TRITA-EPP-82-12

ON THE NEGATIVE RESISTANCE OF DOUBLE LAYERS

M.A. Raadu[1] and M.B. Silevitch[2]

August 1982

1) Dept. of Plasma Physics
Royal Institute of Technology
S-100 44 Stockholm 70, Sweden

2) Dept.of Electrical Engineering
Northeastern University, Boston
MA 02115, USA

ON THE NEGATIVE RESISTANCE OF DOUBLE LAYERS

M.A. Raadu and M.B. Silevitch*

Royal Institute of Technology, Department of Plasma Physics,
S-100 44 Stockholm 70, Sweden

Abstract

It is known that large amplitude oscillations can occur in the current flowing through a plasma diode, typically when a constant potential is applied across the device. Burger (1965) suggested via a computer simulation that the oscillation characteristics was a function of the quantities τ_e and τ_i, namely the respective time for an electron and an ion to cross the electric field region inside the diode. On the rapid time scale τ_e, the self consistent equilibrium configuration, was unstable. Norris (1964) had previously arrived at the same conclusion using analytical arguments. In that work, it was concluded that the instability occurred since the diode acted as a negative resistance on the τ_e scale. A positive feedback effect forced the system away from equilibrium.

Silevitch (1981) used the Burger mechanism to suggest an explanation for the flickering aurora phenomenon. He extended the Norris argument and showed by a variational method that a plausible analytic model for a double layer (DL) behaved as a negative resistance on the τ_e scale. In this present work we re-examine the negative resistance calculation by taking a more detailed account of the constraints which are imposed on the electron distributions that exist in the DL region. Specifically, we shall focus upon the role of the energetic trapped electrons which originate at the high potential side of the DL.

*Permanent address: Dept of Electrical Engineering,
Northeastern University, Boston, MA 02115, U.S.A.

1. Introduction

It is known that large amplitude oscillations can occur in the current flowing through a plasma diode, typically when a constant potential is applied across the device. Burger (1965) suggested via a computer simulation that the oscillation characteristics was a function of the quantities τ_e and τ_i, namely the respective time for an electron and an ion to cross the electric field region inside the diode. On the rapid time scale τ_e, the self consistent equilibrium configuration, was unstable. Norris (1964) had previously arrived at the same conclusion using analytical arguments. In that work, it was concluded that the instability occurred since the diode acted as a negative resistance on the τ_e scale. A positive feedback effect forced the system away from equilibrium.

During the later evolution of the system, Burger found that the internal potential structure developed a negative well near the cathode region. This barrier caused current interruption to occur. The system returned to its initial state in a time $\sim\tau_i$ and the cycle would then repeat. Recently Iizuka et al. (1982a) performed a series of experiments which reproduced the essential features of Burger's simulation.

Silevitch (1981) used the Burger mechanism to suggest an explanation for the flickering aurora phenomenon. He extended the Norris argument and showed by a variational method that a plausible analytic model for a double layer (DL) behaved as a negative resistance on the τ_e scale. As with the case of the plasma diode the oscillating potential structure of the DL should be characterized by a negative potential well and an associated current interruption. Such a current chopping phenomena had earlier been observed by Torvén and Babić (1975) but no detailed theoretical explanation was ever proposed for the effect. Iizuka et al. (1982b) have shown experimentally that the extension of the diode instability mechanism was indeed valid for the DL structures produced by operating a Q-machine in a double ended mode. Additional corroberation of the DL relaxation oscillation mechanism was given by Singh and Schunk (1982) using computer simulation techniques. Both Iizuka et al. (1982b) and Singh and Schunk (1982) have conluded that the period of these oscillations is governed by ion dynamics and is indeed proportional to the time for an ion to traverse a distance which is characterized by the spatial extent of the entire system (i.e. the electrode separation distance). This is in contrast to the suggestion of Silevitch (1981) (henceforth referred to as Paper 1) that the transit distance comprise only the narrow DL region.

In this present work we shall focus upon the dynamics of the DL on the τ_e time scale. We will reexamine the computation of negative resistance given in Paper 1 by taking a more detailed account of the constraints which are imposed on the various electron distributions which exist within the DL region. It should also be emphasized that the DL instability discussed here is a global rather than a local effect. By this we mean that the properties of the external circuit surrounding the DL element play a crucial role in the devlopment of the relaxation oscillations. Thus in a given application it is not enough to determine that the DL behaves as a negative resistor on the τ_e scale. One must also examine how this resistance couples through an external circuit and whether it causes a positive feedback effect.

As in Paper 1 we shall choose a particular steady state DL model and demonstrate that it exhibits a negative resistance on the τ_e time scale. In our case we continue using the Kan and Lee (1980) model introduced in Paper 1. It is intended that the calculation presented here be used as a guide if one wants to investigate the stability of a different DL model that may better describe some particular experimental situation.

A similar but somewhat abbreviated version of this paper, Raadu and Silevitch (1982), will appear as part of the proceedings of a small symposium. We are presenting the results again here because the earlier version will not be widely circulated within the plasma physics community. Moreover we will make extensive use of the material already contained in Paper 1, in order to avoid unneccessary mathematical detail.

In Section 2 we present a summary of the Kan and Lee (1980) DL model. We examine the perturbation of the DL on the τ_e time scale in Section 3. The variational technique given in Paper 1 is generalized and we compute the negative resistance of the DL which results when various constraints are imposed on the electron distribution functions. It is also found that the immobile ions introduce an additional term in the perturbation expansion that was overlooked in Paper 1. We investigate this term in Section 4 and present arguments to show that it has a negligible effect on the results presented here and in Paper 1.

2. Equilibrium model

The equilibrium auroral DL model used in Paper 1 is taken from the work of Kan and Lee (1980). The DL potential structure $\phi(x)$, varies from $\phi(x=0) = 0$ to $\phi(x=d) = \phi_o$. Moreover, as shown in Figure 4 of Paper 1, there are three distinct electron populations associated with the Kan and Lee DL model. These are:

(a) Streaming electrons originating at $\phi(x=0) = 0$

A waterbag velocity distribution is chosen for this population which is simply

$$f_{e1}(\phi) = f_1 = \frac{N_{e1}}{2V_{e1}} , \quad V_\ell(\phi) < v < V_u(\phi) \tag{1}$$

where $V_\ell(\phi) = (2|e|\phi/m)^{\frac{1}{2}}$ and $V_u(\phi) = [4V_{e1}^2 + V_\ell^2(\phi)]^{\frac{1}{2}}$. Here N_{e1} and V_{e1} are the density and streaming velocity of the electrons originating at x = 0. Following Paper 1 we can easily obtain expressions for the electron density, $n_{e1}(\phi)$ and its integral $g_{e1}(\phi) \equiv \int_{\bar{\phi}=0}^{\phi} n_{e1}(\bar{\phi})d\bar{\phi}$ at any point $\phi(x)$ within the DL region. Note that it would be more precise to indicate these quantities as functionals of $f_{e1}(N_{e1}, V_{e1}, \phi, v)$.

(b) Trapped degraded primary and secondary electrons originating at $\phi(x=d) = \phi_o$

Again for these particles we use a waterbag distribution f_{et} of value f_t centered at v = 0 and cut off at $v = \pm(2|e|\phi/m)^{\frac{1}{2}}$. Thus,

$$f_t = \frac{N_{et}}{2}\left(\frac{m}{2|e|\phi_o}\right)^{\frac{1}{2}} \tag{2}$$

where N_{et} is the density of these trapped electrons at x = d. Again simple expressions for $n_{et}(\phi)$ and $g_{et}(\phi)$ can be obtained (see Paper 1).

(c) Trapped low energy electrons originating at $\phi(x=d) = \phi_o$

A Maxwellian distribution is assumed for this population. It is a full range function characterized by the parameters $N_{eo}(\gg N_{e1})$ and $kT_{eo}(\ll|e\phi_o)$ which respectively represent the electron density and thermal energy at x = d.

3. Perturbation Analysis

If the equilibrium DL structure is perturbed on the τ_e time scale then the dynamic resistance of the DL is defined as

$$R_D = \frac{\delta\phi_o}{\delta j}(A)^{-1} . \qquad (3)$$

Here A is the DL cross sectional area and δj ($\delta\phi_o$) represents the electron current density (potential) variations from the equilibrium values

[i.e. $\phi(x) \xrightarrow{\text{perturbation}} \phi^*(x) = \phi(x) + \delta\phi(x)$]. Cleary only the electrons in category (a) contribute to j and thus,

$$\delta j = \delta(|e|N_{e1}V_{e1}) = |e|V_{e1}\delta N_{e1} + |e|N_{e1}\delta V_{e1} \qquad (4)$$

In order to calculate R_D we need the key result

$$0 = \delta(g_{e1} + g_{et} + g_{eo}) \qquad (5)$$

This equation is obtained by first multiplying Poisson's equation by $d\phi/dx$ and then integrating from x = 0 to x = d assuming charge neutrality at both endpoints. Finally, the same procedure is repeated for the perturbed state (ϕ^*) and the two equations are subtracted keeping only first order terms in the variations like $\delta\phi$. It should be noted that to this order in $\delta\phi$ it is not neccessary to impose strict charge neutrality for the perturbed state at x = 0, d. Moreover, a rigorous derivation of (5) would include on the rhs a static ion term δG_{ion} defined by

$$\delta G_i \equiv -\int_{x=0}^{d} dx\, n_i(x)\left[\frac{d\phi^*}{dx} - \frac{d\phi}{dx}\right] \qquad (6)$$

where $n_i(x)$ is the ion density profile in the unperturbed state. In Paper 1 this term was neglected. An argument justifying this approximation is presented in the final portion of this paper.

In order to obtain an expression for R_D we need to specify in detail those constraints that apply during the initial disruption. To illustrate this consider electron population (a). Eq. (4) defines a relation between δN_{e1} and δV_{e1}. Another is needed. For example, in Paper 1 it was assumed that N_{e1} and V_{e1} were independent and so either $\delta N_{e1} = 0$ (i.e. strict charge neutrality) or $\delta V_{e1} = 0$. Perhaps a more realistic constraint would be to impose the condition $\delta f_1 = 0$. This would imply from

Eq. (1)

$$0 = \delta N_{el} - \frac{N_{el}}{V_{el}} \delta V_{el} \quad . \tag{7}$$

From Eqs. (4) and (7) we find

$$\delta N_{el} = \frac{1}{2} \frac{\delta j}{|e|V_{el}} \quad , \quad \delta V_{el} = \frac{1}{2} \frac{\delta j}{|e|N_{el}} \quad . \tag{8}$$

The expression for R_D is now obtained by expanding Eq. (5) as:

$$0 = [\frac{\partial g_{el}}{\partial N_{el}} \delta N_{el} + \frac{\partial g_{el}}{\partial V_{el}} \delta V_{el} + \frac{\partial g_{et}}{\partial N_{et}} \delta N_{et}$$

$$+ \frac{\partial g_{eo}}{\partial N_{eo}} \delta N_{eo} + \Delta \delta\phi_o] \tag{9}$$

where $\Delta \equiv \frac{\partial g_{el}}{\partial \phi_o} + \frac{\partial g_{et}}{\partial \phi_o} + \frac{\partial g_{eo}}{\partial \phi_o}$. Let us first follow Paper 1 by assuming $\delta N_{el} = \delta N_{et} \equiv \delta N_{eo} \equiv 0$. Note that these constraints would not impose strict charge neutrality at $x = d$. Using Eqs. (4) and (9) we obtain the resistance R_1 given by

$$R_1 = -(\Delta N_{el} \; |e|A)^{-1} \; \frac{\partial g_{el}}{\partial V_{el}} \tag{10}$$

If we replace $\delta N_{el} = 0$ by condition (7) we find the DL negative resistance R_2 will have the value R_2 where

$$R_2 = -(2A|e|\Delta)^{-1} [\frac{\partial g_{el}}{\partial N_{el}} \frac{1}{V_{el}} + \frac{\partial g_{el}}{\partial V_{el}} \frac{1}{N_{el}}] \tag{11}$$

For the parameters of the auroral example in Paper 1 we find that the two terms in the bracket have roughly the same magnitude and hence $R_2 \approx R_1$.

Let us now generalize the constraints on δN_{et} and δN_{eo}. An obvious first step is to assume strict charge neutrality at $\phi = \phi_o$. This would imply

$$-\delta n_{el}(\phi_o) = \delta N_{et} + \delta N_{eo} \tag{12}$$

For simplicity let us also still assume $\delta N_{el} = \delta N_{eo} = 0$. From Eqs. (4) and (12) we find that

$$\delta N_{et} = -\{(N_{el}|e|)^{-1} \; \frac{\partial n_{el}}{\partial V_{el}} \delta j + \frac{\partial n_{el}}{\partial \phi_o} \delta \phi_o\} \tag{13}$$

the DL negative resistance will now have the value R_3 given by:

$$R_3 = -(N_{el}|e|A)^{-1}[\Delta - \frac{\partial g_{et}}{\partial N_{et}} \frac{\partial n_{el}}{\partial \phi_o}]^{-1}[\frac{\partial g_{et}}{\partial V_{el}} - \frac{\partial g_{et}}{\partial N_{et}} \frac{\partial n_{el}}{\partial V_{el}}] \quad (14)$$

For the parameters of the auroral DL of Paper 1 we find that only the $\delta n_{el}/\delta V_{el}$ correction term is important and

$$R_3 \approx \frac{1}{4} R_1 \quad (15)$$

Moreover, if we assumed that is was primarily the low energy population which reacted to maintain charge neutrality (i.e. $\delta N_{et} = 0$, $\delta N_{eo} = -\delta n_{el}(\phi_o)$), then the resulting negative resistance would be approximately equal to R_1. A result similar to Eq. (15) is obtained if we relax the charge neutrality constraint and assume instead $\delta f_t = \delta N_{eo} = 0$.

From the above discussion we conclude that the response of the trapped particle populations will have quite an important effect upon the value of DL negative resistance. According to the theory in Paper 1 a smaller value of negative resistance could quench the DL disruption on the τ_e time scale. To test this hypothesis we can envisage an experiment which allows the controlled injection of trapped electrons. One could then study the DL disruption characteristics as a function of the trapped electron distributions.

4. Approximate Treatment of Unperturbed Ions

Before concluding this paper we will justify the neglect of δG_i (Eq. (6)) in Eq. (5). The unperturbed electric field of a strong DL structure is primarily nonzero in an interior region of space $\Delta x (<d)$. Under the assumption that the perturbed electric field $(\frac{-d\phi^*}{dx})$ is also confined to essentially the same Δx region we can rewrite Eq. (6) as,

$$\delta G_i \simeq \bar{n}_i \int_0^d dx \frac{d}{dx} (\phi^* - \phi) = \bar{n}_i \, \delta\phi_o .$$

Here $\bar{n}_i$ is a mean value of unperturbed ion density in the Δx region. It is given by the expression,

$$\bar{n}_i = \frac{1}{\phi_o} \int_{x=0}^d dx \, n_i(x) \frac{d\phi}{dx} = \frac{1}{\phi_o} G_i(\phi_o) .$$

For the auroral DL model discussed in Paper 1, it can be shown that $\bar{n}_i \sim 0.1 \, (\partial g_e/\partial \phi_o)$ and so for this case δG_i can indeed be neglected in Eq. (5).

Acknowledgement. The authors would like to thank L.P. Block, C.-G. Fälthammar, L. Lindberg and S. Torvén for their stimulating discussions during the course of this work. This work was supported by the Natural Science Research Council and for one of us (M.B. Silevitch) also by AFOSR grant 78-3731.

References

Burger, P., Theory of large amplitude oscillations in the one-dimensional cesium thermionic converter, J. Appl. Phys., 36, 1938, 1965.

Iizuka, S., Michelsen, P., Rasmussen, J.J., Schrittwieser, R., Hatakeyama, R., Saeki, K. and Sato, N., Dynamics of a potential barrier formed on the tail of a moving double layer in a collisionless plasma, Phys. Rev. Lett., 48, 145, 1982a.

Iizuka, S., Michelsen, P., Rasmussen, J.J., Schrittwieser, R., Hatakeyama, R., Saeki, K., Sato, N., Dynamical evolution of double layers in a collisionless plasma, Proceedings of the 1982 Int. Conf. on Plasma Physics, Göteborg, June 9-15, p. 134, 1982b.

Kan, J.R. and Lee, L.C., On the auroral double layer criterion, J. Geophys. Res., 85, 788, 1980.

Norris, W.T.,Nature of spontaneous oscillations in a cesium diode energy converter, J. Appl. Phys., 35, 3260, 1964.

Raadu, M.A. and Silevitch, M.B., On the negative resistivity of double layers, paper presented at the Symposium on Plasma Double Layers, Risø, June 16-18 (to be published as a Risø report), 1982.

Silevitch, M.S., On a theory of temporal fluctuations in the electrostatic potential structures associated with auroral arcs, J. Geophys. Res., 86, 3573, 1981.

Singh, N. and Schunk, R.W., Current carrying properties of double layers and low frequency auroral fluctuations, Geophys. Res. Lett 9, 446, 1982.

Torvén, S. and Babić, M., Current chopping space charge layers in a low pressure arc plasma, Twelfth Int. Conf. on Phenomena in Ionized Gases, p. 124, 1975.

TRITA-EPP-82-12

Royal Institute of Technology, Department of Plasma Physics
S-100 44 Stockholm 70, Sweden

ON THE NEGATIVE RESISTANCE OF DOUBLE LAYERS

M.A. Raadu and M.B. Silevitch*

August 1982, 8 pp., in English

Abstract

It is known that large amplitude oscillations can occur in the current flowing through a plasma diode, typically when a constant potential is applied across the device. Burger (1965) suggested via a computer simulation that the oscillation characteristics was a function of the quantities τ_e and τ_i, namely the respective time for an electron and an ion to cross the electric field region inside the diode. On the rapid time scale τ_e, the self consistent equilibrium configuration, was unstable. Norris (1964) had previously arrived at the same conclusion using analytical arguments. In that work, it was concluded that the instability occurred since the diode acted as a negative resistance on the τ_e scale. A positive feedback effect forced the system away from equilibrium.

Silevitch (1981) used the Burger mechanism to suggest an explanation for the flickering aurora phenomenon. He extended the Norris argument and showed by a variational method that a plausible analytic model for a double layer (DL) behaved as a negative resistance on the τ_e scale. In this present work we re-examine the negative resistance calculation by taking a more detailed account of the constraints which are imposed on the electron distributions that exist in the DL region. Specifically, we shall focus upon the role of the energetic trapped electrons which originate at the high potential side of the DL.

Key words: Double Layers, Stability, Negative Resistance, Relaxation Oscillations, Stationary Ions.

*Permanent adress: Dept. of Electrical Engineering, Northeastern University, Boston, MA 02115, U.S.A.