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II. Relativistic-Particle Quantum Mechanics

(Applications and Approximations)

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In this lecture I hope to show that relativistic-particle quantum mechanics with direct interactions is a useful tool for building models applicable to hadron systems at intermediate energies. To do this I will first describe a class of models designed to incorporate nucleon-nucleon interactions, pion production, absorption and scattering into a single dynamical framework without dressing the nucleons with pion clouds.[1,2] The second major topic concerns electromagnetic interactions. In the first lecture (referred to as I in the following) I specifically excluded long-range forces and zero-mass particles. Since many of the experimental data in hadron physics involve electromagnetic interactions this limitation is a major defect which must be addressed.

The elementary particles of the NNπ model are the nucleon, the isobar and the pion. Let $\mathcal{H}_N$, $\mathcal{H}_\Delta$ and $\mathcal{H}_\pi$ be the Hilbert spaces of the corresponding one-particle states. The Hilbert space of states under consideration is then

$$\mathcal{H} = \mathcal{H}_N \otimes \mathcal{H}_N \oplus \mathcal{H}_N \otimes \mathcal{H}_\Delta \oplus \mathcal{H}_N \otimes \mathcal{H}_N \otimes \mathcal{H}_\pi \quad . \tag{1}$$

The interactions are such that the Δ decays into a pion and a nucleon. The physical particles are the nucleons, the pion and the deuteron. The space is therefore

$$\mathcal{H}_f = \mathcal{H}_{fN} \otimes \mathcal{H}_{fN} \oplus \mathcal{H}_{fd} \otimes \mathcal{H}_\pi \oplus \mathcal{H}_{fN} \otimes \mathcal{H}_{fN} \otimes \mathcal{H}_{f\pi} \quad . \tag{2}$$

The generators G_o and G_f are defined on these spaces in the obvious manner. Following the general scheme of I we first construct $G_{NN,\pi}$ and $G_{N\pi,N}$ for the partitions (NN)π and (Nπ)N. Next we need mass operators $\tilde{M}_{NN,\pi}$ and $\tilde{M}_{N\pi,N}$ which commute with $\vec{X}_o$ and are scattering equivalent to $M_{NN,\pi}$ and $M_{N\pi,N}$.

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Let $\vec{p}_a$ and $\vec{p}_b$ be the momenta of the two nucleons. States in $\mathcal{H}_{NN} := \mathcal{H}_N \otimes \mathcal{H}_N$ are represented by functions $\psi(\vec{p}_a, \vec{p}_b)$, or equivalently by functions of $\vec{P}_{NN}$ and $\vec{k}_a$, where

$$\vec{P}_{NN} = \vec{p}_a + \vec{p}_b \tag{3}$$

and

$$k_a = L(\vec{P}_{NN}/M^o_{NN}) \; p_a \quad . \tag{4}$$

Spin variables will be suppressed throughout in order to simplify the notation. The Bakamjian-Thomas construction of M_{NN} is straightforward, i.e.

$$(\vec{P}'_{NN} \; \vec{k}'_a | M_{NN} | \vec{k}_a \vec{P}_{NN}) = \delta(\vec{P}'_{NN} - \vec{P}_{NN})(\vec{k}'_a | \hat{M} | \vec{k}_a) \quad , \tag{5}$$

where $\hat{M}_{NN}$ is independent of $\vec{P}_{NN}$,

$$(\vec{k}' | \hat{M}_{NN} | \vec{k}) = 2(\vec{k}^2 + m_N^2)^{\frac{1}{2}} \, \delta(\vec{k}' - \vec{k}) + (k' | v_{NN} | \vec{k}) \quad . \tag{6}$$

The wave operator $\Omega_{NN\pm}$ for nucleon-nucleon scattering is

$$\Omega_{NN\pm} = \Omega_{\pm}(M_{NN}, M^o_{NN}) = \underset{t \to \pm\infty}{\text{s-lim}} \; e^{iM_{NN}t} \, e^{-iM^o_{NN}t} \quad , \tag{7}$$

$$(\vec{P}'_{NN} k'_a | \Omega_{NN\pm} | \vec{k}_a \vec{P}_{NN}) = \delta(\vec{P}'_{NN} - \vec{P}_{NN})(\vec{k}'_a | \hat{\Omega}_{NN\pm} | \vec{k}_a) \quad . \tag{8}$$

The wave matrix $\hat{\Omega}_{NN\pm}$ can be obtained from $\hat{v}_{NN}$ by solving a Lippmann-Schwinger equation. In the presence of a pion spectator we have

$$(\vec{p}'_\pi P'_{NN} \vec{k}'_a | M_{NN} | k_a \vec{P}_{NN} \vec{p}_\pi) = \delta(\vec{p}'_\pi - \vec{p}_\pi)(\vec{P}'_{NN} \vec{k}'_a | M_{NN} | \vec{k}_a \vec{P}) \quad . \tag{9}$$

The generators are then additive,

$$H_{NN,\pi} = H_{NN} + H_\pi \quad , \tag{10}$$

$$\vec{K}_{NN,\pi} = \vec{K}_{NN} + \vec{K}_\pi \quad , \tag{11}$$

and the wave operators remain unchanged,

$$\Omega_{NN,\pi\pm} = \Omega_{NN\pm} \quad , \tag{12}$$

where

$$(\vec{p}\,'_\pi \vec{P}\,'_{NN} \vec{k}\,'_a | \Omega_{NN\pm} | \vec{k}_a \vec{P}_{NN} \vec{p}_\pi) = \delta(\vec{p}\,'_\pi - \vec{p}_\pi)\delta(\vec{P}\,'_{NN} - \vec{P}_{NN})(\vec{k}\,'_a | \hat{\Omega}_{NN\pm} | \vec{k}_a) \quad . \tag{13}$$

Instead of representing states by functions of $\vec{k}_a, \vec{P}_{NN}$ and $\vec{p}_\pi$, we may choose as independent variables $\vec{k}_a, \vec{P}$ and $\vec{q}_\pi$, where $\vec{q}_\pi$ is defined by

$$q_\pi = L(\vec{P}/M_o) p_\pi \quad . \tag{14}$$

The virtue of this choice is that $\vec{X}_o$ is represented by $i\nabla_P$

$$\vec{X}_o \; \psi(\vec{k}_a, \vec{P}, \vec{q}_\pi) = i\nabla_p \; \psi(\vec{k}_a, \vec{P}, \vec{q}_\pi) \quad , \tag{15}$$

and that $\tilde{M}_{NN,\pi}$ defined by

$$\tilde{M}_{NN,\pi} := (\vec{q}_\pi^{\,2} + \tilde{M}_{NN}^2)^{\frac{1}{2}} + (\vec{q}_\pi^{\,2} + m_\pi^2)^{\frac{1}{2}} \quad , \tag{16}$$

where

$$(\vec{q}\,'_\pi \vec{P}\,' \vec{k}\,'_a | \tilde{M}_{NN} | \vec{k}_a \vec{P} \vec{q}_\pi) = \delta(\vec{q}\,'_\pi - \vec{q}_\pi)\delta(\vec{P}\,' - \vec{P})(\vec{k}\,'_a | \hat{M}_{NN} | \vec{k}_a) \tag{17}$$

commutes with $\vec{X}_o$.

The operators $M_{NN,\pi}$ and $\tilde{M}_{NN,\pi}$ are defined by stipulating that they vanish on $\mathcal{H}_{NN}$ and on $\mathcal{H}_{N\Delta} := \mathcal{H}_N \otimes \mathcal{H}_\Delta$. It follows from (16) and (17) that

$$\tilde{\Omega}_{NN,\pi\pm} = \tilde{\Omega}_{NN\pm} \tag{18}$$

and

$$(\vec{q}\,'_\pi \vec{P}\,' \vec{k}\,'_a | \tilde{\Omega}_{NN\pm} | \vec{k}_a \vec{P} \vec{q}_\pi) = \delta(\vec{q}\,'_\pi - \vec{q}_\pi)\delta(\vec{P}\,' - \vec{P})(\vec{k}\,'_a | \hat{\Omega}_{NN\pm} | \vec{k}_a) \quad . \tag{19}$$

Since $\vec{q}'_\pi = \vec{q}_\pi$ follows from $\vec{p}'_\pi = \vec{p}_\pi$ and $\vec{P}'_{NN} = \vec{P}_{NN}$ if and only if $|\vec{k}'_a| = |\vec{k}_a|$, we have

$$S_{NN} = \Omega^\dagger_{NN+}\ \Omega_{NN-} = \tilde{\Omega}^\dagger_{NN+}\ \tilde{\Omega}_{NN-} \quad , \tag{20}$$

and the unitary operator $A(NN,\pi)$,

$$A(NN,\pi) := \tilde{\Omega}_{NN\pm}\ \Omega^\dagger_{NN\pm} \tag{21}$$

transforms $M_{NN,\pi}$ into $\tilde{M}_{NN,\pi}$,

$$\tilde{M}_{NN,\pi} = A(NN,\pi)\ M_{NN,\pi}\ A^{-1}(NN,\pi) \quad . \tag{22}$$

For the cluster consisting of nucleon a and the pion we define

$$\vec{P}_{a\pi} := \vec{p}_a + \vec{p}_\pi \quad , \tag{23}$$

$$\vec{k}_\pi := L(\vec{P}_{a\pi}/M^o_{a\pi})p_\pi \quad . \tag{24}$$

State vectors in $\mathcal{H}_N \oplus \mathcal{H}_\Delta \oplus \mathcal{H}_N \otimes \mathcal{H}_\pi$ are represented by 3-component functions $\psi_N(\vec{p}_a)$, $\psi_\Delta(p_\Delta)$, $\psi_{N\pi}(\vec{P}_{a\pi},\vec{k}_\pi)$. The operator M_a is given by the block matrix

$$\hat{M}_{a\pi} = \begin{pmatrix} m_N & 0 & 0 \\ 0 & m_\Delta & v_\Delta(\vec{k}_\pi) \\ 0 & v^*_\Delta(\vec{k}_\pi) & W(\vec{k}_\pi)\delta(\vec{k}'_\pi - \vec{k}_\pi)(\vec{k}'_\pi|\ v_{N\pi}|\vec{k}_\pi) \end{pmatrix} \tag{25}$$

where

$$W(\vec{k}) = (\vec{k}^2 + m^2)^{\frac{1}{2}} + (\vec{k}^2 + m_N^2)^{\frac{1}{2}} \quad . \tag{26}$$

If we add a spectator nucleon b the Hilbert space is

$$\mathcal{H} = (\mathcal{H}_N \oplus \mathcal{H}_\Delta \oplus \mathcal{H}_N \otimes \mathcal{H}_\pi) \otimes \mathcal{H}_N \quad , \tag{27}$$

$G_{a\pi}$ is defined in the obvious manner and

$$G_{a\pi,b} = G_{a\pi} + G_b \quad . \tag{28}$$

It follows that

$$\Omega_{a\pi,b\pm} = \Omega_{a\pi\pm} \quad . \tag{29}$$

As an operator on functions $\vec{k}_\pi, \vec{P}$ and $\vec{q}_b$, defined by

$$q_b = L(\vec{P}/M_o)p_b \quad , \tag{30}$$

$\vec{X}_o$ is again $i\nabla_P$. The mass operator $\tilde{M}_{a\pi,b}$ defined by

$$\tilde{M}_{a\pi,b} := (\vec{q}_b^{\,2}+\tilde{M}_{a\pi}^2)^{\frac{1}{2}} + (\vec{q}_b^{\,2}+m_N^2)^2 \tag{31}$$

commutes with $\vec{X}_o$ if

$$(\vec{q}_\pi{}'\vec{P}'|\tilde{M}_{a\pi}|\vec{P}\vec{q}_\pi) := \delta(\vec{P}'-\vec{P})\delta(\vec{q}_\pi{}'-\vec{q}_\pi)\hat{M}_{a\pi} \tag{32}$$

where $\hat{M}_{a\pi}$ is the block matrix (25). The representations $\tilde{G}_{a\pi,b}$ and $\tilde{G}_{a\pi,b}$ are scattering equivalent.

The complete mass operator $\tilde{M}$ is then

$$\tilde{M} = \tilde{M}_{NN,\pi}+\tilde{M}_{a\pi,b}+\tilde{M}_{b\pi,a} - 2\ M_o+V_o+V'' \quad , \tag{33}$$

where V_o is a two-body interaction in $\mathcal{H}_{NN} \oplus \mathcal{H}_{N\Delta}$ and vanishes in $\mathcal{H}_{NN\pi}$, and V'' is a three-body interaction in $\mathcal{H}_{NN\pi}$ with transition matrix elements to $\mathcal{H}_{NN}$. Betz and Lee[2] have fitted the parameters of a model of this type to pion-nucleon scattering and to both elastic and inelastic nucleon-nucleon scattering. The application to pion-deuteron scattering produced reasonable results.

We now come to the problem of electromagnetic interactions. What can be done to combine the quantum electrodynamics of photons and electrons with a direct-interaction hadron model? Is it possible to add to the Hamiltonian the standard interaction of the form

$$H' = \int d^3x\; j_h^{\mu}(\vec{x}) A_{\mu}(\vec{x}) \tag{34}$$

where $j_h^{\mu}(\vec{x})$ is a hadron current density? The following lemma should be useful.

Lemma: Assume that $\vec{J},\vec{P},H,\vec{K}$ satisfy the Poincaré commutation relations and define

$$H' := \int d^3x\; \eta(\vec{x}) \quad , \tag{35}$$

$$K' := \int d^3x\; \vec{x}\; \eta(\vec{x}) \quad , \tag{36}$$

where $\eta(0)$ commutes with $\vec{J}$ and $\vec{K}$,

$$[\vec{J},\eta(0)] = [\vec{K},\eta(0)] = 0 \quad , \tag{37}$$

and

$$\eta(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\; \eta(0)\; e^{-i\vec{P}\cdot\vec{x}} \quad . \tag{38}$$

Then the generators $\vec{J},\vec{P},H+H'$, $\vec{K}+\vec{K}'$ satisfy the commutation relations (I.3)-(I.9) provided $\vec{K}'$ commutes with H' and the components of $\vec{K}'$ commute with each other,

$$[\vec{K}',H'] = \frac{1}{2} \int d^3x \int d^3x' (\vec{x}-\vec{x}')[\eta(\vec{x}),\eta(\vec{x}')] = 0 \tag{39}$$

$$[K_i,K_j] = \frac{1}{2} \int d^3x \int d^3x' (\vec{x}\times\vec{x}')[\eta(\vec{x}),\eta(\vec{x}')] = 0 \quad . \tag{40}$$

From (35), (37) and (38) it follows that

$$[\vec{P},H'] = [\vec{J},H'] = 0 \tag{41}$$

and from (35), (36), (37) and (38) we have

$$[J_i,K_j'] = i \sum_k \varepsilon_{ijk}\; K_k' \quad , \tag{42}$$

and

$$[K_i',P_k] = i\delta_{ik}\; H' \quad . \tag{43}$$

From (38) and (I.9) it follows that

$$[K_i, \eta(\vec{x})] = x_i[H, \eta(\vec{x})] \quad , \tag{44}$$

and hence

$$[K_i, H'] + [K_i', H] = 0 \quad . \tag{45}$$

Thus Eq. (39) is necessary and sufficient for

$$[(\vec{K}+\vec{K}'), (H+H')] = i\,\vec{P} \tag{46}$$

to hold. From (44) it follows that

$$[K_i, K_k'] + [K_i', K_k] = 0 \ . \tag{47}$$

Thus Eq. (40) is necessary and sufficient for

$$[(K_i+K_i'), (K_k+K_k')] = i \sum_m \varepsilon_{ikm} J_m \tag{48}$$

to hold. Obviously local commutativity, $[\eta(\vec{x}), \eta(\vec{x})] = 0$, is sufficient for (39) and (40).

For practical purposes the conditions (39) and (40) can be ignored for the large number of applications where H' is a perturbation and the first order is sufficient. An example is high-energy electron-nucleus scattering in the one-photon exchange approximation.

Let G_{em} and G_h be respectively the Poincaré generators of quantum electrodynamics (electrons, positrons and photons) and for a system of hadrons with direct interactions. Then the operator $\eta(\vec{x})$,

$$\eta(\vec{x}) = j^\mu(\vec{x}) A_\mu(\vec{x}) \tag{49}$$

satisfies (37) if $j^\mu(\vec{x})$ is a hadron current density satisfying

$$U_h(\Lambda) j^\mu(0) U_h^{-1}(\Lambda) = \Lambda^\mu{}_\nu \, j^\nu(0) \tag{50}$$

and $A_\mu(\vec{x})$ is the Maxwell field,

$$U_{em}(\Lambda)\, A^\mu(0) U_{em}^{-1}(\Lambda) = \Lambda^\mu{}_\nu\, A^\nu(0) \quad . \tag{51}$$

No general prescription is known for the construction of the current density for directly interacting hadrons. Approximate solutions can be attained by formal expansion in inverse powers of the velocity of light.[3,4] Classical theory suggests that the construction of a covariant conserved current may be related to particle position operators satisfying the world-line conditions. Canonical coordinates cannot satisfy the world-line conditions exactly[5] but they can be satisfied approximately[6] in a formal expansion in inverse powers of the velocity of light to order $1/c^2$. The approximate construction of covariant conserved currents and the approximate world-line conditions are indeed closely related in that approximation but the approximations do not seem to point to an exact relation.

A word of caution is in order concerning expansions in powers of $1/c^2$. The velocity of light is a convenient tage, but its power does not by itself measure the size of terms in the expansion. The relevant physical quantities are the velocities of the particles. In a classical theory the expansion is justified if the velocities of all particles are small compared to the velocity of light everywhere on each orbit. In a quantum mechanical theory the expansion is in powers of the unbounded operator $\vec{p}^2/(mc)^2$ $(c=1)$. An expansion of $(\vec{p}^2+m^2)^{\frac{1}{2}}$ in powers of $\vec{p}^2/m^2$ must be justified by restrictions on acceptable states ψ . The error of a nonrelativistic approximation

$$\|((m^2+\vec{p}^2)^{\frac{1}{2}} - m - \frac{\vec{p}^2}{2m})\psi\| \tag{52}$$

may be acceptably small. The errors of successive improvements

$$\|((m^2+\vec{p}^2)^{\frac{1}{2}} - m - \frac{\vec{p}^2}{2m} + \frac{1}{8}\frac{\vec{p}^4}{m^3})\psi\|$$

and

$$\|((m^2+\vec{p}^2)^{\frac{1}{2}} - m - \frac{\vec{p}^2}{2m} + \frac{1}{8}\frac{\vec{p}^4}{m^3} - \frac{1}{16}\frac{\vec{p}^6}{m^5})\psi\|$$

may or may not be successively smaller. Momentum-space wavefunctions typically decrease as some power of the momentum for large p. Depending on the nonrelativistic approximation may be quite adequate, but the improved versions are much worse. Or perhaps the first relativistic correction is still an improvement. The moral of this story is simple: Quit while your are ahead! Don't press your luck! Also it may be legitimate to expand in powers of some momenta and not others. In the applications of the NNπ model discussed earlier the pion velocities are usually relativistic, baryon velocities are usually but not always nonrelativistic.

Expansion in powers of $1/c^2$ have been widely used for the purpose of constructing compatible interaction terms for $\vec{K}$ and H without recourse to the Bakamjian-Thomas construction. The procedure has yielded satisfactory results to order $1/c^2$. In that approximation cluster separability, the world line conditions and a reasonable relation to conventional field theories are all closely related.

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