancy forces which are induced by temperature gradients. Ostrach (1) has classified the natural convection problems into two categories : external problems and internal problems. The flow areound heated plate or rod caused by existance of temperature difference between the body and the fluid are external problems. Flow in fluid-filled cavity caused by the temperature differences between the wall and cavity are internal flow problems. Both external and internal flow problems are encountered in nuclear reactor systems.

A brief account of historical developments is given in reference (2) by Dev Vahl Davis. Natural convection problems are coupled problems in the sense that momentum and energy equations are coupled. It means that momentum and energy equations have to be solved together. The direct solution of non-linear differential equations is not possible and hence one has to resort to a numerical technique. The main difficulty in the analysis of free convection problems at high Rayleigh number ($> 10^3$) is coupling of boundary layer and core region in case of internal flow problems. If the boundary layer effect has to be captured which is most essential for high Rayleigh number, one has to use small meshes near the boundary wall.

Finite difference method using an implicit alternating direction method (ADI) has been most popular method to analyse free convection problems. Although finite difference method gives accurate solution over a wide range of Rayleigh numbers but this method is not convenient for irregular confugration of the enclosures. Moreover, non-uniform meshes cannot be used easily near the wall of enclosures.

Finite element method has been used to analyse free convection heat transfer problems by many investigators. Gartling (3) has analysed conjugate heat transfer problems based on velocity-pressure and temperature formulation. Steven (4) has used stream function vorticity least square scheme for analysing free convection problems. Tabarrok and Lin (5), Reddy and Satake (6) have used stream function-vorticity-temperature formulation for solving free convection problems in different types of enclosures. Heinrich at el. (7), Reddy at el. (6, 8, 9) have used penalty function method for solving free convection problems.

The numerical solution of advection (convection) dominated flows poses serious difficulties which are well described by Roache (10). This difficulty is present in the solution of momentum and energy equation. A possible way to overcome these difficulties in context of finite element has been suggested by Zienkiewicz et al. (11), Heinrich (12) and in a better methodological way by Hughes et al. (13).

In the present report penality function approach has been used to solve non-linear coupled momentum equations and energy equation is solved directly. An attempt has been made to establish the accuracy obtainable with coarse mesh compared to fine mesh used by earlier investigators. The program also estimates the local and average heat transfer coefficient in the enclosure. This parameter is required during heat conduction analysis.

2. BASIC EQUATIONS:

Two dimensional steady analysis of incompressible fluid driven by bouyancy forces with in Boussinesq assumption will be considered in this analysis. The axisymmetric form of the equation follows in a straight forward manner. The effect of viscous dissipation and radiative transport have been neglected. The equations will be used in cartesian tensor.

The appropriate mathematical description of fluid motion is derived from balance of linear momentum, which is known as Navier-Stokes equation.

$$\rho u_j \frac{\partial u_i}{\partial x_j} = \rho f_i + \frac{\partial \sigma_{ij}}{\partial x_j} \qquad .....(1)$$

The constitutive equation is

$$\sigma_{ij} = -p\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad .....(2)$$

The condition of fluid incompressibility is enforced by application of principle of mass conservation which results in the continuity equation.

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad .....(3)$$

The transport of thermal energy in the fluid is described by

$$\rho C_p u_j \frac{\partial T}{\partial x_j} + \frac{\partial q_i}{\partial x_i} - Q = 0 \qquad .....(4)$$

The appropriate constitutive equation is

$$q_i = -K\,\frac{\partial T}{\partial x_i} \qquad .....(5)$$

In order to incorporate the effect of temperature change on density in the momentum equation, in the form of buoyancy term, the equation of state for perfect gases is

$$\frac{\rho - \rho_\infty}{\rho_\infty} = - \frac{T - T_\infty}{T_\infty} \qquad \text{.....(6)}$$

The relative change in volume is $\beta(T - T_\infty)$ and the lift force generated by the change is $\rho_\infty g \beta(T - T_\infty)$. Introducing these into the equation (1).

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\rho g_i \beta (T - T_\infty) + \frac{\partial \sigma_{ij}}{\partial x_j} \qquad \text{.....(7)}$$

In the equation (1) to (7) u_i is velocity in x_i coordinate direction, ρ is fluid density, p is pressure, μ is viscosity, T is temperature, T_∞ is reference temperature ρ_∞ is datum density, σ_{ij} is stress tensor, δ_{ij} is kroneker delta, g is gravitational constant, β is coefficient of volume expansion, Cp is specific heat, q_i is heat flux, K is conductivity tensor and Q is volumetric heat generation.

Substituting equation (2) in (1) one gets

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\rho g_i \beta (T - T_\infty) - \frac{\partial p}{\partial x_i} + 2\mu \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \text{.....(8)}$$

It is convenient to express the above equations in non-dimensional form. Introducing the following dimension-less variables.

$$X = x/l, \quad Y = y/l, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$U = \frac{ul}{\alpha}, \quad V = \frac{vl}{\alpha}, \quad P = p\, l^2/\rho\alpha^2 \qquad \text{.....9(a-f)}$$

Where l is characteristic length, Tw is wall temperature, $\alpha = \frac{K}{\rho C_p}$ is thermal diffusitivity and $\nu = \mu/\rho$ is Kinematic Viscosity.

Substituting eqn. (7) into (3) and (8) one gets

$$\frac{\partial U_i}{\partial X_i} = 0 \qquad \text{.....(10)}$$

$$U_j \frac{\partial U_i}{\partial X_j} = -\frac{\partial P}{\partial X_i} + \nu/\alpha \frac{\partial}{\partial X_j}\left(\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i}\right) - g_i \beta \frac{\ell^3 (T_w - T_\infty)}{\alpha^2} \text{.(11)}$$

Using eqn. (4), (5) and (7).

$$U_j \frac{\partial \theta}{\partial X_j} = \frac{\partial}{\partial X_j}\left(\frac{\partial \theta}{\partial X_j}\right) + \hat{Q} \qquad \text{.....(12)}$$

Again introducing following dimensionless parameters

Rayleigh number, $Ra = \frac{g \beta l^3 (T - T_\infty)}{\nu \alpha}$

Prandtl number, $Pr = \nu / \alpha$; $\nu = \mu/\rho$ and $Q = \frac{Q l^2}{\rho C_p \alpha (T_w - T_\infty)}$

Equation (11) may be recast as

$$U_j \frac{\partial U_i}{\partial X_j} = -\frac{\partial P}{\partial X_i} + P_r \left[\frac{\partial}{\partial X_j}\left(\frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i}\right) - Ra\,\theta\right] \text{....(13)}$$

Now eqns. (10), (13) and (12) have to be solved.

To complete the formulation of the boundary value problems for free convection, a suitable set of boundary conditions are required. The present study is restricted for following boundary conditions on . For hydrodynamic part of the problem either velocity component is specified or surface traction must be zero.

$$U_i = U_i^s \quad \text{on} \quad \Gamma_1$$

$$\sigma^*_{ij} \; n_j = 0 \quad \text{on} \quad \Gamma_2$$

The thermal part of the problem requires either temperature is specified or heat flux is zero.

$$\theta = \theta^s \quad \text{on} \quad \Gamma_3$$

$$\frac{\partial \theta}{\partial X_j} n_j = 0 \quad \text{on} \quad \Gamma_4$$

Where σ_{ij}^{*} is non-dimensional stress tenser $\sigma_{ij}L^2 / p\alpha^2$, n_j are the components of the outward normal to the boundary and $\Gamma_f = \Gamma_1 + \Gamma_2$ in total boundary enclosing fluid and $\Gamma_\theta = \Gamma_3 + \Gamma_4$ is total boundary enclosing energy transfer region.

3.0 PENALTY FUNCTION FORMULATION :

In penalty function approach the constraint is imposed through the addition of a penalised constraint error term to the weak form of the basic equations (a detailed presentation of Navier-Stokes eqn. can be found in ref. (13) and ref. (7)). In the above formulation pressure is eliminated and hence the remaining variables are only velocity and temperature. Let W_j denote the test or weighting function for velocity, then the weak form of eqn. (13) is

$$\int_V W_j U_j \frac{\partial U_i}{\partial x_j}\, dV + P_r \int_V \frac{\partial W_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) dV +$$

$$\lambda \int_V \frac{\partial W_i}{\partial x_i} \frac{\partial U_j}{\partial x_j}\, dV - R_a P_r \int_V W_i\, \theta\, dV = 0 \quad .....(14)$$

Similarly introducing test function W for the temperature the weak form of eqn. (12), neglecting Q is

$$\int_V \left[W U_j \frac{\partial \theta}{\partial x_j} + \frac{\partial W}{\partial x_j} \frac{\partial \theta}{\partial x_i} \right] dV = 0 \quad .....(15)$$

The following interpolation of the variable U, V and θ over the element 'e' are

$$U = \sum_{i=1}^{node} N_i \; U_i$$

$$V = \sum_{i=1}^{node} N_i \; V_i$$

$$\Theta = \sum_{i=1}^{node} N_i \; \Theta_i \qquad \text{.....(16 a-c)}$$

Where N_i is shape function and U_i, V_i and Θ_i are nodel velocities and temperature. Substituting 16 (a-c) into equ.(14) and using Galerkin's approach (i.e. setting test function as shape function) one gets following martrix equations.

$$\underset{\approx}{C} \cdot \underset{\sim}{V} + \underset{\sim}{N}(V) = \underset{\sim}{F} \qquad \text{..(17)}$$

and

$$\left(\underset{\approx}{K} + \underset{\approx}{K_V}\right) \underset{\sim}{T} = \underset{\sim}{G} \qquad \text{..(18)}$$

Where

$$\underset{\approx}{C} = \mathop{\mathsf{A}}_{e=1}^{nel} \underset{\approx}{C^e} \; ; \; \underset{\sim}{N}(V) = \mathop{\mathsf{A}}_{e=1}^{nel} \underset{\sim}{N^e}(V)$$

$$\underset{\sim}{F} = \mathop{\mathsf{A}}_{e=1}^{nel} \underset{\sim}{F^e} \; , \; \underset{\approx}{K} = \mathop{\mathsf{A}}_{e=1}^{nel} \underset{\approx}{K^e} \; , \; \underset{\approx}{K_V} = \mathop{\mathsf{A}}_{e=1}^{nel} \underset{\approx}{K_V^e}$$

$$\underset{\sim}{G} = \mathop{\mathsf{A}}_{e=1}^{nel} \underset{\sim}{G^e}$$

A is assembly operater

$$\underset{\approx}{C^e_{ab}} = \int_V \underset{\approx}{B_a^T} \cdot \underset{\approx}{D_\lambda} \cdot \underset{\approx}{B_b} \, dV + \int_V \underset{\approx}{B_a^T} \cdot \underset{\approx}{D_\mu} \cdot \underset{\approx}{B_b} \, dV \qquad \text{.....6}$$

$$\underset{\sim}{N}(V) = \int_V N_a \; U_j \frac{\partial U_i}{\partial x_j} \, dV$$

.....19(a-1)

$$\underset{\approx}{K}^{e}_{ab} = \int_V \bar{\underset{\sim}{B}}_a^T \cdot \underset{\approx}{D} \cdot \bar{\underset{\sim}{B}}_b \, dV \, ; \; (\underset{\approx}{K}_v)_{ab} = \int_V N_a U_j \frac{\partial N_b}{\partial x_j} \, dV$$

$$\underset{\sim}{G}^{e} = \int_V N_a^e \quad Q \quad dv$$

$$\underset{\sim}{F}^{e} = Ra \; Pr \int_V N_a^e \quad \Theta \quad dv$$

$$\underset{\approx}{B}^{e}_{a} = \begin{bmatrix} N_a^e{}_{,1} & 0 \\ 0 & N_a^e{}_{,2} \\ N_a^e{}_{,2} & N_a^e{}_{,1} \end{bmatrix} \; ; \quad \bar{\underset{\sim}{B}}_a = \begin{Bmatrix} N_{a,1} \\ N_{a,2} \end{Bmatrix}$$

$$\underset{\approx}{D}_\lambda = \lambda \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underset{\approx}{D}_\mu = Pr \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$N_a^e{}_{,1} = \frac{\partial N_a^e}{\partial x_1}$$

4. SOLUTION PROCEDURE

Equations (17) and (18) are non-linear and pose considerable difficulties in getting the solution. In this report an incremental Newton-Raphson Scheme has been used where Rayleigh number is used as "load parameter". Within each load level eqn. (17) is iterated until convergence is achieved. The converged solution is used to solve non-linear energy equation (18). Nodel temperature output is used in the next load step for eqn. (17). Before starting the iteration, energy equation is first solved neglecting the advection term. The first load level is initilized by solving momentum equation neglecting the non-linear part.

The non-linear finite element eqn.(17) at a load step can be written as

$$R_i\ (u) = C_{ij}\ U_j + N_j\ (u) - F_i = 0 \quad \ldots(20)$$

Expanding equation (20) with the help of Taylor series and retaining only the first order terms, one gets,

$$R_i(u) = R_i(U^o + U) = R_i(U^o) + \frac{\partial R_i(U^o)}{\partial U_j} \Delta U_j + \quad = 0 \quad ..(21)$$

The equation (21) can be recasted as

$$\underset{\approx}{C^*}ij\ \underset{\sim}{\Delta Uj} = \underset{\sim}{\Delta F}_i \qquad \ldots.(22)$$

where

$$\underset{\approx}{C^*}ij = \frac{\partial R_i(U^o)}{\partial U_j} = \underset{\approx}{C\,ij} + \frac{\partial N_i(U^o)}{\partial U_j} \quad \ldots.(23)$$

and

$$\underset{\sim}{\Delta F_i} = \underset{\sim}{R_i}(U^o) = \underset{\sim}{F_i} - \underset{\approx}{Ci_j}\ \underset{\sim}{U_j^o} - \underset{\sim}{N_i}(U^o) \quad \ldots(24)$$

In a typical load level, the iterates are computed from eqn. (22) with latest approximation to the solution. The equation (23) represents the unbalance force and $\partial N_i(U^o)/\partial U_j$ is known as tangent convection matrix. The element tangent convection matrix has to be evaluated for each element and assembled to calculate $\underset{\approx}{C^*}$ matrix. The element tangent convection matrix is

$$\frac{\partial N_i^e(U^o)}{\partial U_j} = \int_V N_a \left(\frac{\partial U_i}{\partial x_j} N_b + \delta_{ij} U_K \frac{\partial N_b}{\partial x_K} \right) dV. \quad ..(25)$$

It is well known that in Newton-Raphson method error decreases with square of the error at the previous step. In other words, the rate of convergence is of second order.

5.0 SAMPLE PROBLEMS:

5.1 FREE CONVECTION IN SQUARE ENCLOSURE:

Free Convection in the rectangular enclosures formed by two parallel plates maintained at different temperatures with insulated horizontal plates have been analysed by several investigators. There are numerous references available in literature on this problem and would be difficult to site all of them here. A square cavity has been chosen in present study to establish the accuracy with coarse mesh. The analysis was performed using 3 x 3 uniform mesh as compared to 4 x 4 uniform mesh by Heinrich (7). In both the analysis biquadratic elements have been used. The same problem was analysed by Reddy at el. (6) using 10 x 10 liner mesh for $Ra = 10^3$, 10^4 and 12 x 12 for $Ra = 10^5$. Tabarrok at el (5) have also analysed the same problem using 200 triangular element (Stream-vorticity approach). The present analysis was carried out for three different Rayleigh numbers 10^3, 10^4 and 10^5 keeping Prandtl number equal to unity in all the three cases.

Since most of the previous investigators have compared Nusselt number, vorticity and stream values, it is felt that its importance has no meaning (except Nusselt number) in subsequent heat conduction analysis. Hence, in this study importance was given to local and average heat transfer coefficients not the vorticity and stream functions values. Fig.1 shows the vertical velocity profile and Fig.2 shows the temperature along the centre of the cavity for all the three cases. The heat transfer coefficient for above three cases are shown in Fig.3.

5.2 FREE CONVECTION IN TRIANGULAR ENCLOSURE:

This problem has been taken from ref.(14), is that of a two-dimensional enclosures with right triangular cross-section. Fluid motion is set up by heating the hypotenuse (while cooling the horizontal base), The third side represents the common boundary between the two identical halves of the cavity. This side has been considered as being adiabatic. The mesh and boundary conditions imposed for this problem are shown in Fig.4. The H/B (Height/Base) ratio in present analysis was 0.25. The Grashof number was based on B.i.e. $Gr = g\beta B^3 \theta/\nu\alpha$. The study was performed for Pr number equals to 0.733 and for varying Groshof number 4000, and 16,000. . The isotherms for various Gr number are shown in Fig.4. At relatively low Gr of 4000, the effect of convection on the isotherm is only slight. When Gr is raised to 16000 the change in isotherm is hardly noticed. This situation in similar to pure conduction. The results compare well with finite difference (14). The variation of Nu number with x/B is shown in Fig.5. This also matches very well with ref.(14).

5.3 FREE CONVECTION IN THE ANNULUS BETWEEN HORIZONTAL CONCENTRIC CYLINDER

Natural convection between horizontal concentric isothermal cylinder was analysed by many investigators (Ref.15) using experimental set with air, water and oil. They have performed many experiments for different flow regims for different values of Grashof number and diameter ratio of cylinders.

The present study was made for $L/D_i = 0.8$, where L is the annulus gap width and Di is the outside diameter of the inner cylinder. The boundary condition imposed in the present problem are two impermeable isothermal walls at congtrant radii and two vertical sysmmetry at $\phi = 0$ and $\phi = 180.0$ as shown in Fig.6. The velocity boundary conditions are also shown in Fig.6. The dimension less temperature equals to zero at the outer cylinder and unity at the inner cylinder. It is assumed that at the line of symmetry the angular d[illegible]vative of the temperature is assumed to be zero. Kuehn and Goldstien have analysed this problem with 304 nodes using finite difference method. The solution was obtained for Rayleigh number $Ra = \frac{\rho \rho g L^3 (T_i - T_o)}{\mu \alpha}$, 10^3 and 10^4 for fixed value of Prandtle number of 0.7. At low Ra number velocities are small to affect the temperature distribution, which remains essentially in pure conduction. The isotherm begin to resemble ecentric circles near a Rayleigh number of 10^3, as can be seen from Fig.7, with further increase in Rayleigh number the temperature distribution becomes distorted. This results in increase in heat transfer coefficients.

A plot of isothermal line at Rayleigh number 10^4 shown in Fig.7 and matches well with ref. (15).

6. CONCLUSION:

Compared to U-V-P-T formulation and stream-vorticity-temperature formulation, penality function formulation is simple and computationally competative. Incremental Newton-Raphons method employed in this study is effective and efficient. From this study it is established that very fine mesh is not required for a low Rayleigh number considered in this study. The upwinding finite element may be necessary to avoid oscillations for higher Rayleigh numbers.

7. ACKNOWLEDGEMENT:

The author wishes to thank Shri R.L. Sehgal for preparation of data for the above problems.

8. REFERENCES:

1.0 Ostrach, S. "Natural Convection in Enclosures," Advances in Heat Transfer, 8, 161-227, 1972.

2.0 De Vahl Davis, G. "Laminar Natural Convection in an Enclosed Rectangular Cavity," Int.J. Heat Mass Transfer, 11, 1675-1693, 1968.

3.0 Gartling, D.K., "Convective Heat Transfer Analysis By Finite Element Method", Computer Methods in Applied Mechanics and Engineering, 12, 365-382, 1977.

4.0 Stevern, G.P."Finite Element Solution for a Fluid Subjected to Thermal and Bouyancy Effects," Numberical Simulation of Fluid Motion, ed. John - Noye, 551-566, 1978.

5.0 Tabarrok, B. and Lin, R.C., "Finite Element Analysis of Free Convection Flows," Int. J. of Heat and Mass Transfer, 20, 945-952, 1977.

6.0 Reddy, J.N. and Satake, Akio, "A comparision of a Penalty Finite Element Model with the Stream Function-Vorticity Model of Natural Convection, "Journal of Heat Transfer, 102, 659-666, 1980.

7.0 Heinrich, J.C., Marshall, R.S. and Zienkiewicz, O.C., "Penalty Function Solution of Coupled Convective Conductive Heat Transfer," Int. Conf. on Numberical Methods in Laminar and Turbulent Flow, Swansea, July, 1978.

8.0 Reddy, J.N., and Mamidi, D.R. "Penality Velocity-Stream Function Finite Element Model for Free Convection Heat Transfer Problems, "Recent Advances in Engineering Science, R.L. Sierakowski (ed.) University of Florida, Gainesville, 381-386, 1978.

9.0 Reddy, J.N., "Penalty Finite Element Methods for the Solutionsof Advection and Free Convection Flow", Finite Element Method in Engineering, A.P. Kabakia and V.A. Pulmano (eds.), The University of New South Wales, Sydney, Australia, p 583-598, 1979.

10.0 Roache, P.J. "Computational Fluid Dynamic Harmosa Publisher, 1972.

11.0 Heinrich, J.C., Huyakorn, P.S., Zienkiewicz, O.C. and Mitchell, A.R. "An Upwind Finite Element Schemes for Two-Dimensional Convective. Transport Equation, "Int. J. for Numberical Methods in Engg. 11, 131-143, 1977.

12.0 Heinrich, J.C. "On Quadratic Elements in Finite Element Solutions of Steady-state convection-Diffusion Equation, "Int. J. for Num. methods in Engineering, 15, 1041-1052, 1980.

13.0 Hughes, T.J.R., Lin, W.K., and Brooks, A. "Review of Finite Element Analysis of Incompressible Viscous Flows by the Penalty Function Formulation," J. of Computational Physics 30, 1. 1-16, 1979.

14.0 Akinsete, V.A. and Coleman, T.A., "Heat Transfer By Steady Laminar Free Convection Within Triangular Enclosures," Numerical Methods in Thermal Problems, R.W. Lewis and K. Morgan (eds.), Pineridge Press, Swansea, U.K. 259-268, 1979.

15.0 Kuehn, T.H. and Goldstein, R.J., "An Experimental and Theoretical Study of Natural Convection in the Annulus Between Horizontal Concentric Cylinders", J. Fluid Mech. 74, 4, 695-719, 1976.

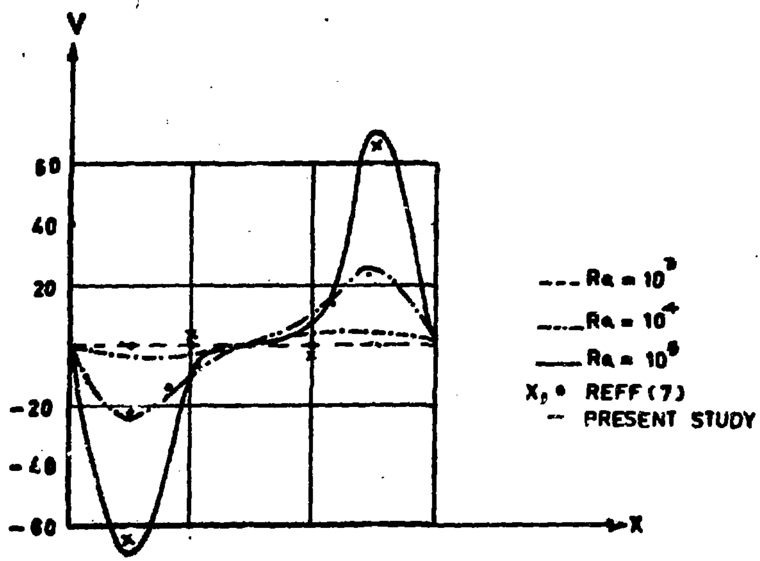

FIG. 1 MIDHEIGHT VERTICAL VELOCITY DISTRIBUTION

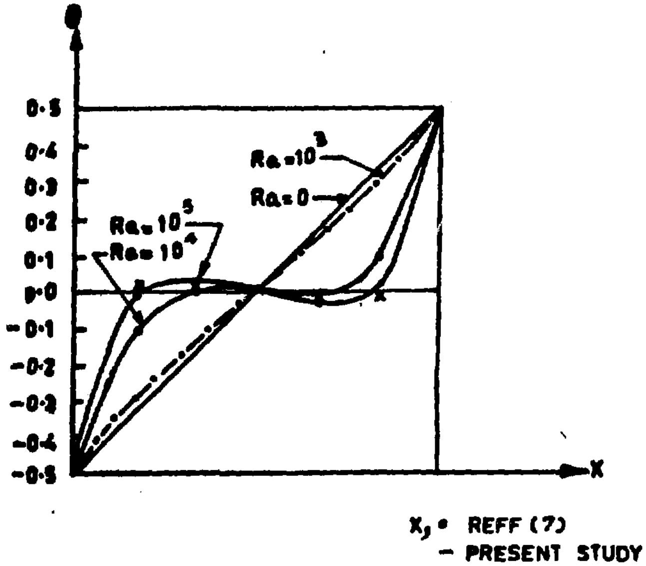

FIG.: 2 MIDHEIGHT TEMPERATURE DISTRIBUTION

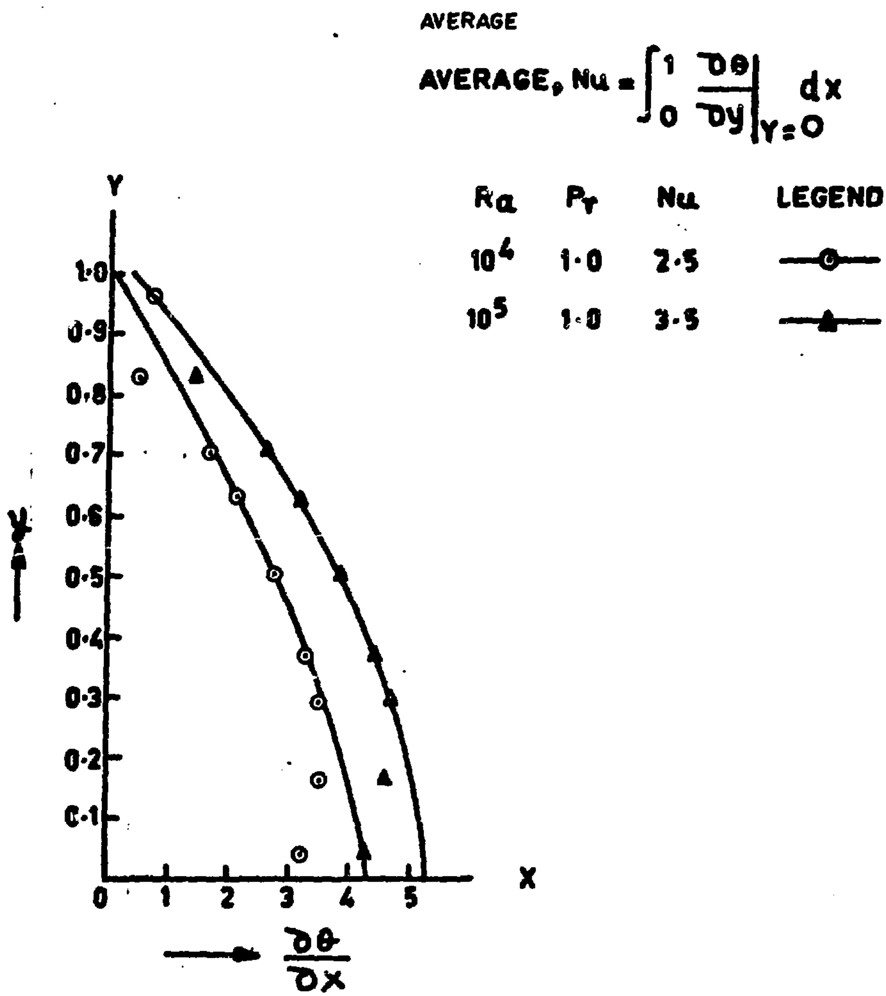

FIG. 3 TEMPERATURE GRADIENT ALONG HOT SURFACE

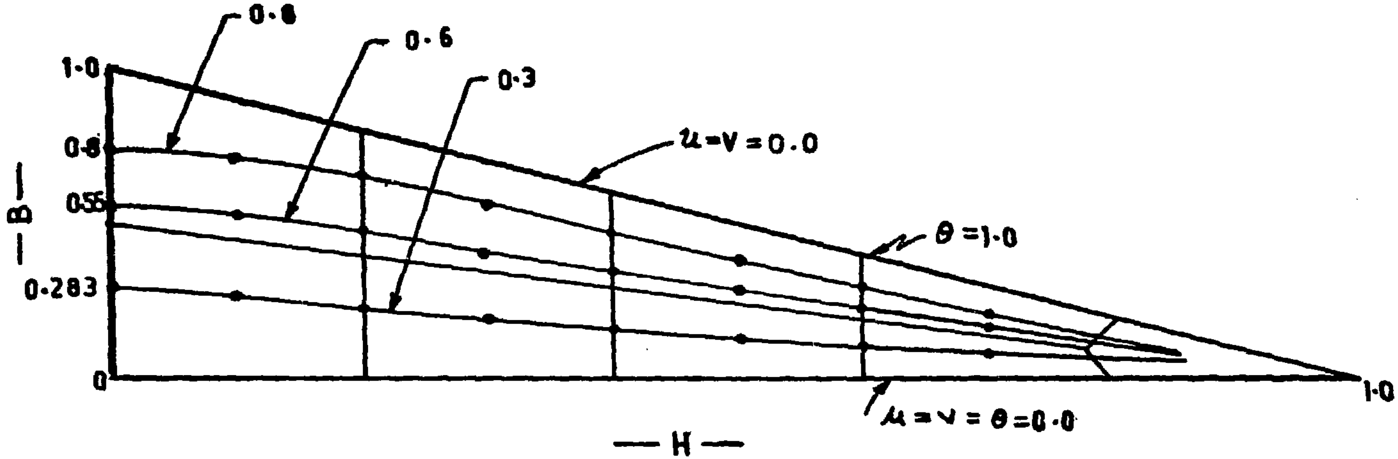

FIG. 4· ISOTHERM LINES FOR H/B = 0·25, GR (B) = 4000 & 16000

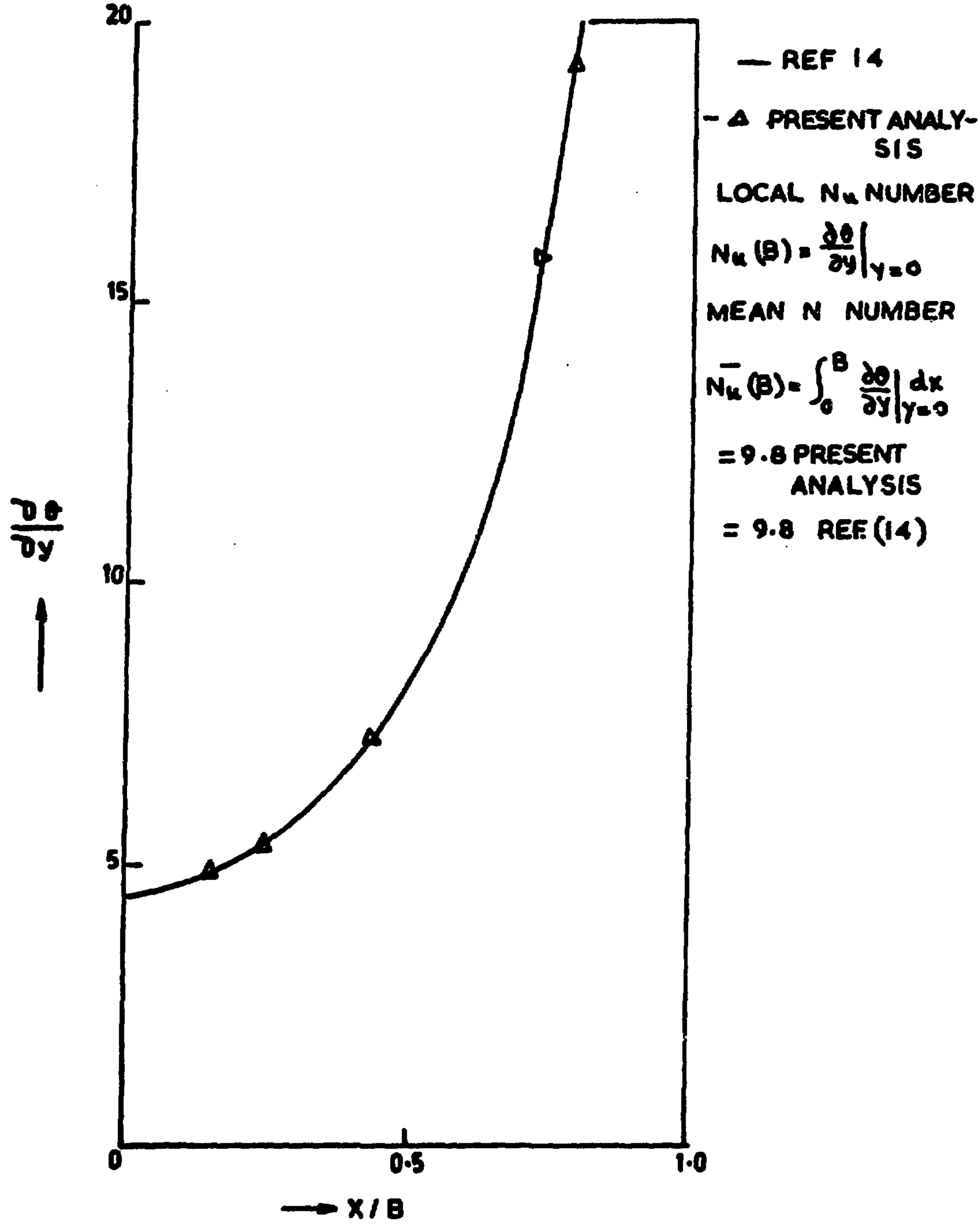

FIG.5. TEMPERATURE GRADIENT ALONG COLD SURFACE

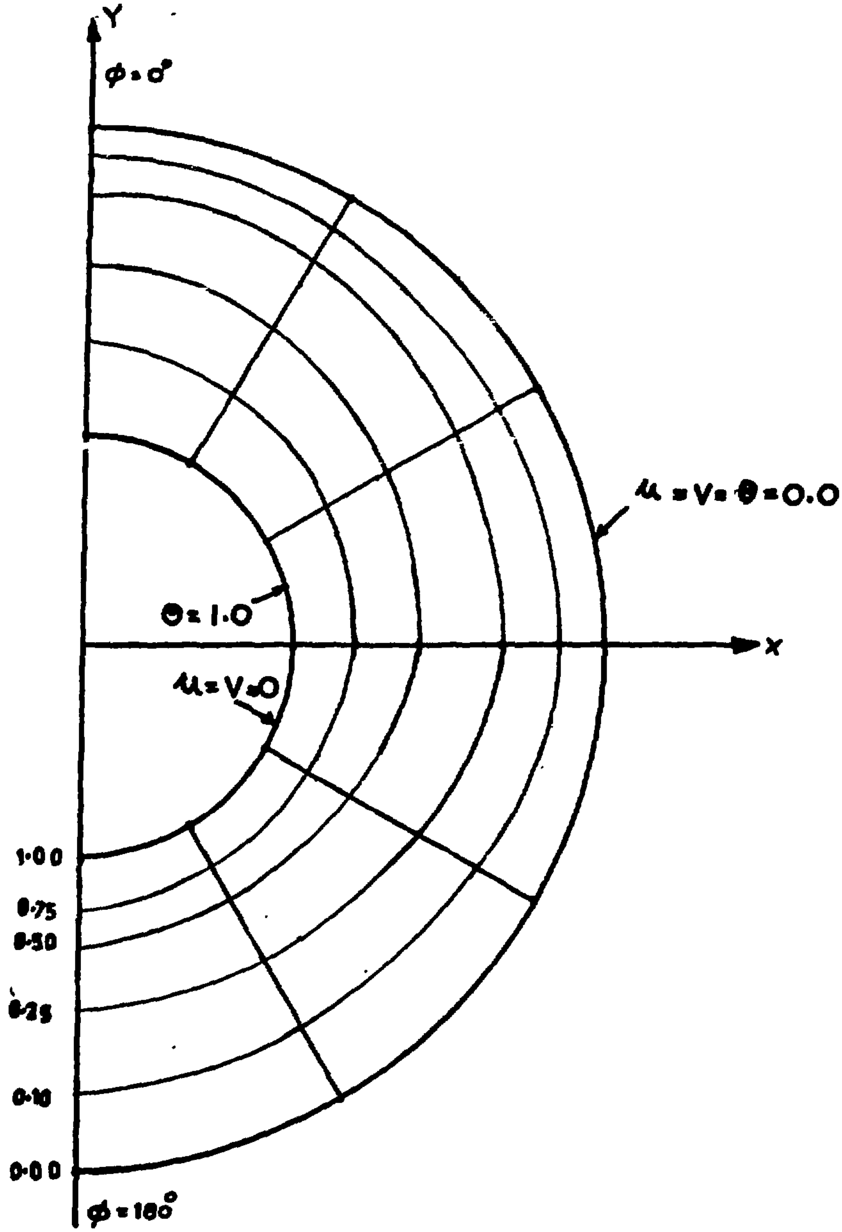

FIG. : 6. ISOTHERMS FOR $R_a = 10^3$, $P_r = 0.7$, $L/D_i = 0.8$

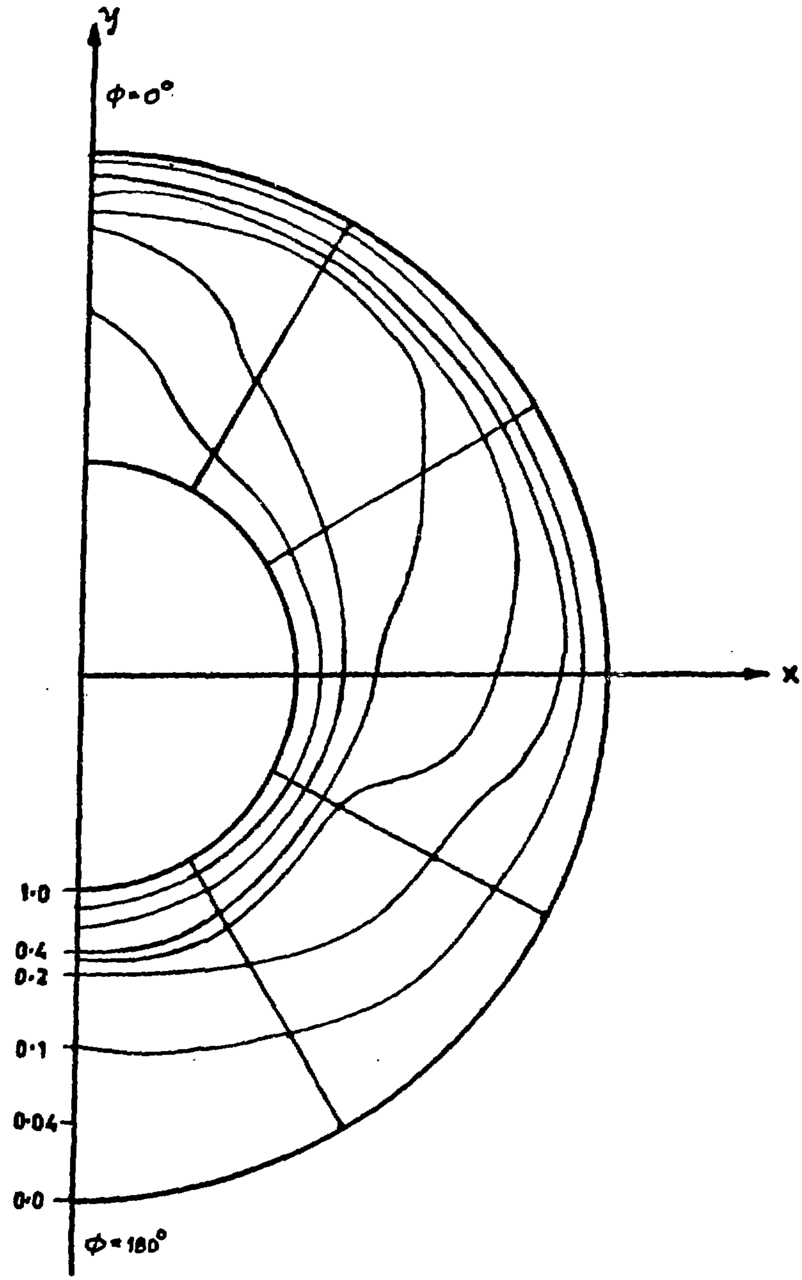

FIG.: 7, ISOTHERMS FOR $Ra = 10^4$, $Pr = 0.7$, $L/D_i = 0.8$