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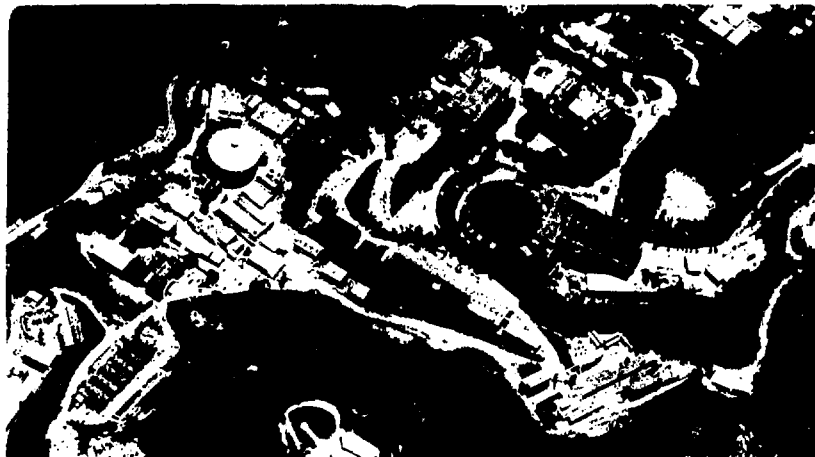
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AN ANALYSIS OF FORWARD AND NEAR-FORWARD ELASTIC-
SCATTERING AMPLITUDES FOR pp AND $\bar{p}p$ COLLISIONS

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AN ANALYSIS OF FORWARD AND NEAR-FORWARD
ELASTIC-SCATTERING AMPLITUDES FOR pp AND $\bar{p}p$ COLLISIONS^{*†}

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Abstract

We will present the results of two recently published (1983) papers by M.M. Block and R.N. Cahn^(1,2), which analyze for $\bar{p}p$ and pp elastic scattering the ρ values (ratios of the real to the imaginary parts of the forward nuclear scattering amplitudes), the total (hadronic) cross sections σ , and the b values, the nuclear slope parameters. The predictions of the analyses, from $\sqrt{s} > 5$ GeV, is compared with the recently measured values of σ and b at the SPS Collider. The analysis has also been redone to include new ISR data available from R211 at $\sqrt{s} = 62.5$ GeV, in order to estimate "odderon" contributions, i.e., contributions from odd amplitudes with unconventional (non-Reggeon) energy dependence. Limits of $\sim 1\%$ are placed on these amplitudes. Our analysis has been extrapolated up to 100 TeV, to give σ , ρ and b predictions for cosmic ray and future collider energies.

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The high energy behavior (defined here as $\sqrt{s} > 5$ Gev) of total cross sections σ , cross section differences ($\Delta\sigma = \sigma(pp) - \sigma(\bar{p}p)$), ρ values (ratio of the real to imaginary portion of the forward hadronic amplitude), and b values, the nuclear slope parameters, for both pp and $\bar{p}p$ elastic scattering, have long been one of the central concerns of high energy physics. As we note in our papers analyzing these quantities (1,2), the recent operation of the ISR with both $\bar{p}p$ and pp collisions has dramatically extended the energy domain of possible measurements (up to $\sqrt{s} = 52.8$ Gev at the time of publication). Recently, R211 has measured the cross section, σ , ρ and b at $\sqrt{s} = 62.5$ Gev for pp , and $\bar{p}p$, and UA4(4) has measured $\sigma(\bar{p}p)$ and $b(\bar{p}p)$ (at low $|t|$) at $\sqrt{s} = 540$ Gev, which we heard reported this morning. In this note, we have incorporated these new ISR results, which represent the end of a 25 year era in comparing pp and $\bar{p}p$ elastic data, into our analysis in order to test for the possible presence of "odderons". The "odderon", a name coined by Prof. Nicolescu, is an odd amplitude (of a non-Reggeon variety) with an unconventional energy dependence, and we will come back to it later on.

Analyticity is a central theme of asymptotic energy behavior, and is of course essential to our analysis. However, rather than introducing analyticity via the form of dispersion relations, we exploit it in a more direct and simple fashion by writing out explicit amplitudes of the proper analytic behavior and symmetry (in the high energy domain, where the amplitudes, far from poles, are smooth functions of the center-of-mass energy, \sqrt{s}). As shown in Fig. 1, it is convenient to introduce even and odd amplitudes, M_{\pm} . Our goals are three-fold:

- (1) To parameterize σ , ρ , b concisely and precisely, for both $\bar{p}p$ and pp data, from $\sqrt{s} > 5$ Gev to $\sqrt{s} = 62.5$ Gev.
- (2) To extrapolate these fits to collide energies $\sqrt{s} = 540$ Gev ($\sigma = 71$ mb) and the Tevatron collider $\sqrt{s} = 2000$ Gev ($\sigma = 100$ mb) and indeed, to $\sqrt{s} \sim 20$ Tev ($\sigma = 175$ mb), the cosmic ray domain that will be discussed later by Prof. Yodh(3).

- (3) To see if the forward (hadronic) pp and $\bar{p}p$ amplitudes become equal at very high energies. For example, how well do the data of today exclude the possibility of a constant cross section difference?

As detailed in our paper⁽¹⁾, we get an excellent fit to all available data, putting in Fig. 2, $C = a = 0$. C is the equivalent of a subtraction constant in dispersion relations, and turns out to be negligible, whereas setting $a=0$ means the cross section asymptotically continues to rise with a $\ln^2 s$ behavior, its limiting value due to the Froissart bound. All data are fitted⁽¹⁾ with the 5 constants A, B, so, D, α given in Table 1, Fit 1. The explicit formulae for σ and ρ are given in Fig. 2. This note has updated our published analysis by including the recent experimental results at $\sqrt{s} = 62.5$ Gev, presented by R211⁽⁴⁾ this morning. The values of A, B, so, D, α given by Fit 2, Table 1, remain unchanged, with a new $\chi^2/d.f. = 88.7/76$. The predictions⁽¹⁾ for σ and ρ are shown in Fig. 3 and 4, respectively, compared to the experimental data and are the upper curves. In Fig. 3, the total cross section value at $\sqrt{s} = 540$ Gev, $(1 + \rho^2) \sigma = 71 \pm 7$ mb, as reported this morning by UA4⁽⁵⁾, is shown. We used the value of $\rho = 0.2$ (Fig. 4) in appending the value $\sigma = 68.3 \pm 7$ mb to Fig. 3. Also shown in Fig. 3 and 4 are the dotted curves (lower) corresponding to a non-zero value of a in Fig. 1. It is an attempt to see if the measured values of σ at today's energies, which are dominated by a $\ln^2 s$ behavior, are indeed compatible with a total cross section that eventually will flatten and go to a constant value. Its parameters are given in Table 1, Fit 2. Clearly, the present experimental results are too inaccurate now to decide. A much more sensitive place to look is in the ρ value, and a reasonable measurement of ρ at the SPS might soon settle this important issue. The prediction of 175 mb for $\sqrt{s} = 20$ TeV also allows cosmic ray data to verify whether cross sections continue rising* as $\ln^2 s$. The nuclear slope parameter $b(\bar{p}p)$ has been measured at small t by UA4⁽⁵⁾ and UA1⁽⁶⁾, and was also presented

*The prediction for $a \neq 0$ is ~ 110 mb at this energy.

this morning. They obtained the values $b = 17.6 \pm 1.0$ and 17.1 ± 1.0 (Gev/c)⁻². We show in Fig. 4 our parameterization of the nuclear slopes $b(pp)$ and $b(\bar{p}p)$, in terms of even and odd slopes $b^+ = C_+ + D_+ \ln s + E_+ \ln^2 s$ and $b^- = C_- + D_- \ln s$, for very small $|t|$. By fitting the low energy data up to ISR energies⁽²⁾, we obtain the values $C_+ = 10.94 \pm .53$, $D_+ = -.09 \pm .18$, $E_+ = 0.44 \pm .016$, $C_- = 23.2 \pm 1.6$ and $D_- = .94 \pm .16$. Our analysis predicts $b_{pp} = 16.7 \pm 0.9$ (Gev/c)⁻² at $\sqrt{s} = 540$ Gev, and it is excellent agreement with the new measurements. The fitted curve is shown in Fig. 5, along with the experimental values.

We now turn our attention the "odderons", the non-Reggeon odd amplitudes whose strengths are denoted by $E^{(0)}$, $E^{(1)}$ and $E^{(2)}$ in Fig. 1. We note that odderon⁽⁰⁾ gives no contribution to the total cross section as $s \rightarrow \infty$, but does not allow $\Delta\sigma$ to go to zero as $s \rightarrow \infty$. Odderon⁽¹⁾ gives both finite $\Delta\sigma$ and $\Delta\rho$ as $s \rightarrow \infty$, whereas odderon⁽³⁾ actually causes $\Delta\sigma$ to diverge as $\ln s$ as $s \rightarrow \infty$. Of course, all 3 possibilities are allowed by the Pomeranchuk theorem, which for cross sections rising as $\ln^2 s$, only states that the ratio of $\sigma_{pp}/\sigma_{\bar{p}p} \rightarrow 1$ as $s \rightarrow \infty$. The statement that a cross section, rising asymptotically as $\ln^2 s$, "saturates the Froissart bound" is perhaps semantically misleading. Although the energy rise due to the Froissart bound can go no faster than $\ln^2 s$, the (experimental) coefficient of $\ln^2 s$ is only ~1% of the allowed strength of the $\ln^2 s$ term. This perhaps is one of the deeper mysteries remaining in our understanding of strong interactions in the high energy domain.

Our original analysis⁽¹⁾, as shown in Table I, indicates that all 3 coefficients $E^{(0)}$, $E^{(1)}$ and $E^{(2)}$ of the odderons are only about two standard deviations from zero (each taken separately). When compared to the dominant even amplitude, the odderon amplitude is at most ~1%. Of course, if compared to the odd Reggeon amplitude, which vanishes with increasing \sqrt{s} , they will become large by definition since they do not vanish with increasing energy. In particular, to give a scale to these results, it is instructive to examine the constant $\Delta\sigma$ arising from odderon⁽¹⁾, i.e., $\Delta\sigma = \frac{\pi}{2} E^{(1)} = -.16 \pm .06$ mb., as $s \rightarrow \infty$. This would suggest that there is a crossover in pp and $\bar{p}p$ cross

sections at very high energies, with the pp cross section eventually $.16 (\pm .06)$ mb higher. Since $\sqrt{s} = 62.5$ Gev is the upper end of the ISR and is the highest energy where, for the next decade or so, one can compare elastic pp and $\bar{p}p$ scattering, we refit all of the experimental data at lower energies along with the new R211 data⁽⁴⁾, for odderon⁽¹⁾. The result is essentially indistinguishable from that given in Table I, with only the coefficient D changing from -40.8 ± 1.8 to -40.9 ± 1.8 , and $\chi^2/d.f. = 80.6/75$. Thus, our original conclusions^(1,2) that the odderon contributions are exceedingly small is reinforced by the new experimental data.

There is currently discussion underway to plan for an ultra-high energy collider of 20 Tev on 20 Tev, using $\bar{p}p$ collisions. If the cross section continues to rise as $\ln^2 s$, we predict $\sigma = 200$ mb, $\rho = .16$ and $b = 29$ (Gev/c)⁻².

References

- 1) M.M. Block and Robert N. Cahn, Phys. Lett. 120B, 224 (1983).
- 2) M.M. Block and Robert N. Cahn, Phys. Lett. 120B, 229 (1983).
- 3) G. Yodh, Cosmic Ray Results, this conference.
- 4) M. Bottje, R211 results, this conference.
- 5) Sanguinetti - UA4 results, this conference.
- 6) Hodges, UA1 results, this conference.

Acknowledgments

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Fit Type	A(mb)	B(mb)	s_0 (GeV ²)	D(mb) GeV ^{2-2α}	α	a	C(mb) GeV ²	E(mb)	χ^2 d.f.
1) Simple (a=0, C=0)	41.77 ± 0.04	0.68 ± 0.01	343. $\pm 8.$	-39.0 ± 1.7	0.48 ± 0.01	-	-	-	86.7/73
2) Constant asymptotic cross-section (a \neq 0, C=0)	41.74 ± 0.04	0.66 ± 0.02	338. $\pm 8.$	-38.7 ± 1.6	0.49 ± 0.01	0.0050 ± 0.0031	-	-	84.0/72
3) With subtraction constant (a=0, C \neq 0)	41.77 ± 0.04	0.68 ± 0.01	344. $\pm 8.$	-39.2 ± 1.8	0.48 ± 0.01	-	5.0 ± 10.6	-	86.5/72
4) Odderon 0	41.77 ± 0.04	0.69 ± 0.01	345. $\pm 8.$	-41.7 ± 2.4	0.46 ± 0.02	-	-	-0.26 ± 0.13	82.6/72
5) Odderon 1	41.74 ± 0.04	0.69 ± 0.01	350. $\pm 8.$	-40.8 ± 1.8	0.49 ± 0.01	-	-	-0.10 ± 0.04	80.1/72
6) Odderon 2	41.70 ± 0.05	0.66 ± 0.01	356. $\pm 10.$	-35.2 ± 2.2	0.50 ± 0.02	-	-	-0.04 ± 0.02	81.8/72

TABLE I - Parameters for the best fits to the cross-section and ρ values for pp and $\bar{p}p$ data. The parameters a and C are set to zero except in Fits 2 and 3. The odd amplitude for the first three fits is given by Eq. (5). For the last three fits the odd amplitude is a sum of this term and one term from among the three Odderons.

$$M_{\pm} = [M_{\bar{p}p} \pm M_{pp}] / 2$$

$$\sigma = -\text{Im } M/s \quad \text{and} \quad \rho = \text{Re } M / \text{Im } M$$

$$M_{+} = -is \left[A + \frac{B(\ln s/s_0 - i\pi/2)^2}{1 + a(\ln s/s_0 - i\pi/2)^2} \right] + C$$

$$M_{-} = Ds^{\alpha} e^{i\pi(1-\alpha)/2} \quad (\text{Normal Amplitude})$$

(Odderons)

$$M_{-}^{(0)} = E_s$$

$$M_{-}^{(1)} = E_s (\ln s/s_0 - i\pi/2)$$

$$M_{-}^{(2)} = E_s (\ln s/s_0 - i\pi/2)^2$$

$$M_{- \text{tot}} = M_{-} + M_{-}^{(j)}, \text{ where } j=0, 1, \text{ or } 2.$$

Fig. 1 Decomposition of forward scattering amplitudes for pp and $\bar{p}p$ elastic collisions into even and odd amplitudes, including "odderon" contributions.

for $C=E=0$ and for real A, B, D, α , and s_0 :

$$\sigma_{pp} = A + B(\ln^2 s/s_0 - \pi^2/4) + Ds^{\alpha-1} \cos(\pi\alpha/2)$$

$$\sigma_{\bar{p}p} = A + B(\ln^2 s/s_0 - \pi^2/4) - Ds^{\alpha-1} \cos(\pi\alpha/2)$$

$$\rho_{pp} = \frac{\pi B \ln s/s_0 + Ds^{\alpha-1} \sin(\pi\alpha/2)}{\sigma_{pp}}$$

$$\rho_{\bar{p}p} = \frac{\pi B \ln s/s_0 - Ds^{\alpha-1} \sin(\pi\alpha/2)}{\sigma_{\bar{p}p}}$$

Fig. 2 Formulae for total cross sections σ and real to imaginary ratios ρ , for pp and $\bar{p}p$.

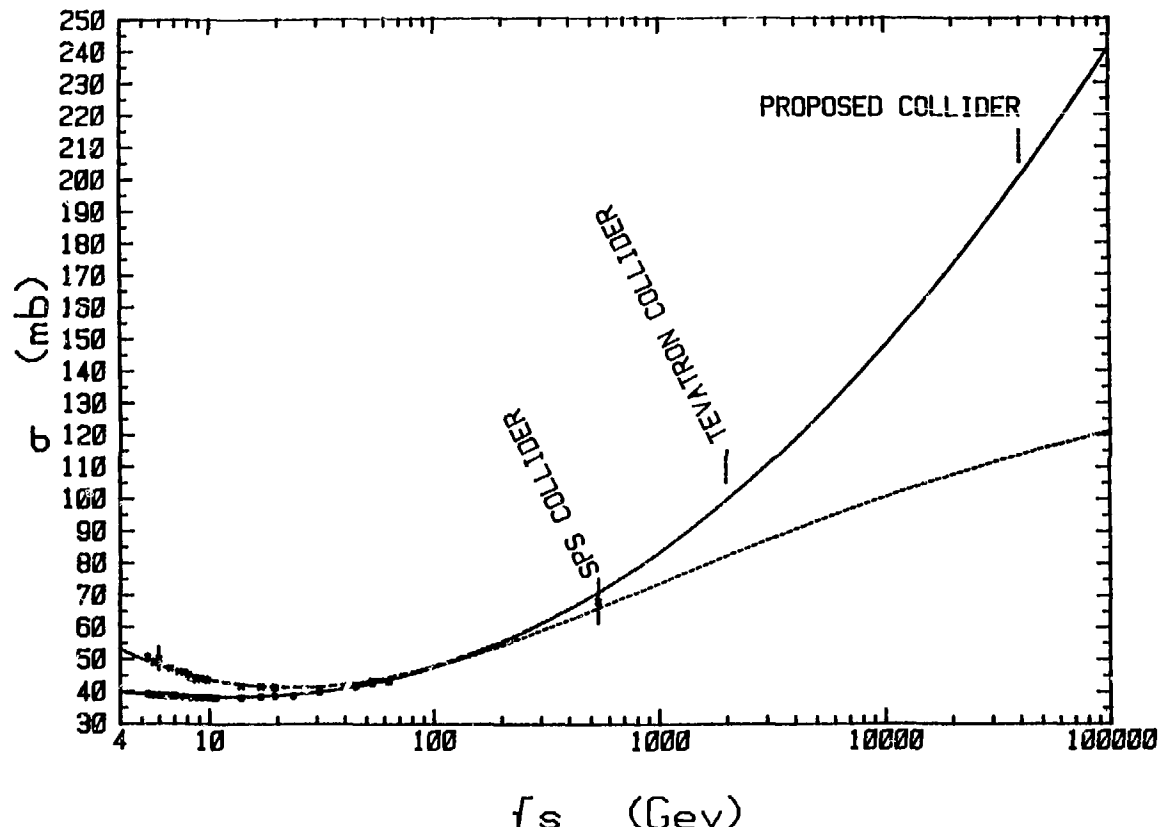


Fig.3 Cross section predictions from $\sqrt{s} = 5$ Gev to $\sqrt{s} = 100$ Tev, for both pp and $\bar{p}p$. The upper (solid) curve is Fit 1, from Table 1, ($a=0$), whereas the lower (dotted) curve is Fit 2, Table 1.

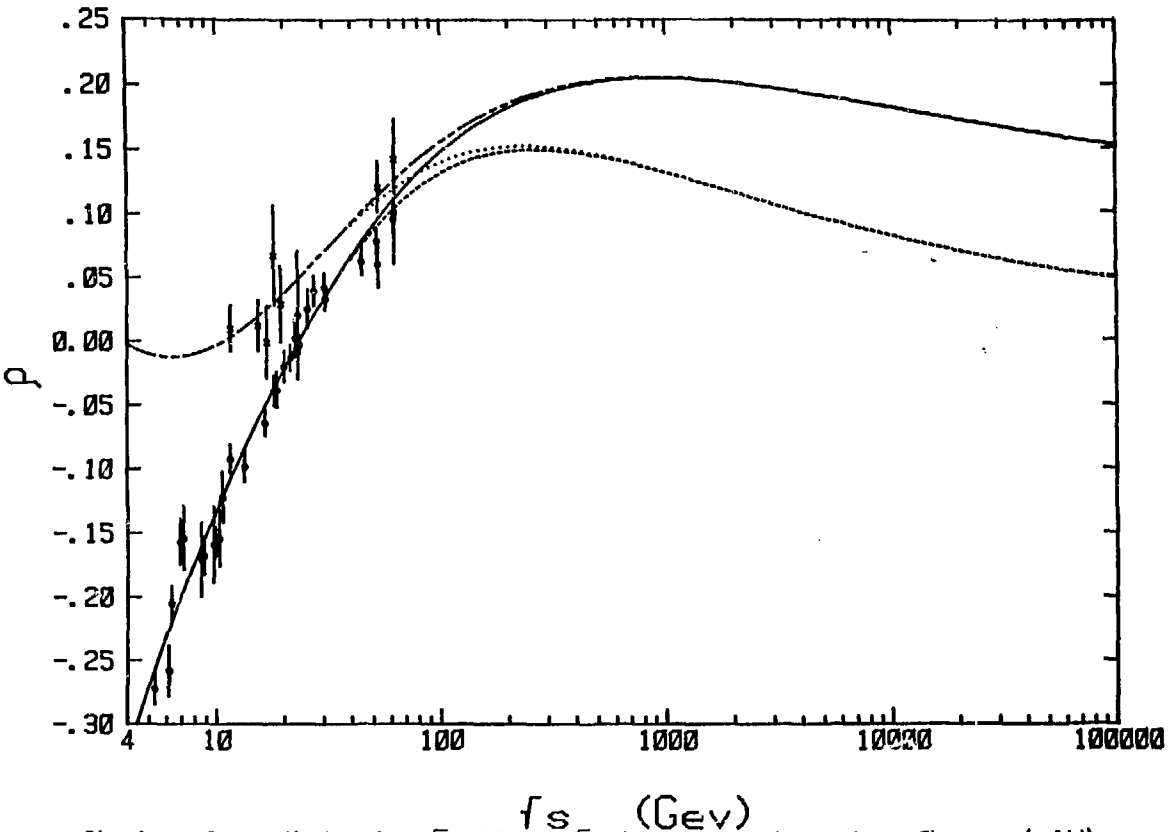


Fig. 4 ρ -value predictions from $\sqrt{s} = 5$ GeV to $\sqrt{s} = 100$ TeV for both pp and pp. The upper (solid) curve is Fit 1, from Table 1, whereas the lower (dotted) curve is Fit 2, Table 1.

$\sigma = p$ $x = \bar{p}$

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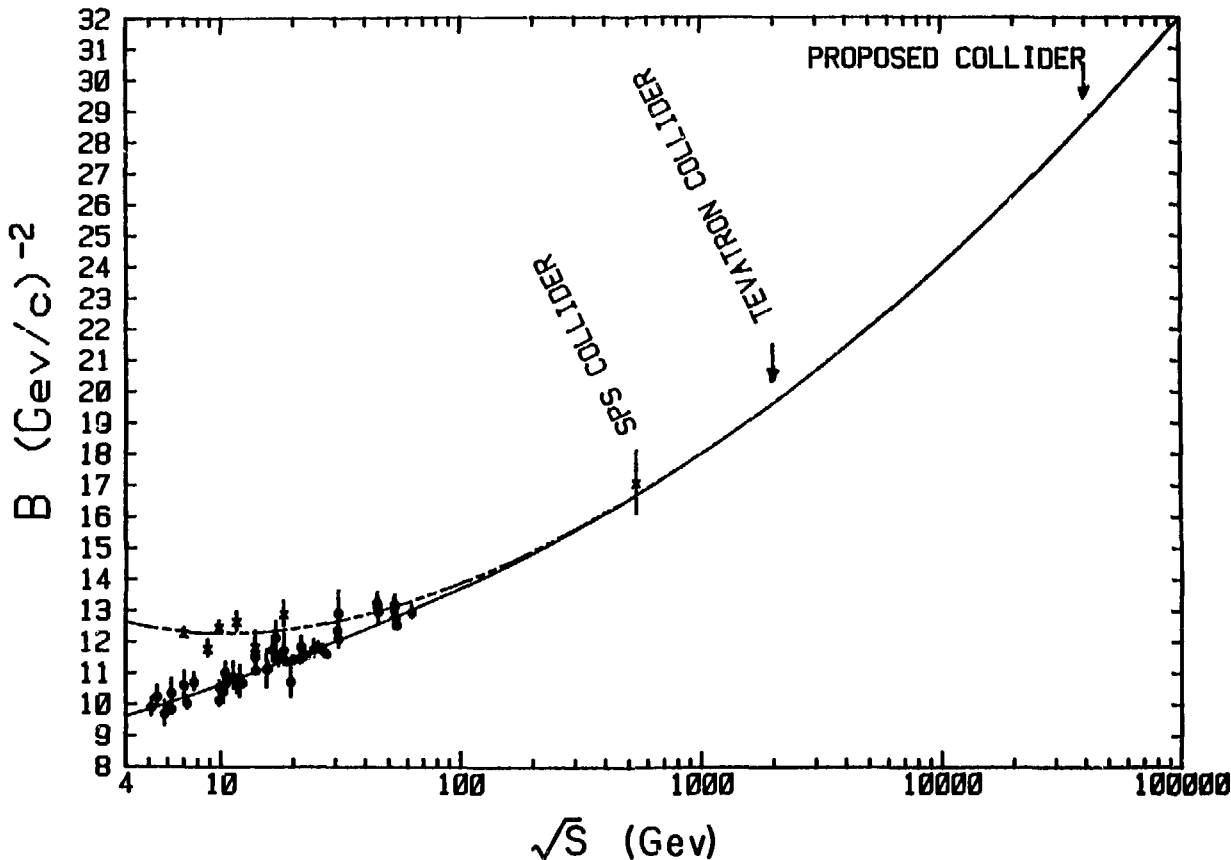


Fig. 5 The predictions for b , the nuclear slope parameter, from $\sqrt{s} = 56$ GeV to $\sqrt{s} = 100$ TeV, for both pp and $\bar{p}p$.

NEAR-FORWARD ELASTIC SCATTERING

$$d\sigma/dt = |M_+ f_+(s, t) \pm M_- f_-(s, t)|^2,$$

with + for $\bar{p}p$ and - for pp , and where :

$$f_{\pm}(s, t) = \exp[b_{\pm}(s) t/2],$$

and

$$b_+(s) = C_+ + D_+ \ln(s) + E_+ \ln^2(s),$$

$$b_-(s) = C_- + D_- \ln(s).$$

Fig. 6 Formulae for nuclear slope parameters b for pp and $\bar{p}p$ elastic scattering.