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EXPERIMENTAL STUDIES OF EQUILIBRIUM IN A LINEAR EXTRAP Z-PINCH DISCHARGE

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Experimental Studies of Equilibrium in a Linear Extrap Z-Pinch Discharge

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Abstract

Experimental studies of the radial equilibrium condition in linear Extrap discharges have been carried out. Bennet like scaling, $NT \propto I_p^2$, has been observed where N is the plasma line density, T is the temperature and I_p is the plasma discharge current.

1. Extrap Experiment

Experimental studies of the radial equilibrium condition in linear Extrap discharges have been carried out. In the Extrap configuration [1-3], the plasma discharge is basically a linear Z-pinch, but the axial plasma current channel is along the axial zero magnetic field strength line of a linear octupole magnetic field. The octupole field is produced by currents in four rods.

The experimental device is shown in Fig.1 and the parameters of the experiment are given in Table 1.

The discharge is initiated by applying a voltage difference between the electrodes. Breakdown occurs along a thin channel close to the zero-point in the magnetic field [4]. After breakdown, the discharge rises to full amplitude in about 5 μsec. After this rise, the current is held essentially constant by the external circuit which drives the discharge.

Theoretical discussions of the equilibrium and stability of the Extrap discharge are discussed in References [2], [5] and [6].

In this paper we discuss experimental studies of the radial pressure balance. During the constant-current phase of the discharge, the pressure balance can be described by a relation which is essentially a Bennet relation [7]. The scaling of the plasma discharge pinch radius, a, and the axial plasma density, n_o, and temperature, T_o, with the total plasma current, I_p, has the form

$$n_o T_o a^2 \propto I_p^2 \qquad (1)$$

This scaling implies that the radial plasma pressure is balanced by the pinch ($\vec{J} \times \vec{B}$) force which is produced by the plasma current.

We begin the discussion with a review of the Bennet relation in Sec. 2. The experimental method is discussed in Secs. 3 and 4 and the results are presented in Sec. 5.

2. Radial Pressure Balance

In a cylindrical coordinate system, the MHD pressure balance, without an external magnetic field, has the form

$$\frac{\partial p}{\partial r} = - j_z B_\theta . \tag{2}$$

This balance together with the relevant component of Maxwell's equations,

$$\frac{1}{r}\frac{\partial}{\partial r}\,(rB_\theta) = \mu j_z \tag{3}$$

are the basis for the Bennett relation. We assume that the current can be expressed in terms of the current on axis j_o and some function of r, called $f(r)$, which gives

$$j_z(r) = j_o f(r) . \tag{4}$$

The total plasma current is then given by

$$I_p = 2\pi j_o \int_o^a r f dr . \tag{5}$$

Equations (2), (3) and (4) are combined to form a second order differential equation for $p(r)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{r}{f}\,\frac{\partial p}{\partial r}\right] = j_o^2 f . \tag{6}$$

The boundary conditions are taken to be

$$\left[\frac{\partial p}{\partial r}\right]_{r=0} = 0 \tag{7}$$

and

$$p(a) = 0 \tag{8}$$

Equation (6) is integrated twice, incorporating the boundary conditions, to give

$$\int_0^a rp\,dr = \frac{\mu_o j_o^2}{2} \int_0^a r f(r) F(r)\, dr \tag{9}$$

where

$$F(r) = \int_0^r r' f(r')\, dr' \tag{10}$$

Incorporating the definition of I_p from Eq. (5) gives

$$\int_0^a rnkT\,dr = \frac{\mu_o I_p^2}{32\pi^2} . \tag{11}$$

where we have used $p = 2nkT$.

3. Experimentally Measured Parameters

Equation (11) is the basis for experimental scaling studies of the radial pressure balance. The relationship we would like to examine is then just

$$\int_0^a rnTdr \propto I_p^2 \tag{12}$$

which is Eq.(11) without the constants and the factors introduced by profile effects. The profile effects will be discussed later. Now we continue with a discussion of the experimental method used to check the scaling relationship given in Eq.(12). The relation described a plasma where the radial pressure force is balanced by the pinch ($\vec{j} \times \vec{B}$) force.

The product nT integrated over the cross-sectional area of the discharge, which is the left-hand-side of Eq.(12), cannot be measured directly. However, measurements of the line integrated electron density and the line integrated intensity of emitted Balmer spectral radiation can be used to estimate the integral. The line-of-sight for both measurements is perpendicular to the axis of the discharge.

The line-integrated electron density is measured with an interferometer and is expressed in terms of a phase shift ϕ,

$$\phi = c_n \int_0^a ndr \tag{13}$$

where c_n is a constant dependent on the wavelength of the interferometer.

The temperature is evaluated using a measurement of the emitted intensity of a Balmer spectral line. The excited state, m, giving rise to a given Balmer line is in local thermodynamic equilibrium with the continuum and is populated by recombination for the typical densities and temperatures of the Extrap discharges discussed here. Also, the discharge is optically thin to the spectral radiation. The emitted density of the spectral line is then expressed as follows,

$$I_m \propto n^2 T^{-3/2}$$

in regions where we have $n \gtrsim 10^{21} m^{-3}$ and $T \gtrsim 2$ eV.

This intensity is observed using a filter and a lens and fiber optic system which, in effect, integrates this emitted intensity along a line-of-sight so that the observed signal becomes

$$S_m \approx c_m \int_0^a n^2 T^{-3/2} dr \qquad (14)$$

where c_m is a constant dependent on spectral constants and efficiencies of the optics and detector.

In summary, we see that we would like to measure the quantity, in Eq.(12)

$$\int_0^a rnTdr$$

to verify the scaling predicted by the radial pressure balance, but instead we measure the integrals in Eqs.(13) and (14). If we assume that the density and temperature profiles are of the form

$$n = n_o f_n(r)$$

$$T = T_o f_T(r)$$

then Eqs. (12)-(15) can be combined to give

$$\phi^{7/3} S_m^{-2/3} a^{1/3} \propto n_o T_o a^2 \propto I_p^2 . \qquad (16)$$

This relation is essentially an expression of the radial pressure balance. However the left-hand-side is in terms of the measured parameters ϕ and S_m and has only a very weak dependence on the plasma radius a.

4. Profile Effects on Scaling Measurements

The radial pressure balance is studied by examining the scaling of $\phi^{7/3} S^{-2/3}$ versus I_p. However, if the density and temperature profiles change shape as I_p is varied, the proportionality constant in Eq. (16) changes and errors are introduced. Information on the magnitude of possible errors due to profile changes can be estimated by examining a few standard cases. We now return to the expression for the current profile given in Eq. (4). If we have

$$f(r) = 1 - (r/a)^{\alpha} \qquad (17)$$

then Eq. (5) becomes

$$I_p = \pi a^2 j_o \frac{\alpha}{(\alpha+2)} \tag{18}$$

and Eq. (11) becomes

$$\int_o^a rnkTdr = \frac{\mu_o I_p^2}{32\pi^2} \, . \tag{19}$$

In addition, the pressure profile can be solved for and has the form [5]

$$nkT = \frac{\mu_o I_p^2}{8\pi^2 a^2} \left\{ \frac{\alpha+3}{\alpha+1} - \left[\left(\frac{\alpha+2}{\alpha}\right)^2 - \frac{2(\alpha+4)}{\alpha^2} \left(\frac{r}{a}\right)^\alpha + \frac{2(\alpha+2)}{\alpha^2(\alpha+1)} \left(\frac{r}{a}\right)^{2\alpha} \right] \left(\frac{r}{a}\right)^2 \right\} . \tag{20}$$

We now use Eqs. (13), (14), (19) and (20) to determine the effects of profile shapes on the proportionality constant inherent in Eq. (16). Three cases are summarized in Table 2. The three cases are basically a constant-current profile case, a surface-current case and a peaked-current profile case. The temperature is assumed constant for all cases. The factor indicating the change in the proportionality constant in Eq. (16)

is shown in the fifth column. The factor ranges from 0.50 to 0.38 which is a rather small variation considering the large change in the current profile for these three cases.

We conclude that, if the plasma pressure profile is determined by a balance between the radial pressure force and the pinch force, the basic scaling in Eq. (16) should be observed. Specifically, we should see

$$\phi^{7/3} S^{-2/3} \propto I_p^2.$$

If this scaling is seen, it does not prove conclusively that the pressure balance is as assumed since other models could give the same scaling. However if the scaling breaks down, it is evidence that the discharge is not contained by the pinch force.

5. Experimental Measurements of the Radial Pressure Balance

We now turn to data taken on the linear Extrap device. In Figs. 2a and 2b, we show plots of the quantity $(\phi^{7/3} S_\gamma^{-2/3})$ versus I_p at two different times during the discharge. The other parameters of the experiment are given in Table 1. In the experiment, the plasma current was varied from 3kA to 21 kA. In Fig. 2a, which shows the scaling at 10 μsec into the discharge, we see that $\phi^{7/3} S_\gamma^{-2/3}$ is proportional to I_p^2, which is consistent with the pinch model, over the range $6\ \text{kA} < I_p < 17\ \text{kA}$. For lower values of I_p, the scaled parameter falls below the proportionality curve. This is probably due to the fact that the power input is very low for this low current discharge and the discharge is probably not fully ionized. For higher values of I_p, the scaled parameter also tends to fall below the proportionality curve. As discussed in Reference [3], these high current discharges are not stable, which explains the deviation.

In Fig. 2b, the scaling is shown for the same discharges but at a time 20 μsec into the discharge. For $I_p = 6$ kA and 10 kA, the parameter is essentially unchanged. However for $I_p > 10$ kA, the scaled parameter has fallen well below the proportionality curve seen for the 10-μsec case. This indicates that the pinch model of the radial pressure balance is not applicable for discharge currents $I_p > 10$ kA for time scales longer than 10 μsec for this case where the rod current was $I_v = 30$ kA.

These observations are consistent with stability studies described in Reference [3]. In these studies a stability limit is defined which is equal to the maximum ratio $f_c = I_p/I_v$, for stable discharges. The criterion for stability is based on fluctuation levels in the plasma. This limit is a function of the parameters of the experiment including discharge length, filling pressure and wall material. The limit falls in the range $\frac{1}{4} < f_c < \frac{1}{2}$ which is consistent with the observations here. The conclusion is that the breakdown of the pinch equilibrium is due to instabilities which cause fluctuations. In addition, an axial magnetic field has been observed to develop when the fluctuations appear.

6. Magnitude of the Plasma Pressure

Up to this point, we have only discussed scaling. Some observations about the magnitude of the plasma pressure can be made as well. The interferometer calibration factor c_n is explicitly known, but the spectral intensity calibration factor c_m is difficult to evaluate. We therefore do not use the spectral intensity measurements. Instead, the pressure balance in Eq. (19) and the interferometry relationship in Eq. (13) are used and we assume a constant current profile. We then solve for T_o which becomes

$$kT_o = \frac{c_n}{\phi} \frac{\mu_o I_p^2}{12\pi^2 a}$$

If we take values for ϕ at 10 μsec with I_p = 10 kA and assume that the plasma radius a is the distance from the axis to the separatrix given by

$$a = \frac{a_v}{\sqrt{2}} \left(\frac{I_p}{I_v}\right)^{1/4}$$

where a_v and I_v are defined in Table 1, the calculated value for T_o is 5 eV. This is an average temperature over the entire cross-section of the discharge. The value is quite reasonable and is consistent with earlier observations [3].

Acknowledgements

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7. References

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Table 1. Parameters of the linear Extrap experiments.

radial distance to rods, a_v	28 mm
radial distance to vacuum vessel wall	70 mm
electrode separation	220 mm
rod current, I_v	30 kA
filling pressure	20÷200 mtorr
plasma current	3÷21 kA
plasma density	10^{21}÷$10^{22}m^{-3}$
electron temperature	5÷20 eV

Table 2. Summary of Profile Effects

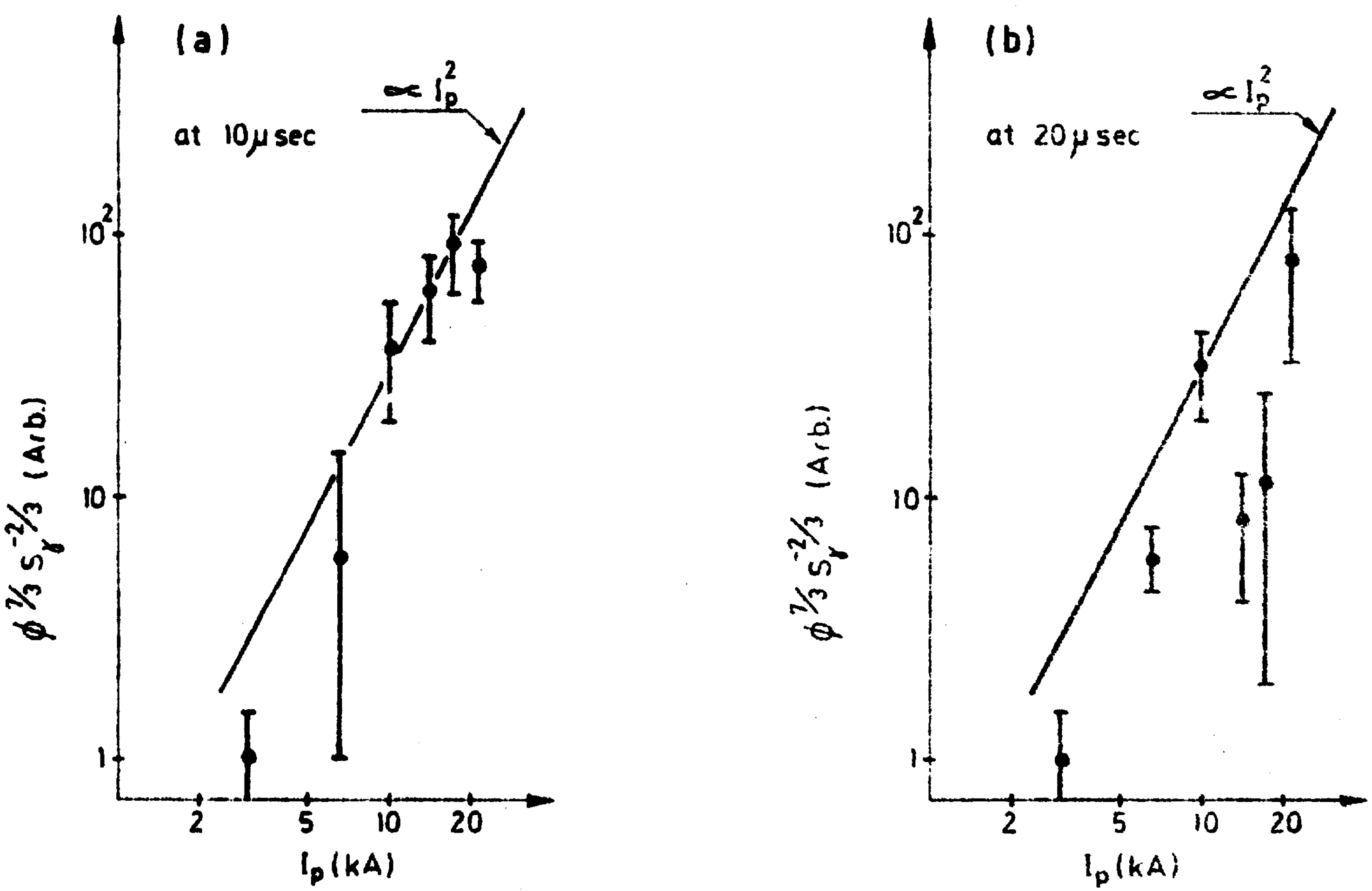
case	current profile	temperature profile	density profile	$\dfrac{a^{-1/3}\int_0^a rnT\,dr}{\left[\int_0^a n\,dr\right]^{7/3}\left[\int_0^a n^2T^{-3/2}\,dr\right]^{-2/3}}$	n_oT_o
constant current	$j = j_o$	$T = T_o$	$n = n_o(1-(\frac{r}{a})^2)$	0.42	$\dfrac{\mu_o I_p^2}{8\pi^2 a^2}$
surface current	surface	$T = T_o$	$n = n_o$	0.50	$\dfrac{\mu_o I_p^2}{16\pi^2 a^2}$
peaked current	$j = j_o(1-\frac{r}{a})$	$T = T_o$	$n=n_o\left\{1-\frac{9}{2}(\frac{r}{a})^2+5(\frac{r}{a})^3-\frac{3}{2}(\frac{r}{a})^4\right\}$	0.38	$\dfrac{\mu_o I_p^2}{4\pi^2 a^2}$

Figure Captions.

Fig.1. Linear Extrap experiment device.

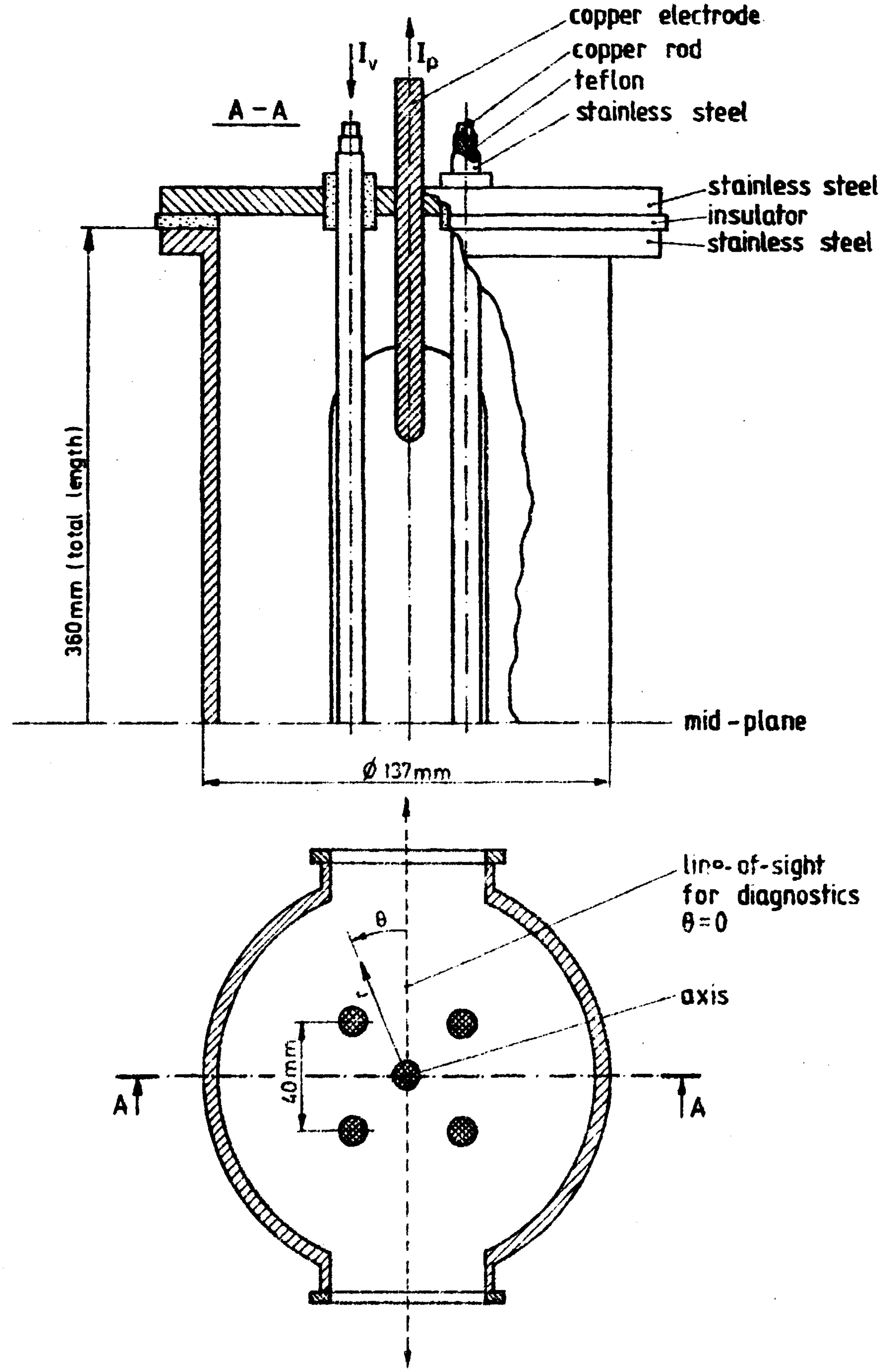
Fig.2. Scaling of the plasma pressure, expressed in the measured parameters, versus plasma current. The measured parameters are the line-of-sight integrated density $\phi \propto \int n d\ell$ and the line-of-sight integrated intensity of emitted Balmer H_γ light $S_\gamma \propto \int n^2 T^{-3/2} d\ell$. Bennet-like scaling is seen at 10 μsec, but the scaling is not maintained for currents above 10 kA at 20 μsec.

Fig.1



(a)
at 10μsec
∝ I_p^2
$\phi^{1/3} S_\gamma^{-2/3}$ (Arb.)
10^2
10
1
2
5
10
20
I_p (kA)
(b)
at 20μsec
∝ I_p^2
$\phi^{1/3} S_\gamma^{-2/3}$ (Arb.)
10^2
10
1
2
5
10
20
I_p (kA)

Fig. 2



copper electrode
copper rod
teflon
stainless steel
I_v
I_p
A – A
stainless steel
insulator
stainless steel
360 mm (total length)
mid-plane
Ø 137 mm
line-of-sight
for diagnostics
θ=0
θ
r
axis
40 mm
A
A

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Key words: Z-Pinch equilibrium, Extrap configuration, Bennet Relation.