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THE CRITICAL IONIZATION VELOCITY  
MECHANISM FOR THE CASE OF GAS  
MIXTURES

Michael A Raadu

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Department of Plasma Physics  
Royal Institute of Technology  
S-100 44 Stockholm, Sweden

THE CRITICAL IONIZATION VELOCITY MECHANISM FOR THE CASE OF GAS MIXTURES

Michael A Raadu

Royal Institute of Technology, Department of Plasma Physics,  
S-100 44 Stockholm, Sweden

Abstract

The theory of the critical ionization velocity mechanism is discussed. In the case of gas mixtures the critical velocity is expected to depend on the ionization cross sections. An analytic approximation is introduced which can be used to set limits on a generalized expression for the critical velocity of gas mixtures.

## 1. Introduction

For a single component neutral gas in relative motion with respect to a magnetized plasma, the critical ionization velocity is defined by equating the kinetic energy of a neutral atom in the plasma frame of reference with its ionization energy<sup>1</sup>. When the relative velocity exceeds the critical velocity,  $v_c = \sqrt{2 eV_i/m_a}$  ( $V_i$ , ionization potential;  $m_a$ , mass of the neutral atom), rapid ionization occurs and the relative motion is braked. This interaction has been demonstrated in numerous experiments which confirm the definition of the critical velocity<sup>2,3</sup>. The experimental results<sup>4,5</sup> support a theory for the critical velocity interaction in which the free energy of newly formed ions moving with respect to the magnetized plasma may be used to energize the electrons leading to further ionization<sup>6</sup>. The braking of the relative motion may be seen as a consequence of an anomalous friction between the new ions and the plasma due to the growth of the modified two-stream instability which is also responsible for electron acceleration<sup>6,7</sup>. The interaction may also be treated from a macroscopic point of view applying energy and momentum balance and introducing an efficiency factor for the transfer of ion free-energy to the electrons<sup>8</sup>.

From experiments with neutral gas mixtures<sup>9</sup> the critical velocity is found to depend on the relative proportions of the components. There is a tendency for the heavier component or that with the lowest ionization potential to dominate. A natural generalization of the definition of the critical velocity is to equate the free energy supply associated with the formation of new ions to the loss rate through ionization<sup>10</sup> so that

$$\alpha \frac{1}{2} m_{a1} v_c^2 + (1-\alpha) \frac{1}{2} m_{a2} v_c^2 = \alpha c V_{i1} + (1-\alpha) e V_{i2} \quad (1)$$

where the subscripts 1,2 refer to two components and  $c$  is the fractional ionization rate for component one. Axnäs<sup>9</sup> has found reasonable agreement between this expression and experiments, by calculating  $\alpha$  on the assumption that the electron temperature agrees with the experimental value of the total ionization frequency. In general  $\alpha$  depends both on the ionization cross sections and on the distribution function of those electrons with sufficient energy to

ionize. To define a generalized critical velocity therefore requires some extra hypothesis for the form of the electron distribution function. It is also convenient to find some suitable analytical approximation for the ionization cross sections.

## 2. Analytical Approximation for the Ionization Cross-Section

Using the classical cross-section for electron impact ionization, the rate coefficient for a Maxwellian electron distribution, temperature  $T$ , is of the form

$$S = S_0 \theta^{\frac{1}{2}} E_2(\theta) \quad (2)$$

where  $\theta = eV_i/kT$ . Using the large order representation of the exponential integral  $E_2$  a crude approximation is then

$$S = S_1 \theta^{\frac{1}{2}} e^{-\theta}/(\theta+\eta) \quad (3)$$

where  $\eta = 2$ . An expression of this type has appeared in the literature<sup>11</sup>, however here we note that it in fact corresponds exactly to a cross-section of the form

$$\sigma = \sigma_1 [1 - \exp(-\eta(\epsilon-1))]/(\eta\epsilon) \quad (4)$$

where  $\epsilon = W/eV_i$ ,  $W$  is the electron energy and  $S_1 = \sigma_1 \sqrt{8 eV_i/\pi m_e}$ . For a suitable  $\eta$  (e.g.  $\eta = 0.1834$  gives a maximum at  $4 V_i$ ) this is a better approximation than the classical one. Many analytical approximations have been proposed<sup>12,13</sup>, but the exact correspondence between the simple expressions  $S$  and  $\sigma$  should be particularly advantageous.

## 3. The Critical Velocity for a Mixture of Two Gases

In the generalized definition of the critical velocity for two neutral components, Eq. (1), the fractional ionization rate  $\alpha$  is equal to  $(1+\beta)^{-1}$  where  $\beta = n_2 S_2/n_1 S_1$  and  $n_{1,2}$ ,  $S_{1,2}$  are the densities and rate coefficients of the neutral gas components. If we now assume a Maxwellian electron distribution and cross-sections of the form given by Eq. (4) the rate coefficients are given by Eq. (3) and

$$\beta = \frac{n_2 \sigma_2}{n_1 \sigma_1} \exp[-(V_{i2} - V_{i1}) e/kT] \left\{ \frac{1 + nkT/eV_{i1}}{1 + nkT/eV_{i2}} \right\} \quad (5)$$

This is a monotonic increasing function of  $T$  if  $V_{i2} > V_{i1}$ , and  $0 < \beta < n_2 \sigma_2 V_{i2} / n_1 \sigma_1 V_{i1}$ . The ratio  $\sigma_2 / \sigma_1$  may be replaced by the ratio of the maximum values in view of the assumed functional dependence, Eq. (4). There are corresponding limits on the critical velocity,  $v_c$ , given by Eq. (1) which may now instead be written

$$v_c = v_{c2} \left\{ \frac{\beta + V_{i1}/V_{i2}}{\beta + m_1/m_2} \right\}^{\frac{1}{2}} \quad (6)$$

where  $v_{c2}$  is the critical velocity of the second component. The same extreme values corresponding to  $\beta = 0$  ( $v_c = v_{c1}$ ) and to  $\beta = n_2 \sigma_2 V_{i2} / n_1 \sigma_1 V_{i1}$  are obtained for the case of mono-energetic electrons.

To fix the value of  $\beta$  and hence the critical velocity  $v_c$  for a Maxwellian electron distribution as considered here the temperature  $T$  must be determined. To do this would in general require an integration of the energy and momentum balance equations for the critical velocity interaction. It can be argued that the electrons are energized until the driving instabilities are quenched or cease to heat the electrons efficiently<sup>6,7</sup>. In the case of the modified two-stream instability we may estimate  $T$ , generalizing previous results<sup>7</sup>, by

$$kT = \frac{1}{2} m^* v_c^2$$

where  $m^*$  is an effective electron mass such that

$$\omega_{pe}^{*2} = \omega_{p1}^2 + \omega_{p2}^2$$

and  $\omega_{p1,2}$  are the plasma frequencies of the ions produced from the neutral gas components. If the ion density ratio is estimated by  $\beta$  and the total density by  $n_e$  the temperature  $T$  may be determined.

#### 4. Conclusions

The critical velocity for a gas mixture is expected to depend on the ionization cross-sections of the components and the electron energy distribution. Even in the case of a Maxwellian electron distribution, the temperature  $T$  must be determined by considering details of the interaction. The complexity of these relationships should increase the value of critical velocity experiments in gas mixtures for testing general theories of the critical velocity interaction.

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References

1. Alfvén, H., 1954: On the Origin of the Solar System, Oxford U.P., Oxford, England.
2. Danielsson, L., 1973: *Astrophys. Space Sci.*, 24, 459.
3. Raadu, M.A., 1980: in *Relation between Laboratory and Space Plasmas*, ed.H.Kikuchi, D.Reidel Publ.Co., Dordrecht, Holland.
4. Danielsson, L., 1970: *Phys. Fluids*, 13, 2288.
5. Danielsson, L. and Brenning, N., 1975: *Phys. Fluids*, 18, 661.
6. Sherman, J.C., 1972: Nobel Symp. No.21, *From Plasma to Planets*, ed.A.Elvius, Almqvist & Wiksell, Stockholm, p. 315.
7. Raadu, M.A., 1978: *Astrophys. Space Sci.*, 55, 125.
8. Petelski, E.F., Fahr, H.J., Ripken, H.W., Brenning, N., and Axnäs, I., 1980: *Astron. and Astrophys.*, 87, 20.
9. Axnäs, I., 1978: *Astrophys. Space Sci.*, 55, 139.
10. Raadu, M.A., 1976: in Axnäs, 1978.
11. McWhirter, R.W.P., 1965: in *Plasma Diagnostic Techniques*, ed. Huddlestone and Lennart, Academic Press, New York, p. 221.
12. Drawin, H.-W., 1961: *Z. Physik*, 164, 513.
13. Drawin, H.-W., 1968: in *Plasma Diagnostics*, ed.W. Lochte-Holtgreven, North-Holland, Amsterdam, p. 842.

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Key words: Critical ionization velocity, Critical velocity, Ionization, Ionization cross section, Plasma-neutral gas interaction, Gas mixtures, Modified two-stream instability.