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Acoustic Loading Effects on Oscillating Rod Bundles

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This paper is concerned with the analytical study of the interaction between an infinite acoustic medium and a cluster of circular rods. The acoustic field due to oscillating rods and the acoustic loading on the rods are first solved in a closed form. The acoustic loading is then used as a forcing function for rod responses, and the acousto-elastic couplings are solved simultaneously. Numerical examples are presented for several cases to illustrate the effects of various system parameters on the acoustic reaction force coefficients. The effect of the acoustic loading on the coupled eigenfrequencies are discussed.

Introduction

Acoustic radiation and scattering due to oscillating structures and acoustic loading on the structures are companion effects in the interaction of fluid and solid. The motion of one or more structures in the acoustic medium will cause disturbances to be radiated and scattered into the fluid as sound waves. These sound waves may again excite the structures into motion. The interaction repeats itself over and over, resulting in a coupling effect between the motion of the fluid and the structure.

In the past, analytical studies of acousto-elastic vibration of slender rods were primarily made based on a single rod consideration [1-5]. For a group of elastic rods in a dense acoustic medium, the motion of a rod will excite the surrounding ones because of the fluid coupling; hence, there are many coupled modes and the rods will respond as a group rather than as a single rod [6]. The object of this paper is to provide a general methodology to solve the coupled motion between an infinite acoustic medium and a group of circular elastic[1] rods.

Acoustic Loading on Oscillating Rods

Consider a group of K circular cylinders[2] oscillating harmonically in an inviscid and adiabatically compressible fluid medium at rest, as shown in Fig. 1. Assuming that the disturbances caused by the motion of the cylinders are small enough such that the linearization theory is applicable, and that the cylinders are infinitely long and their axes are parallel so that the axial wave propagation is negligible, then the motion of the perturbed fluid is described by a two-dimensional linear acoustic wave equation [7]

$$\nabla^2\Phi = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \tag{1}$$

[1] Herein, "elastic" is deduced from the "Bernoulli-Eulerian" theory.
[2] Hereafter, the word "cylinder" will be used to denote the oscillating rod.

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Note: Appropriate SI Conversion Units — 1 in. = 2.54 cm, 1 ft = 0.3048 m

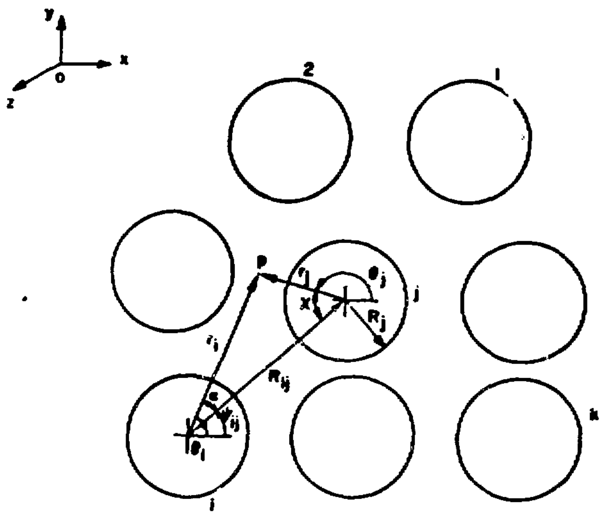

Fig. 1 A group of circular cylinders oscillating in an infinite compressible fluid

and the associated acoustic pressure is obtained as

$$p = \rho_\infty \frac{\partial\Phi}{\partial t} \tag{2}$$

where Φ is the perturbed velocity potential of the fluid, c the speed of sound of the unperturbed medium, ρ_∞ the density of the undisturbed fluid, and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

in two-dimensional cylindrical coordinates.

The appropriate boundary conditions are: 1) the normal component of the fluid velocity at the boundary of a cylinder is equal to the vibrational velocity of the cylinder in that direction, and 2) the wave has to satisfy the Sommerfeld radiation condition.

The solution to equation (1) for the velocity potential of the perturbances caused by the motion of K circular cylinders is composed of the partial fields due to each cylinder, namely

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$$\Phi = \sum_{i=1}^{K} \Phi_i \tag{3}$$

where

$$\Phi_i(r_i, \theta_i, t) = \sum_{m=0}^{\infty} (A_{im} \cos m\,\theta_i + B_{im} \sin m\,\theta_i)\, H_m^{(1)}(kr_i)\, e^{-i\omega t} \text{ [3]} \tag{4}$$

which includes all the possible radiation and scattering of the acoustic fields generated by the oscillating cylinder, numbered i, with other cylinders being kept stationary. $H_m^{(1)}(kr_i)$ is the Hankel function of the first kind of order m with argument kr_i, and (r_i, θ_i) are the circular coordinates associated with cylinder i. When kr_i becomes large, $H_m^{(1)}(kr_i)e^{-i\omega t}$ behaves as outgoing waves; hence, the Sommerfeld radiation condition is satisfied.

The total field Φ in equation (3) is written in terms of the local circular coordinates of each cylinder. To satisfy the boundary condition at the surface of a particular cylinder, the total field is expressed in terms of the circular coordinates of that cylinder by using the transformation of the cylindrical wave functions [8]. In doing so and using the boundary condition at each cylinder, one can reduce the problem of the acoustic radiation and scattering to an infinite set of algebraic equations for the unknown coefficients A_{im} and B_{im}. By truncating the index m to a finite number, the resulting equations can be solved for A_{im} and B_{im} by a high-speed digital computer.

Knowing A_{im} and B_{im}, the total field Φ is solved and the related quantities such as the pressure distribution, the reaction force, and the acoustic impedance, etc., can be computed accordingly. The components of the net reaction force acting on cylinder i per unit length can be written in a matrix form [9] as

$$\{F_j\} = [\gamma_{jl}^R]\{\ddot{w}_l\} + \omega[\gamma_{jl}^I]\{\dot{w}_l\} \tag{5}$$

where $\{\ddot{w}_l\}$ and $\{\dot{w}_l\}$ are, respectively, the column matrices of the acceleration and velocity of the oscillating cylinder, j and l are from 1 to $2K$. Matrix $[\gamma_{jl}^R]$ is called added-mass matrix and $[\gamma_{jl}^I]$ is called acoustic radiation-damping matrix, since the effect of the first term in equation (5) is to increase the inertias of the vibrating cylinders and the second term is to diminish the vibration amplitudes of the cylinders. The square matrices are proved symmetrical and dependent on the wavenumber, the cylinder radius, and the distance and orientation between cylinders [9].

Numerical results for the added-mass coefficients and the acoustic radiation damping coefficients for two cylinders of different radii in a row and for three cylinders and seven cylinders of equal radii arranged in arrays are worked out and shown in Figs. 2 to 5.

The results plotted in Figs. 2 and 3 show for small dimensionless wavenumbers both the added-mass and acoustic radiation-damping coefficients are monotonically decreasing when the gap-to-radius ratio increases, and eventually the self added-mass coefficients approach unity and the mutual added-mass coefficients and the acoustical radiation-damping coefficients become zero as the gap-to-radius ratio becomes sufficiently large ($G/R > 0(1)$). The trend of the variation of these coefficients with the change of the gap-to-radius ratio is consistent with the results obtained from incompressible flow theory [10] and experiments [11]. For moderate and large dimensionless wavenumber, the variation of these force coefficients with respect to a change of gap-to-radius ratio is not simply monotonic because complicated interference effects (or coupling effects) between waves and cylinders occur.

For a fixed value of gap-to-radius ratio, the variation of the acoustic reaction force coefficients with respect to a change of dimensionless wavenumbers can be seen from Figs. 4 and 5. For infinitesimal dimensionless wavenumbers ($kR \leq 0.03$), the added-mass coefficients are close to those obtained by incompressible flow theory [10], and the acoustic radiation-damping coefficients are practically zero. This implies that if the radius of the cylinder is very small compared to the acoustic wavelength, which is typical in many practical cases.

[3] In the combination of $i\omega$, i is defined as $\sqrt{-1}$.

Nomenclature

- A_{im}, B_{im} = undetermined constants in equation (4)
- c = speed of sound of the unperturbed fluid
- $[C_{jl}]$ = damping matrix
- EI = flexural rigidity of rod
- $\{F_j\}$ = column matrix of acoustic reaction force
- f, g = external forces on rod in x and y directions
- G = gap between adjacent rods
- $H_m^{(1)}$ = Hankel function of the first kind of order m
- $[I_{jl}]$ = unit matrix
- k = acoustic wavenumber
- K = total number of rods
- $[K_{jl}]$ = stiffness matrix
- m = mass of rod per unit length
- $[M_{jl}]$ = inertia matrix
- P = acoustic pressure
- $\{q_{ln}(t)\}$ = column matrix of normal coordinates
- r = radial coordinate
- R = radius of rod
- t = time
- u, v = x and y-components of rod displacement
- $\{\dot{w}\}, \{\ddot{w}\}$ = column matrices of velocity and acceleration of rod, respectively
- z = axial coordinate
- α_{ik}, σ_{ik} = coefficients of x-component of reaction force on rod i due to the acceleration of rod k in x and y directions, respectively
- β_{ik}, τ_{ik} = coefficients of y-component of reaction force on rod i due to acceleration of rod k in x and y directions, respectively
- $\eta(t), \xi(t)$ = normal coordinates
- $[\gamma_{jl}^I]$ = acoustic radiation-damping matrix
- $[\gamma_{jl}^R]$ = added-mass matrix
- θ = angular coordinate
- ρ_∞ = density of unperturbed fluid
- Φ = velocity potential of perturbed fluid
- ω = circular frequency of oscillation
- $\zeta(z)$ = normal mode of rod in vacuo
- ∇^2 = Laplacian operator

Subscripts

- i, k = refer to rod number, i.e., 1, 2, . . . , K
- j, l = are from 1 to $2K$
- m = refers to radial mode of acoustic wave
- n = refers to axial mode of flexural wave in rod

Superscripts

- I = imaginary part of complex number
- R = real part of complex number

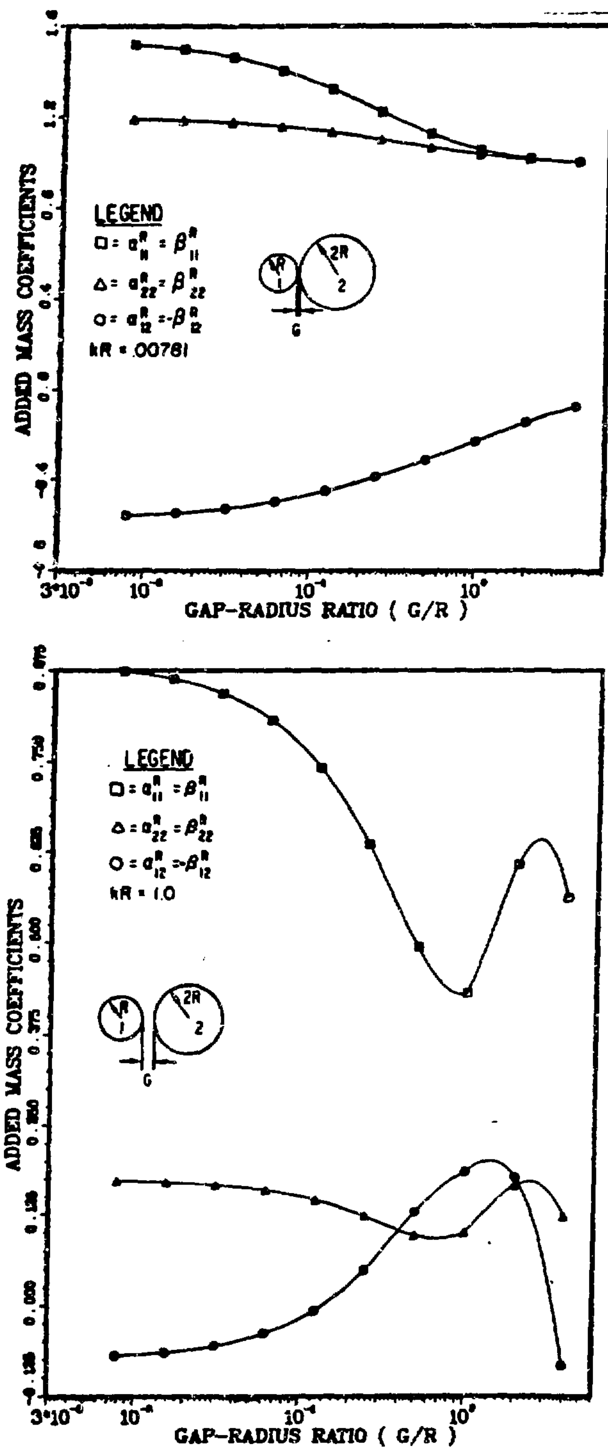

Fig. 2 Added-mass coefficients versus gap-radius ratio for different dimensionless wavenumbers

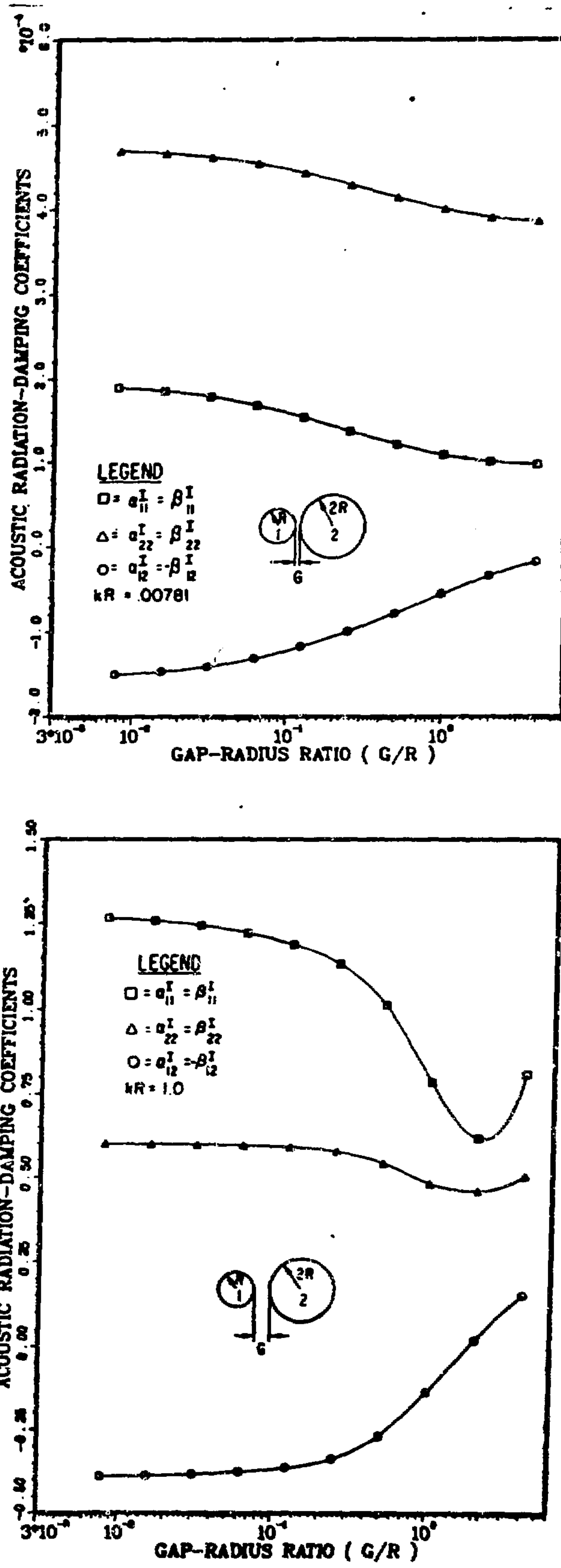

Fig. 3 Acoustic radiation-damping coefficients versus gap-radius ratio for different dimensionless wavenumbers

the effect of the added mass is dominant and the values of the added mass obtained by the incompressible potential theory can be used as valid approximation. As the dimensionless wavenumber increases, the absolute values of both the added mass and acoustic radiation-damping coefficients increase, reach to their maxima, and then decrease rapidly when the dimensionless wavenumber is increased further. When $kR \geq 1$, these coefficients oscillate with respect to changes in the

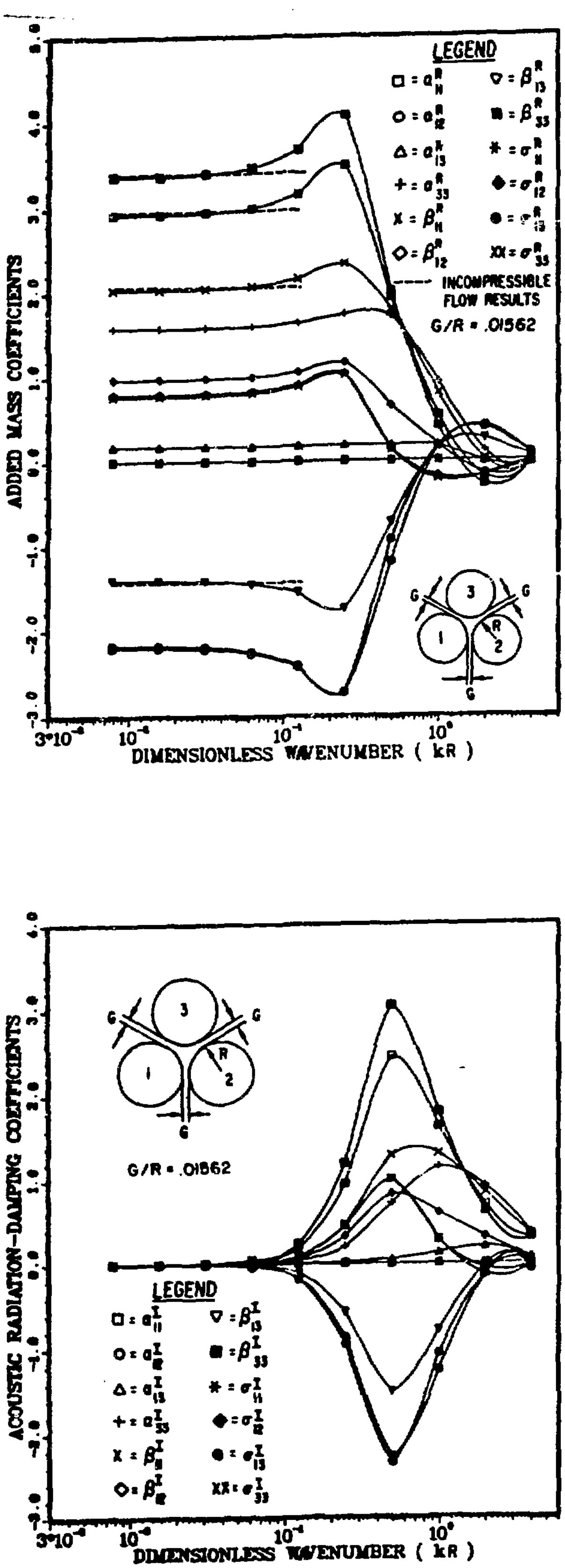

Fig. 4 Acoustic reaction force coefficients of three-cylinder array versus dimensionless wavenumber

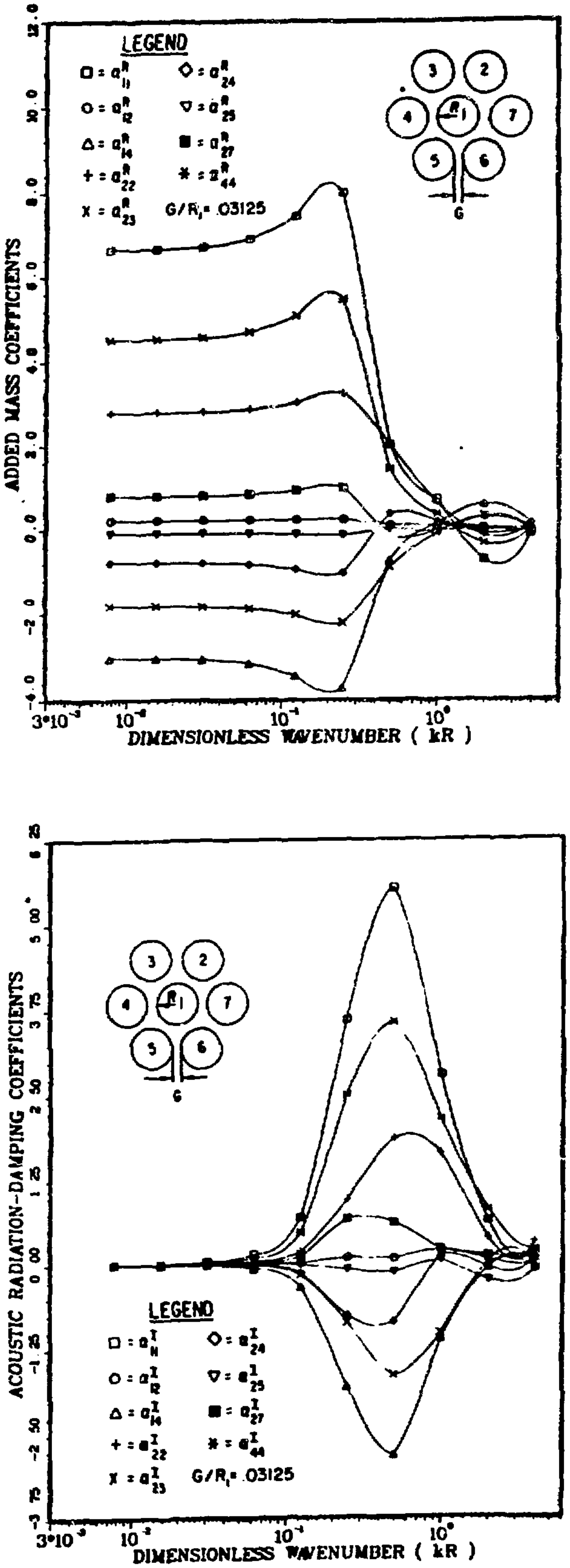

Fig. 5 Acoustic reaction force coefficients of seven-cylinder array versus dimensionless wavenumber

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dimensionless wavenumber, and their absolute values are generally small.

Vibration of a Cluster of Circular Rods in an Infinite Acoustic Medium

The acoustic reaction force derived in the previous section is used here as a forcing function for the vibration of a group of rods in an infinite acoustic medium. As shown in Fig. 1, each rod can move in the x and y directions. The equations of motion for rod i are

$$E_iI_i\frac{\partial^4 u_i}{\partial z^4}+m_i\frac{\partial^2 u_i}{\partial t^2}=f_i \quad (6)$$

in the x direction, and

$$E_iI_i\frac{\partial^4 v_i}{\partial z^4}+m_i\frac{\partial^2 v_i}{\partial t^2}=g_i \quad (7)$$

in the y direction, where E_iI_i is the flexural rigidity, m_i the mass per unit length, u_i, v_i the transverse displacements of rod i, and f_i and g_i, respectively, the external forces on the rod in the x and y directions. If no other forces are involved in the acousto-elastic system, the force on the rod will be only the hydrodynamic reaction of the acoustic medium.

Assuming all rods have the same types of boundary conditions in the x and y directions, then the normal modal functions in both directions will be the same. Using modal analysis, $u_i\ (z,t)$ and $v_i\ (z,t)$ can be expressed as

$$u_i(z,t)=\sum_{n=1}^{\infty}\xi_{in}(t)\zeta_n(z) \quad (8)$$

$$v_i(z,t)=\sum_{n=1}^{\infty}\eta_{in}(t)\zeta_n(z) \quad (9)$$

where $\xi_{in}(t)$ and $\eta_{in}(t)$ are the normal coordinates, and $\zeta_n(z)$ the normal mode of rods in vacuo. Substituting expressions (8,9) into equations (6,7) and using expression (5) for the acoustic reaction force, multiplying the results by $\zeta_m(z)$, and integrating over the range of the span of a rod, we have the following set of ordinary equations in $\xi_{in}(t)$ and $\eta_{in}(t)$

$$[M_{jl}]\{\ddot q_{in}\}+[C_{jl}]\{\dot q_{in}\}+[K_{jl}]\{q_{in}\}=\{0\} \quad (10)$$

where

$$[M_{jl}] = [\gamma^R_{jl}]+m_j[I_{jl}],\ j \text{ and } l = 1,2,\ldots,2K,$$

$$[C_{jl}] = [\gamma^I_{jl}],$$

$$[K_{jl}] = m_j\omega^2_{jn}[I_{jl}], \quad (11)$$

$$\{q_{in}(t)\} = \begin{Bmatrix}\xi_{in}(t)\\ \eta_{in}(t)\end{Bmatrix},\ i=1,2,\ldots,K,$$

$[I_{jl}]$ is a unit matrix,

and

$$n=1,2,3,\ldots,\infty.$$

Due to acoustic coupling, $\xi_{in}(t)$ is coupled with $\eta_{in}(t)$ through the inertia and damping terms for rods oscillating in a stationary acoustic medium. For each value of n, there are $2K$ equations in $\xi_{in}(t)$ and $\eta_{in}(t)$. Equation (10) is the basic equation used to study dynamics of the self-excited acousto-elastic system of a single rod or bundles of rods.

For harmonic motion of rods, we can write

$$q_{in}=\hat q_{in}e^{-i\omega t} \quad (12)$$

$\hat q_{in}$ being complex constants indicating the vibrational amplitudes and phases. Substituting expression (12) into equation (10) and manipulating the result, we have

$$[-[M_{jl}]\omega^2-[C_{jl}]i\omega+[K_{jl}]]\{\hat q_{in}\}=\{0\} \quad (13)$$

The only nontrivial solutions for $\hat q_{in}$ are obtained under the condition that the determinant of the coefficients of equation (13) vanishes. This condition then leads to an eigenvalue equation (or frequency equation) for the acousto-elastic system, from which the eigenfrequencies ω_n can be obtained.

Since the acoustic reaction force coefficients are dependent on wavenumber ω_n/c, the frequency equation is highly transcendental. An iteration scheme is, therefore, needed to determine ω_n when the geometric arrangement of rod is known. Once the admissible frequency is obtained, the ratios of $\hat q_{in}/\hat q_{in}$ can be determined from equation (13) and hence can the corresponding mode be obtained.

Instead of solving the transcendental equations for frequencies and modes, the following example is presented to illustrate the effect of acoustic loading on the system parameters of the vibration of circular rods.

The speeds of sound in the air and in the water at room temperature are, respectively, about 110 ft/s and 4877 ft/s. The radii R of the fuel rods in a reactor are of the order of 0.5 in. For low frequency vibration, say $f < 200$ Hz, the values of dimensionless wavenumber kR are 0.0476 and 0.0108 for rod/air and rod/water interactions. As shown in the previous section, the acoustic reaction force is independent of oscillation frequency and its real part (i.e., the added mass) is more dominant than its imaginary part (i.e., the radiation damping) for $kR \le 0.03$. The effect of acoustic loading in this case is to reduce the values of natural frequencies of the coupled system. The coefficient matrix of equation (13) can be diagonalized to obtain natural frequencies and normal modes of the system since $[C_{jl}]$ is vanishingly small for infinitesimal dimensionless wavenumbers.

For high frequency vibration, say $f > 10^4$ Hz, the values of kR are 4.76 and 1.08 for rod/air and rod/water interactions, respectively. Both the added mass and the acoustic radiation damping are very small for dimensionless wavenumbers greater than unity. However, the acoustic radiation damping is more dominant than the added mass because a small accession in mass will not significantly alter the natural frequencies while a small value of damping will cause energy to be radiated. The effect of acoustical loading in this case is to change the total damping of the system. The coupled frequencies of the system are approximately the same as in vacuo, but the corresponding modal shapes are altered by acoustic radiation damping.

For vibration of moderate frequencies such that the dimensionless wavenumber kR being of order of unity, the interaction between the motion of the rods and the fluid becomes very complicated. The coupled frequencies and modal configurations must be determined by solving the transcendental equations. The acoustic loading alters both the frequencies and modal shapes of the acousto-structure system.

Conclusions

This paper provides a general method of analysis for acoustic loading effect on circular rods oscillating in an infinite compressible fluid medium. The hydrodynamic reaction force due to the acoustic field generated by the moving rods is first solved, and the interaction between the motion of the rods and the fluid is then presented.

When the acoustic wavelength is large compared with the cylinder radius (i.e., $kR << 0(1)$), the added mass is more dominant than the radiation damping, and both are independent of oscillation frequency. Incompressible potential flow theory thus provides a good approximation for the fluid

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loading for low-frequency vibration. On the other hand, when the acoustic wavelength is small compared with the cylinder radius (i.e., $kR > O(1)$), the radiation damping is more dominant than the added mass, and both are small.

For low-frequency oscillation, the effect of the acoustic loading is to alter the vibration characteristics, especially to reduce the natural frequency of the acousto-elastic system. For high-frequency oscillation, the acoustic loading does not change the coupled frequency, but the modal shape of the system.

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