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NUMERICAL DOUBLE LAYER SOLUTIONS
WITH IONIZATION

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Abstract

Maxwell's equation $\text{div } \bar{D} = \rho$ in one dimension is solved numerically, taking ionization into account. Time independent anode sheath and double layer solutions are obtained. By varying voltage, neutral gas pressure, temperature of the trapped ions on the cathode side and density and temperature of the trapped electrons on the anode side, diagrams are constructed that show permissible combinations of these parameters. Results from a recent experiment form a subset. Distribution functions, the Langmuir condition, some scaling laws and a possible application to the lower ionosphere are discussed.

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Introduction

In recent experiments (Torvén and Andersson 1979, Torvén and Lindberg 1980, Andersson 1981), double layers have been studied in magnetized plasmas under conditions where a low but not negligible rate of ionization has been essential for sustaining the plasma on the anode side where there has been no plasma source. Andersson (1981) showed that a partial understanding can be gained from a numerical time independent solution of Maxwell's equation $\text{div}\bar{D} = \rho$ in one dimension when ionization both in the layer and the plasma is taken into account, but apart from this no theoretical models seem to have been published for this kind of double layer. Langmuir (1929) certainly discussed double layer formation due to ionization, and Andrews and Allen (1971) constructed a model for a constriction double layer, but they considered ionization only outside the layer.

Other theoretical models usually refer to collision-free conditions with plasma sources on both sides. For reviews on double layers, see Block (1978), Carlqvist (1979) and Torvén (1979).

The purpose of this paper is to present more systematically numerical time-independent solutions of Maxwell's equation $\text{div}\bar{D} = \rho$ in one dimension when ionization also within the layer has been considered. Several parameters are varied within ranges relevant to the experiment (Andersson 1981). The stability of these solutions against perturbations is not discussed here; a time independent solution may however be a suitable starting-point for a later stability analysis.

Assumptions and method

Maxwell's equation $\text{div}\bar{D} = \rho$ gives in a one-dimensional, time-independent case

$$\frac{d^2V}{dz^2} = -\frac{\rho}{\epsilon_0} = -\frac{e}{\epsilon_0}(n_1 - n_e) \quad (1)$$

where the space charge ρ is determined by the densities of both accelerated and trapped particles (table 1). As in Andersson (1981) we assume a waterbag distribution for the accelerated electrons from the cathode side at $V=0$ (fig. 4; table 1) giving

$$n_{eacc}(V) = \frac{2 n_{ec}}{\sqrt{(1 + \sqrt{v_{ec}/v_{de}})^2 + v/v_{de}} + \sqrt{(1 - \sqrt{v_{ec}/v_{de}})^2 + v/v_{de}}} \quad (2)$$

and for the electrons trapped on the anode side

$$n_{eth}(V) = n_{eth}(V_a) \exp((V-V_a)/V_{ea}) \quad (3)$$

Analogously, the density of ions trapped on the cathode side is

$$n_{ith}(V) = n_{ith}(0) \exp(-V/V_{ic}) \quad (4)$$

and the density of accelerated ions, created with zero velocity by ionization due to the accelerated electrons¹⁾, is given by

$$n_{iz}(V) = \frac{i_e}{e} \left(\frac{m_i}{2e}\right)^{1/2} \int_V^{V_a} \frac{dz/d\phi}{\lambda_{iz}(\phi) (\phi-V)^{1/2}} d\phi \quad (5)$$

assuming that the potential is monotonic from V to V_a and time-independent. The electron current density is $i_e = n_{ec} e (2eV_{de}/m_e)^{1/2}$ (table 1), and the electron mean free path (in m) for ionization is $\lambda_{iz} = 1.33/(pP_1)$ where $P_1(V)$ is the average number of ionizing collisions per electron and cm at 1 torr, 0°C, and p is the pressure in Pa (at 0°C; Brown 1959). P_1 (for Ar^+) is approximated by the broken line in fig. 2a (inset). To get the correct space charge from (5) when also Ar^{2+} is taken into account, $\sqrt{2} P_1(Ar^{2+})$ has been

1) Thermal electrons are not assumed to ionize.

added, giving the full line in fig. 2a (inset) as an approximation.

Formula (5) can be derived by integrating the ion distribution function

$$f_{iz}(v) = \frac{i_e}{ev\lambda_{iz}} \frac{dz}{dv} = \frac{i_e m_i}{e^2 \lambda_{iz}} \frac{dz}{dV} \quad (6)$$

When a gas atom is ionized, also a new electron is created. This electron is supposed to be thermalized quickly and contribute to n_{eth} , unless an anode sheath (with $n_{eth} = 0$) is considered, when it disappears quickly into the anode due to its low mass and contributes little to n_e .

Due to the integral (5), the integration of Poisson's equation (1) is started at $V = V_a$ (usually by a $\rho = 0$ approximation discussed later) and repeated a number of times, varying V_{ic} (or $dV/dz(V_a)$ for an ordinary anode sheath) until an acceptable solution is obtained. Simultaneously, n_{ith} is corrected for n_{iz} so that $\rho(V=0) = 0$.

Results

a. Solutions without trapped electrons

Let us first look at some ordinary anode sheath solutions with $n_{eth} = 0$ (fig. 1) as the double layer in the experiment (Andersson 1981) evolves from such a sheath. $V_a = 35V$; $p = 0.0565$ Pa (curve a) and the values in table 2 refer to this experiment. A sheath solution can be found also when p is increased to 0.3 Pa (curve b), but not any more at 0.35 Pa; curves c-f show some unsuccessful attempts. Velocity distributions at $V = 0$; 15 and 30 V for curve a are shown in fig. 4 for electrons ($f_{eacc}(v)$) and ions due to ionization ($f_{iz}(v)$, broken line; calculated from formula (6)).

Another kind of anode sheath solution (also with $n_{eth} = 0$) is

attempted in fig. 2. In this case the integration is started with a small electric field so chosen that n_{iz} from formula (5) makes $\rho = 0$ after the first step Δz . At $p = 0.0565$ Pa (curve a-d) levelling out at $V = 0$ is impossible; in trying to reach $V = 0$ the program automatically increases V_{ic} so much that the Bohm condition is violated and the wrong sign of ρ is obtained. If p is increased to 0.21 Pa (curve e) successful solutions are obtained, but not any more when $p = 0.3$ Pa (curve f).

Curve e in fig. 2 is by definition a double layer (although it is attached to the anode) with $n_{eth} = 0$. How this is possible is seen in fig. 2b; ionization in the volume allows n_i to increase above n_e when V decreases which is not possible for a strong collision-free double layer. If α (table 1) is calculated for curve e, $\alpha = 1.30$ is obtained (whereas $\alpha = 1$ is the so called Langmuir condition). This, together with values from other cases, is also found in fig. 8 discussed later.

b. Solutions with trapped electrons

If a trapped electron population with density n_{eth} and temperature V_{ea} is included on the anode side, double layer solutions can be found at lower pressures, fig. 3. Here $p = 0.0565$ Pa as in the experiment (Andersson 1981), and $V_{ea} = 5V$ (also reasonable for this experiment). The density on the anode side has been varied (by varying n_{eth}) from $n_a = 1 \times 10^{15} \text{ m}^{-3}$ (curve a) to $2 \times 10^{15} \text{ m}^{-3}$ (curve b) and $3 \times 10^{15} \text{ m}^{-3}$ (curve c). Curve b, whose value of n_a corresponds best to the experiment, is the only successful solution. For this curve, $\alpha = 0.89$, and the velocity distributions $f_{eacc}(v)$ and $f_{iz}(v)$ (full lines) are found in fig. 4. The high proportion of ions with low energy (relative to $V_{ea} = 5V$) at $V = 30V$ (curve c), still more pronounced closer to $V_a = 35V$, may seem to violate the Bohm condition. However, this condition has

been derived for a collisionless case when the densities are functions of the potential only; hence it is strictly speaking not applicable when there is volume ionization (it is applicable on the cathode side, however).

When n_a is so high relative to $n_{eacc}(V_a)$, it is difficult to start the integration immediately at $V = V_a$. The difficulty is shown schematically for the first few steps enlarged in fig. 3a, inset. The three starting steps are so chosen that (1) gives $\rho < 0$ (implying $V'' > 0$, equation (1)), (2) gives $\rho = 0$ ($V'' = 0$) and (3) gives $\rho > 0$ ($V'' < 0$). The following steps are a consequence of equation (1) and the integral (5); however, the integration cannot continue as soon as $V' < 0$ for a step due to the integral (5). A successful start should be between case (2) and (3), but the "window" is in reality extremely narrow and difficult to find. Therefore, a $\rho = 0$ solution with several steps has been used as a good approximation before the integration proper; the transition is marked by dots on the curves in fig. 3. The $\rho = 0$ solution is always kept as short as possible; when $n_{eth} = 0$ as in fig. 2, one short $\rho = 0$ step suffices due to its higher V' in this case. By varying the length of this solution in fig. 3 it has been checked that no appreciable error is introduced in the double layer solution. Oscillations of n_{iz} often appear at the transition however, and sometimes even derail the integration by making $V' < 0$.

To trap the thermal electrons on the anode side in fig. 3, a reversed electric field is needed at $z > 1.6$ cm. This is easily provided by a mirror solution, terminated by the anode at a suitable potential $V_A < V_a$. In the experiment, there is also an intermediate plasma where the ion current is radial (Andersson 1981), but this cannot be included in this one-dimensional model.

c. Variation of the parameters

Calculations similar to those in fig. 1 were used in Andersson

(1981) to establish an upper limit for ordinary anode sheath solutions when pressure and voltage are varied; this limit is reproduced here¹ as curve 1 in fig. 5. These parameters have now been varied also for the double layer solutions; $n_{eth} = 0$ (as in fig. 2) gives curve 2 in fig. 5, and $n_a = 2 \times 10^{15} \text{ m}^{-3}$, $V_{ea} = 5\text{V}$ (as in fig. 3, curve b) gives curve 4 in fig. 5. These values for curve 4 are from the experiment (Andersson 1981), and the agreement with the experimental curve 3 is also good above 0.05 Pa. The discrepancy at lower pressure is not surprising; in the experiment these double layers were too unstable to allow measurements, and e.g. a lower n_a could be the explanation.

That double layer solutions with a given voltage can be obtained at different pressures by varying n_a is shown in fig. 6 for $V_a = 35\text{V}$ and 100V . The explanation is that varying n_a changes the length of the structure (as in fig. 3a), thereby changing the ion production, and this can be compensated for by changing the pressure. No upper limit is known for n_a ; however, numerical instabilities after the transition from the $\rho = 0$ solution to the integration proper make it increasingly difficult to get solutions when n_a is increased and p decreased.

An increase of the temperature of the trapped electrons has the same effect as an increase of n_a , and can thus be counterbalanced by a decrease of the pressure, or, if the pressure is fixed, by a decrease of n_a , fig. 7. Above $V_{ea} = 5\text{V}$ a new effect prevails: the thermal electrons penetrate deep into the double layer, thereby diminishing the positive space charge, and this must be counterbalanced by increasing n_a .

In fig. 8, finally, it has been tested to what extent the Langmuir condition $\alpha = 1$ is fulfilled by plotting $\alpha(V_a)$. The symbols are from fig. 5 and give $\alpha(V_a)$ for double layers with $n_{eth} = 0$ (curve 1), and with $n_a = 2 \times 10^{15} \text{ m}^{-3}$, $V_{ea} = 5\text{V}$

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(curve 2). Some combinations of p and V_a in fig. 5 that do not give double layers, but nevertheless solutions that extend below $V = 15V - V_{de}$ (fig. 2, inset), thus giving a definite value of α , have also been included. For double layers with $V_a = 35V$ the two bars a and b show how α varies if n_a and V_{ea} are varied as in figs. 6 and 7 respectively. Deviations from the Langmuir condition, especially at low voltage, are seen, as in Andrews and Allen (1971). One would expect α to approach unity at higher voltage, but at least for curve 1 this is not the case. This may be due to the fact that this kind of double layer can never become "strong" in one respect; an appreciable portion of the ions will always be created within the layer and thus have a lower energy than corresponds to the double layer voltage.

Scaling laws

Although some of the parameters have been kept constant (table 2), it should nevertheless be possible to extend these results to other conditions (at least approximately) by using the following scaling laws.

- a. From formula (1) - (5) it can be seen that if for a given successful double layer solution $1/z^2$, n_{ec} , n_{ith} , n_{eth} and p^2 are all multiplied by a constant k , then all the k 's cancel and a new double layer solution has been found (this is scaling by the Debye length together with a necessary adjustment of p for the ionization).
- b. Similarly, a new double layer solution can be found by multiplying $1/p$ and $P_i \sqrt{m_i/m_e}$ by the same constant. This means a new gas, and although a gas with $P_i(V) = \text{const.} \times P_i(V)$ for argon may not exist, an approximate scaling should nevertheless be possible in some cases.
- c. Furthermore, if V , V_a , V_{ic} , V_{ea} , V_{de} , V_{ec} and z^2 are multiplied by k and $\sqrt{m_i/m_e} p P_i(V)$ is replaced by $(\sqrt{m_i/m_e} p P_i(kV))/k$, a new double layer solution is found. Alternatively, n_{ec} , n_{ith} and n_{eth} instead of z^2 can be multiplied by k if $\sqrt{m_i/m_e} p P_i(V)$ is replaced by $(\sqrt{m_i/m_e} p P_i(kV))/\sqrt{k}$. Again, this means a new gas, and although a gas with these proper-

ties may not exist, a rough scaling may be possible by observing that for higher double layer voltages most of the ionization occurs at energies where P_i (V) does not vary much.

Discussion

The calculations above have shown that double layer solutions are possible for a wide range of parameters when the ions on the anode side are created by ionization due to the accelerated electrons, and that the double layers in a recent experiment (Andersson 1981) form a subset. Some approximations have been made in that ionization due to thermal electrons has been disregarded, as has the truncation of the maxwellian distribution due to the loss of sufficiently energetic particles, and in that a waterbag has been assumed for the accelerated electrons. Simple estimations show that the errors thus introduced are negligible, however. More serious is the fact that in the experiment we have a 3-dimensional, time dependent phenomenon whereas this model is one-dimensional and time independent. Again, the agreement with the experiment shows that this model is a meaningful approximation locally at $r = 0$, although it cannot explain the transition from an anode sheath to a double layer. The fact that it was not possible to get a stable double layer closer than ≈ 10 cm from the anode (excessive ions were then accelerated radially) raises the question of the stability of these solutions, however, and of what would happen in a physical situation when also the ions are magnetized and cannot escape radially (as in e.g. an ionospheric application). It is hoped that future research will answer this question.

Finally, let us make a rough attempt to scale these double layers to ionospheric conditions, disregarding the differences in plasma composition as we are only interested in orders of magnitude here. With $n_e \approx 10^{15} \text{ m}^{-3}$ and a double layer voltage of 200 - 35 V we can have $p \approx 0.005 - 0.05 \text{ Pa}$ (fig. 5), which means a neutral gas density $n_n \approx 10^{18} - 10^{19} \text{ m}^{-3}$. In the ionosphere this value of n_n is found at an altitude of about 115 - 100 km where, however, n_e is only $\approx 10^9 - 10^{11} \text{ m}^{-3}$ (see

e.g. Jones and Rees 1973 or Valley 1965). If we scale our solutions to $n_n \approx 10^{16} \text{ m}^{-3}$ (case a under "Scaling laws" above), we get $n_e \approx 10^9 - 10^{11} \text{ m}^{-3}$, and in this case both n_n and n_e agree with the densities at an altitude of 200 km, the lower value of n_e being applicable in the night. On the other hand, if we go to much higher altitudes than this, n_n becomes too low relative to n_e for this kind of double layer. According to Albert and Lindstrom 1970, there are indeed indications from rocket experiments of double layers of 100 - 200 V at altitudes of 200 - 300 km. If so, our simple scaling shows at least that ionization due to the accelerated electrons may be important, although conditions in the ionosphere are much more complicated. A more realistic model has to be time dependent and take the actual chemical composition, among other things, into account.

In conclusion, double layer solutions with ionization have been found in agreement with experimental results, and simple scaling suggests a possible application to the lower ionosphere. An understanding of the stability and time evolution of these double layers requires time dependent models, however.

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TABLE 1 - DEFINITIONS DL = Double layer, A = Anode, C = Cathode

$V_e = kT_e/e = m_e v_e^2/2e$: Electron temperature for a maxwellian distribution

V_{ea} : V_e for trapped electrons on A side of a DL

V_{ic} : temperature for trapped ions on C side of a DL or A sheath

$V_{de} = m_e v_{de}^2/2e$: v_{de} = electron drift velocity on C side of a DL or A sheath

$V_{ec} = m_e v_{ec}^2/2e$: Electron "temperature" for a waterbag distribution on C side; refer to f_{eacc} in fig. 4

m_e ; m_i : Electron and ion masses

i_e ; i_i : Electron and ion current densities

$$\alpha = (i_e/i_i) (m_e/m_i)^{1/2}$$

n_{ec} : Electron density on C side

n_a : Density on A side of DL at V_a

n_{iz} : Ion density due to ionization

n_{ith} : Density of thermal ions from C side

n_{eacc} : Density of accelerated electrons

n_{eth} : Density of thermal electrons from A side

$$\rho = e(n_i - n_e) = e(n_{iz} + n_{ith} - n_{eacc} - n_{eth})$$

V_a : Maximum potential on A side (relative to C side)

V_A : Anode potential ($V_A = V_a$ only if $n_{eth} = 0$)

TABLE 2 - Parameters that are not varied

$$V_{de} = 1.6 \text{ V}; \quad V_{ec} = 0.4 \text{ V}$$

$$(m_i/m_e)^{1/2} = 269.8 \text{ (argon)}$$

$$n_{ec} = 1.5 \times 10^{15} \text{ m}^{-3}$$

P_i (ionization probability) for argon as in fig. 2a, inset

Figure captions

Fig. 1: a) Attempts to find ordinary anode sheath solutions at different neutral gas pressures. The integrations have started at $V = 35$ V and dV/dz has been varied to find solutions that level out at $V = 0$. This is possible at $p = 0.0565$ Pa (curve a) and 0.3 Pa (curve b), but not at 0.35 Pa (c-f; shifted 0.1 cm for clarity). $V_{ic} = 1$ V; see table 1 and 2.

b) Electron and ion densities for the curves a, b and e in fig. 1a (shifted for clarity).

Fig. 2: a) Attempts to find double layer-like anode sheath solutions ($n_{eth} = 0$) by choosing dV/dz for the first step at $z = 2$ cm so that it gives $\rho = 0$. In this case V_{ic} has been varied to find a solution that levels out at $V = 0$. At $p = 0.0565$ Pa (curve a-d) this is not possible; V_{ic} has been increased from 1 V (a) to 5.62 V (b), 7.50 V (c) and 7.53 V (d). At 0.21 Pa (curve e, $V_{ic} = 1.43$ V) levelling out at $V = 0$ is possible, but not any more at 0.3 Pa (curve f, $V_{ic} = 0.11$ V). See table 1 and 2.

Inset: Approximation of ionization probability versus electron energy for argon (explained in the text).

b) Electron and ion densities for curve e in fig. 2a.

Fig. 3: a) Attempts to find double layer solutions with trapped electrons on the anode side. n_{eth} has been varied so that n_a (table 1) at 35 V is $1 \times 10^{15} \text{ m}^{-3}$ (curve a), $2 \times 10^{15} \text{ m}^{-3}$ (curve b) and $3 \times 10^{15} \text{ m}^{-3}$ (curve c). Levelling out at $V = 0$ is possible for curve b with $V_{ic} = 1.67$ V, but not for curve a ($V_{ic} = 1$ V - 5.62 V) or c ($V_{ic} = 1$ V - 0.11 V). $10^{-3} \lambda_{iz}$ (λ_{iz} = electron

mean free path for ionization) at 35 V and 10 Debye lengths (λ_D) for $n_{ec} = 1.5 \times 10^{15} \text{ m}^{-3}$, $V_e = 5 \text{ V}$ are given for comparison. $p = 0.0565 \text{ Pa}$; $V_{ea} = 5 \text{ V}$; see table 2.

Inset: The difficulty of starting the integration at $V_a = 35 \text{ V}$, explained in the text.

- b) Electron and ion densities for curve b in fig. 3a. The dots on the curves indicate the transition from the $\rho = 0$ solution to the integration proper (also found in fig. 3a).

Fig. 4: Velocity distributions for accelerated electrons (f_{eacc}) and for ions due to ionization (f_{iz}) at $V = 0$ (curve a), 15 V (b) and 30 V (c). The full lines in f_{iz} refer to the double layer in fig. 3a, curve b, and the broken line to the anode sheath in fig. 1a, curve a at the same pressure.

Fig. 5: Permissible potential differences versus neutral gas pressure for anode sheaths and double layers. Region A: No solutions that level out at $V = 0$ exist. B: Only ordinary anode sheaths are possible. C: Anode sheaths and double layers are possible. D: Only ordinary anode sheaths are possible due to lack of ionization.

The symbols denote attempts to find solutions.

I: Double layers with $n_{eth} = 0$; II: Double layers with $n_a = 2 \times 10^{15} \text{ m}^{-3}$ and $V_{ea} = 5 \text{ V}$. a) Levelling out is possible at $V > 0$ only; b-d) successful solutions that level out at $V = 0$ with V_{ic} within the indicated ranges (in volts); e) the curves cannot level out. In addition to the theoretical curves 1, 2 and 4 an experimental curve (3) is given. Other parameters: Table 2.

- Fig. 6: Density on the anode side versus neutral gas pressure for double layers with $V_a = 100$ V (I) and 35 V (II). a-e as in fig. 5. $V_{ea} = 5$ V; see table 2.
- Fig. 7: Density on the anode side versus temperature of the trapped electrons for double layers with $V_a = 35$ V; $p = 0.0565$ Pa; table 2. Symbols as in fig. 6.
- Fig. 8: $\alpha = (i_e/i_i)(m_e/m_i)^{1/2}$ versus double layers voltage. $\alpha = 1$ is the Langmuir condition. The symbols are from fig. 5; curve 1 refers to $n_{eth} = 0$ and curve 2 to $n_a = 2 \times 10^{15} \text{ m}^{-3}$, $V_{ea} = 5$ V. Other values are obtained if n_a or V_{ea} is varied; bar a shows the variation of α (for $V_a = 35$ V) for $0.31 \leq n_a \leq 2.5$ ($\times 10^{15} \text{ m}^{-3}$), $V_{ea} = 5$ V, and bar b for 0.5 V (top) $\leq V_{ea} \leq 10$ V (bottom), $n_a = 2 \times 10^{15} \text{ m}^{-3}$.

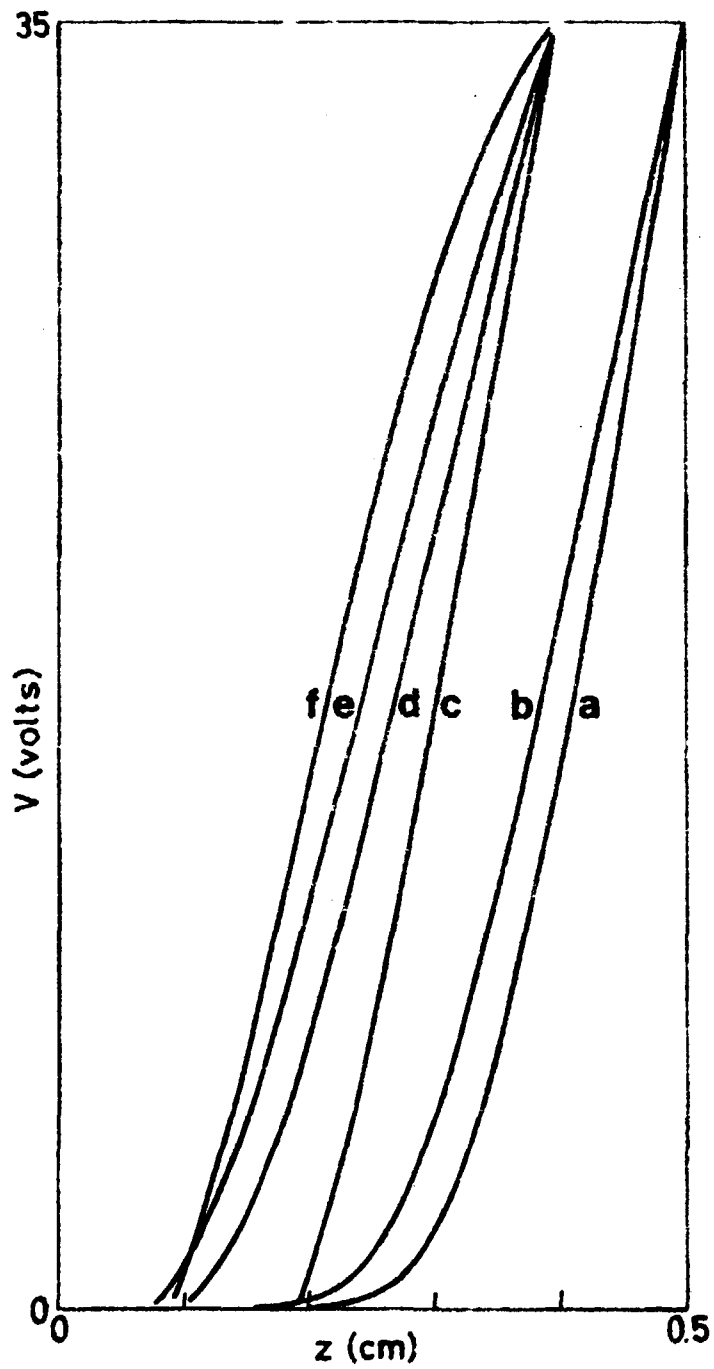


Fig. 1a

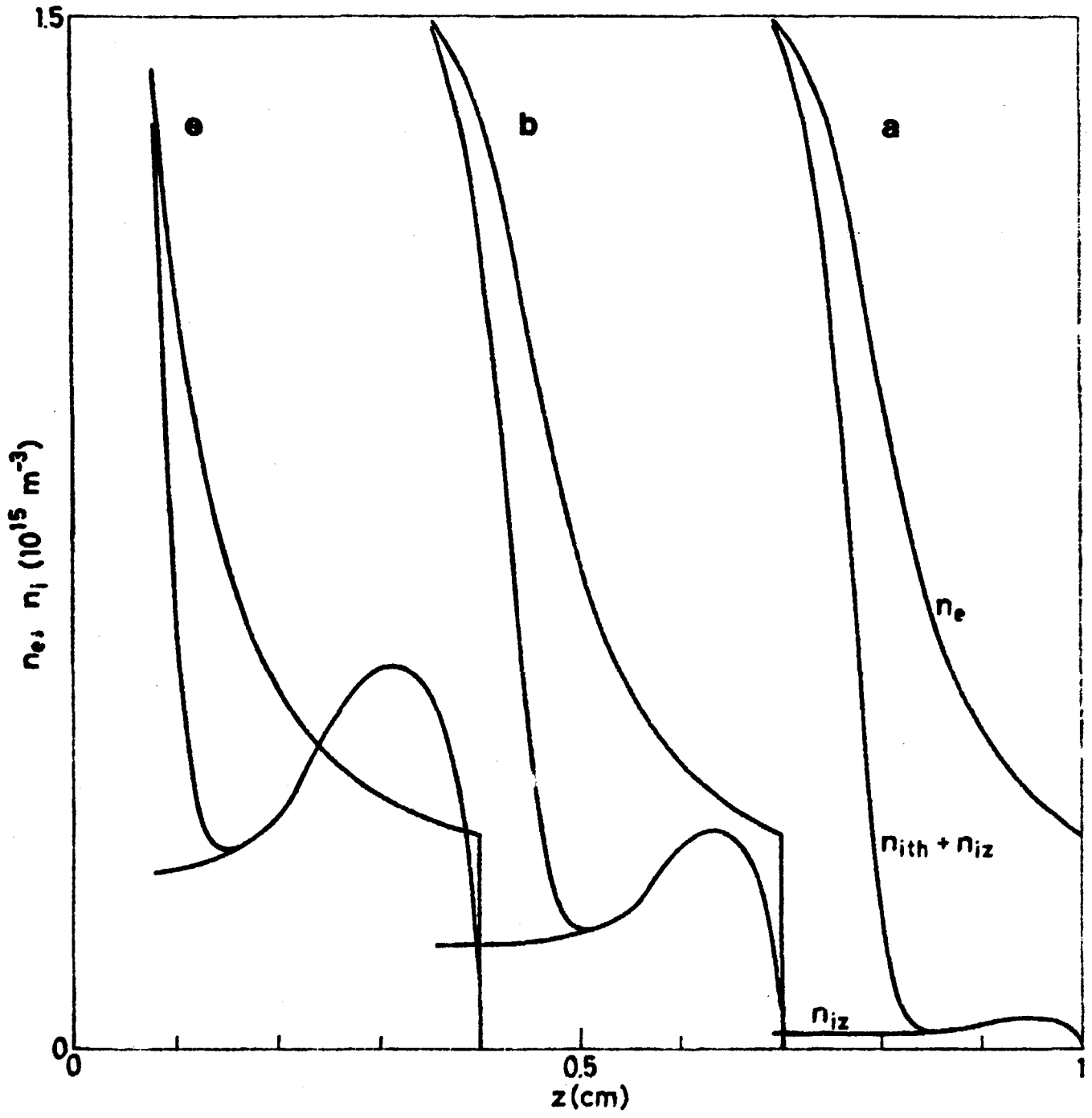


Fig. 1b

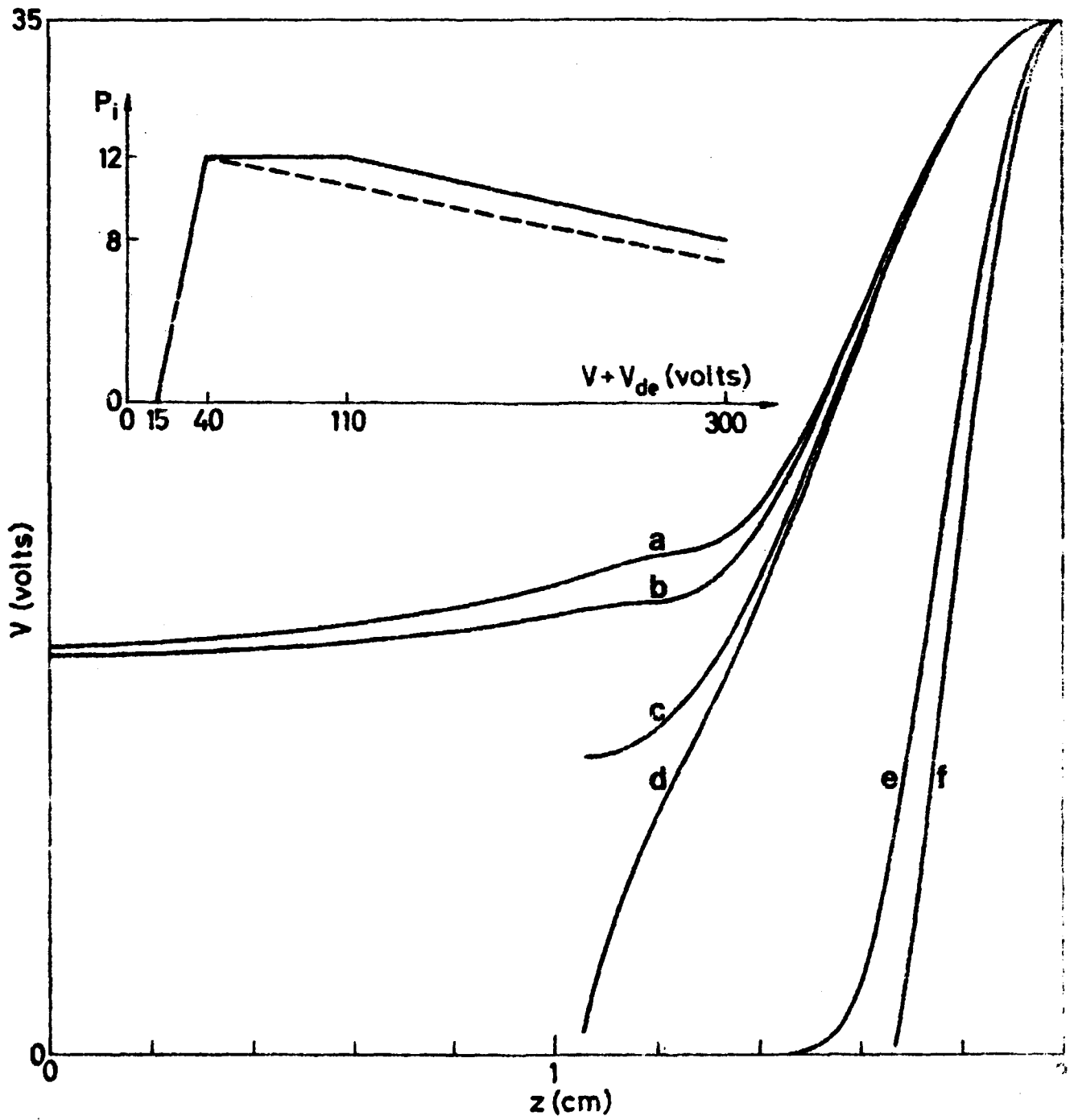


Fig. 2a

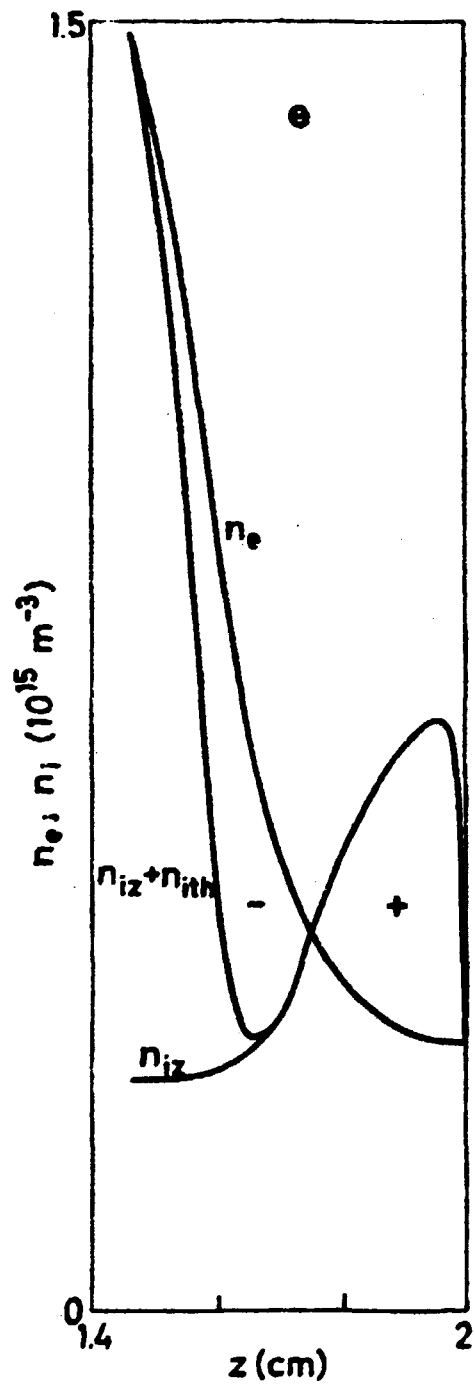


Fig. 2b

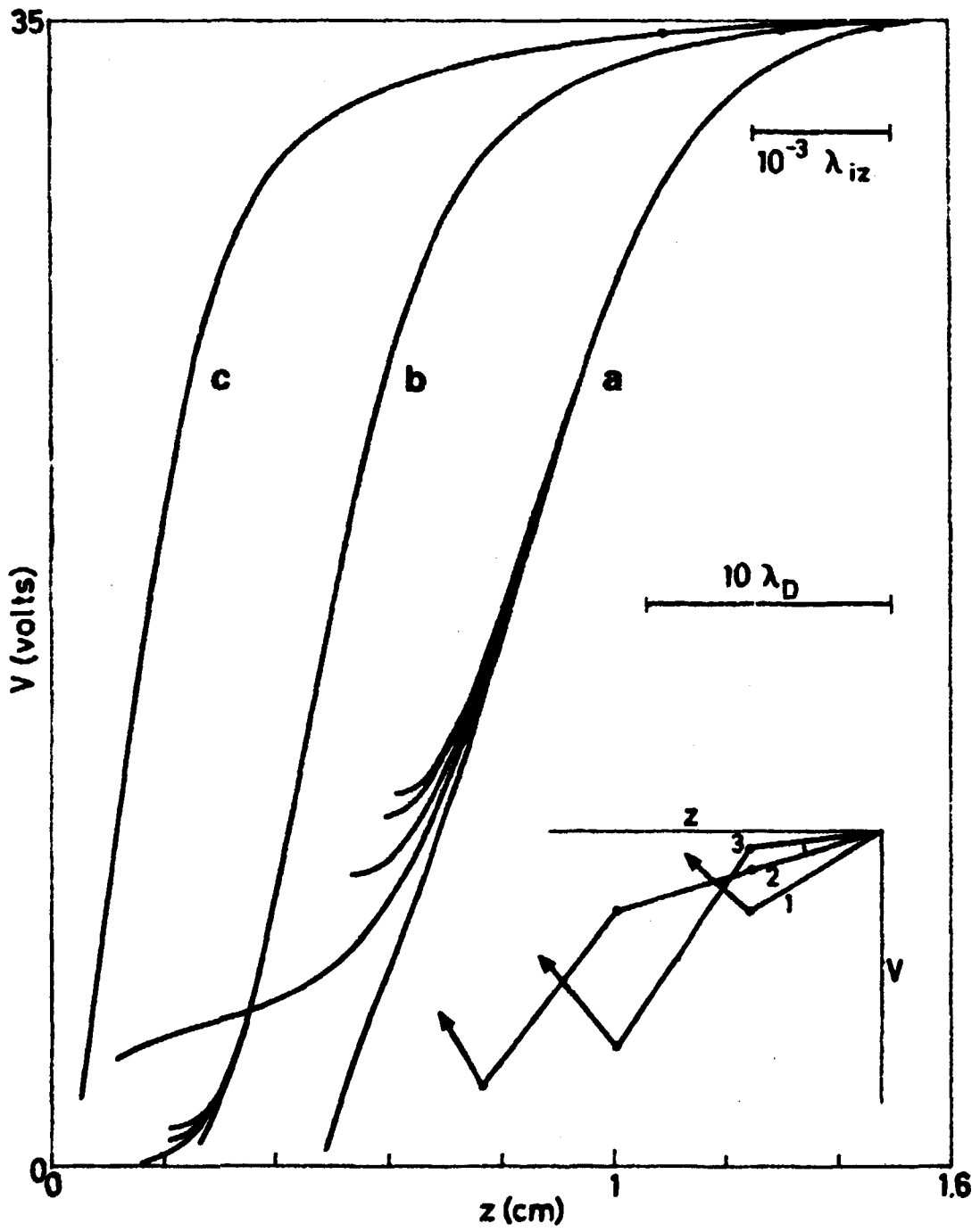


Fig. 3a

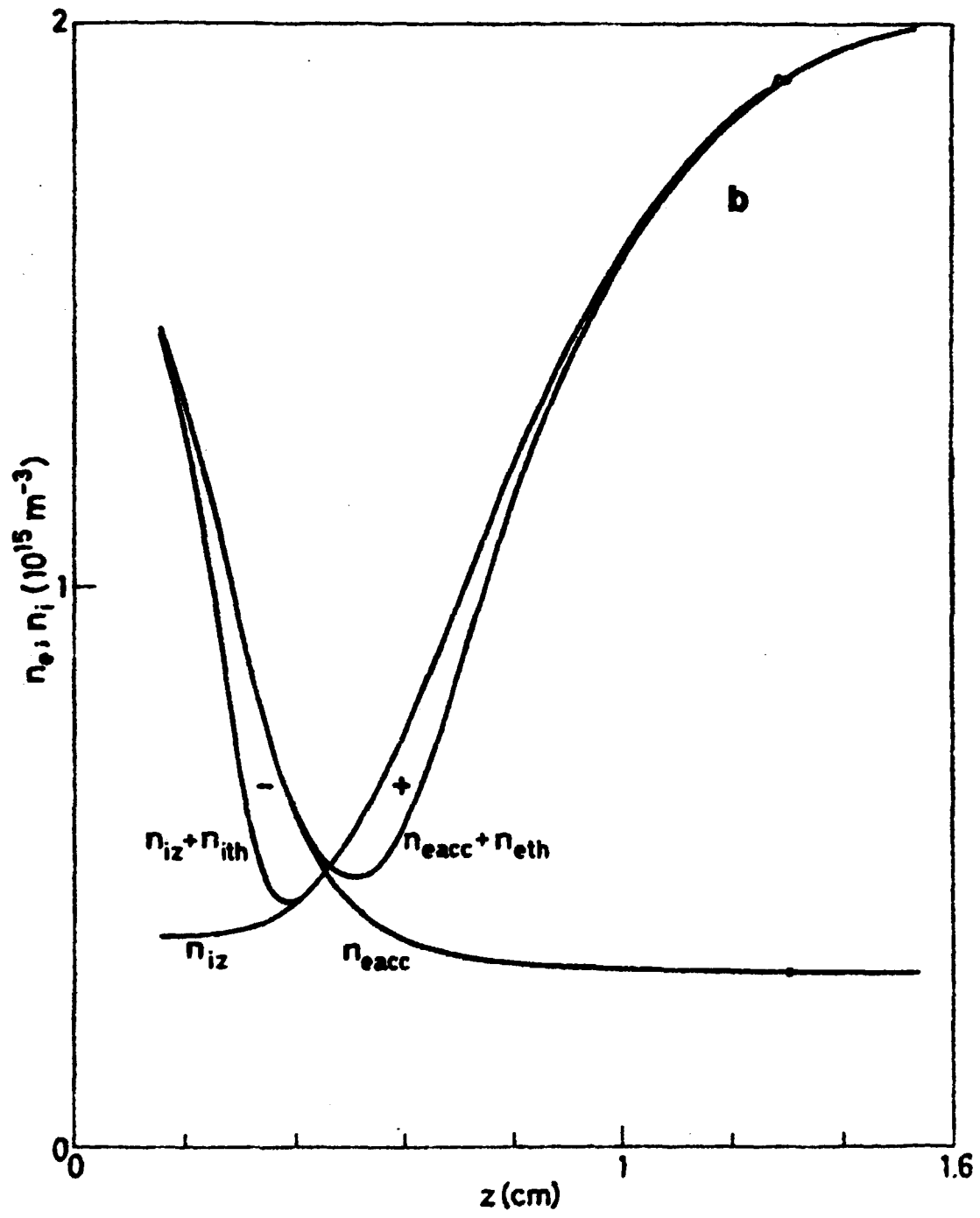


Fig. 3b

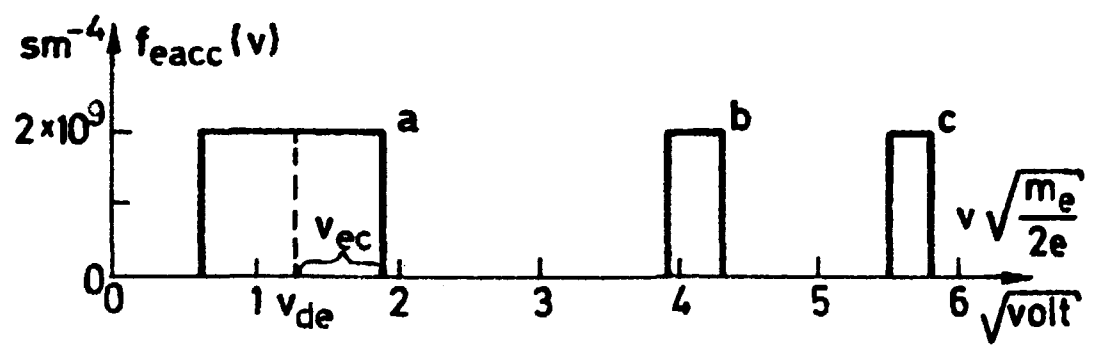
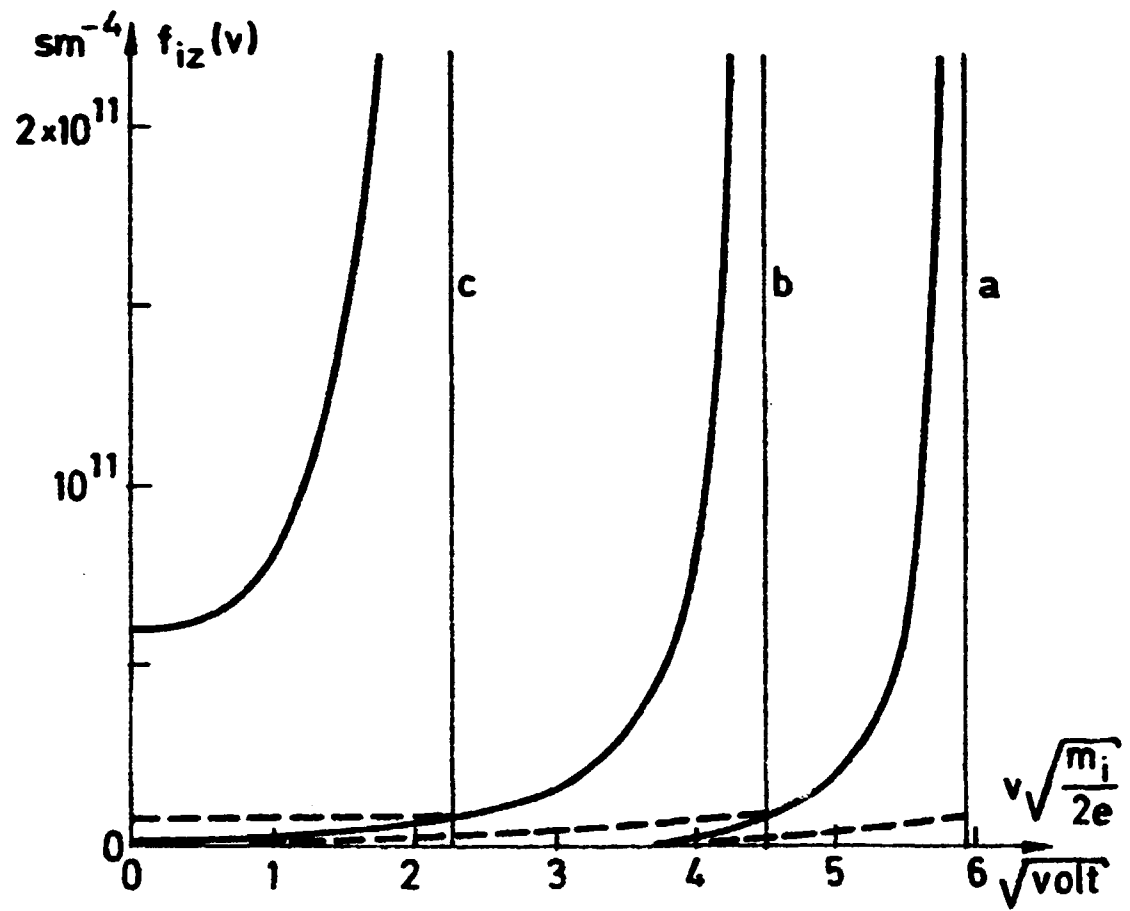


Fig. 4

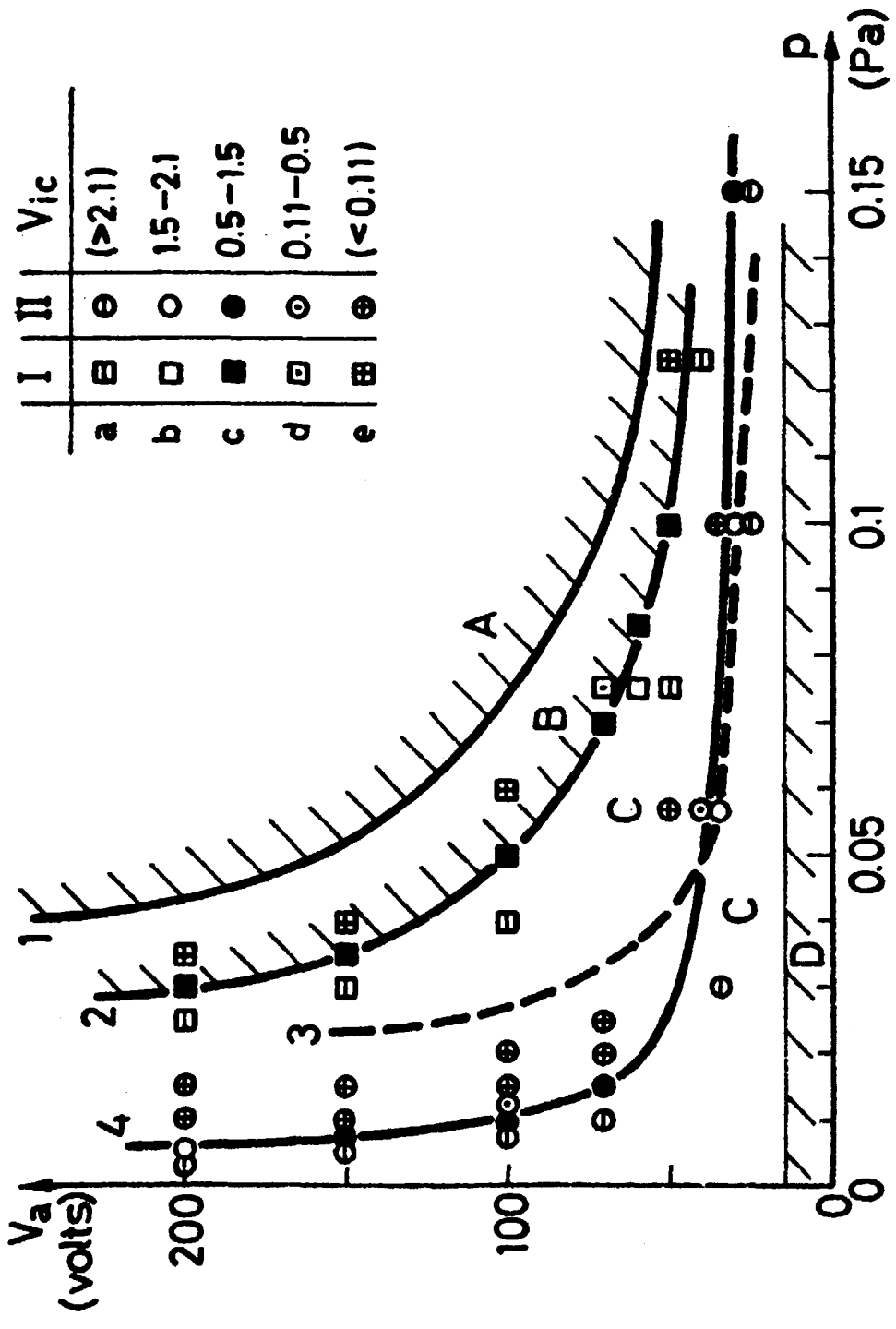


Fig. 5

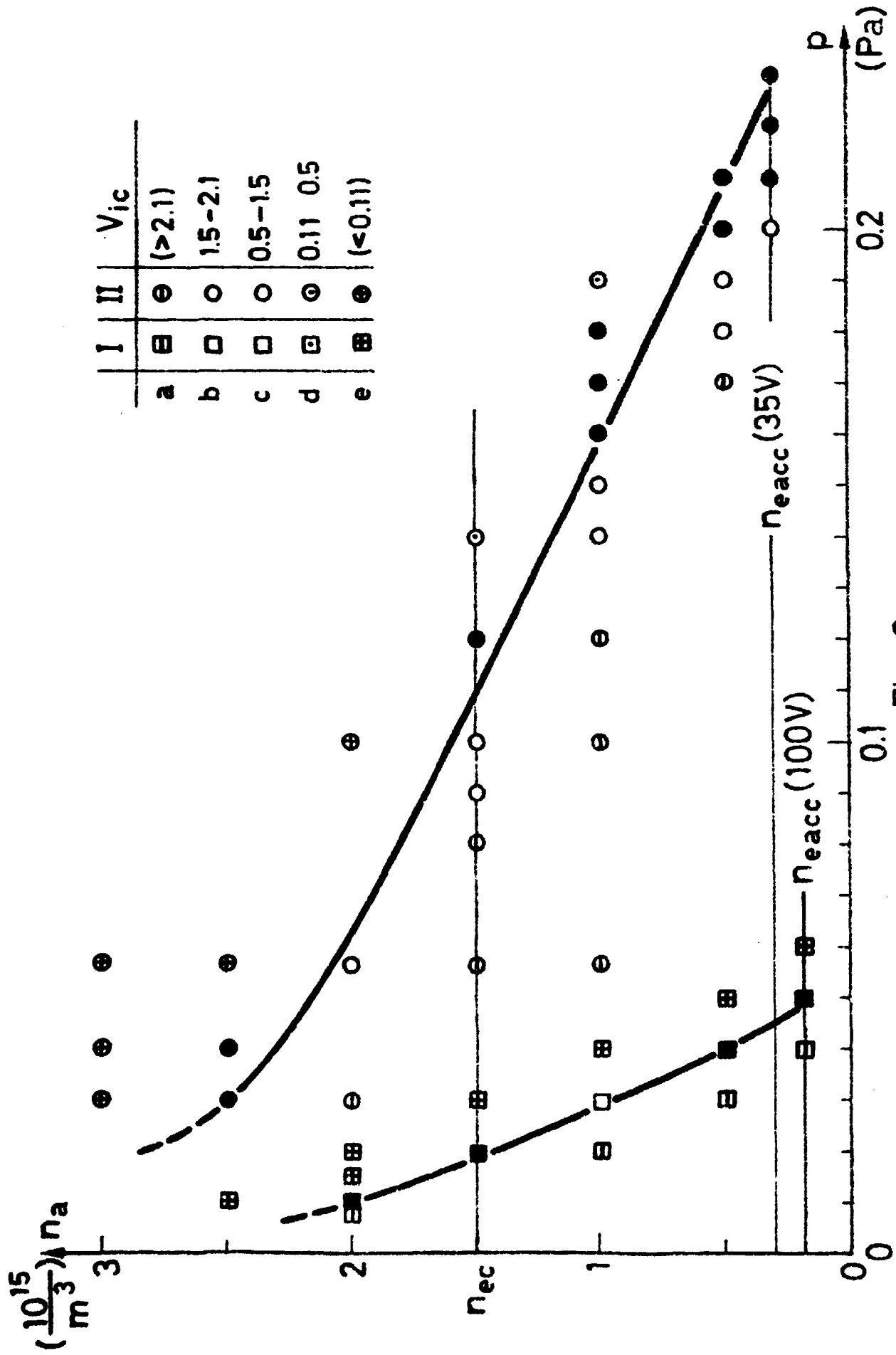


Fig. 6

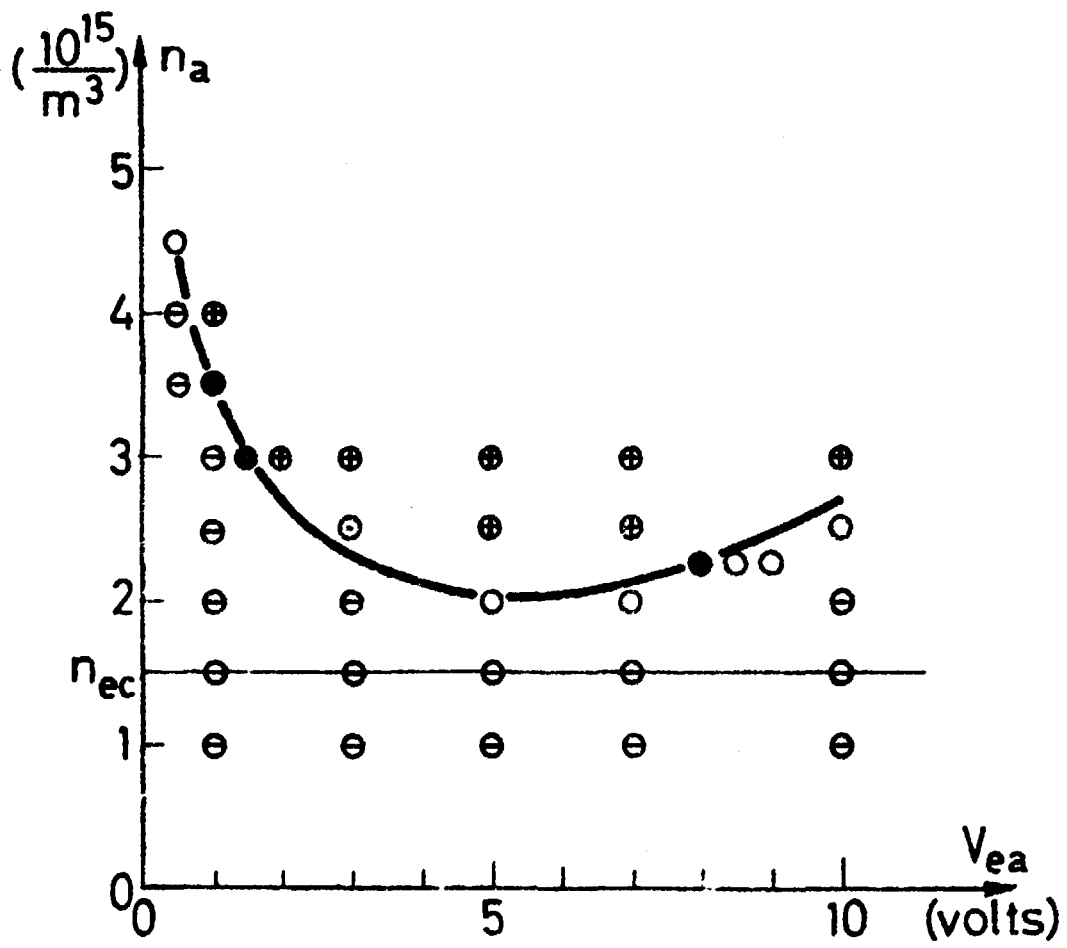


Fig. 7

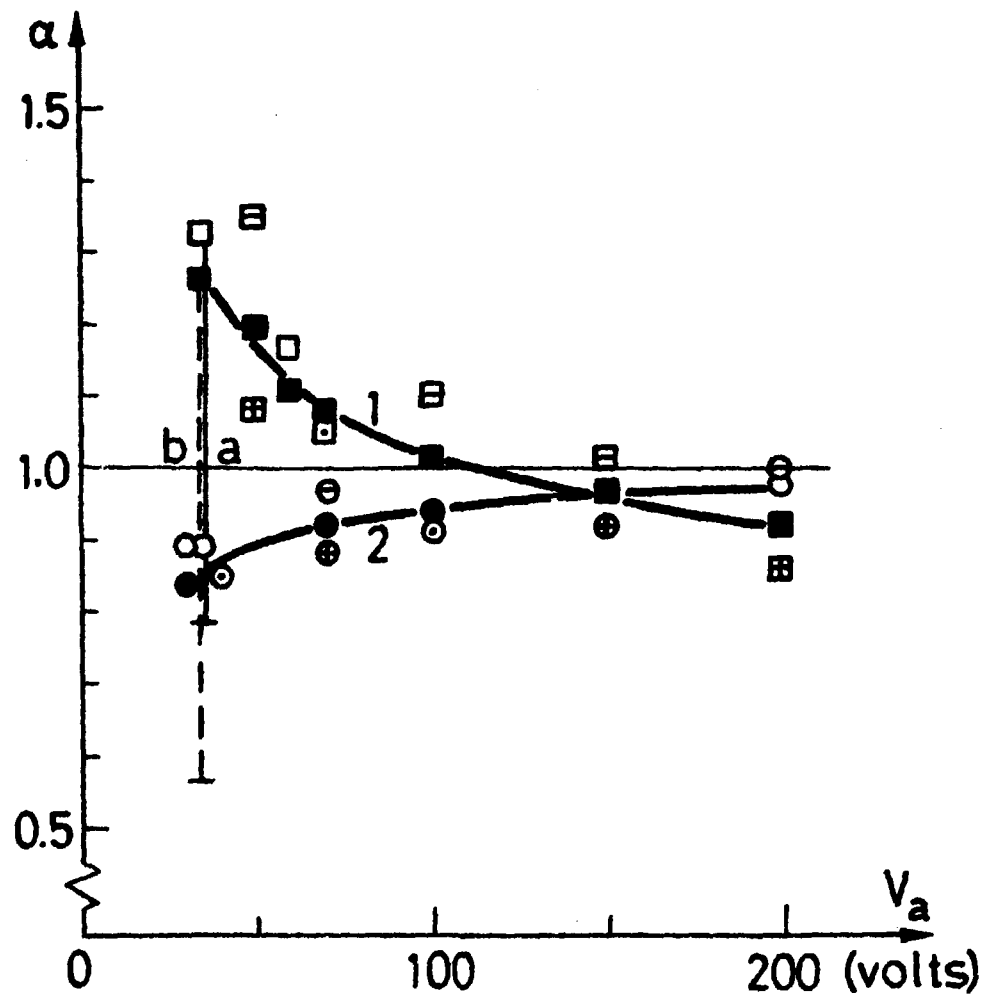


Fig. 8

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Maxwell's equation $\text{div } \bar{D} = \rho$ in one dimension is solved numerically, taking ionization into account. Time independent anode sheath and double layer solutions are obtained. By varying voltage, neutral gas pressure, temperature of the trapped ions on the cathode side and density and temperature of the trapped electrons on the anode side, diagrams are constructed that show permissible combinations of these parameters. Results from a recent experiment form a subset. Distribution functions, the Langmuir condition, some scaling laws and a possible application to the lower ionosphere are discussed.

Key words: Double layer, Ionization, Ionosphere.