IC/82/57
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

# TOWARDS AN EFFECTIVE BILOCAL THEORY FROM QCD IN A BACKGROUND *

J.A. Magpantay
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

Using the path integral, we show how we can get background gauge invariant bilocals (to be identified with mesons) from QCD in a non-trivial ground state. We discuss in this paper mainly the formal manipulations, especially how to deal with the zero modes. We also discuss the large N limit of QCD in a background.

MIRAMARE - TRIESTE
May 1982

* To be submitted for publication.

## I. INTRODUCTION

The problem we will try to address to in this paper is how we can derive an effective field theory for mesons starting from QCD [1]. The physical idea we will invoke is that low-energy hadron physics just might be given by QCD in a non-trivial ground state [2] which we will identify as the background field. This means that hadrons are formed via the interactions of gluons and quarks in a non-trivial ground state. The mathematical machinery we will use is the path-integral formulation and the background decomposition of a field [3].

The organization of this paper is as follows: In Sec.II we discuss the background field decomposition and the physical significance of the background. In Sec.III, we deal with the zero mode problem. We show how to restore the global symmetries before we carry out the introduction of bilocals. This guarantees that the bilocal theory (and by identification the mesons) will respect the global symmetries. In Sec.IV we carry out the introduction of bilocals and derive the bilocal effective action. In Sec.V, we discuss the interesting limit of large N. We conclude with some comments. The appendix carries a discussion of the approximation used in the Faddeev-Popov determinant.

## II. THE QCD GROUND STATE AND THE BACKGROUND FIELD DECOMPOSITION

We will assume the following scenario. Pure Yang-Mills theory will determine the QCD ground state. This ground state characterized by $\tilde{A}_\mu(x)$ is a solution of the classical field equation

$$\left.\frac{\delta S_{YM}}{\delta A_\mu}\right|_{A=\hat{A}} = 0 . \tag{1}$$

We will then assume that the path integral is dominated by this classical configuration. The "gluon" is then a quantum or gaussian fluctuation about this classical field $\tilde{A}_\mu$ .

In general the classical field $\tilde{A}_\mu(x)$ will depend on some parameters of the global symmetries of the pure Yang-Mills theory, i.e. $\tilde{A}_\mu = \tilde{A}_\mu(x,w)$ and $w$ collectively represents the parameters. In this case, we do have zero modes to contend with because the classical solution $\tilde{A}_\mu(x,w)$ breaks the symmetry corresponding to $w$, i.e.

$$\left.\frac{\delta^2 S_{YM}}{\delta A^2}\right|_{A=\tilde{A}} \frac{\delta \tilde{A}}{\delta \omega} = 0. \tag{2}$$

These zero modes have to be dealt with properly to restore the (broken) global symmetries in the quantum theory. This is done by the collective co-ordinate method [4).

Now, when we add the fermions, we will assume that the ground state will not be perturbed at all. The fermions and gluons will then evolve and interact in the presence of this background. The result of this fermion-gluon interaction in this background is hopefully the observed hadron physics in low energy regime.

The above picture is the physical scenario that we will assume for QCD. Now let us look at the technical aspect of expanding about a background. This means writing the full potential $A_\mu$ as

$$A_\mu = \tilde{A}_\mu + a_\mu \quad , \tag{3}$$

where we will identify the $a_\mu$ as the gluon field. Using the gauge transformation of the full $A_\mu$, i.e.

$$A'_\mu = \Omega A_\mu \Omega^{-1} - \frac{i}{g}(\partial_\mu \Omega)\Omega^{-1}, \tag{4}$$

we find that we can assign the following transformations:

$$\tilde{A}_\mu \to \Omega \tilde{A}_\mu \Omega^{-1} - \frac{i}{g}(\partial_\mu \Omega)\Omega^{-1} \quad , \tag{5a}$$

$$a_\mu \to \Omega a_\mu \Omega^{-1} \quad . \tag{5b}$$

This is the so-called background gauge transformations and its advantage lies in the fact that when we make the expansion

$$S[\tilde{A}+a] = S[\tilde{A}] + \frac{1}{2}\frac{\delta^2 S}{\delta \tilde{A}^2} a^2 + \frac{1}{3!}\frac{\delta^3 S}{\delta \tilde{A}^3} a^3 + \frac{1}{4!}\frac{\delta^4 S}{\delta \tilde{A}^4} a^4, \tag{6}$$

each term is separately invariant under (5a) and (5b).



When we fix the gauge, we want a gauge-fixing term that is covariant under (5a) and (5b), the reason being that we want $\mathcal{L}_{g.f.}$ to be background invariant. This requirement is satisfied by the background gauge condition

$$f[\tilde{A}, a] = D_\mu(\tilde{A})\, a_\mu \quad . \tag{7}$$

The corresponding Faddeev-Popov determinant is evaluated by using the so-called "quantum" gauge transformation (in infinitesimal form)

$$\tilde{A}_\mu \to \tilde{A}_\mu \quad ; \quad a_\mu \to a_\mu + \frac{1}{g} D_\mu(\tilde{A}+a)\Lambda \quad . \tag{8}$$

The resulting Faddeev-Popov determinant is

$$\Delta_f = \det\left(D_\mu(\tilde{A})\, D_\mu(\tilde{A}+a)\right) \quad , \tag{9}$$

which is invariant under (5a) and (5b). We will approximate this by

$$\Delta_f = \det\left(D_\mu(\tilde{A})\, D_\mu(\tilde{A})\right) \quad , \tag{10}$$

which is also invariant under (5a) and (5b). This approximation is discussed in the appendix.

In the following sections, whenever we speak of gauge invariance, we refer to invariance under background gauge transformations (5a) and (5b). Also, we will not specify the background field keeping the discussion as general as possible. As for the actual form of $\tilde{A}_\mu$, it is hoped that the resulting theory for mesons which critically depends on the background will lead to the determination of $\tilde{A}_\mu$.

## III. COLLECTIVE CO-ORDINATES AND RESTORATION OF THE GLOBAL SYMMETRIES

We will start from the path-integral for QCD (SU(N) with fermions in the fundamental representation)

$$W = \int dA\, d\psi\, d\bar{\psi}\; \Delta_f\, \delta(f)\, \exp.i\int d^4x \left\{-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ + gA\!\!\!/)\psi\right\} . \tag{11}$$

Using the background decomposition (3) and keeping up to $O(a^2)$ due to the assumption of $\tilde{A}$ being a dominant field configuration

$$W \sim \int da\, d\psi\, d\bar{\psi}\, \det(D_\mu(\hat{A}) D_\mu(\hat{A}))\, \exp i \int d^4x \Big\{ g\bar{\psi}\not{a}\psi + \frac{1}{2} a^a_\mu(x)\Theta^{ab}_{\mu\nu}(x)\, a^b_\nu(x) + \bar{\psi}(i\not{\partial} + g\hat{\not{A}})\psi \Big\} , \tag{12}$$

where

$$\Theta^{ab}_{\mu\nu} = \frac{\delta^2 S}{\delta\hat{A}^a_\mu \delta\hat{A}^b_\nu} + \frac{1}{\alpha}(D_\mu(\hat{A}) D_\nu(\hat{A}))^{ab}$$

$$= \Big[ g_{\mu\nu} D_\lambda(\hat{A}) D_\lambda(\hat{A}) - (1-\frac{1}{\alpha}) D_\mu(\hat{A}) D_\nu(\hat{A}) - 2ig F_{\mu\nu}(\hat{A}) \Big]^{ab} . \tag{13}$$

We already stated in Sec.II that if the classical solution breaks any of the global symmetries of the pure Yang-Mills action, then there will be a zero mode $\partial\tilde{A}/\partial w_i$ for each of the broken symmetries. However, in evaluating (12), the relevant zero modes are those of the operator $\theta$ defined by (13). The zero modes of $\theta$ are actually given by

$$z^a_\mu(x) = \frac{\delta\tilde{A}^a_\mu}{\delta w} + \frac{1}{g} D^{ab}_\mu(\hat{A})\Lambda^b , \tag{14a}$$

and $\Lambda^b$ is solved by imposing

$$D^{ab}_\mu(\hat{A})\, z^b_\mu = 0 . \tag{14b}$$

This means that the relevant zero mode is just $\partial\tilde{A}/\partial w$ gauge rotated so as to satisfy the background gauge condition.

We will assume for simplicity that the zero modes are square-integrable in $R^4$. Even if they are not, we can always introduce cut-offs and recover the infinite volume limit at the end.

The zero modes in (12) are taken into account via collective co-ordinate technique. The result is



$$W \sim \int (\prod_j dw_j)\, M(w)\, d\psi\, d\bar{\psi}\, \det{}'(D_\mu(\hat{A}) D_\mu(\hat{A}))(\det{}'\theta)^{-1/2}\, e^{iS_{YM}(\hat{A})}$$
$$\times \exp i \int d^4x\, d^4y \Big\{ \bar{\psi}(x)\big[(i\not{\partial} + g\hat{\not{A}})\delta^4(x,y)\big]\psi(y)$$
$$- \frac{1}{2} g^2 \lambda^a_{ij}\lambda^b_{kl}\, \tilde{G}^{ab}_{\mu\nu}(x,y)\, \bar{\psi}_i(x)\gamma_\mu\psi_j(x)\, \bar{\psi}_k(y)\gamma_\nu\psi_l(y) \Big\} , \tag{15}$$

where det' indicates zero eigenvalues excluded and

$$M(w) = \Big[\det\big(\langle z_i(x) | z_j(x)\rangle\big)\Big]^{-1/2} \det\Big(\langle \frac{\delta\tilde{A}}{\delta w_i} | z_j \rangle\Big) . \tag{16}$$

The Green function $\tilde{G}^{ab}(x,y)$ is well-defined because it does not include the zero modes. It satisfies the equation

$$\Theta^{ab}_{\mu\nu}(x)\, \tilde{G}^{bc}_{\nu\lambda}(x,y) = \delta^{ac} g_{\mu\lambda}\delta^4(x-y) - \sum_j z^a_\mu(x)\, z^c_\lambda(y) . \tag{17}$$

At this point, we may introduce bilocals $\phi(x,y) \sim \bar{\psi}(x)\,\psi(y)$ and convert (15) into a bilocal expression. However, there are problems that we will meet in doing this. First, we still have to restore the global symmetries by doing the collective co-ordinates or else we will end up with a bilocal theory that does not respect the global symmetries. Once the bilocals are introduced, the collective co-ordinate integrals will be difficult to do because the collective co-ordinate $w$ (which appears in $\tilde{A}(x,w)$ and $\tilde{G}$ ) will be inside certain complicated terms. Thus recoving back the global symmetries will be very difficult.

To see this in detail, consider the eigenfunctions of the operator $i\not{\partial} + g\hat{\not{A}}$, i.e.

$$(i\not{\partial} + g\hat{\not{A}})\, f_\alpha(x) = \lambda_\alpha\, f_\alpha(x) . \tag{18}$$

In cases when the background has non-trivial topology, i.e. characterized by integer number

$$\mathcal{M} = -\frac{1}{16\pi^2}\int d^4x \; \mathrm{tr}\left[F_{\mu\nu}(\hat{A})\, {}^*F_{\mu\nu}(\hat{A})\right], \tag{19}$$

we know that the operator $i\partial\!\!\!/ + g\tilde{A\!\!\!/}$ have zero modes [5]. We can express the quartic term in terms of bilocal fields $\phi(x,y)$, i.e.

$$\exp.-\frac{1}{2}g^2\int d^4x\, d^4y\, \lambda^a_{ij}\lambda^b_{kl}\,\tilde{G}^{ab}_{\mu\nu}(x,y;\hat{A})\,\bar\psi_i(x)\gamma_\mu\psi_j(x)\,\bar\psi_k(y)\gamma_\nu\psi_l(y)$$

$$= \int d\phi(x,y)\,\exp\left\{-\frac12\int d^4x\,d^4y\left[\phi^2(x,y) - \bar\psi(x)\,\mathcal{H}(x,y)\,\phi(x,y)\,\psi(y)\right]\right\}$$

and $\mathcal{H}(x,y)$ is chosen such that we get the quartic term after completing the squares. Substituting this in (15) and integrating out the fermions, we will get

$$W \sim \int d\phi(x,y)\int\Big(\prod_j dw_j\Big)\, M(w)\, \det{}'\big(D_\mu(\hat{A})D_\mu(\hat{A})\big)\big(\det{}'\Theta\big)^{-1/2} e^{iS_{YM}(\hat{A})}$$

$$\times \prod_\beta \int d^4x\, d^4y \; \mathop{\mathrm{tr}}_{\text{color}}\Big(\bar{f}^{(0)}_\beta(x)\,\mathcal{H}(x,y)\,\phi(x,y)\,f^{(0)}_\beta(y)\Big)$$

$$\times \prod_\alpha{}' \int d^4x\, d^4y \; \mathop{\mathrm{tr}}_{\text{color}}\Big(\bar{f}_\alpha(x)\big[D\!\!\!/(\hat{A})\,\delta^4(x-y) + \mathcal{H}(x,y)\,\phi(x,y)\big] f_\alpha(y)\Big)$$

$$\times \exp.-\frac12\int d^4x\, d^4y\; \phi^2(x,y)\,. \tag{20}$$

The terms appearing in (20) are: $f^{(0)}_\beta$ is a zero mode of $D\!\!\!/(\tilde{A})$, $f_\alpha(x)$ is a non-zero mode of $D\!\!\!/(\tilde{A})$. The collective co-ordinate parameters are implicit in $f^{(0)}_\beta(x)$, $f_\alpha(x)$ and $\mathcal{H}(x,y)$. It is clear from (20) that the collective co-ordinate integration or restoring the global symmetries cannot be carried out in a way that will give us an effective bilocal action.

What we propose to do then is carry out the collective co-ordinate integration first to restore all the global symmetries in the non-local, four-Fermi theory given by (15).

The zero mode integration involves the following:

$$\int\Big(\prod_j dw_j\Big)\,\bar{M}(w)\,\exp.\Big\{ig\int d^4x\,\bar\psi(x)\,\tilde{A\!\!\!/}(x,w)\,\psi(x)$$
$$-\frac{i}{2}g^2\int d^4x\, d^4y\, \lambda^a_{ij}\lambda^b_{kl}\,\tilde{G}^{ab}_{\mu\nu}(x,y)\,\bar\psi_i(x)\gamma_\mu\psi_j(x)\,\bar\psi_k(y)\gamma_\nu\psi_l(y)\Big\}, \tag{21}$$



where

$$\bar{M}(w) = M(w)\, e^{iS_{YM}(\hat{A})}\, \det{}'\big(D_\mu(\hat{A})D_\mu(\hat{A})\big)\big(\det{}'\Theta\big)^{-1/2}. \tag{22}$$

Expanding (21) in powers of $(\bar\psi\psi)^n$, we will get something like

$$C\Big\{1 + \sum_{n=1}^{\infty} g^n \int d^4x_1 \ldots d^4x_n\, \Xi^{a_1\ldots a_n}_{\mu_1\ldots\mu_n}(x_1,\ldots x_n)$$
$$\times\big(\lambda^{a_1}\ldots\lambda^{a_n}\big)\,\bar\psi(x_1)\gamma_{\mu_1}\psi(x_1)\ldots\ldots\bar\psi(x_n)\gamma_{\mu_n}\psi(x_n)\Big\} \tag{23}$$

where the $\Xi$'s are basically given by integrals over the collective co-ordinates of terms containing $\tilde{A}_\mu(x,w)$ and $\tilde{\bar{G}}^{ab}_{\mu\nu}(x,y;w)$.

Eq.(23) will not be useful if we cannot exponentiate it back to the form

$$\exp.\Big\{ig\int d^4x\,\bar\psi(x)\,B\!\!\!/(x)\,\psi(x) - \frac{i}{2}g^2\int d^4x\, d^4y\,\lambda^a_{ij}\lambda^b_{kl}$$
$$\times g_{\mu\nu}\,\bar{G}^{ab}(x,y)\,\bar\psi_i(x)\gamma_\mu\psi_j(x)\,\bar\psi_k(y)\gamma_\nu\psi_l(y)\Big\}. \tag{24}$$

The reason we want the form (24) is so that we can carry out the introduction of the bilocals, except that the fermions will see an effective background field $B_\mu(x)$.

One case when we can express (23) in the form given by (24) is when we have factorization in the collective co-ordinate integration.

A. <u>Factorization in the collective co-ordinate integration</u>

Although this situation is highly improbable, we will discuss it because of some interesting results. Let us define the collective co-ordinate averaged field $b_\mu(x)$

$$b_\mu(x) = \frac{1}{C}\int\Big(\prod dw_j\Big)\,\bar{M}(w)\,\tilde{A}_\mu(x,w) = \langle\tilde{A}_\mu(x,w)\rangle_w\,, \tag{25a}$$

where $C$ is given by the $(\bar\psi\psi)^0$ term, i.e.

$$C = \int (\prod_j d\omega_j)\, \bar{M}(\omega) \qquad (25b)$$

We say that factorization exists if

$$b_{\mu_1}(x_1) \ldots b_{\mu_m}(x_m) = \langle \tilde{A}_{\mu_1}(x_1,\omega) \ldots \tilde{A}_{\mu_m}(x_m,\omega) \rangle_\omega \ . \qquad (26)$$

Equating (23) to (24), we find that

$$B_\mu(x) = b_\mu(x) \ , \qquad (27)$$

provided $g_{\mu\nu}\bar{G}^{ab}(x,y)$ is determined consistently.

Now to order $(\bar{\Psi}\psi)^2$ in (23) and (24) we find

$$\int(\prod_j d\omega_j)\, \bar{M}(\omega) \int d^4x\, d^4y\, \lambda^a_{ij} \lambda^b_{kl}\, \bar{\Psi}_i(x) \gamma_\mu \Psi_j(x)\, \bar{\Psi}_k(y) \gamma_\nu \Psi_l(y)\, \tilde{G}^{ab}_{\mu\nu}(x,y;\tilde{A})$$

$$= \int d^4x\, d^4y\, \lambda^a_{ij} \lambda^b_{kl}\, \bar{\Psi}_i(x) \gamma_\mu \Psi_j(x)\, \bar{\Psi}_k(y) \gamma_\nu \Psi_l(y)\, g_{\mu\nu} \bar{G}^{ab}(x,y) \ . \qquad (28)$$

Because of factorization, we can prove that

$$\langle \tilde{G}^{ab}_{\mu\nu}(x,y;\hat{A}) \rangle_\omega = G^{ab}_{\mu\nu}(x,y;B) = g_{\mu\nu} \bar{G}^{ab}(x,y) \ . \qquad (29)$$

To prove the first step in (29), we go back to Eq.(17) and expand $\theta^{ab}_{\mu\nu}$

$$\left\{ \delta^{ab}\left[g_{\mu\nu}\partial^2 - \left(1-\frac{1}{\alpha}\right)\partial_\mu\partial_\nu\right] + g\, Q^{ab}_{\mu\nu}(\tilde{A}) \right\} \tilde{G}^{bc}_{\nu\lambda}(x,y) =$$

$$\delta^{ac} g_{\mu\lambda}\, \delta^4(x,y) - \sum \mathfrak{z}^a_\mu(x)\, \mathfrak{z}^c_\lambda(y)$$

where

$$Q^{ab}_{\mu\nu}(\hat{A}) = -f^{abc}\Big\{ 2 g_{\mu\nu} \tilde{A}^c_\alpha \partial_\alpha - \left(1-\frac{1}{\alpha}\right)\left(\hat{A}^c_\mu \partial_\nu + \tilde{A}^c_\nu \partial_\mu\right)$$

$$+ g_{\mu\nu}(\partial_\alpha \tilde{A}^c_\alpha) - \left(1-\frac{1}{\alpha}\right)(\partial_\mu \hat{A}^c_\nu) + 2 F^c_{\mu\nu}(\tilde{A}) \Big\}$$

$$+ g f^{acd} f^{cbe}\left[ g_{\mu\nu} \hat{A}^d_\alpha \tilde{A}^e_\alpha - \left(1-\frac{1}{\alpha}\right) \tilde{A}^d_\mu \tilde{A}^e_\nu \right]$$

We can formally expand $\tilde{G}^{ab}_{\mu\nu}$ in terms of the Green function $\mathcal{G}_{\mu\lambda}(x,y,\alpha)$ defined by

$$\left[g_{\mu\nu}\partial^2 - \left(1-\frac{1}{\alpha}\right)\partial_\mu\partial_\nu\right] \mathcal{G}_{\nu\lambda}(x,y) = g_{\mu\lambda}\, \delta^4(x,y) \ . \qquad (30)$$

The expansion is given by iteration, i.e.

$$\tilde{G}^{ab}_{\mu\nu}(x,y) = \delta^{ab}\mathcal{G}_{\mu\nu}(x,y;\alpha) - \int d^4z\, \mathcal{G}_{\mu\lambda}(x,z;\alpha)\left[ Q^{ac}_{\lambda\delta}(z)\, \tilde{G}^{cb}_{\delta\nu}(z,y) - \sum_{\mathfrak{z}} \mathfrak{z}^a_\mu(z)\, \mathfrak{z}^b_\nu(y) \right]$$

$$= \delta^{ab}\mathcal{G}_{\mu\nu}(x,y,\alpha) - \int d^4z\, \mathcal{G}_{\mu\lambda}(x,z) \sum_{\mathfrak{z}} \mathfrak{z}^a_\mu(z)\, \mathfrak{z}^b_\nu(y)$$

$$- \int d^4z\, \mathcal{G}_{\mu\lambda}(x,z,\alpha)\, Q^{ab}_{\lambda\delta}(z,\hat{A})\, \mathcal{G}_{\delta\nu}(z,y;\alpha) + \ldots\ldots \qquad (31)$$

Substituting (31) in (28), we will get

$$\int d^4x\, d^4y\, \lambda^a_{ij} \lambda^b_{kl}\, \bar{\Psi}_i(x) \gamma_\mu \Psi_j(x)\, \bar{\Psi}_k(y) \gamma_\nu \Psi_l(y) \Big\{ \delta^{ab}\mathcal{G}_{\mu\nu}(x,y;\alpha)$$

$$- \int d^4z\, \mathcal{G}_{\mu\lambda}(x,z;\alpha) \sum_{\mathfrak{z}} \mathfrak{z}^a_\mu(z)\, \mathfrak{z}^b_\nu(y) - \int d^4z\, \mathcal{G}_{\mu\lambda}(x,z;\alpha)\, Q^{ab}_{\lambda\delta}(z;\hat{A})\, \mathcal{G}_{\delta\nu}(z,y;\alpha)$$

$$+ \ldots\ldots \Big\} .$$

However, we have the general consistency requirement

$$\int d^4y\, \lambda^b_{kl}\, \bar{\Psi}_k(y) \gamma_\nu \Psi_l(y)\, \mathfrak{z}^b_\nu(y) = 0 \ ,$$

which says that using the non-singular Green function is only consistent if the source is orthogonal to the zero mode [6]. Thus, we can forget about all the zero mode terms in (28). Doing the collective co-ordinate integration now and using factorization, we will just substitute $B_\mu$ for every $\tilde{A}_\mu$ in (31) and drop the zero modes. This proves the first part of (29). The $g_{\mu\nu}$ structure of $G^{ab}_{\mu\nu}$ must then follow because Lorentz invariance is restored after doing the Lorentz collective co-ordinates.

After establishing (29) in $(\bar{\psi}\psi)^2$, it follows from factorization that

$$\left\langle \tilde{G}^{a_1 b_1}_{\mu_1\nu_1}(x_1,y_1;\tilde{A}) \cdots\cdots \tilde{G}^{a_m b_m}_{\mu_m \nu_m}(x_m,y_m;\tilde{A}) \right\rangle_\omega$$

$$= g_{\mu_1\nu_1}\bar{G}^{a_1 b_1}(x_1,y_1) \cdots\cdots g_{\mu_m\nu_m}\bar{G}^{a_m b_m}(x_m,y_m) \ . \qquad (32)$$

This completes the consistency of equating (23) to (24).

There is an interesting trivial case where there is factorization and that is when

$$B_\mu(x) = 0 \ . \qquad (33)$$

This will give $G^{ab}_{\mu\nu}(x,y;\ B=0) = \delta^{ab} g_{\mu\nu}(x,y;\alpha)$. The $g_{\mu\nu}$ structure can easily be arrived at by taking $\alpha = 1$ Eq.(15) becomes

$$W \sim \int d\psi d\bar{\psi}\ \exp. i\int d^4x\, d^4y \Big\{ \bar{\psi}(x)\, i\partial\!\!\!/\, \delta^4(x,y)\,\psi(y)$$

$$- \tfrac{1}{2} g^2 \lambda^a_{ij}\lambda^a_{k\ell}\, \bar{\psi}_i(x)\gamma_\mu\psi_j(x)\,\bar{\psi}_k(y)\gamma_\mu\psi_\ell(y)\ \mathcal{g}(x-y)\Big\} , \qquad (34)$$

where $\mathcal{g}(x-y)$ is just the Coulomb potential in Euclidean four dimensions, i.e. $|x-y|^{-2}$. When (34) is expressed in terms of bilocal fields, the resulting meson physics will not see at all the non-trivial ground state. This does not seem too promising.

So the relevant question is, can we have factorization in the collective co-ordinate integration and at the same time have a non-trivial $B_\mu(x)$? The answer lies in the following observations. First, under gauge transformation



$$B_\mu(x) \to \Omega B_\mu \Omega^{-1} - \frac{i}{g}(\partial_\mu \Omega)\Omega^{-1} \ . \qquad (35a)$$

Because of factorization

$$D_\mu(B)\, F_{\mu\nu}(B) = 0 , \qquad (35b)$$

$$n(B) = -\frac{1}{16\pi^2}\int d^4x\ \mathrm{tr}\left[F_{\mu\nu}(B)\ {}^*F_{\mu\nu}(B)\right] = n(\hat{A}) \ . \qquad (35c)$$

Thus, if $\tilde{A}_\mu(x,w)$ has a non-trivial topology, then $B_\mu(x)$ cannot be zero because $n(B) = n(\tilde{A}) \neq 0$. On the other hand, if we used an $\tilde{A}$ with trivial topology, i.e. $n(\tilde{A}) = 0$, then $n(B) = 0$ and by (35a) we can always do a topology non-changing gauge transformation to have $B_\mu = 0$. This will give us the case given by (34). Eq.(35b) says $B$ itself is a classical solution.

B. <u>Absence of factorization in the collective co-ordinate integration</u>

If we do not have factorization, i.e. (26) is not satisfied, Eq.(23) can be written as

$$C \exp\Big\{ ig\int d^4x\, \bar{\psi}(x) B\!\!\!/(x)\psi(x) - \tfrac{1}{2}g^2\int d^4x\, d^4y\ \lambda^a_{ij}\lambda^b_{k\ell}\,\bar{G}^{ab}(x,y)\,\bar{\psi}_i(x)\gamma_\mu\psi_j(x)\,\bar{\psi}_k(y)\gamma_\mu\psi_\ell(y)\Big\}$$

$$\times \exp\Big\{ i\sum_{m=3}^{\infty} g^m \int d^4x_1 \cdots d^4x_m\ \mathcal{P}^{a_1\cdots a_m}_{\mu_1\cdots\mu_m}(x_1,\dots,x_m)(\lambda^{a_1}\cdots\lambda^{a_m})$$

$$\times(\bar{\psi}(x_1)\gamma_{\mu_1}\psi(x_1))\cdots(\bar{\psi}(x_m)\gamma_{\mu_m}\psi(x_m))\Big\} \ . \qquad (36)$$

Equating (36) to (23), we determine $B_\mu(x)$, $g_{\mu\nu}\bar{G}^{ab}(x,y)$, $\mathcal{P}^{a_1\cdots a_m}_{\mu_1\cdots\mu_m}$ in terms of $\Xi^{a_1\cdots a_m}_{\mu_1\cdots\mu_m}$. The order $(\bar{\psi}\psi)$ and $(\bar{\psi}\psi)^2$ will determine $B_\mu$ and $g_{\mu\nu}\bar{G}^{ab}$. However, when we go to $O((\bar{\psi}\psi)^3)$ and higher, the equations get more complicated. For example

$$B_\mu(x) = b_\mu(x) = \frac{1}{C}\int\Big(\prod_j dw_j\Big)\bar{M}(w)\ \tilde{A}_\mu(x,w) \ , \qquad (37a)$$

$$g_{\mu\nu}\bar{G}^{ab}(x,y) = i\left[B^a_\mu(x)B^b_\nu(y) - \frac{1}{C}\int\Big(\prod dw\Big)\bar{M}(w)\,\tilde{A}^a_\mu(x,w)\,\hat{A}^b_\nu(y,w)\right]$$

$$+ \frac{1}{C}\int\Big(\prod_j dw_j\Big)\bar{M}(w)\ \tilde{G}^{ab}_{\mu\nu}(x,y;\tilde{A}) \ , \qquad (37b)$$

$$\mathcal{P}^{abc}_{\mu\nu\lambda}(x,y,z) = \frac{1}{6}\Big\{\Big[\frac{1}{C}\int\prod_j dw_j\, \bar{M}(w)\, \tilde{A}^a_\mu(x)\, \tilde{A}^b_\nu(y)\, \hat{A}^c_\lambda(z) - B^a_\mu(x)\, B^b_\nu(y)\, B^c_\lambda(z)\Big] + \frac{\lambda}{2}\Big[B^a_\mu(x)\, g_{\nu\lambda}\, \bar{G}^{bc}(y,z) - \frac{1}{C}\int\prod dw_j\, \bar{M}(w)\, \hat{A}^a_\mu(x)\, \tilde{G}^{bc}_{\nu\lambda}(y,z;\hat{A})\Big]\Big\} . \tag{37c}$$

Note that in the limit of factorization, all the $\mathcal{P}^{a_1\ldots a_m}_{\mu_1\ldots\mu_m} \to 0$, (37b) gives the old result and we recover all the results in Sec.III.A.

We will assume that the order $(\bar\psi\psi)^3$ terms and higher in (36) can be treated perturbatively. This means we have the appropriate background so as to render these terms perturbative corrections. We will look at the following:

$$W_0 \sim \int d\psi\, d\bar\psi\, \exp\, i\int d^4x\, d^4y \Big\{ \bar\psi(x)\left[(i\partial + gB)\,\delta^4(x-y)\right]\psi(y) - \frac{i}{2}g^2 \lambda^a_{ij}\lambda^b_{k\ell}\, \bar{G}^{ab}(x,y)\, \bar\psi_i(x)\gamma_\mu\psi_j(x)\, \bar\psi_k(y)\gamma_\mu\psi_\ell(y) \Big\} . \tag{38}$$

When we express (38) in terms of the bilocals, we can guarantee that the bilocal theory will respect the global symmetries.

We just emphasize at this point that (38) is the starting point for both cases of with or without factorization. The difference is that with factorization (38) is exact while without factorization we have to consider the $(\bar\psi\psi)^3$ terms and higher given in (36) as perturbative corrections. Secondly, with factorization, $B_\mu(x)$ has non-trivial topology if $\tilde{A}_\mu(x,w)$ has non-trivial topology (see (35c)). This does not have to be the case when we do not have factorization.

## IV. DERIVATION OF THE BILOCAL THEORY

We will carry out this process for the no factorization case and with $B_\mu(x)$ and $\bar{G}^{ab}$ given by (37a) and (37b). We want the no factorization case so that there is a possibility that even if $\tilde{A}_\mu(x,w)$ has non-trivial topology, i.e. $n(\tilde{A}) \neq 0$, we may have $n(B) = 0$. Then, we do not have to deal



with zero modes of the operator $i\partial\!\!\!/ + gB\!\!\!/$. So our first and second requirement for the background is no factorization and

$$n(B) = -\frac{1}{16\pi^2}\int d^4x\, \mathrm{tr}\, F_{\mu\nu}(B)\, {}^*F_{\mu\nu}(B) = 0 . \tag{39}$$

The third requirement on the background is that we can treat perturbatively the terms proportional to $\mathcal{P}^{a_1\ldots a_m}_{\mu_1\ldots\mu_m}(x_1,\ldots,x_n)$. With these three assumptions, we can now carry out the introduction of bilocals in (38).

The quartic term is explicitly gauge-invariant. To prove this, we note that since $g^{ab}_{\mu\nu}$ is covariant under (5), then $\tilde{G}\,(x,y)$ is also covariant

$$\tilde{G}\,(x,y;\hat{A}) \to \Omega_{adj}(x)\, \tilde{G}(x,y;\hat{A})\, \Omega^{-1}_{adj}(y) . \tag{40}$$

Using (5a),(35a)and(40) in (37b) we find that $\bar{G}^{ab}(x,y)$ is also covariant. Since the fermions transform according to $\psi(x) \to \Omega_{fund.}(x)\,\psi(x)$, then we find that the quartic term is explicitly invariant.

To extract the gauge invariant bilocals in this quartic term, we need to do Fierz reordering , i.e.

$$\bar\psi_i(x)\gamma_\mu\psi_j(x)\, \bar\psi_k(y)\gamma_\mu\psi_\ell(y) = \sum_\alpha \left(\bar\psi_i(x)\Gamma_\alpha\psi_\ell(y)\right)\left(\bar\psi_k(y)\Gamma_\alpha\psi_j(x)\right),$$

$$\Gamma_\alpha = \left\{ 1,\ i\gamma_5,\ \frac{i}{\sqrt{2}}\gamma_\mu,\ \frac{1}{\sqrt{2}}\gamma_\mu\gamma_5 \right\} . \tag{41}$$

Furthermore, we need to rewrite $\lambda^a_{ij}\lambda^b_{k\ell}\,\bar{G}^{ab}(x,y)$. There are $(N^2-1)^2$ terms in $\bar{G}^{ab}$. We define

$$\lambda^a_{ij}\lambda^b_{k\ell}\,\bar{G}^{ab}(x,y) \equiv Q_{i\ell}(x,y)\,Q_{kj}(y,x) - \frac{1}{N}R_{ijk\ell}(x,y) . \tag{42}$$

Eq.(42) solves for Q and R in terms of $\bar{G}^{ab}$. There are $N^2$ terms in $Q_{ij}$ and $N^4$ terms in $R_{ijk\ell}$. There are only $(N^2-1)^2$ equations in (42) because the $\lambda^a$'s are traceless. We will get then $2N^2$ constraints, i.e.

$$\sum_\ell \left[ Q_{i\ell}(x,y)\,Q_{\ell j}(y,x) - \frac{1}{N}R_{ij\ell\ell}(x,y) \right] = 0 , \tag{43a}$$

$$\sum_i \left[ Q_{i\ell}(x,y) Q_{ki}(y,x) - \frac{1}{N} R_{iik\ell}(x,y) \right] = 0. \tag{43b}$$

This will give us $N^2-1$ more unknowns than equations. This means we are free to add $N^2-1$ more constraints (and we will impose them on the R's) provided the gauge transformation consistency of (42) is maintained.

The left-hand side of (42) under background gauge transformation goes into

$$\lambda^a_{ij} \lambda^b_{k\ell} \left( \delta^{aa'} - f^{aa'c} \Lambda^c(x) \right) \bar{G}^{a'b'}(x,y) \left( \delta^{b'b} + f^{b'bd} \Lambda^d(y) \right) \tag{44a}$$

Eq.(42) is consistent if Q and R transform as tensors under the fundamental, i.e.

$$Q_{i\ell}(x,y) \to \left( \delta_{ii'} - i \lambda^c_{ii'} \Lambda^c(x) \right) Q_{i'\ell'}(x,y) \left( \delta_{\ell'\ell} + i \lambda^d_{\ell'\ell} \Lambda^d(y) \right), \tag{44b}$$

$$R_{ijk\ell}(x,y) \to \left( \delta_{ii'} - i \lambda^a_{ii'} \Lambda^a(x) \right) \left( \delta_{j'j} + i \lambda^b_{j'j} \Lambda^b(x) \right) R_{i'j'k'\ell'}(x,y)$$
$$\times \left( \delta_{kk'} - i \lambda^c_{kk'} \Lambda^c(y) \right) \left( \delta_{\ell'\ell} + i \lambda^d_{\ell'\ell} \Lambda^d(y) \right). \tag{44c}$$

Thus, the gauge transformed fields will also satisfy (42) if Q and R transform according to (44b) and (44c), respectively.

The reader may argue why express $\lambda^a_{ij} \lambda^b_{k\ell} \bar{G}^{ab}$ in the form given by (42). The reason is because we can construct gauge-invariant mesons by using (41) and (42). This is the quantity

$$\phi_\alpha(x,y) = \bar{\psi}_i(x) \Gamma_\alpha R_{i\ell}(x,y) \psi_\ell(y) \quad . \tag{45}$$

This represents a quark and anti-quark at two points connected by a coloured bilocal "string-like" object. The colour of Q is due to the background field and we note that the net colour at each end is zero. Another reason for (42) is that in the limit $\bar{G}^{ab} \simeq \delta^{ab} \bar{G}(x,y)$ (which will most probably happen after the internal global symmetry is restored) we find that



$$\lambda^a_{ij} \lambda^b_{k\ell} \delta^{ab} \bar{G}(x,y) = \left( \delta_{i\ell} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{k\ell} \right) \bar{G}(x,y), \tag{46}$$

and we easily identify $Q_{i\ell}(x,y) = \delta_{i\ell}(\bar{G}(x,y))^{1/2}$ and $R_{ijk\ell}(x,y) = \delta_{ij}\delta_{k\ell}\bar{G}(x,y)$.

We will substitute the above results in (38). Now, we will have to neglect for the moment the $R_{ijk\ell}$ term by arguing that they are down by $\frac{1}{N}$ anyway. This argument may look weak at this point but in the large N limit (which we will discuss in the next section), this is a good approximation.

Introducing the bilocals given by (45), we can write (38), after integrating out the quarks as

$$W_0 \sim \int d\phi_\alpha(x,y) \exp \Big\{ -i \operatorname*{tr}_{\substack{\text{Dirac} \\ \text{color} \\ xy}} \ln \Big[ (i\partial + g \not{B}) \delta^4(x,y)$$
$$+ g Q(x,y) \sum_\beta \Gamma_\beta \phi_\beta(y,x) \Big] - \frac{1}{2} \int d^4x \, d^4y \sum_\beta \phi^2_\beta(x,y) \Big\}. \tag{47}$$

The bilocal effective action is then given by

$$\Gamma = -\frac{1}{2} \int d^4x \, d^4y \sum_\beta \phi^2_\beta(x,y) - i \operatorname*{tr}_{\substack{\text{Dirac} \\ \text{Color} \\ x,y}} \ln \Big[ (i\partial + g\not{B}) \delta^4(x,y)$$
$$+ Q(x,y) \sum_\beta \Gamma_\beta \phi_\beta(y,x) \Big] \quad . \tag{48}$$

Once we have (47) or (48) we can carry out the standard procedure to derive an effective field for the bilocals.

To end this section we reiterate the main steps that led to (47). First, we assumed no factorization in the collective co-ordinate integration and n(B) = 0. Second, we assumed that the $\mathcal{P}^{a_1 \ldots a_m}_{\mu_1 \ldots \mu_m}$ dependent terms are perturbative corrections. Third, we used the 1/N behaviour to also treat the $R_{ijk\ell}$ term perturbatively.

Of all the three assumptions, the third is the least acceptable in QCD because N = 3. The first and second on the other hand can be used as a constraint on the possible $\tilde{A}_\mu(x,w)$.

## V. THE LARGE N LIMIT

From the work of 't Hooft [7] and as expounded by Witten [8], perturbative QCD is dominated by planar graphs with just one complete fermion loop at the boundary in the limit of large N. In the 1+1 world, this set of graphs can be summed exactly to give the meson spectrum. In four dimensions, this is not the case because of the three and four gluon couplings.

What we will do here is to carry out the programme discussed in II to IV for large N. The first thing to do is to pinpoint the N dependence. To agree with the perturbative expansion we rescale $g \to \frac{g}{\sqrt{N}}$. The field equation thus becomes

$$\partial_\mu\left(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu - \frac{ig}{\sqrt{N}}[\tilde{A}_\mu, \tilde{A}_\nu]\right) - \frac{ig}{\sqrt{N}}\left[\tilde{A}_\mu, \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu - \frac{ig}{\sqrt{N}}[\tilde{A}_\mu, \tilde{A}_\nu]\right] = 0 . \tag{49}$$

It is erroneous to drop the non-linear terms in the limit $N \to \infty$. By rescaling the fields

$$\tilde{A}_\mu(x) = \sqrt{N}\ A'_\mu(x) \tag{50}$$

we will get

$$\sqrt{N}\left\{\partial_\mu\left(\partial_\mu A'_\nu - \partial_\nu A'_\mu - ig[A'_\mu, A'_\nu]\right) - ig\left[A'_\mu, \partial_\mu A'_\nu - \partial_\nu A'_\mu - ig[A'_\mu, A'_\nu]\right]\right\} = 0 . \tag{49'}$$

The solution of (49'), i.e. the $A'_\mu(x,w)$ will now be explicitly N independent. We note that (50) is precisely the form of instantons for SU(N).

We can repeat everything that was done in Secs.III and IV, only we have to substitute $A'_\mu(x,w)$ instead of $\tilde{A}_\mu(x,w)$ in $\theta^{ab}_{\mu\nu}(x)$, $\tilde{G}^{ab}_{\mu\nu}$, etc. The difference comes in the term $g\bar{\psi}A\!\!\!/\psi \to \frac{g}{\sqrt{N}}\bar{\psi}A\!\!\!/\psi$. This will induce the quartic term

$$-\frac{g^2}{2N}\int d^4x\, d^4y\, \lambda^a_{ij}\lambda^b_{kl}\,\tilde{G}^{ab}_{\mu\nu}(x,y;A')\,\bar{\psi}_i(x)\gamma_\mu\psi_j(x)\,\bar{\psi}_k(y)\gamma_\nu\psi_l(y) .$$



The derivation of the bilocal theory starts from

$$W \sim \int d\psi\, d\bar{\psi} \int\left(\prod_j dw_j\right) M(w)\, \det{}'(D_\mu(A')D_\mu(A'))\,(\det{}'\theta)^{-1/2}\, e^{iS_{YM}(\tilde{A})}$$
$$\times \exp i\int d^4x\, d^4y \left\{\bar{\psi}(x)\left[(i\partial\!\!\!/ + gA\!\!\!/')\,\delta^4(x,y)\right]\psi(y)\right.$$
$$\left. -\frac{1}{2}g^2\lambda^a_{ij}\lambda^b_{kl}\left(\frac{1}{N}\right)\tilde{G}^{ab}_{\mu\nu}(x,y;A')\,\bar{\psi}_i(x)\gamma_\mu\psi_j(x)\,\bar{\psi}_k(y)\gamma_\nu\psi_l(y)\right\} . \tag{51}$$

Note that this is very much like (15), the only difference being the 1/N factor that goes with $\tilde{G}^{ab}_{\mu\nu}(x,y;A')$. Integrating out the collective co-ordinates, we will get an expression like (36). The coefficients of the $(\bar{\psi}\psi)^n$ terms are given by

$$C = \int\left(\prod dw_j\right)\bar{M}(w) , \tag{52a}$$

$$B'_\mu(x) = \frac{1}{C}\int\left(\prod dw_j\right)\bar{M}(w)\, A'_\mu(x,w) , \tag{52b}$$

$$g_{\mu\nu}\bar{G}^{ab}(x,y) = i\left[B'^a_\mu(x)B'^b_\nu(y) - \frac{1}{C}\int\left(\prod dw_j\right)\bar{M}(w)\,A'^a_\mu(x,w)A'^b_\nu(y,w)\right]$$
$$+\frac{1}{C}\left(\frac{1}{N}\right)\int\left(\prod dw_j\right)\bar{M}(w)\,\tilde{G}^{ab}_{\mu\nu}(x,y;A') , \tag{52c}$$

$$P^{abc}_{\mu\nu\lambda}(x,y,z) = \frac{1}{6}\left[\frac{1}{C}\int\left(\prod dw_j\right)\bar{M}(w)\,A'^a_\mu(x,w)\,A'^b_\nu(y,w)\,A'^c_\lambda(z,w)\right.$$
$$\left. - B'^a_\mu(x)B'^b_\nu(y)B'^c_\lambda(z)\right] + \frac{i}{2}\left[B'^a_\mu(x)\bar{G}^{bc}_{\nu\lambda}(y,z)\right.$$
$$\left. -\frac{1}{C}\left(\frac{1}{N}\right)\int\left(\prod dw_j\right)\bar{M}(w)\,A'^a_\mu(x,w)\,\tilde{G}^{bc}_{\nu\lambda}(y,z;A')\right] . \tag{52d}$$

and so on.

What we have to do is to look for the N dependence of the quantities given in (52a) to (52d). Aside from the obvious 1/N that goes with every $\tilde{G}^{ab}$, there will be other N dependence. Firstly, the collective co-ordinates can be separated into space-time part (maximium of 15 parameters for the conformal group) and internal global symmetry (in this case the subgroup of SU(N) which does notleave the classical solution invariant). Thus, M(w) given by (16a) will have an N dependence which will go like $(N)^{cN}$ where c is a positive constant. Secondly, the term

$$\det{}'(D_\mu(A')D_\mu(A'))(\det{}'\theta)^{-1/2}\, e^{iS_{YM}(\tilde{A})}$$

will also have an N dependence. As an example, for instantons in SU(N) we find [8]

$$\det{}'(D_\mu(A')D_\mu(A'))(\det{}'\theta)^{-1/2}\, e^{iS_{YM}(\tilde{A})} =$$

$$\cdot \exp\left[\alpha(1) - 2(N-2)\alpha(\tfrac{1}{2}) - N\left(\frac{8\pi^2}{g^2} - \frac{11}{3}\ln(\mu_0\rho)\right)\right] . \tag{53a}$$

Furthermore, after integrating out the internal global parameters, one finds that [9]

$$C = \left(\frac{4}{\pi^2}\right)\frac{e^{[\alpha(1)-2(N-2)\alpha(1/2)]}}{(N-1)!\,(N-2)!}\int \frac{d^4z\, d\rho}{\rho^5}\left(\frac{4\pi^2}{g^2}N\right)^{2N} e^{-N\left(\frac{8\pi^2}{g^2}-\frac{11}{3}\ln(\mu_0\rho)\right)} . \tag{53b}$$

It is sufficient to say that the instanton example tells us that the N behaviour of the quantities defined by (52a) and (52d) is rather involved. Thus, we cannot say offhand if we can neglect the $\mathcal{P}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}$ terms. Further discussions about the instanton will be left for the future.

In this paper we will use the "known" behaviour of mesons in the limit of large N to serve as a guideline to determine the constraints on the background field $\tilde{A}_\mu$. Mesons in the large N limit are free and non-interacting. This means that (51) and (52) must yield an effective bilocal theory such that after taking the colour trace, we get the form [10]

$$W = \int d\phi_\alpha(x,y)\, \exp -(N\Gamma(\phi)) , \tag{54a}$$

$$\Gamma(\phi) = \frac{1}{2}\int d^4x\, d^4y \sum_\beta \phi^2_\beta(x,y) + i \operatorname*{tr}_{\substack{\text{Dirac}\\ x,y}} \ln\Big[(i\partial + gB)\delta^4(x,y) + g\,Q(x,y)\sum_\beta \Gamma_\beta \phi_\beta(y,x)\Big] , \tag{54b}$$



where B(x) and Q(x,y) no longer have colour indices.

This means that the background field $\tilde{A}_\mu(x,w) = \sqrt{N}\, A'_\mu(x,w)$ must satisfy the following

i) $$\left(B'_\mu(x)\right)_{ij} = \lambda^a_{ij}\, B'^a_\mu(x) = \delta_{ij} B_\mu(x) , \tag{55a}$$

i.e. the effective field seen by the fermion is colour diagonal. Since trace $(\lambda^a)$ is zero, the only possibility is $B'_\mu(x) = 0$.

ii) Furthermore, Eq.(52b) must give

$$\bar{G}^{ab}(x,y) = \delta^{ab} Q^2(x,y) . \tag{55b}$$

iii) The large N behaviour must yield

$$\mathcal{P}^{a_1\ldots a_n}_{\mu_1\ldots\mu_n}(x_1,\ldots x_n) \sim \left(\frac{1}{N}\right)^{n+1} , \tag{55c}$$

at least so that we can drop all these terms in (36).

If we have all these requirements, we will get the effective action given by (54b) which guarantees that for large N, mesons are not interacting.

There is a trivial solution to (55a,b,c). This is when we have factorization and $n(A') = 0$. In this case $\mathcal{P}^{a_1\ldots a_m}_{\mu_1\ldots\mu_m}$ is exactly zero and Q(x,y) is the square-root of the Coulomb field.

## VI. DISCUSSION

In this paper we put forward the idea that low-energy hadron physics might be given by QCD in a non-trivial ground state. We discussed the manipulations that will lead to an effective theory for bilocals which will be interpreted as mesons. In particular, we discussed in detail the method of dealing with zero modes which led us to a non-local four-Fermi theory with all the original global symmetries of the theory restored.

The important point is that even without knowing the actual collective co-ordinates, we can give the general result of the collective co-ordinate integration just from the fermion structure. Once we have the non-local four-Fermi theory given by (38), we are able to introduce the bilocal object $\phi_\alpha(x,y)$ which is a quark and anti-quark connected by a "string" which depends on the background.

The bilocal effective action (48) can now be used as the starting point in deriving the field theory for mesons [11]. The input from QCD are the gauge covariant quantities $\tilde{S}(B)$ and $Q(x,y)$. However, we can see that by expanding (48) these quantities will always appear in a gauge invariant manner, i.e. $\mathrm{tr}(\tilde{S}Q)$, $\mathrm{tr}(\tilde{S}Q\tilde{S}Q)$. Thus the resulting theory will describe gauge invariant bilocals, This programme can then be turned around, i.e. by comparing with the observed meson physics, we will determine the QCD ground state. However, we will not particularly see the $\tilde{A}_\mu(x,w)$ but the global symmetry restored quantities $B_\mu(x)$ and $\bar{\tilde{G}}^{ab}(x,y)$.

Finally, we discussed the large N limit. First, we showed that the classical solutions are of the form $\tilde{A}_\mu(x,w) = \sqrt{N}\,A'(x,w)$. Then we showed that the actual N dependence of the quantites $B_\mu$ and $\bar{G}$ are complicated and will crucially depend on the background field. Instead, we used the "known" behaviour of mesons in the limit of large N to determine the constraints on the background field. These constraints are given by Eqs.(55a,b,c,).

We hope to report in the future the result of using this formalism with a particular background field. All that this entails is evaluating the quantities given in (37a,b,c).


## ACKNOWLEDGMENTS

The author takes pleasure in acknowledging discussions with Drs. C. Mukku and M.A. Namazie. He thanks Professor N.S. Craigie, Drs. W. Nahm and W.A. Sayed for reading the manuscript and suggesting improvements, and is deeply grateful to Professor G. Veneziano for pointing out an error and a discussion on $(\frac{1}{N})$ physics. He is grateful to Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.




## APPENDIX

Suppose that we do not make the approximation (10) for the Faddeev-Popov determinant. We then have to evaluate

$$W = \int d\psi\, d\bar{\psi}\, da\ \det\left(D_\mu(\hat{A})D_\mu(\tilde{A}+a)\right) \exp. i\int d^4x \Big\{ \tfrac{1}{2} a\Theta a - g\bar{\psi}\not{a}\psi + \bar{\psi}(i\not{\partial}+g\hat{\not{A}})\psi \Big\}. \tag{A.2}$$

Now, we have two equivalent options:

i) Expand $\det(D_\mu(\tilde{A})\,D_\mu(\tilde{A}+a))$ in terms of ghosts. However, this will lead to a fermion ghost mixing. This is not desirable because the identification of bilocals ("mesons") will involve ghost fields.

ii) Or equivalently, we can expand $\det(D_\mu(\tilde{A})\,D_\mu(\tilde{A}+a))$ consistently up to order $a^2$. This will have an advantage of not introducing fictitious particles and the resulting theory can be expressed in terms of bilocals .

$$\det(D_\mu(\tilde{A})D_\mu(\tilde{A}+a)) = \det\mathcal{D}\ \exp.\mathrm{tr}\ln(1+X)\ , \tag{A.3}$$

where

$$\mathcal{D} = D_\mu(\tilde{A})\,D_\mu(\tilde{A})\ , \tag{A.4}$$

$$X^{ab} = g^2\mathcal{D}^{-1ac}(x,y)\,f^{cde}f^{efb}\tilde{A}^f_\mu(y)\,a^d_\mu(y)\ . \tag{A.5}$$

Using (A.3) and (A.5) we find that (A.1) can be written as (up to order $a^2$)

$$W = \int da\, d\psi\, d\bar{\psi}\ e^{iS_{YM}(\tilde{A})}(\det\mathcal{D})\ \exp. i\Big\{\int d^4x\,\bar{\psi}(i\not{\partial}+g\tilde{\not{A}})\psi + \int d^4x\ a^a_\mu(x)\,\beta^a_\mu(x) + \int d^4x\, d^4y\ a^a_\mu(x)\,\bar{\Theta}^{ab}_{\mu\nu}(x,y)\,a^b_\nu(y)\Big\}, \tag{A.6}$$

where

$$\beta^a_\mu(x) = -g\lambda^a_{ij}\bar{\psi}_i(x)\gamma_\mu\psi_j(x) + \chi^a_\mu(x)$$

$$= -g\lambda^a_{ij}\bar{\psi}_i(x)\gamma_\mu\psi_j(x) + g^2\left(\int d^4y\, \mathcal{D}^{-1bc}_{(y,x)}\right) f^{cae} f^{efb} \tilde{A}^f_\mu(x), \tag{A.7}$$

$$\bar{\Theta}^{ab}_{\mu\nu}(x,y) = \Theta^{ab}_{\mu\nu}(x)\,\delta^4(x,y) - g^4\left[\int d^4z\, \mathcal{D}^{-1dc}_{(z,x)}\mathcal{D}^{-1d'c'}_{(z,y)}\right]$$

$$\times f^{cae} f^{efd'} f^{c'be'} f^{e'f'd} \tilde{A}^f_\mu(x)\tilde{A}^f_\nu(y) \; . \tag{A.8}$$

It is due to the extra complicated terms in (A.7) and (A.8) that we approximated (9) by (10).

We can still carry out the introduction of bilocals even with the above complications. Integrating out the $a_\mu$ (it is doubtful if $\bar{\Theta}$ will still have zeromodes because of its complicated bilocal structure) we find

$$W = \int d\psi d\bar{\psi}\, e^{iS_{eff.}(\hat{A})} \exp\left\{ i\int d^4x\, \bar{\psi}(x)\left[i\partial + g\mathcal{C}(x)\right]\psi(x) \right.$$

$$\left. - \frac{1}{2} g^2 \int d^4x\, d^4y\, \lambda^a_{ij}\lambda^b_{kl}\,\hat{G}^{ab}_{\mu\nu}(x,y)\,\bar{\psi}_i(x)\gamma_\mu\psi_j(x)\,\bar{\psi}_k(y)\gamma_\nu\psi_l(y) \right\}, \tag{A.9}$$

where

$$S_{eff.}(\hat{A}) = S_{YM}(\hat{A}) + \mathrm{tr}\ln\mathcal{D} - \frac{1}{2}\mathrm{tr}\ln\bar{\Theta} + \frac{i}{2}\int d^4x\, d^4y\, \chi^a_\mu(x)\,\hat{G}^{ab}_{\mu\nu}(x,y)\,\chi^b_\nu(y), \tag{A.10}$$

$$C^a_\mu(x) = \tilde{A}^a_\mu(x) + \int d^4y\, \hat{G}^{ab}_{\mu\nu}(x,y)\,\chi^b_\nu(y) \; , \tag{A.11}$$

and $\hat{G}$ is defined by

$$\int d^4y\, \bar{\Theta}^{ab}_{\mu\nu}(x,y)\,\hat{G}^{bc}_{\nu\lambda}(y,z) = \delta^{ac} g_{\mu\lambda}\,\delta^4(x,z) \; . \tag{A.12}$$

Since $e^{iS_{eff}(\tilde{A})}$ is independent of $\psi$, we can factor it out and include in the normalization. We can now introduce bilocals in (A.9). However, the resulting meson bilocals are more complicated than those given in Sec.IV. This is the reason we used (10) instead of (9) in Eq.(12).



REFERENCES


1) H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Letters 47B, 365 (1973).

2) Some references that deal with this concept in one form or another are C. Callan, R. Dashen and D. Gross, Phys. Rev. D17, 2717 (1978); G.K. Savvidy , Phys. Letters 71B, 133 (1977); H. Pagels and E. Tomboulis, Nucl. Phys. B143, 485 (1978); M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B147, 385, 448 (1979).

3) For a review and references see L.F. Abbott, TH-3113-CERN.

4) J.L. Gervais, A. Jevicki and B. Sakita, Phys. Rev. D12, 1038 (1975); G. 't Hooft, Phys. Rev. D14, 3432 (1976); C. Bernard, UCLA/79/TEP/12, Talk presented at the Canadian Mathematical Society Summer Research Institute (1979); R.J. Crewther, Schladming lectures (1978).

5) See the review by R. Jackiw, C. Nohl and C. Rebbi, Lecture at Summer Institute on Particles and Fields (1977).

6) See for example R. Courant and P. Hilbert, Methods of Mathematical Physics, Vol.I (Interscience Publishers, Inc., New York 1953), p.355;

7) G. 't Hooft, Nucl. Phys. B72, 461 (1974); B75, 461 (1974).

8) E. Witten, Nucl. Phys. B160, 57 (1979).

9) C. Bernard, Phys. Rev. D19, 3013 (1979).

10) See for example E. Witten, Ref.7.

11) Other works on this line are given by H. Kleinert, Phys. Letters 62B, 429 (1976); V.N. Pervushin and D. Ebert, Theor. Math. Phys. 36, 759 (1979).

CURRENT ICTP PUBLICATIONS AND INTERNAL REPORTS

IC/82/1 INT.REP.* TH.M. EL-SHERBINI - Excitation mechanisms in singly ionized krypton laser.

IC/82/2 INT.REP.* CHR.V. CHRISTOV, I.I. DELCHEV and I.Z. PETKOV - On direct mechanism of light-particle emission in incomplete-fusion reactions

IC/82/3 F. CLARO and V. KUMAR - Phase diagrams for a square lattice with two- and three-body interactions.

IC/82/4 E. ROMAN, G. SENATORE and M.P. TOSI - A simple model for the surface energy of ionic crystals.

IC/82/5 INT.REP.* G.A. CHRISTOS - "Loop the loop".

IC/82/6 H.R. KARADAYI - L-R asymmetry in GUT's.

IC/82/7 INT.REP.* YASUSHI FUJIMOTO and JAE HYUNG YEE - The electron magnetic moment at high temperature.

IC/82/8 INT.REP.* T.S. SANTHANAM - Charge structure of quarks and the number of valence quarks in the nucleon.

IC/82/9 INT.REP.* TH.M. EL-SHERBINI and A.A. RAHMAN - Auto-ionizing states in MgI.

IC/82/10 INT.REP.* K.P. JAIN and G.S. JAYANTHI - Quasibound exciton-LO phonon intermediate state in multi-phonon Raman scattering of semiconductors.

IC/82/11 T. SRIVASTAVA - A two-component wave equation for particles of spin $\frac{1}{2}$ and non-zero rest mass (Part II).

IC/82/12 INT.REP.* T.S. SANTHANAM and S. MADIVANANE - The structure of the Hamiltonian in a finite-dimensional formalism based on Weyl's quantum mechanics.

IC/82/13 J.A. MAGPANTAY, C. MUKKU and W.A. SAYED - Hot gauge theories in external electromagnetic fields.

IC/82/14 H.R. KARADAYI - The derivation of the conventional basis for the simple Lie algebra generators.

IC/82/15 H.R. KARADAYI - SO(14) unification of 3+1 families.

IC/82/16 W. NAHM - The construction of all self-dual multimonopoles by the ADHM method.

IC/82/17 B. GRUBER and T.S. SANTHANAM - Indecomposable representations for parabose algebra.

IC/82/18 R. PERCACCI and S. RANDJBAR-DAEMI - Kaluza-Klein theories on bundles with homogeneous fibres - I.

IC/82/19 W. ANDREONI, M. ROVERE and M.P. TOSI - Co-ordination of heterovalent cation impurities in molten salts.

IC/82/20 INT.REP.* L. SMRČKA - A "quadratized" augmented plane wave method.

IC/82/21 V. de ALFARO, S. FUBINI and G. FURLAN - Some remarks about quantum gravity.

IC/82/22 G. GHABRIER, G. SENATORE and M.P. Tosi - Ionic structure of solutions of alakali metals and molten salts.

THESE PREPRINTS ARE AVAILABLE FROM THE PUBLICATIONS OFFICE, ICTP, PO BOX 586, I-34100 TRIESTE, ITALY



IC/82/23 INT.REP.* SUN KUN OH - Mass splitting between $B^+$ and $B^0$ mesons.

IC/82/24 INT.REP.* A. BREZINI - Self-consistent study of localization near band edges.

IC/82/25 INT.REP.* C. PANAGIOTAKOPOULOS - Dirac monopoles and non-Abelian gauge theories.

IC/82/26 CAO CHANG-qi and DING XING-fu - Intermediate symmetry $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ and the SU(N) unification series.

IC/82/27 H.B. GHASSIB and S. CHATTERJEE - $^4$He-impurity effects on normal liquid $^3$He at low temperatures - I: Preliminary ideas and calculations.

IC/82/28 C. PANAGIOTAKOPOULOS, ABDUS SALAM and J. STRATHDEE - Supersymmetric local field theory of monopoles.

IC/82/29 INT.REP.* G.A. CHRISTOS - Concerning the proofs of spontaneous chiral symmetry breaking in QCD from the effective Lagrangian point of view.

IC/82/30 INT.REP.* M.I. YOUSEF - Diffraction model analyses of polarized $^6$Li elastic scattering.

IC/82/31 L. MIZRACHI - Electric-magnetic duality in non-Abelian gauge theories.

IC/82/32 INT.REP.* W. MECKLENBURG - Hierarchial spontaneous compactification.

IC/82/33 C. PANAGIOTAKOPOULOS - Infinity subtraction in a quantum field theory of charges and monopoles.

IC/82/34 INT.REP.* M.W. KALINOWSKI, M. SEWERYNSKI and L. SZYMANOWSKI - On the F equation.

IC/82/35 INT.REP.* H.C. LEE, LI BING-AN, SHEN QI-XING, ZHANG MEI-MAN and YU HONG - Electroweak interference effects in the high energy $e^+ + e^- \rightarrow e^+ + e^-$ + hadrons process.

IC/82/36 G.A. CHRISTOS - Some aspects of the U(1) problem and the pseudoscalar mass spectrum.

IC/82/37 C. MUKKU - Gauge theories in hot environments: Fermion contributions to one-loop.

IC/82/38 INT.REP.* W. KOTARSKI and A. KOWALEWSKI - Optimal control of distributed parameter system with incomplete information about the initial condition.

IC/82/39 INT.REP.* M.I. YOUSEF - Diffraction model analysis of polarized triton and $^3$He elastic scattering.

IC/82/40 INT.REP.* S. SELZER and N. MAJLIS - Effects of surface exchange anisotropy in Heisenberg ferromagnetic insulators.

IC/82/41 INT.REP.* H.R. HAROON - Subcritical assemblies, use and their feasibility assessment.

IC/82/42 W. ANDREONI and M.P. TOSI - Why is AgBr not a superionic conductor?

IC/82/43 N.S. CRAIGIE and J. STERN - What can we learn from sum rules for vertex functions in QCD.

IC/82/44 INT.REP.* ERNEST C. NJAU - Distortions in power spectra of digitised signals - I: General formulations.