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P. Carlqvist

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Department of Plasma Physics
Royal Institute of Technology
S-100 44 Stockholm, Sweden

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P. Carlqvist
Royal Institute of Technology, Department of Plasma Physics,
S-100 44 Stockholm, Sweden

Abstract

A model of a strong, time-independent, and relativistic double layer is studied. Besides double layers having the electric field parallel to the current the model also describes a certain type of oblique double layers. The "Langmuir condition" (ratio of ion current density to electron current density) as well as an expression for the potential drop of the double layer are derived. Furthermore, the distributions of charged particles, electric field, and potential within the double layer are clarified and discussed. It is found that the properties of relativistic double layers differ substantially from the properties of corresponding non-relativistic double layers.

1. Introduction

Electrostatic double layers represent a phenomenon which may now be considered well-established in laboratory plasmas. The double layer can briefly be described as a local region in the plasma being capable of sustaining a high potential drop, ϕ_{DL}. Inside the double layer positively and negatively charged particles are accelerated in opposite directions. The dominant forces acting in the double layer are the electrostatic and the inertia forces, while collisional forces are generally of negligible importance. For reviews on double layers see e.g. Block (1978), Carlqvist (1979a), and Torvén (1979).

From experimental work we know that double layers appear in many different forms. On the basis of their physical properties double layers may be classified in several different ways. Depending on their behaviour in time we can, for instance, divide the double layers into time-dependent and time-independent layers. Another way of classifying double layers is in weak and strong layers. The strong layers are characterized by having a potential drop that is much larger than the equivalent thermal potentials of all the particle populations (free and reflected ions and electrons) present in the surrounding plasma. In the weak layers at least one of the particle populations has an equivalent thermal potential that is of the same magnitude as or larger than the potential drop.

Double layers may also be divided into relativistic and non-relativistic layers. The double layer is said to be relativistic when the potential drop is large enough ($\phi_{DL} \gg m_i c^2/Ze$) to accelerate both electrons and ions to relativistic velocities. The layer is non-relativistic when the potential drop is so small ($\phi_{DL} \ll m_e c^2/e$) that it is not capable of accelerating even the electrons to relativistic velocities. All layers having potential drops in the interval in between are termed semi-relativistic.

As mentioned above double layers are now experimentally well established. It is reasonable to expect that just as double layers occur in laboratory plasmas they should also occur in cosmic plasmas. The strongest evidence for the existence of cosmic double layers comes at present from measurements in the ionosphere and magnetosphere of the Earth (see Albert and Lindstrom, 1970; Carlqvist and Boström, 1970; Wescott et al., 1976; Haerendel et al., 1976; Mozer et al., 1977; Temerin et al., 1982). The measurements indicate that double layers with potential drops of the order $\phi_{DL} \approx 10^2 - 10^4$ V may exist at altitudes ranging from a few hundred kilometres up to several thousand kilometres.

It has also been suggested that double layers may occur in the atmosphere of the Sun. Based on the idea that double layers are formed in thin current filaments penetrating the solar atmosphere theories of solar flares (Alfvén and Carlqvist, 1967; Carlqvist, 1969) and solar surges (Carlqvist, 1979b) have been worked out. The potential drops expected across these layers are of the order $\phi_{DL} \approx 10^9 - 10^{11}$ V.

Double layers may furthermore exist on a galactic scale. Thus, Alfvén (1978) has advanced a model for double radio sources (radio galaxies) in which double layers are responsible for the release of the enormous energies connected with these sources. The potential drops of the layers anticipated here are of course even much higher than in the solar case.

From the above examples we see that both relativistic and non-relativistic double layers are thought to occur in cosmic plasmas. The theory of non-relativistic double layers has been developed in a number of papers (see e.g. Langmuir, 1929; Andrews and Allen, 1971; Block, 1972; Knorr and Goertz, 1974; Lee et al., 1977; Hasan and ter Haar, 1978; Levine and Crawford, 1980) whereas the theory of relativistic double layers has not been so well investigated. It is the aim of the present paper to study a simple relativistic double layer model. The results obtained from this model will be discussed in detail and compared with the results found from a corresponding non-relativistic model.

2. The Double Layer Model

We shall consider a simple model of a strong and time-independent double layer of plane geometry. In this model, which has several properties in common with models studied earlier by Langmuir (1929) and Carlqvist (1969), the double layer is confined between two plane surfaces - the anode boundary and the cathode boundary - being parallel to one another and separated by the distance d (see Figure 1). The electrostatic potential is defined to be $\phi = 0$ at the cathode boundary and $\phi = \phi_{DL}$ at the anode boundary so that the potential drop across the double layer is ϕ_{DL}. In order to simplify the calculations we also introduce the quantity $\phi_1 = \phi_{DL} - \phi$.

Two species of particles - positive and negative - are present in the double layer. In the first place we shall assume that the positive particles consist of ions while the negative particles consist of electrons (but in principle also other kinds of particles are conceivable - cf. Sections 5 and 6). The ions are emitted with zero velocity from the anode boundary while the electrons are emitted also with zero velocity from the cathode boundary.

It is supposed that the double layer is penetrated by a uniform magnetic field, $\underline{B}$, being so strong that both ions and electrons are fully magnetized. Hence, all the charged particles in our model are forced to move along the magnetic field lines. The angle between $\underline{B}$ and the normals of the boundaries is ψ. For angles, $\psi \neq 0$, we are dealing with oblique double layers.

Inside the double layer the particles move under the influence of electrostatic, magnetic, and inertia forces, while collisional forces are of negligible importance. After leaving the boundary surfaces the ions and electrons are accelerated in opposite directions by the electric field, $\underline{E} = -\nabla\phi$. The potential distribution in the layer must be fully consistent with the net charge density of the accelerated particles as described by Poisson's equation. In order to simulate a real double layer surrounded by quasi-neutral plasma as well as possible we assume that the electric field is zero at both the anode and the cathode boundaries. This arrangement ensures that no surface charges exist at boundaries of the layer and that the layer as a whole is electrically neutral.

For the further analysis of the model we introduce two orthogonal systems of coordinates - (x,y,z) and (x_1,y_1,z_1). As may be seen from Figure 1 the y- and z-axes coincide with the anode boundary while the x-axis is directed towards the cathode boundary. Similarly the y_1- and z_1-axes are situated in the cathode boundary while the x_1-axis points towards the anode boundary. It is assumed that the magnetic field lines are contained in the $z = z_1$ = constant planes. In addition to the two coordinate systems mentioned above we also introduce a ξ-axis starting from $x = 0$, $y = 0$, $z = 0$ and being parallel with the magnetic field, $\underline{B}$.

3. Some Basic Relationships

Inside the double layer the ions and electrons are accelerated by the electric field in opposite directions being aligned with the magnetic field. The ions move towards the cathode boundary while the electrons move towards the anode boundary. Remembering that the particles start from zero velocity we obtain from the law of conservation of energy

$$Ze\phi_1 = \frac{m_i c^2}{\left(1 - \frac{v_i^2}{c^2}\right)^{1/2}} - m_i c^2 \tag{3.1}$$

for the ions and

$$e\phi = \frac{m_e c^2}{\left(1 - \frac{v_e^2}{c^2}\right)^{1/2}} - m_e c^2 \tag{3.2}$$

for the electrons. Here Ze is the ion charge, c is the velocity of light, v_i and v_e are the velocities of the ions and electrons, and m_i and m_e are the masses. Putting $\phi_i = m_i c^2/Ze$ and $\phi_e = m_e c^2/e$ we find from Equations (3.1) and (3.2) the velocities of the ions and electrons

$$v_i = \frac{c(\phi_1^2 + 2\phi_1\phi_i)^{1/2}}{\phi_1 + \phi_i} \tag{3.3}$$

and

$$v_e = -\frac{c(\phi^2 + 2\phi\,\phi_e)^{1/2}}{\phi + \phi_e} \tag{3.4}$$

defined in the positive ξ direction. It is to be noticed that although the ions and electrons move parallel or anti-parallel to the ξ-axis the gradients of the densities of the particles are directed perpendicular to the boundaries. Hence, the gradient of the potential must also be perpendicular to the boundaries according to Poisson's equation.

The current densities of the ions and of the electrons being aligned with the ξ-axis are

$$i_i = Z e\, n_i\, v_i \tag{3.5}$$

and

$$i_e = -\, e\, n_e\, v_e \tag{3.6}$$

respectively. Combining Equations (3.3) and (3.4) with Equations (3.5) and (3.6) and inserting the densities of the ions and electrons, n_i and n_e, into Poisson's equation we obtain

$$\frac{d^2\phi}{dx^2} = -\frac{e}{\varepsilon_o}(Z\,n_i - n_e) = -\frac{i_i}{\varepsilon_o c}\,\frac{\phi_1+\phi_i}{(\phi_1^2+2\phi_1\phi_i)^{1/2}} + \frac{i_e}{\varepsilon_o c}\,\frac{\phi + \phi_e}{(\phi^2+2\phi\,\phi_e)^{1/2}}\,. \tag{3.7}$$

After multiplying both sides of Equation (3.7) by $2\,d\phi/dx = -\,2\,d\phi_1/dx$ we integrate and get

$$\left(\frac{d\phi}{dx}\right)^2 = \frac{2i_i}{\varepsilon_o c}\,(\phi_1^2 + 2\phi_1\phi_i)^{1/2} + \frac{2\,i_e}{\varepsilon_o c}\,(\phi^2 + 2\,\phi\,\phi_e)^{1/2} - C \tag{3.8}$$

where C is a constant of integration. This equation expresses the momentum balance in the double layer. By means of Equations (3.1) to (3.6) and the electric field $E = -\,d\phi/dx$, Equation (3.8) may be rewritten in the more accessible form

$$n_i v_i^{\,2}\,\frac{m_i}{\left(1 - \frac{v_i^{\,2}}{c^2}\right)^{1/2}} + n_e v_e^{\,2}\,\frac{m_e}{\left(1 - \frac{v_e^{\,2}}{c^2}\right)^{1/2}} - \frac{\varepsilon_o E^2}{2} = C \tag{3.9}$$

(cf. Block, 1972; Lee et al., 1977). Here, the first two terms on the left hand side represent the kinetic beam pressures of the two species of particles while the third term is equivalent to the tension of the electric field.

From Section 2 we recall that the electric field tends to zero at the two boundaries of the double layer. Thus, putting $d\phi/dx = 0$ for $\phi = 0$ and $\phi = \phi_{DL}$ we obtain from Equation (3.8) the constant of integration

$$C = \frac{2i_i}{\varepsilon_o c}\,(\phi^2_{DL} + 2\,\phi_{DL}\phi_i)^{1/2} = \frac{2i_e}{\varepsilon_o c}\,(\phi^2_{DL} + 2\,\phi_{DL}\phi_e)^{1/2} \quad . \qquad (3.10)$$

Combining Equations (3.8) and (3.10) we get the general expression for the electric field

$$-\frac{d\phi}{dx} = \left\{\frac{2i_e}{\varepsilon_o c}\left[\left(\frac{\phi_{DL} + 2\phi_e}{\phi_{DL} + 2\phi_i}\right)^{1/2}(\phi_1^2 + 2\phi_1\phi_i)^{1/2} + (\phi^2 + 2\phi\,\phi_e)^{1/2} - (\phi^2_{DL} + 2\phi_{DL}\phi_e)^{1/2}\right]\right\}^{1/2} . \qquad (3.11)$$

By means of Equation (3.10) we can also calculate the ratio of the ion current density to the electron current density

$$\frac{i_i}{i_e} = \left(\frac{\phi_{DL} + 2\phi_e}{\phi_{DL} + 2\phi_i}\right)^{1/2} . \qquad (3.12)$$

This ratio is usually referred to as the "Langmuir condition". For non-relativistic double layers, implying $\phi_{DL} \ll \phi_e \leq \phi_i$, the current ratio may be approximated by

$$\frac{i_i}{i_e} \approx Z^{1/2}\left(\frac{m_e}{m_i}\right)^{1/2}\left(1 + \frac{1}{4}\,\frac{\phi_{DL}}{\phi_e} - \frac{1}{4}\,\frac{\phi_{DL}}{\phi_i}\right) \approx Z^{1/2}\left(\frac{m_e}{m_i}\right)^{1/2} . \qquad (3.13)$$

Putting $Z = 1$ into this equation we find the same expression for the current ratio as Langmuir (1929) did for his non-relativistic and non-oblique ($\psi = 0$) double layer model carrying electrons and singly ionized ions.

For relativistic double layers, on the other hand, having potential drops, $\phi_{DL} \gg \phi_i \geq \phi_e$, we obtain from Equation (3.12) the current ratio

$$\frac{i_i}{i_e} \approx 1 - \frac{\phi_i}{\phi_{DL}} + \frac{\phi_e}{\phi_{DL}} \approx 1 \ . \tag{3.14}$$

In Figure 2 we show how the current ratio, i_i/i_e, varies with the potential drop, ϕ_{DL}, in two different double layers accelerating a) protons and electrons and b) alpha particles and electrons. In both cases the current ratio increases monotonically with the potential drop from small values in the non-relativistic regime to values close to one in the relativistic regime. The curve referring to the alpha particle/electron layer can also, to a first approximation, be employed to represent the current ratio of all double layers accelerating electrons and heavier atomic nuclei ($Z \geq 3$, where all the electrons around the atoms have been stripped off). The reason for this is that heavier nuclei and alpha particles have approximately the same values of ϕ_i.

It is to be noticed that the angle, ψ, between the magnetic field and the normals of the double layer never enters into the equations derived. This implies that all the equations are valid for any value of ψ, provided the assumption that the charged particles are moving along the magnetic field lines is fulfilled. However, it should be kept in mind that the current densities i_i and i_e are aligned with the magnetic field.

4. The Non-Relativistic Double Layer

Before we turn to study the relativistic double layer more in detail we shall briefly discuss the non-relativistic double layer. Putting $\phi \leq \phi_{DL} << \phi_e \leq \phi_i$ and neglecting small terms we may simplify Equation (3.11) to

$$\frac{d\phi}{dx} = -\left[\frac{2^{3/2}\phi_e^{1/2} i_e}{\varepsilon_o c} \left(\phi^{1/2} + \phi_1^{1/2} - \phi_{DL}^{1/2}\right)\right]^{1/2} . \tag{4.1}$$

This equation is identically the same as the equation deduced by Langmuir (1929) for his non-relativistic and non-oblique ($\psi = 0$) double layer model. The identity is not fortuitous but depends on the fact that the angle, ψ, does not enter into our equations.

By integrating Equation (4.1) numerically Langmuir was able to show that the relationship between the potential drop, ϕ_{DL} , the electron current density, i_e, and the thickness, d , of the double layer is

$$i_e d^2 = C_1 \frac{4\epsilon_o}{9} \left(\frac{2e}{m_e}\right)^{1/2} \phi_{DL}^{3/2} . \tag{4.2}$$

The constant C_1 was estimated to be 1.860. Later Raadu (1980) has derived the slightly higher value, $C_1 = 1.865$.

In Section 3 we found that the ratio between the ion current density and the electron current density for the non-relativistic double layer is $i_i/i_e = Z^{1/2}(m_e/m_i)^{1/2}$. Combining this current ratio, the total current density, $i = i_e + i_i$, and Equation (4.2) we obtain for our model

$$id^2 = C_1 \frac{4\epsilon_o}{9} \left[1 + Z^{1/2}\left(\frac{m_e}{m_i}\right)^{1/2}\right]\left(\frac{2e}{m_e}\right)^{1/2} \phi_{DL}^{3/2} . \tag{4.3}$$

From Equation (4.1) Langmuir could also calculate the potential distribution, $\phi(x)$, within the double layer. The result, which is applicable to our double layer model as well, is illustrated in Figure 3. Also shown in Figure 3 are the consistent distributions of the electric field, $E(x)$, and of the densities of the positive and negative charges, $Zn_i(x)$ and $n_e(x)$, respectively.

5. The Relativistic Double Layer

We now turn to the relativistic double layer having a potential drop $\phi_{DL} >> \phi_i \geq \phi_e$. In this kind of layer both ions and electrons acquire relativistic energies. After leaving the anode and cathode boundaries the ions and electrons are rapidly accelerated up to velocities close to c. This acceleration occurs in two relatively narrow regions near the boundaries. The particles then continue to move with velocities close to c through the major part of the double layer while gaining further energy.

In our model having a plane geometry the densities of the particles are inversely proportional to their velocities (see Equations(3.5) and (3.6)). Hence, it is clear that the density of the ions, n_i , must be large close to the anode boundary while the density of the electrons, n_e , must be large close to the cathode boundary. In the region between the two density spikes both n_i and n_e are comparatively low and nearly constant. This region of low and constant density will be referred to as the relativistic region. In Figure 4a we have displayed the quantities, Zn_i and n_e, proportional to the absolute values of the space charge densities of the ions and electrons, as functions of the x-coordinate.

The potential drop. In the region close to the anode boundary where $\phi_1 \ll \phi_{DL}$ Equation (3.1) may be approximated by

$$\frac{d\phi_1}{dx} \approx \left\{ \frac{2i_i}{\varepsilon_o c} \left[(\phi_1{}^2 + 2\phi_1\phi_i)^{1/2} - \phi_1 \right] \right\}^{1/2} . \tag{5.1}$$

It can be shown that in the region where the ions are non-relativistic, implying $\phi_1 \ll \phi_i$, the solution of Equation (5.1) is

$$i_i x^2 \approx \frac{2^{1/2}4}{9} \frac{\varepsilon_o c}{\phi_i{}^{1/2}} \phi_1{}^{3/2} . \tag{5.2}$$

This relation is identical to the well-known diode formula derived by Child (1911). For larger values of ϕ_1 satisfying $\phi_i \ll \phi_1 \ll \phi_{DL}$ integration of Equation (5.1) leads to

$$i_i x^2 \approx \frac{\varepsilon_o c}{2\phi_i} \phi_1{}^2 . \tag{5.3}$$

Similarly, in the region close to the cathode boundary where $\phi \ll \phi_{DL}$ Equation (3.11) can be reduced to

$$\frac{d\phi}{dx_1} \approx \left\{ \frac{2i_e}{\varepsilon_o c} \left[(\phi^2 + 2\phi\phi_e)^{1/2} - \phi \right] \right\}^{1/2} . \tag{5.4}$$

In the non-relativistic regime of the electrons, $\phi \ll \phi_e$, the solution of Equation (5.4) approaches the diode formula for electrons

$$i_e x_1{}^2 \approx \frac{2^{1/2}4}{9} \frac{\varepsilon_o c}{\phi_e{}^{1/2}} \phi^{3/2} \tag{5.5}$$

as given by Langmuir (1913). For $\phi_e \ll \phi \ll \phi_{DL}$ integration yields

$$i_e x_1^2 \approx \frac{\varepsilon_o c}{2\phi_e} \phi^2 \quad . \tag{5.6}$$

We now consider the potential interval, $\phi_a \leq \phi \leq \phi_b$, where the limits satisfy the inequalities, $\phi_e \ll \phi_a \ll \phi_{DL}$, and , $\phi_i \ll \phi_{DL} - \phi_b \ll \phi_{DL}$. In this interval, which covers most of the potential drop of the double layer, we may expand the terms of Equation (3.11) in series and obtain

$$\frac{d\phi}{dx} \approx - \left[\frac{2i_e}{\varepsilon_o c} \left(\phi_e + \frac{\phi_i - \phi_e}{\phi_{DL}} \phi \right) \right]^{1/2} \quad . \tag{5.7}$$

If we integrate this equation from ϕ_a to ϕ_b, corresponding to the space coordinates x_a and x_b, and remember that $i_e \approx i_i \approx i/2$ we find

$$x_a - x_b \approx \left(\frac{4\varepsilon_o c}{i} \right)^{1/2} \frac{\phi_{DL}}{\phi_i - \phi_e} \left[\left(\phi_e + \frac{\phi_i - \phi_e}{\phi_{DL}} \phi_b \right)^{1/2} - \left(\phi_e + \frac{\phi_i - \phi_e}{\phi_{DL}} \phi_a \right)^{1/2} \right] . \tag{5.8}$$

With $\phi_a/\phi_{DL} \approx 0$ and $\phi_b \approx \phi_{DL}$ Equation (5.8) can be approximated by

$$x_a - x_b \approx \left(\frac{4\varepsilon_o c}{i} \right)^{1/2} \frac{\phi_{DL}}{\phi_i^{1/2} + \phi_e^{1/2}} \quad . \tag{5.9}$$

Inserting $\phi_1 = \phi_{DL} - \phi_b$ and $x = x_b$ into Equation (5.3) and $\phi = \phi_a$ and $x_1 = d - x_a$ into Equation (5.6) and comparing with Equation (5.9) we find that $x_a - x_b$ is much larger than both x_b and $d - x_a$. Hence, we can put $x_a - x_b \approx d$ into Equation (5.9) and obtain

$$id^2 \approx \frac{4\varepsilon_o c \, \phi_{DL}^2}{(\phi_i^{1/2} + \phi_e^{1/2})^2} \quad . \tag{5.10}$$

This equation describes the relationship between the total current density, i , the thickness, d , and the potential drop, ϕ_{DL} , of the relativistic double layer. In Figure 5 ϕ_{DL} is shown as a function of id^2 for two double layers carrying a) protons and electrons and b) alpha particles and electrons. The diagram covers both the relativistic and the non-relativistic regimes.

A striking feature of the figure is how similar the two curves are. The potential drops of the two double layers (taken for the same values of id^2) never differ by more than a factor of ≈ 1.5.

It should be noticed that no constraint has been put on the ratio ϕ_i/ϕ_e for the derivation of Equation (5.10). Hence, this equation is valid also for $\phi_i = \phi_e = \phi_Q$ for which case it reduces to $id^2 \approx \epsilon_o c\ \phi_{DL}^2/\phi_Q$. Examples of such somewhat odd double layers are a) layers carrying positive and negative ions and b) layers carrying electrons and positrons.

For more normal double layers carrying electrons and positive ions ϕ_i is generally much larger than ϕ_e. Equation (5.10) may then be simplified to

$$id^2 \approx \frac{4\epsilon_o c\ \phi_{DL}^2}{\phi_i} \quad (5.11)$$

The charge distribution. In view of the fact that the potential drop across the relativistic double layer is directly proportional to the thickness of the layer (cf. Equations (5.10) and 5.11)) one might be inclined to believe that the charge distribution of the layer should generally be similar to that of a parallel plate capacitor of infinite extent. Such an interpretation is, however, not correct. We may instead describe the charge distribution of the relativistic double layer as follows: Due to different masses of the ions and electrons the two density spikes close to the boundaries of the double layer (see Figure 4) do not contain the same amount of positive and negative charge. The electrons are accelerated more easily than the ions are and consequently the density spike of the electrons is thinner than the density spike of the ions. This implies that the density spike of the electrons contains less charge than that of the ions.

Quantitatively we can calculate the net charge content in each of the two density spikes by making use of Gauss' law

$$\iiint_V \rho \, dV = \varepsilon_o \oiint_S \underline{E} \cdot d\underline{S} \quad . \tag{5.12}$$

In the plane geometry of our model Equation (5.12) transforms into

$$q = \varepsilon_o (E_1 + E_2) \tag{5.13}$$

where q denotes the net charge per unit surface of the spike and E_1 and E_2 are the electric fields emerging from each side of the spike. From Equation (5.1) we find for $\phi_i << \phi_1 << \phi_{DL}$, corresponding to a point well outside the ion density spike but still relatively close to the anode boundary,

$$E_1 = - \frac{d\phi}{dx} = \frac{d\phi_1}{dx} \approx \left(\frac{i \, \phi_i}{\varepsilon_o c} \right)^{1/2} \quad . \tag{5.14}$$

Remembering that the electric field is zero at the anode boundary, implying $E_2 = 0$, we obtain by inserting E_1 from Equation (5.14) into Equation (5.13) the net charge per unit area of the ion density spike

$$q_{is} \approx \left(\frac{\varepsilon_o \, i \, \phi_i}{c} \right)^{1/2} \tag{5.15}$$

Correspondingly we find the net charge per unit area of the electron density spike to be

$$q_{es} \approx - \left(\frac{\varepsilon_o \, i \, \phi_e}{c} \right)^{1/2} \quad . \tag{5.16}$$

From Equations (5.15) and (5.16) we see that the ratio of the net charges of the ion and electron density spikes is $q_{is}/q_{es} \approx - (\phi_i/\phi_e)^{1/2}$. For normal mass ratios of ions and electrons the charge content of the ion density spike is therefore much larger than the charge content of the electron density spike. The condition for zero electric field at the boundaries of the double layer, however, requires that the double layer be electrically neutral as a whole. A negative charge has therefore to be present in the relativistic region between the two density spikes.

We can obtain a measure of this negative charge by considering the charge density given by Equation (3.7). If we insert the ion current density defined by Equation (3.11) in Equation (3.7) we get

$$\rho = e(Zn_i - n_e) = \frac{i_e}{c}\left[\left(\frac{\phi_{DL} + 2\phi_e}{\phi_{DL} + 2\phi_i}\right)^{1/2} \frac{\phi_1 + \phi_i}{(\phi_1^2 + 2\phi_1\phi_i)^{1/2}} - \frac{\phi + \phi_e}{(\phi^2 + 2\phi\phi_e)^{1/2}}\right] . \quad (5.17)$$

For double layers satisfying $\phi_{DL} >> \phi_i > \phi_e$ it turns out that this charge density adopts a nearly constant value

$$\rho \approx -\frac{i}{2c}\left(\frac{\phi_i - \phi_e}{\phi_{DL}}\right) \quad (5.18)$$

throughout the relativistic region (see Figure 4). The slight deviation of the ion charge density from the electron charge density in this region is primarily due to the fact that the ion and electron current densities are not exactly equal (cf. Equation (3.14)).

Since the charge density given by Equation (5.18) exists over a distance that is approximately equal to the thickness of the double layer, d , the total charge per unit area of the relativistic region is

$$q_{rr} \approx \rho\, d \approx -\left(\frac{\varepsilon_o\, i}{c}\right)^{1/2} (\phi_i^{1/2} - \phi_e^{1/2}) \quad . \quad (5.19)$$

This charge exactly cancels the sum of the charges in the ion and electron spikes as required (cf. Equations (5.15) and (5.16)).

Idealized picture. Using the results above we may extract the following idealized picture of the charge distribution in the relativistic double layer: The ion and electron density spikes represent two thin regions close to the boundaries of the layer having positive and negative net charges. These charges may in a simplified description be considered as surface charges (see Figure 6a, b). For a double layer carrying ions and electrons the positive surface charge dominates largely over the negative surface charge. In the relativistic region between the surface charges the charge density is relatively low, negative, and constant.

In the idealized picture of the double layer shown in Figure 6 there are three charges, q_{is}, q_{es}, and q_{rr}. For this layer the electric field may be divided into two components. One component consists of a nearly uniform field with the strength, $-q_{es}/\varepsilon_o$, corresponding to the net charge density of the electron density spike (Figure 6c, d). The other component consists of a field which decreases approximately linearly from a maximum value of, $(q_{is} + q_{es})/\varepsilon_o$ near the anode boundary $(x \approx 0)$ to zero close to the cathode boundary $(x \approx d)$. The two electric field components taken together are completely consistent with the potential drop given by Equation (5.10).

From the above discussion we may distinguish two extreme types of relativistic double layers. Firstly there is the normal type of layer accelerating electrons and heavy ions implying $\phi_i \gg \phi_e$ (see Figure 7). Here, the uniform component of the electric field vanishes which means that the total electric field decreases nearly linearly from the maximum value, $E_m = q_{is}/\varepsilon_o$, at $x \approx 0$ to zero at $x = d$. Such an electric field corresponds to the potential drop given by Equation (5.11). Secondly there is the type of layer where $\phi_i = \phi_e = \phi_Q$. In this layer the total electric field is uniform.

By means of the idealized pictures shown in Figures 6 and 7 we can also see how the potential drop of the double layer, ϕ_{DL}, depends on the thickness, d. We consider two double layers, I and II, carrying the same kinds of particles and the same current density, i , but having different thicknesses (in Figures 6 and 7 the thicknesses differ by a factor of two). The "surface charges" near the boundaries are then equal, $q_{is_I} = q_{is_{II}} = q_{is}$ and $q_{es_I} = q_{es_{II}} = q_{es}$, for both of the layers and so are the uniformly distributed charges, $q_{rr_I} = q_{rr_{II}} = q_{rr}$. This means that the magnitude of the electric field has to be the same in the two layers. The distribution of the electric field is only stretched in proportion to the thickness. Hence, we find that the potential drop must generally be directly proportional to the thickness of the double layer in accordance with Equations (5.10) and (5.11).

6. Comparison of Relativistic and Non-Relativistic Double Layers

From what has been said above it is clear that the properties of the relativistic double layers differ in many ways from the properties of the non-relativistic double layers. We shall here briefly compare the two types of layers yielded by our model.

Double layers with $\phi_i/\phi_e >> 1$. We first consider double layers carrying ordinary ions and electrons implying $\phi_i/\phi_e >> 1$. In the non-relativistic regime such layers possess distributions of positive and negative charge units, $Zn_i(x)$ and $n_e(x)$ respectively, that are symmetric to one another about the centre of the layers at $x = d/2$ (Figure 3a). Hence, the surface of charge neutrality (dashed-dotted line in Figure 3) separating the positive and negative space charges is situated at the middle of the layers. As a consequence of the symmetry of the charges the distribution of the electric field, $E(x)$, is also symmetric with respect to $x = d/2$ (Figure 3b).

In the relativistic regime a similar symmetry does not exist. The surface of charge neutrality is here displaced towards the anode boundary (dashed-dotted line in Figure 4).

Another difference between the relativistic and the non--relativistic double layers appears in the Langmuir condition. In the non-relativistic layer the electron current density is, as is evident from Equation (3.13), much larger than the ion current density while in the relativistic layer the two current densities are about equal (Equation (3.14)).

From Equation (4.3) we see that the relationship between the current density, the thickness, and the potential drop of the non-relativistic double layer may be written as $id^2 \propto \phi_{DL}^{3/2}$. This is the same dependence (but for a constant factor) that is valid for a space-charge limited current in a plane diode (Child, 1911; Langmuir, 1913). For the relativistic layer, on the other hand, Equation (5.11) yields $id^2 \propto \phi_{DL}^2$. The exponent of ϕ_{DL} thus changes from 3/2 for the non-relativistic layer to 2 for the relativistic layer (cf. also Figure 5).

Double layers with $\phi_i/\phi_e = 1$. In double layers where $\phi_i = \phi_e$ the absolute values of the charge to mass ratio of the positive and negative particles building up the layers are equal. As a result of this equality such layers are always symmetric with respect to the centre of the layers irrespective of the value of ϕ_{DL}. For the same reason the ion and electron current densities are also always the same. Furthermore, the relationship between the current density, the thickness, and the potential drop may be expressed as $id^2 \propto \phi_{DL}^{3/2}$ in the non-relativistic regime and $id^2 \propto \phi_{DL}^2$ in the relativistic regime (just as for the layers where $\phi_i >> \phi_e$).

7. Conclusions

In order to obtain as straightforward results as possible we have in this investigation chosen to study a very simple double layer model. It is then not astonishing that the model will also be subject to some limitations. One obvious limitation consists for instance in the fact that the model is one-dimensional and accordingly of infinite extent. This non-physical property makes it impossible to obtain a fully consistent description of the magnetic field only by using the currents through the double layer. Hence, we must assume that the magnetic field is also produced by outer means.

Other limitations of the model are that there are only two populations of particles present in the layer (free positive and free negative particles), the densities of which tend to infinity at the emitting boundary surfaces. However, these limitations are probably not very serious, at least not as long as only strong double layers are considered (cf. Hasan and ter Haar, 1978; Carlqvist, 1979a).

As was pointed out in Section 1 double layers may be divided into time-dependent and time-independent layers. The model we have studied is primarily intended to describe layers of the latter type. Nevertheless, under certain conditions the model may also be relevant for time-dependent layers. If the relative change of the double layer is small in a time period corresponding to the maximum passage time of the charged particles across the layer, the layer will be quasi-steady from the point of view of the particles. Under such conditions our model may also fairly well describe the structure of the time-dependent double layer.

Before we can say that we understand the physics of double layers reasonably well we must be able to answer the following three questions: 1) What does the structure of the double layer look like? 2) How and why are double layers formed in plasmas? 3) How do double layers develop in time? In this connection it is important to stress what our double layer model can explain and what it cannot explain. From the above study it is clear that our model is capable of describing the structure of the double layer, both as regards the distributions of the charged particles and as concerns the variation of the electric field. By means of the model we have also been able to obtain relationships between various quantities such as the ion current density, the electron current density, the thickness, and the potential drop of the double layer.

Our model can, however, not tell us what potential drop or what thickness a double layer carrying a given current density will adopt. Thus, it is clear that our model can offer an answer to question 1) but not to the questions 2) and 3).

If we want to find the answers to the two latter questions it is not enough to study the double layer as an isolated phenomenon. Instead the investigation must be extended to include both the plasma that surrounds the double layer and the electric circuit in which the double layer is an integral part. On the one hand, it is necessary to take into account the surrounding plasma since it interacts closely with the double layer through fields and particles (Carlqvist, 1979a, 1979b). On the other hand, the electric circuit is of vital importance since it provides the voltage source and influences decisively the formation and development of the double layer (Alfvén, 1977, 1981). When the surrounding plasma and the external circuit are incorporated into the problem the complexity of the calculations unfortunately increases substantially. We must therefore conclude that, although we now have a great deal of knowledge of the physics of double layers, much work remains to be done before we can say that the double layer phenomenon is fully understood.

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Figure Captions

Fig. 1 Model of a strong and steady double layer of plane geometry. The double layer has the thickness, d, and sustains the potential drop, ϕ_{DL}. A current, i, consisting of an ion current, i_i, and an electron current, i_e, passes through the layer. The ions giving rise to, i_i, are emitted with zero velocity from the anode boundary while the electrons giving rise to, i_e, are emitted, also with zero velocity, from the cathode boundary. Inside the double layer the ions and electrons are accelerated in opposite directions by an electric field which is consistent with the accelerated charges themselves as described by Poisson's equation. It is assumed that the double layer is penetrated by a uniform magnetic field, B, being so strong that both ions and electrons are forced to move along the magnetic field lines.

Fig. 2 Ratio of the ion current density to the electron current density, i_i/i_e, as a function of the potential drop, ϕ_{DL}, given for two different double layers consisting of a) protons and electrons and b) alpha particles and electrons. The current ratios of the two layers differ noticeably for $\phi_{DL} << m_e c^2/e$ (non-relativistic regime) but both tend to one for $\phi_{DL} >> m_i c^2/Z e$ (relativistic regime).

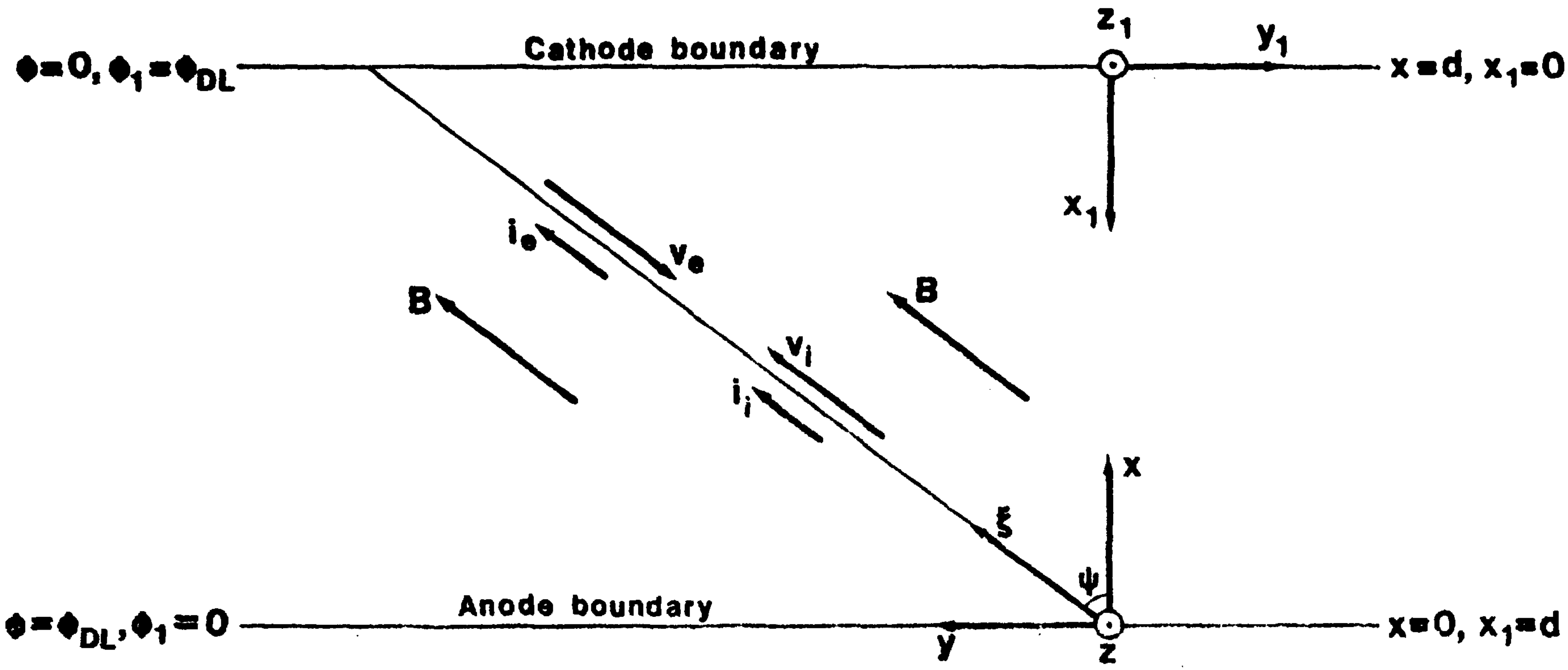
Fig. 3 Distributions of a) the densities of positive charges, $Z n_i(x)$, (dashed curve) and of negative charges, $n_e(x)$, (solid curve), b) the electric field, E(x), and c) the potential, $\phi(x)$, in a non-relativistic double layer of plane structure. The scales of the axes are all linear. The dashed-dotted line separates regions of opposite charge polarity. As is shown by the diagrams there is a prominent symmetry of the layer.

Fig. 4 Distributions of a) the densities of positive charges, $Z n_i(x)$, (dashed curve) and of negative charges, $n_e(x)$, (solid curve), b) the electric field, E(x), and c) the potential, $\phi(x)$, in a relativistic double layer of plane structure. The scales of the axes are all linear. The dashed-dotted line separates regions of opposite charge polarity. It is to be noticed that the relativistic double layer is strongly unsymmetrical.

Fig. 5 Potential drop, ϕ_{DL}, as a function of, $i\,d^2$, given for two different double layers consisting of a) protons and electrons and b) alpha particles and electrons. For small values of $i\,d^2$ ($i\,d^2 \ll 10^3$ A, implying non-relativistic double layers) the potential curves of the two layers are almost identical. For larger values of $i\,d^2$ ($i\,d^2 \gg 10^8$ A, implying relativistic double layers) the potential curves are clearly separated. It is to be noticed that the slopes of the curves are different in the relativistic and in the non-relativistic regimes.

Fig. 6 Idealized picture of two relativistic double layers carrying the same current density and with a finite value of ϕ_i/ϕ_e. Frames a) and c) refer to one double layer of thickness d_I while frames b) and d) refer to another double layer of thickness d_{II}. The charge distributions in the layers, $\rho(x)$, are shown in frames a) and b). There are two surface charges at the boundaries of each of the layers and a distributed charge of low density in between. The surface charges are equal in the two layers while the densities of the distributed charges differ. In frames c) and d) the consistent distributions of the electric field, $E(x)$, are displayed. Each of these distributions may be considered to be composed of one rectangular distribution and one triangular distribution.

Fig. 7 Idealized picture of two relativistic double layers carrying the same current density and with an infinite value of ϕ_i/ϕ_e. Frames a) and c) refer to one double layer of thickness, d_I, while frames b) and d) refer to another double layer of thickness d_{II}. The charge distributions in the layers are shown in frames a) and b). There is one surface charge at the anode boundary of each of the layers and a distributed charge of low density in between. The surface charges are equal in the two layers while the densities of the distributed charges differ. In frames c) and d) the consistent distributions of the electric field, $E(x)$, are depicted. Each of these distributions is of a triangular shape.



ϕ=0, ϕ1=ϕDL
Cathode boundary
z1
y1
x=d, x1=0
x1
ie
ve
B
vi
ii
B
x
ξ
ψ
ϕ=ϕDL,ϕ1=0
Anode boundary
y
z
x=0, x1=d

Fig. 1

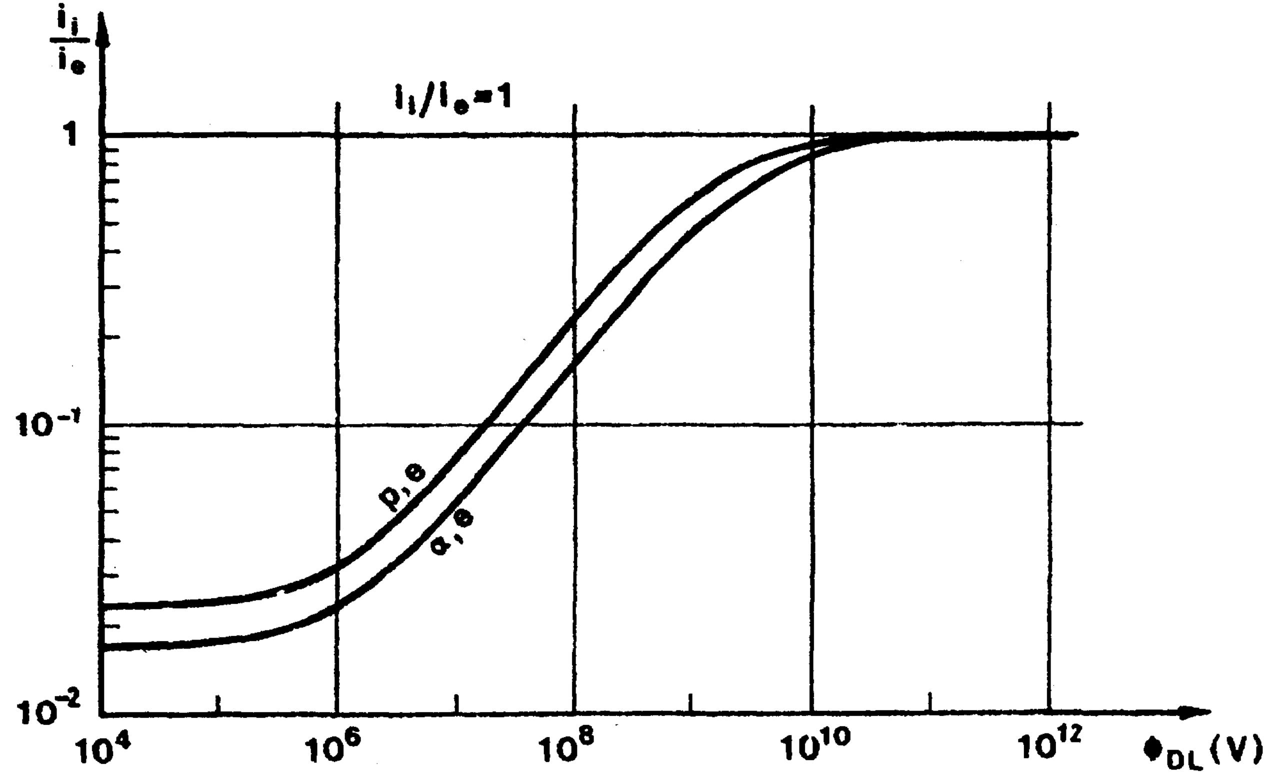


Fig. 2

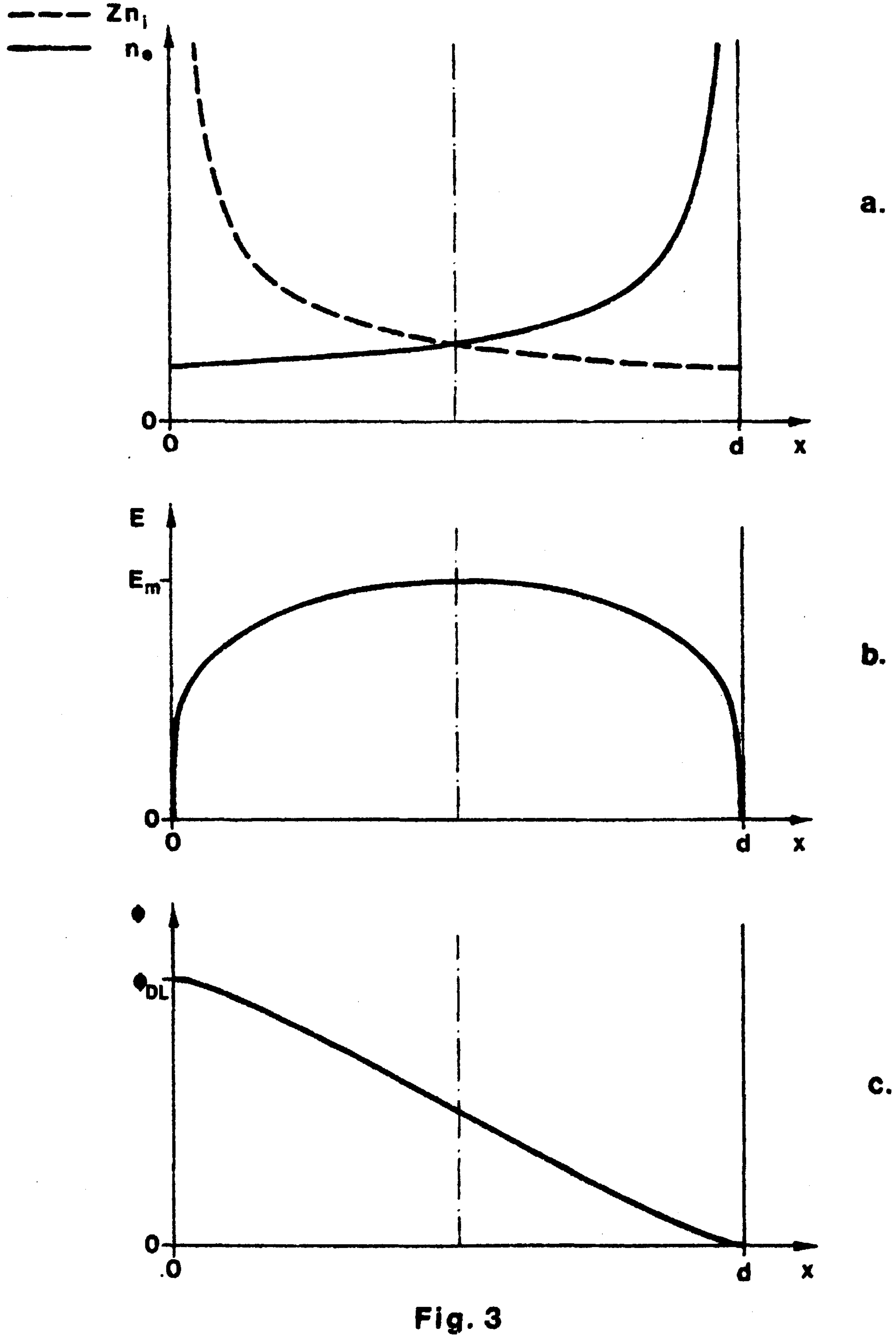


Fig. 3

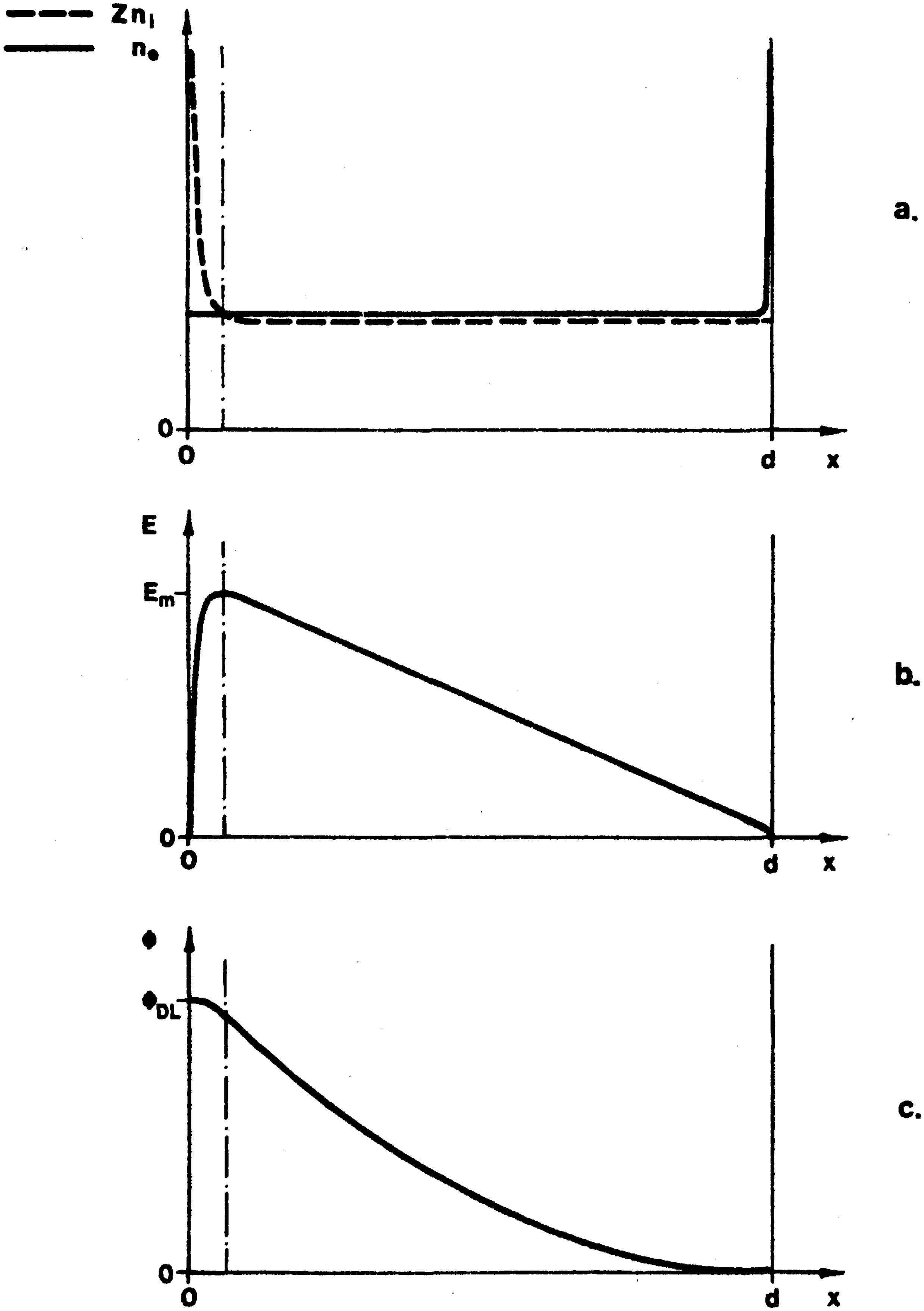
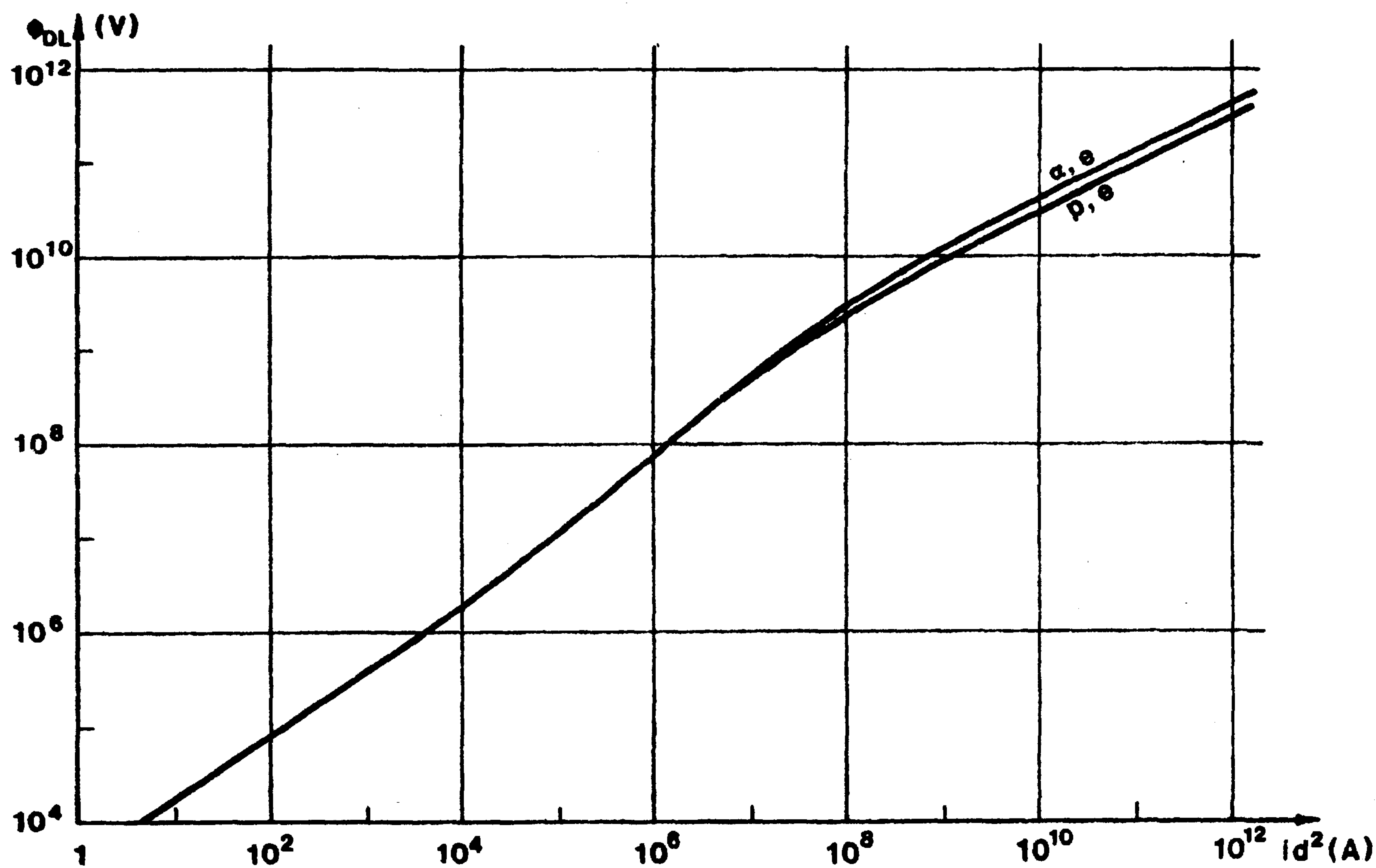
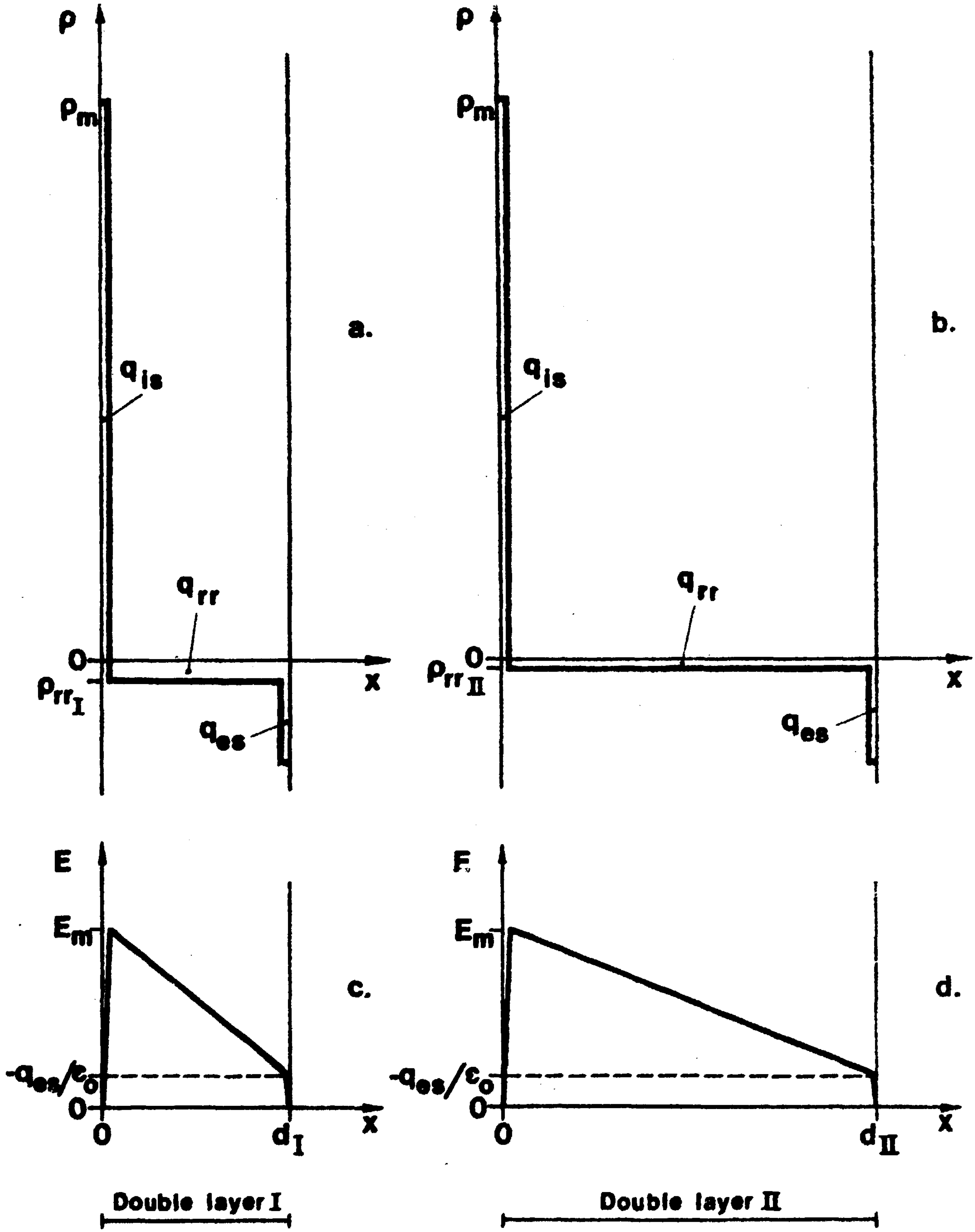


Fig. 4



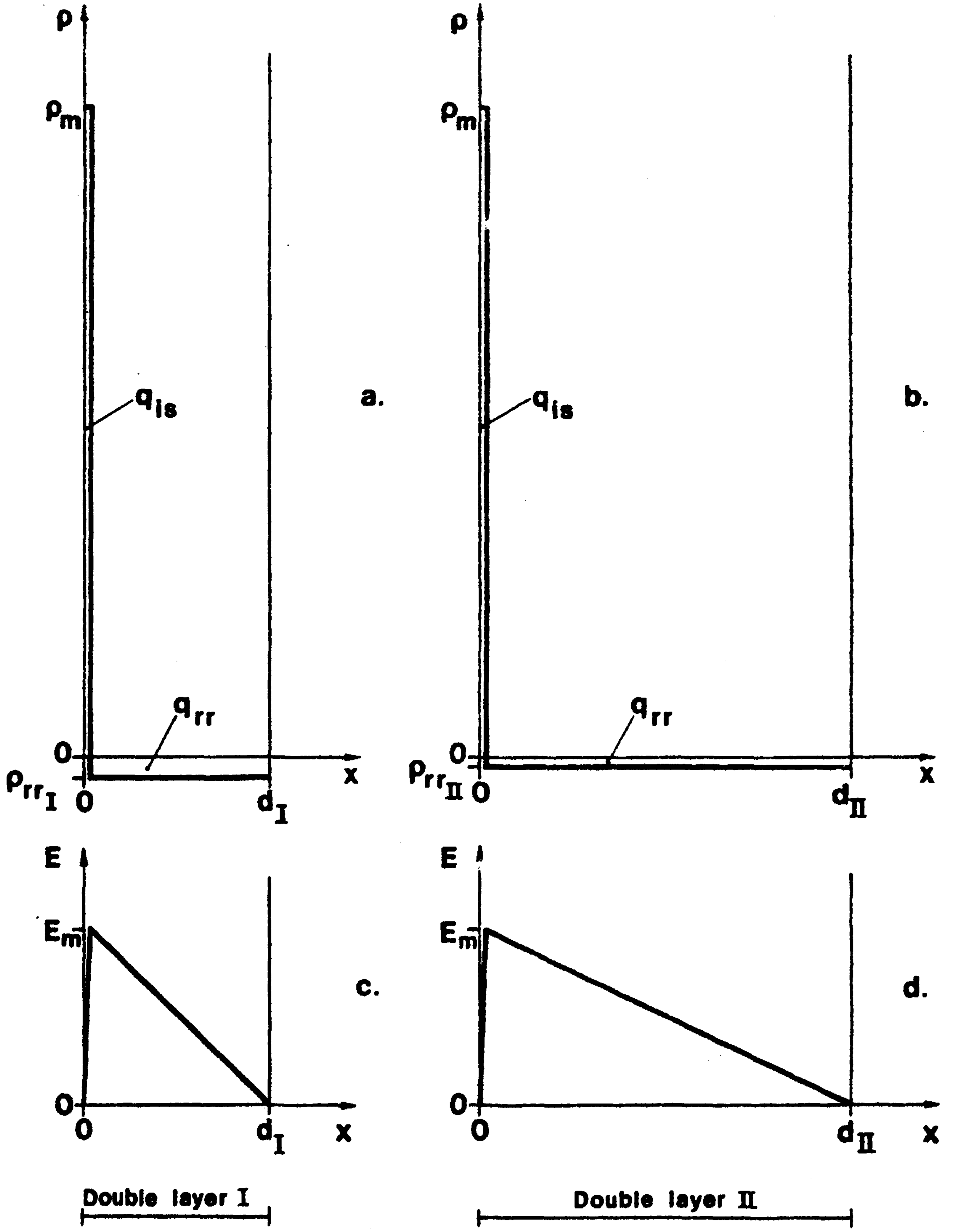
Φ_{DL} (V)
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
1
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
10^{12}
Id^2(A)
α, e
p, e

Fig. 5



ρ
ρ_m
q_{is}
q_{rr}
0
ρ_{rr_I}
q_{es}
x
a.
ρ
ρ_m
q_{is}
q_{rr}
0
$\rho_{rr_{II}}$
q_{es}
x
b.
E
E_m
$-q_{es}/\varepsilon_0$
0
0
d_I
x
c.
E
E_m
$-q_{es}/\varepsilon_0$
0
0
d_{II}
x
d.
Double layer I
Double layer II

Fig. 6



ρ
ρm
qis
qrr
0
ρrrI
0
dI
x
a.
ρ
ρm
qis
qrr
0
ρrrII
0
dII
x
b.
E
Em
0
0
dI
x
c.
E
Em
0
0
dII
x
d.
Double layer I
Double layer II

Fig. 7

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Royal Institute of Technology, Department of Plasma Physics,
S-100 44 Stockholm, Sweden

ON THE PHYSICS OF RELATIVISTIC DOUBLE LAYERS

P. Carlqvist
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A model of a strong, time-independent, and relativistic double layer is studied. Besides double layers having the electric field parallel to the current the model also describes a certain type of oblique double layers. The "Langmuir condition" (ratio of ion current density to electron current density) as well as an expression for the potential drop of the double layer are derived. Furthermore, the distributions of charged particles, electric field, and potential within the double layer are clarified and discussed. It is found that the properties of relativistic double layers differ substantially from the properties of corresponding non-relativistic double layers.

Key words: Double layer, High potential drop, Langmuir condition, Particle accelerator, Relativistic particles, Cosmic Plasma, Solar flares.