An Analytic Distorted Wave Approximation for Intermediate Energy Proton Scattering

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Abstract: An analytic Distorted Wave approximation has been developed for use in analyses of intermediate energy proton inelastic scattering from nuclei. Applications are made to analyse 402 and 800 MeV data from the isoscalar and isovector $1^+$ and $2^+$ states in $^{12}$C and to the 800 MeV data from the excitation of the $2^-$ (8.88 MeV) state in $^{16}$O. Comparisons of predictions made using different model two-nucleon t-matrices and different models of nuclear structure are given.
1. Introduction.

It has long been realised that nuclear reaction data initiated by nucleons with energies less than 100 MeV was of limited use in determining properties of nuclear structure. Not only was the nuclear structure content of the reaction analyses diffused by folding with a two nucleon t-matrix whose characteristics were largely unknown but also distortion effects in the entrance and exit channel functions were significant and necessary to successfully predict data. Furthermore analyses of select transitions for which nuclear spectroscopy was well understood provided information about the two nucleon t-matrix but only for a small range of low momentum transfers (0 to 2 fm"1 typically). Consequently there has been much interest in intermediate energy (> 100 MeV) data for not only are distortion effects in analyses less severe than for low energy reactions but also data can be obtained at higher momentum transfer values. Furthermore, simplifying approximation techniques, such as the Impulse approximation, are expected to be of use with increasing projectile energies and the nuclear two-nucleon t-matrices should be not far removed, if at all, from those of two free nucleons. Even so, a variety of model t-matrices have been promulgated in the literature and the validity of the impulse approximation questioned.

Herein we present a model for intermediate energy reaction analyses, and for inelastic scattering in particular, that makes use of
the nature of distortion in optical model wave functions
(McCarthy and Pursey, 1961; Amos, 1967; Janus and McCarthy, 1974; Bruce, 1981). At intermediate energies the usual distorted wave functions may be approximated, at least in the surface region of a nucleus, by attenuated plane waves. Such approximations have attendant analytical advantages when used in derivation of reaction amplitudes and thus yield the Analytic Distorted Wave Approximation (A.D.W.A.) This approach allows use of complex, energy and density dependent two-nucleon t-matrices having central, tensor and spin-orbit characteristics in analyses of transition data without necessitating the Impulse Approximation and its concomitant on-shell limitation whilst engendering evaluations as facile as those made under the Impulse approximation.

The ADWA for intermediate energy inelastic proton scattering from nuclei is developed in the next section and applied to analyse select reaction data from $^{12}$C taken at 402 MeV (Haji-Saeid et al., 1982) and at 800 MeV (Blanpied et al., 1978). The method is used with the 400 MeV $l^+$ transition data to compare three model t-matrices and denoted by L-F (Love and Franey, 1981), P-W (Picklesimer and Walker, 1978) and Paris (Nakano and von Geramb, 1981). This comparison confirms use of this ADWA and of select reaction data to differentiate between alternative t-matrix properties. The L-F force is then used to analyse 800 MeV data and the results further demonstrate a capability of this approach to differentiate between alternative nuclear structure-model properties.
Relevant details of the model t-matrices are given in Section 3 whilst those of the nuclear structure models and the results of calculations made using the ADWA are presented thereafter.

2. Theory.

Using relativistic kinematics, inelastic proton scattering differential cross sections are determined by

\[ \frac{d\sigma}{d\Omega}(\text{mb}/\text{sr}) = \left(\frac{S}{J_A^{\text{CP}}}\right)^{\frac{1}{2}} \sum_{i_1 f_1} |T_{if}|^2 \]  

(1)

wherein all quantities are specified in the centre of mass system for the scattering of a proton (labels 'i') from a target (labels 'A') leaving a residual excited nucleus (labels 'B') and a scattered proton (labels 'f'). All quantities are of standard form namely,

with \( \mathcal{M} = c = 1, \)

\[ \tilde{s} = (E_A^* + E_i^*)^2 = \text{invariant mass}, \]
\[ p_i = \left[\lambda(s,m_1^2,M_A^2)/\tilde{s}\right]^{\frac{1}{2}}, \]
\[ E_i = (\tilde{s} + m_1^2 - M_A^2)/2\tilde{s}^{\frac{1}{2}}, \]

and

\[ \lambda(x,y,z) = (x-y-z)^2 - 4yz. \]  

(2)

Other momenta and energies are similarly defined and, with \( a \) denoting \((2a+1)\), the remaining part of Equation (1) is the sum (over asymptotic spin projection values) of transition probabilities.
We will assume that the transition amplitudes have the form

\[ T_{if} = N \langle x_f^{(-)}(0) \psi_B^{(1..N)}(0) | t(01) | x_i^{(+)}(0) \psi_A^{(1..N)}(0) \rangle \]  

wherein \( t(ab) \) is a two nucleon t-matrix by which the nuclear transition between states \( J_A \) and \( J_B \) is effected and \( x_s^{(2)} \) are the relative motion wave functions of the projectile.

Standard algebra then enables Equation (3) to be recast independent of what model of (microscopic) nuclear structure is chosen (Amos et al., 1978), as

\[ T_{if} = \sum_{j_1 j_2 I} S^{(a)}(j_1 j_2; J_A J_B; I) M_{j_1 j_2} \]  

wherein

\[ S^{(a)}(j_1 j_2; J_A J_B; I) = S^{(a)}_{j_1 j_2} = \langle \psi_J_B | [a_{j_2}^+ \times a_{j_1}]_I | \psi_J_A \rangle \]  

are spectroscopic amplitudes (\( a \) denoting proton/neutron) for the transition that weight the two particle matrix elements

\[ M_{j_1 j_2} = \sum_{m_1 m_2 N} (-1)^{j_1 m_1} \langle j_1 j_2 m_1 m_2 | I-N \rangle \]

\[ \times (j_B^{-1}) \langle J_A I_A N | J_B V \rangle \langle x_f^{(-)}(0) \phi_{j_2 m_2}^{(1)}(0) | t(01) | x_i^{(+)}(0) \phi_{j_1 m_1}^{(1)}(0) \rangle. \]
5.

in which $\phi_{jm}(r)$ are the wave functions of the struck bound nucleon, chosen herein to be harmonic oscillator wave functions ($\hbar\omega = 14.9, 13.9$ MeV for $^{12}\text{C}, ^{16}\text{O}$ respectively). The projectile relative motion wave functions, $\chi^{(\pm)}$, are most commonly represented by Distorted Wave (Optical Model) functions which then necessitates use of partial wave expansions in the development of the matrix elements. Simplifying coordinate transformations are thus precluded. For intermediate energies, however, the distortion effects in the spatial region of most relevance for inelastic scattering transition probabilities, the nuclear surface, can be well represented by attenuated plane wave functions (Amos, 1966; Janus and McCarthy, 1974; Bruce, 1981) namely

$$|\chi^{(+)}(0)\rangle = N_1 \exp(i\vec{k}_1 \cdot \vec{r}_0) |\tau_1\rangle |\nu_1\rangle$$

with

$$\vec{k}_1 = (a+ib)\vec{k}.$$  

The parameters $a$ and $b$ may be estimated from a local WKB approximation and/or by fitting scattering data (Janus and McCarthy, 1974) with the normalisation factor, $N_1$, set to yield unit amplitudes for the continuum functions at the nuclear surface. A better scheme to choose the parameter values is to use the attenuated plane wave function in the Born approximation integral with the appropriate optical model potential and thereby fit elastic scattering data (Bruce, 1981). Whatever be the
method of fixing parameters, the form of the attenuated plane waves facilitates a transformation of coordinates in the two particle matrix elements to give

\[ M_{j_1j_2} = \sum_{m_1m_2N} (-)^{j_1m_1} \langle j_1j_2m_1m_2|I-N \rangle \left( I_B \right)^{-1} \]

\[ \times \langle J_A | N \rangle \langle J_B \rangle \langle I_f \rangle \]

\[ \times \langle t_{j_f} \rangle \langle j_{2f} \rangle \langle \exp(ik_{f} \cdot r) | t(r) | \exp(ik_{f} \cdot r) \rangle \]

\[ \times \langle \exp(ik_{j_1} \cdot r_1) \phi_{j_2m_2} (1) \rangle \langle \exp(ik_{j_1} \cdot r_1) \phi_{j_1m_1} (1) \rangle \]

\[ \langle \psi_{i_1} \rangle \langle \psi_{i_2} \rangle \]

(8)

Thus the t-matrix and nuclear structure aspects of intermediate energy reaction amplitudes separate (as with the impulse approximation) and standard angular momentum algebra with a t-matrix of the form

\[ t(01) = \sum_{ST} \langle \psi_T | S_T | \psi_{01} \rangle \]

\[ + \sum_{T} \langle \psi_T | S_{01} | \psi_{01} \rangle \]

\[ + \sum_{T} \langle \psi_T | S_{01} | \psi_{01} \rangle [L \cdot S / h^2] p^T \]

yield "analytic" matrix elements, and hence the analytic Distorted Wave approximation (A,D.W.A.), of
7.

\[ T_{\text{cent}}^{\text{if}} = \sum_{\ell_1 \ell_2 j_1 j_2} N_1 N_F S^{(a)} \left( 2\pi i l \right) \left( - \right)^{S+L+1+j_1-j_2} \hat{\ell} \hat{\ell} \hat{j} \hat{j} [2 \hat\ell_1 \hat{\ell}_2 \hat{j}_1 \hat{j}_2/J_B]^{1/2} <v_{\text{cent}}^{\text{ST}}(Q)>_0 \]

\[ <J_A^{\nu} A^{\nu}|J_B^{\nu} B^{\nu}> <J_A^{\nu} A^{\nu}|J_B^{\nu} B^{\nu}> \left( \begin{array}{c} \frac{1}{2} \frac{1}{2} A \\ \frac{1}{2} \frac{1}{2} S \end{array} \right) \]

\[ <R_{\ell_2 j_2} R_{\ell_1 j_1}(Q)>_L <\ell_2^{\alpha} | T M_i^{\alpha} > <\ell_1^{\alpha} | T M_i^{\alpha} > \]

\[ <\ell_1 \ell_0 | \ell_2 0> <\ell_0 \ell_1 | I-N> \left( \begin{array}{c} \frac{1}{2} \frac{1}{2} \ell \\ \frac{1}{2} \frac{1}{2} \ell \end{array} \right), \quad (10a) \]

\[ T_{\text{tens}}^{\text{if}} = \sum_{\ell_1 \ell_2 j_1 j_2} N_1 N_F S^{(a)} \left( 8\pi i l \right) \left( - \right)^{S+L+1+j_1-j_2} \hat{\ell} \hat{\ell} \hat{j} \hat{j} [3 \hat\ell_1 \hat{\ell}_2 \hat{j}_1 \hat{j}_2/J_B]^{1/2} <v_{\text{tens}}^{\text{ST}}(Q)>_2 R_{\ell_2 j_2} R_{\ell_1 j_1}(Q)>_L, \quad (10b) \]

\[ T_{\text{so}}^{\text{if}} = \sum_{\ell_1 \ell_2 j_1 j_2} N_1 N_F S^{(a)} \left( 6\pi i l \right) \left( - \right)^{S+L+1+j_1-j_2} \hat{\ell} \hat{\ell} \hat{j} \hat{j} [2 \hat\ell_1 \hat{\ell}_2 \hat{j}_1 \hat{j}_2/J_B]^{1/2} <\ell_1 \ell_0 | \ell_2 0> \]

\[ <\ell_1^{\alpha} | T M_i^{\alpha} > <\ell_2^{\alpha} | T M_i^{\alpha} > <\ell_0^{\delta} | \ell_1^{\delta} > \quad (10c) \]

and

\[ <v_{\text{tens}}^{\text{ST}}(Q)>_2 <R_{\ell_2 j_2} R_{\ell_1 j_1}(Q)>_L, \quad (10b) \]

\[ <J_A^{\nu} A^{\nu}|J_B^{\nu} B^{\nu}> <J_A^{\nu} A^{\nu}|J_B^{\nu} B^{\nu}> \left( \begin{array}{c} \frac{1}{2} \frac{1}{2} \ell \\ \frac{1}{2} \frac{1}{2} \ell \end{array} \right) \]

\[ <\ell_1 \ell_0 | \ell_2 0> <\ell_0 \ell_1 | I-N> \left( \begin{array}{c} \frac{1}{2} \frac{1}{2} \ell \\ \frac{1}{2} \frac{1}{2} \ell \end{array} \right), \quad (10a) \]

\[ <\ell_1^{\alpha} | T M_i^{\alpha} > <\ell_2^{\alpha} | T M_i^{\alpha} > <\ell_0^{\delta} | \ell_1^{\delta} > \quad (10c) \]

\[ <v_{\text{tens}}^{\text{ST}}(Q)>_2 <R_{\ell_2 j_2} R_{\ell_1 j_1}(Q)>_L, \quad (10b) \]

and

\[ T_{\text{so}}^{\text{if}} = \sum_{\ell_1 \ell_2 j_1 j_2} N_1 N_F S^{(a)} \left( 6\pi i l \right) \left( - \right)^{S+L+1+j_1-j_2} \hat{\ell} \hat{\ell} \hat{j} \hat{j} [2 \hat\ell_1 \hat{\ell}_2 \hat{j}_1 \hat{j}_2/J_B]^{1/2} <\ell_1 \ell_0 | \ell_2 0> \]

\[ <\ell_1^{\alpha} | T M_i^{\alpha} > <\ell_2^{\alpha} | T M_i^{\alpha} > <\ell_0^{\delta} | \ell_1^{\delta} > \quad (10c) \]

\[ <v_{\text{tens}}^{\text{ST}}(Q)>_2 <R_{\ell_2 j_2} R_{\ell_1 j_1}(Q)>_L, \quad (10b) \]
The summations extend over the quantum numbers $L, T, I, j_1, j_2, I, F$ and $\alpha$ and the radial expectation values are

\[ <G(Q)>_k = \int j_k(Qr) G(r) r^2 dr, \quad (11) \]

involving the complex momentum transfer

\[ Q = \tilde{k}_1 - \tilde{k}_f \quad (12) \]

and which, for intermediate energies (and thus negligible $q$-values insofar as parameter variation is concerned) gives the 'magnitude'

\[ Q = (a+ib) \left(2m_1^2 - 2E_r E_f + 2p_1 p_f \cos \theta_{sc} \right)^{1/2} \quad (13) \]

that appears in the argument of the spherical Bessel functions of Equation (11). The structure radial expectation values in Equations (10) involve functions $R_{kj}(r)$ which are the bound state (harmonic oscillator) radial wave functions. Relevant steps in the derivation of the matrix elements of Equations (10) are given in the Appendix.

Thus the A.D.W.A. for inelastic scattering gives matrix elements that are facile for computation and in which momentum transfer properties of the two nucleon $t$-matrix and momentum transfer properties of nuclear spectroscopy effectively are separated. Given validity of the A.D.W.A. then, it is a convenient scheme to study one or the other of those momentum transfer properties.
Naturally, a collective model prescription of intermediate energy scattering can also be entertained. Using the A.D.W.A., it is a simple exercise to deduce that for excitations of states $\psi_{J_B^T}$ from zero spin ground states

$$
\frac{d\sigma}{d\Omega} \propto (\beta_{J_B} R_0)^2 \left| \left\langle \hat{a}_U / \hat{a}_r (Q) \right| J_B \right|^2
$$

wherein $U(r)$ is the usual optical model potential with $R_0$ the nuclear radius and $\beta_{J_f}$ the deformation parameter.


In the calculations to be reported, three model t-matrices have been used and their momentum transfer properties, as may be reflected in data from inelastic scattering on nuclei, are displayed in figures 1 through 10. In all cases the Love-Franey (L-F) model t-matrix properties are displayed by continuous lines. Properties of the Picklesimer-Walker (P-W) model t-matrix are displayed by the long dashed lines whilst those of a model t-matrix based upon the Paris two-nucleon interaction (Paris) are shown by the short dashed lines. All of these model t-matrices are complex and by using the form

$$
t(r) = t_R(r) - it_L(r)
$$

with real values for $q$, the momentum functions $4\pi <r^L t_R(q)>_L$ and $4\pi <r^L t_L(q)>_L$ are presented in figures 1 through 10 with $L=0$, 1 and 2.
for the central, spin-orbit and tensor components respectively.

These functions, $<t(q)>_L$, can change sign so that negative values are depicted in the diagrams by the inclusion of dots.

All three model t-matrices are combinations of Yukawa functions, i.e.

$$V^{\text{type}}(S)T(r) = \sum_i W^{\text{type}}(i) \exp(-\mu_i r)/\mu_i r$$

(16)

with complex weights, $W(i)$. In the L-F model t-matrix, those weights are also (projectile) energy dependent as are those of the Paris model t-matrix. The Paris model t-matrix furthermore has momentum dependences as well, by virtue of density dependent strengths. We will display the momentum properties of that single set corresponding to a local fermi momentum of 1.0 fm$^{-1}$. From the local density equivalence and a density

$$\rho(r) = 0.156/[(1.0 + \exp(r-2.747)/0.57)]$$

this is the local infinite matter t-matrix at a radius of 2.9 fm.

The P-W model t-matrix (Picklesimer and Walker, 1978) was derived by fitting nucleon-nucleon scattering data (differential cross-sections and polarizations) between 50 and 400 MeV. With complex but energy independent coefficients, quite reasonable fits to data were reported, although the sensitivity
of predictions to parameter variation were not. Further the use (Picklesimer and Walker, 1978) of the PW t-matrix in analyses of inelastic scattering data was inconclusive as a test of the t-matrix model because of the oversimplified nuclear structure and plane wave approximation (at 156 MeV) employed.

The L-F model t-matrix (Love and Franey, 1981) was generated in a similar manner to the P-W version albeit that the complex coefficients of the set of Yukawa interactions were allowed to be energy dependent and determined using a $\chi^2$ search procedure to fit the two nucleon scattering amplitude that was deduced from phase shift analyses of data. In that study (Love and Franey, 1981), the uncertainties in the t-matrix were discussed and a number of applications in inelastic proton scattering and charge exchange reaction analyses were made. But as with the Picklesimer-Walker study, nuclear structure input was simplistic.

The third model t-matrix (Paris) to be used was developed from the free nucleon-nucleon, Paris, potential (Lacombe et al., 1980) by Nakano and von Geramb (Nakano and von Geramb, 1981) to obtain a local energy and density dependent t-matrix from the Bethe-Goldstone equation. The result was then mapped onto a combination of Yukawa functions for each local fermi momentum of a Woods-Saxon matter distribution.
Parameter values for the three model t-matrices are not given herein. Both the P-W and L-F sets of parameters are given in the literature whilst those of the Paris model are very extensive and may be obtained from the University of Hamburg.

The three model t-matrices momentum components for a projectile energy of 400 MeV are compared in figures 1 through 8. The L-F t-matrix is that quoted for 425 MeV in fact.

The singlet odd state t-matrices momentum components are displayed in figure 1. This two body spin channel (S=0) has only central force contributions with the L-F and Paris forces being of similar short ranged repulsive character; the L-F t-matrix having a stronger imaginary component. The P-W t-matrix in this channel has an imaginary part quite like that of the Paris model but its real component has a short ranged attractive part that causes very small momentum (real part) components near 3 fm\(^{-1}\). However, as angular momentum and isospin selection minimizes the effect of this channel in transitions, it is unlikely that nuclear reaction data will test such differences. Certainly the transitions considered herein being of spin and/or isospin flip character will not be influenced by this channel to any extent.

The singlet even and triplet even channels (the Serber force part) play more significant roles in transition data analyses and the central force components of those t-matrices
are shown in figures 2 and 3. The real components all are of long range attractive plus short range repulsive character with the Paris model t-matrix having a further very short ranged attraction. The singlet even (real) forces counterbalance of attraction and repulsion occurs approximately at 0.3 0.5 and 0.7 fermi for the P-W, L-F and Paris model t matrices respectively. The L-F and P-W singlet even imaginary components are very similar with again a long range attraction plus short ranged repulsion being the nett effect of the Yukawa combinations whereas that of the Paris model is again a three region quantity with in this case a very weak repulsive region between 0.4 and 0.6 fermi.

The triplet even central forces are also basically a long range attraction with short range repulsion with counterbalance radii of 0.4 fermi for both the L-F and P-W real and imaginary components. The Paris t-matrix in this channel has an additional very short ranged attraction whence counterbalance radii of 0.2 and 0.8 fermi and 0.3 and 0.6 fermi result for the real and imaginary parts respectively. The nett effect so far as momentum components are concerned is that the imaginary parts of all t matrices are similar in comparison to the distinct difference between the real values of the Paris and L-F or P-W forces.

The triplet-odd channel central force components are shown in figure 4. All forces have distinctive momentum
components with the L-F and Paris real components being of very short ranged repulsive character; there being no P-W force of real nature in this channel. The imaginary components of the L-F and P-W forces are similar having short ranged attraction in this channel. The Paris force on the other hand has a short ranged repulsion with longer ranged attraction and a balance radius of about 1 fermi. In fact, the real components do have a long range, but relatively weak, attractive nature.

The momentum components of the tensor forces are displayed in figures 5 and 6 wherein the triplet even and triplet-odd terms are shown respectively. The individual t-matrices have quite distinctive characteristics in both channels with the P-W triplet even channel force being purely real. In the triplet-even channel the L-F force real component is purely attractive whereas the PW and Paris forces have additionally a short ranged repulsion with a balance at 0.2 and 0.6 fermi respectively. The imaginary components of both the L-F and Paris forces have a long range attraction with a short range repulsion to counterbalance at 0.8 and 1.0 fermi respectively. The momentum components of the tensor force t-matrices in the triplet odd channel are similarly disparate with all real components being repulsive, with short range attractions in both the L-F (0.7 fm) and Paris (0.4 fm) cases. The P-W and Paris forces have purely attractive imaginary components whilst the L-F imaginary term has a weak repulsive region between 1.0 and 2.0 fermi.
The momentum components of the spin orbit forces of the three t-matrices are as dissimilar as are those of the tensor forces. They are displayed for the triplet even and triplet odd channels in figures 7 and 8. In the former the P-W and L-F forces are purely attractive (real) and purely repulsive (imaginary). The Paris force on the other hand has also a short range repulsive region, between 0.3 fm and 1 fm in the real component and for all radii less than 0.3 fm for its imaginary component. In the triplet-odd channel the spin-orbit t-matrices have basically a long range attraction for the real parts and a fairly short range repulsive character for the imaginary parts, save for the P-W imaginary term which does not exist. Both the P-W and Paris model t-matrices have a short range repulsive term in their real components with counterbalance radii of 0.6 and 0.4 respectively.

Of the three t-matrices, only the L-F model has been parametrised to match 800 MeV data and the associated momentum components are given in figures 9 and 10. In figure 9, the singlet channels (purely central) values are displayed with the letters R and I denoting the real and imaginary parts respectively. The momentum components of the central, tensor and spin-orbit components of the 800 MeV L-F model t-matrix are shown in figure 10 by the solid, long dash and short dash curves respectively. These momentum distributions are clearly associated with a complicated coordinate space variation of forces. A simplified graphic representation of the coordinate space variation is given in Table 1; small case characters (rep,att)
denoting weak (repulsion, attraction) regions.

4. **Inelastic Scattering Results.**

The ADWA method has been used with the t-matrices described in the previous section to analyse select 402 MeV and 800 MeV incident energy inelastic proton scattering data from $^{12}$C. Specifically the transition data from the excitation of the isoscalar and isovector $1^+$ (at 12.71 and 15.11 MeV excitation) and $2^+$ (at 4.44 and 16.11 MeV) states have been considered. A shell model (S.M.), large basis particle-hole (PHM) and projected Hartree-Fock (PHFBA) models of nuclear structure have been used in these studies to ascertain the spectroscopic amplitudes, $S^{(a)}_j$, required in the analyses. The proton values, $S^{(-b)}_j$, are listed in table 2 for the $1^+$ state transitions and in table 3 for the $2^+$ state transitions. For these cases the neutron amplitudes have the same values modulated by a phase of $(-1)^{J+T}$ for each transition (to a state $|\psi_{jT}\rangle$). Clearly the $1^+$ states excitations are dominated by transitions within the Op shell and, indeed, spectral properties favour the shell model prescription to that of the PHM (Amos, Morrison, Smith and Schmid, 1981). Strong Op shell transitions should also be evident in the $2^+$ states excitations as may be discerned from the tabulation of spectroscopic amplitudes (table 3). But important contributions from other shells can be expected and, from other studies (Amos et al. 1979, 1981), do in fact occur. In particular the projected Hartree Fock model spectroscopy (PHFBA) not only
predicts the B(E2), longitudinal electron scattering form factor (to $q \sim 2 \text{ fm}^{-1}$ at least) and lower energy (60 MeV) inelastic proton scattering cross-sections in good agreement with data but also, to within a scaling, the transverse electron scattering form factor. As such, the PHFBA may be considered the 'correct' spectroscopy against which the other model predictions may be compared, and it will be seen that high energy (800 MeV) (pp') data not only is consistent with the correct spectroscopy but also selects between analyses with different model spectroscopies in equal fashion to low energy data analyses.

In figure 11, ADWA analyses of the $1^+$ transitions initiated by 402 MeV protons are compared with data (Haji-Saeid et al., 1982). Attenuation parameters ($a, b, N$) of 1.05, 0.04 and 0.61 were used to obtain the isovector transition predictions. These were varied to 1.2, 0.015 and 0.83 for the isoscalar transitions simply to improve the fit from poor to the less than satisfactory results displayed. In concert with previous analyses of lower energy data (Love and Franey, 1981; Amos et al., 1981) the isoscalar $1^+$ excitation in $^{12}\text{C}$ remains a mystery. In the calculations (the results of which are depicted by a continuous line for the L-F t-matrix, by the dash-dot line for the P-W t-matrix and by the dash line for the Paris t-matrix) the S.M. nuclear structure was used. The set labelled as $C+T$ in the $12.71 \text{ MeV}$ state analyses were obtained by omitting the spin-orbit
components of each t-matrix. The variation in structure and magnitude of predictions caused by the spin-orbit force is most evident. Thus in part at least the mystery of the isoscalar $1^+$ transition rests with the proper spin-orbit attribute of the two nucleon t-matrix. The tensor and, to a lesser extent, central force components of the t-matrices dominate analyses of the isovector transition, and as with lower energy data analyses, the data is fit quite well. In both cases the L-F t-matrix calculations give the best comparison with data.

At 800 MeV, only the L-F t-matrix has been defined to date hence in the remaining figures in which 800 MeV data is compared with ADWA predictions all such predictions, save for collective model calculations, were obtained using the L-F t-matrix. In fig. 12, the $^{12}\text{C}$ isoscalar and isovector $1^+$ excitation data is compared with ADWA predictions made using the S.M. (solid line) and P.H.M. (broken line) model spectroscopic amplitudes given in Table 2. The ADWA parameter values of 1.05, 0.035 and 0.54 for $a, b$ and $N$ respectively used to obtain the isovector fits were varied to again improve the comparison with the isoscalar transition data. Values of 1.0, 0.06 and 0.35 give the 12.71 MeV results. The isovector data is fit quite well using the SM model structure and the isoscalar results are quite reasonable. The PHM structure results, however, are quite distinct and give poor predictions. Thus such data does differentiate between nuclear structure models. Furthermore, at this projectile energy a variation of structure between that of the Op-shell
SM model and the larger basis PHM model may result in a good fit to data. It suggests use of the 800 MeV data to fix a credible model structure for the isoscalar $1^+$ transition which may then be used in lower energy data analyses to seek improvements to the t-matrices. To complement analyses of the $1^+$ unnatural parity transitions in $^{12}$C as a test of the t-matrix, we analysed as well the data from 800 MeV inelastic proton scattering to the $2^-$ (8.88 MeV) isoscalar state in $^{16}$O and to the $2^+$ (16.11 MeV) isovector state in $^{12}$C. Again the ADWA parameters were adjusted to optimise a fit to data with values of (1.0, 0.074, 0.24) and (1.15, 0.046, 0.45) for (a,b,N) being used to get the $^{16}$O and $^{12}$C predictions displayed in fig.13. Using a standard collective model form factor for the isovector $2^+$ state transition gave the results depicted by the dashed curve. The good fits suggest that the A.D.W.A. method is reasonable for analysis of any spin and/or isospin flip transition. That this is also the case for non spin and isospin flip transitions as well is shown by the good fit to data, at least for low momentum transfer values, of our ADWA calculations made using the 'correct', PHFBA spectroscopy for the excitation of the $2^+$ (4.44 MeV) state in $^{12}$C. The results are displayed in figure 14 wherein the PHFBA spectroscopy results are shown by the continuous line whilst those obtained using the P.H.M. and S.M. spectroscopic amplitudes are depicted by the dash and dash-dot curves respectively. The fit preference of the PHFBA model values over those of the PHM (in shape) and S.M. (magnitude and shape) is quite evident. Such preference was noted in another study of this reaction but with lower
projectile energy (Amos and Morrison, 1979). It is intriguing to observe that the second peak in the measured data can be reproduced by the usual collective model prescription. To do so, however, required that we use an optical potential of \( V_o (20.0 \text{ MeV}) \), \( r_o \) (1.2 fm), \( a_o \) (0.55 fm), \( V_v \) (38.0 MeV) \( \alpha_d \) (1.35 fm) and \( a_d \) (0.45 fm) with a deformation parameter, \( \beta_2 \), of 0.625 and ADWA parameters of (0.85,0.021,0.69) for \((a,b,N)\). Albeit that microscopic form factors involve a folding of the two nucleon \( t \)-matrix, this result suggests that the nuclear structure properties of the transition other than those of the dominant \( p-s-d \) shells may be the significant factor at the high momentum transfer region of this data.

5. Conclusions.

An Analytic Distorted Wave Approximation (ADWA) has been specified for use in analysis of medium energy inelastic proton scattering data. With a simple form for the continuum (protons) wave functions, transition amplitudes involving central, tensor and two-body spin-orbit force components in the two nucleon \( t \)-matrix, have been specified and which are amenable for predicting differential cross-sections even with very large basis nuclear structure information.

Use of this model prescription in analyses of a select set of transition data, to wit excitation of the \( 1^+ \) isoscalar and isovector states in \( ^{12}\text{C} \) at 402 and 800 MeV and
of the $2^+$ isoscalar and isovector states in $^{12}\text{C}$ at 800 MeV, demonstrated not only that a clear definition between complex models of the two nucleon $t$-matrices and between models of nuclear structure existed in the predictions but also that data was fit by the most appropriate predictions. Albeit that details of the model need refining, for example the removal of arbitrariness in the selection of attenuation parameter values, the ADWA method is thus established as a viable method of data analysis competitive with the Distorted Wave Impulse Approximation. Furthermore, by retaining off-shell dependences in the $t$-matrices it is a distinct improvement over the usual form of the DWIA.
References.


23.


Appendix: Derivation of the Matrix Elements.

a) Central forces.

Using the central force component of a t-matrix (Eqn.(9)) in the matrix elements (Eqn.(8)) and expanding the bound state wave functions gives

\[ M_{j_1 j_2}^{(\text{central})} = \sum (-1)^{j_1-m_1} \langle j_1 j_2 \mid m_1 - m_2 \mid i-N \rangle \]

\[ (j_B)^{-1} \langle J_A j_B | N_1 N_f \rangle \langle J_A j_B | S_M \rangle \]

\[ \langle \ell_1 \mu_1 j_1 \mid \ell_2 \mu_2 \mid j_2 \rangle \]

\[ \int \, dr_o \int \, dr_1 \, \exp(i(\vec{r}_o - \vec{r}_f) \cdot \vec{r}) \, V_{\text{cent}}^{ST}(\vec{r}_0) \]

\[ R_{j_1 j_2}^{(1)}(r_1) Y_{\ell_1 \mu_1}^{*}(\Omega_{1}) \, R_{j_2 j_1}^{(1)}(r_1) Y_{\ell_2 \mu_2}(\Omega_{1}) . \]  

(A1)

Transforming to variables \( \vec{r} \) (\( = \vec{r}_0 - \vec{r}_1 = \vec{r}_0 - \vec{r}_1 \)) and expanding the 'plane' waves in multipoles, then yields with

\[ \vec{Q} = \vec{k}_1 - \vec{k}_f , \]  

(A2)
\[ M_{j_1 j_2} (\text{central}) = \sum (J_B)^{1/2} \langle J_A \nu A | J_B \nu B \rangle \]

\[ \langle \psi T_1 \alpha | TM_T \rangle \langle \psi T_2 \alpha | TM_L \rangle N_1 N_f (4\pi)^{3/2} \]

\[ \langle L \rangle^{1/2} \int r^2 dr \ j_0(Qr) \ \nu^{\text{cent}}_{ST}(r) \]

\[ \int r_1^2 dr_1 \ j_L(Qr_1) \ R_{L,2}^{j_2}(r_1) \ R_{L,1}^{j_1}(r_1) \]

\[ \langle \xi_2 \mu_2 \ | Y_{L0}(N_1) \ | \xi_1 \mu_1 \rangle \]

\[ \langle \psi \nu \mu_1 \ | S_{M} \rangle \langle \psi \nu \mu_2 \ | S_{M} \rangle \]

\[ (-)^{j_1 - m_1} \langle j_1 j_2 \mu_1 m_2 | I - N \rangle \langle \xi_1 \frac{1}{2} \mu_1 \ | j_1 m_1 \rangle \]

\[ \langle \xi_2 \frac{1}{2} \mu_2 \ | j_2 m_2 \rangle . \] (A3)

Using the Wigner-Eckart theorem and standard angular momentum algebra then yields the result given as Equation (10a) in the text.

b) Tensor forces.

The tensor force component of the t-matrix can be recast as

\[ t^{\text{tens}} = \sum_{T} \nu^{\text{tens}}_{T}(r) \ p_{(2/3)^{1/2}} \ \hat{r} \cdot (\hat{e}_0, \hat{e}_1) \cdot \hat{e}_2(n_{01}) , \] (A4)

from which one may deduce that
Again with the expansions of the bound state functions, recoupling to two body spin and isospin functions, and using the coordinate transformations, it is straightforward to develop Equation (8) for a tensor force to

\[
M_{J_1 J_2}^{\text{tensor}} = \sum (J_B)_{-\frac{1}{2}}^{-\frac{1}{2}} \langle J_A|v_N|J_B\rangle
\]

\[
N_{f 1}\langle l_1 m_1 | 1 M_S > < l_2 m_2 | 1 M_S > [2(8\pi)^{1/3} (-)^q < 12 M_S q | 1 M_S > (-)^{J_1 - m_1} \langle j_1 j_2 m_1 - m_2 | I-N \rangle < \frac{1}{2} m_{l_1} m_{l_2} | j_1 m_1 > \langle \frac{1}{2} m_{l_2} m_{l_2} | j_2 m_2 > \int dr^2 \exp(i \mathbf{Q} \cdot \mathbf{r}) \ V_{\mathbf{T}}^{\text{tens}}(r) \ Y_{2-q} (\Omega) \int dr_1^2 \exp(i \mathbf{Q} \cdot \mathbf{r}_1) \ R_{l_2 j_2}(r_1) \ R_{l_1 j_1}(r_1) \ Y^{*}_{l_2 m_{l_2}}(\Omega) \ Y_{l_1 m_{l_1}}(\Omega). \]

Multipole expansions of the 'plane' waves, use of the Wigner-Eckart theorem and standard angular momentum algebra then yields the result given as Equation (10b).
c) The spin-orbit force.

The spin orbit force component of the t-matrix may be handled most easily by using cartesian coordinate expansion of

\[
\sum T V^S_T(r) P_T \left( \frac{i}{\hbar} \mathbf{r} \times \mathbf{\hat{r}} \right) \cdot \mathbf{S},
\]

whence

\[
t(s-o) \exp(i\mathbf{\hat{r}} \cdot \mathbf{r}) |SM_\uparrow>.
\]

Then with \( \mathbf{\hat{Q}} \) defining the z-axis, integrals of the form

\[
\int \exp(i\mathbf{\hat{Q}} \cdot \mathbf{r}) V^S_T(r) \ 0 \rightarrow \downarrow
\]

result in the matrix elements, for which only \( I_z \) is non vanishing. Thus the effective part of t(s-o) above, given \( \mathbf{\hat{Q}} \) as the z-axis and the scattering plane as the x-z plane (whence \( K_y = 0 \)), is

\[
t(s-o) \exp(i\mathbf{\hat{r}} \cdot \mathbf{r}) |SM_\uparrow>
\]

and the matrix elements (Equation (8)) become
M_{j_1 j_2} (spin-orbit) = \sum (-)^{j_1 - m_1} <j_1 j_2 m_1 m_2 | I_N> (J_B)^{-\frac{1}{2}} 

\langle J_A^I v_A^N | J_B^I v_B^N \rangle \sum_f <\lambda_f^J \lambda_{I_f}^I | TM_f^> 

\langle \lambda_{f_1}^J \lambda_{f_2}^J | S'M' \rangle <\lambda_{f_1}^J \lambda_{f_2}^J | SM_S^> 

\langle \ell, \frac{1}{2} \frac{1}{2} m_1 \mu_1 | j_1 m_1 \rangle <\ell, \frac{1}{2} \frac{1}{2} m_2 \mu_2 | j_2 m_2 \rangle 

K_1(x) <S'M'_S | (S_y^J \phi) | SM_S^> 

\int dr_{01} \int dr_1 \exp(-i \hat{\mathbf{K}} \cdot \mathbf{r}_0) \mathbf{R}_{\ell j_2 j_2} (r_1) \mathbf{Y}_{\ell_2 m_2} (\Omega_1) 

r_{01} \cos \theta_{01} \mathbf{V}^{SO} (r_{01}) \exp(i \hat{\mathbf{K}}_1 \cdot \mathbf{r}_1) \exp(i \hat{\mathbf{K}}_1 \cdot \mathbf{r}_1) 

\mathbf{R}_{\ell j_1 j_1} (r_1) \mathbf{Y}_{\ell_1 m_1} (\Omega_1) . 

Transforming the variables and using 

\langle S'M'_S | (S_y^J \phi) | SM_S^> 

= \langle S'M'_S | i(2)^{-\frac{1}{2}} (S_+ + S_-) \phi | SM_S^> 

= \delta_{SS'} \delta_{S_1 S_1} \sum_\xi \xi \langle i1M_\xi | i1M_\xi' \rangle 

then yields
Standard algebra then yields the results given in the text as
Equation (10c), when for high energies and small q-values, $K_1$ and $K_f$ equate to give

$$K_1(x) = K_f(x) = \left( K_1^2 - Q^2/4 \right)^{1/2}$$

With this last condition, the results quoted are then also obtained if a more symmetric form, of

$$t_{(so)} = \sum_T b_T^T \left\{ v_T^{(so)}(r) \hat{T} \cdot \hat{S}/\hat{R}^2 + \hat{T} \cdot \hat{S}/\hat{R}^2 v_T^{(so)}(r) \right\},$$

were to be used in the development.
Table 1: Schematic of the coordinate space variations of the components of the 800 MeV L-F t-matrix.

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<th>Real/Imag.</th>
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Table 2: Proton spectroscopic amplitudes for the excitation of the isoscalar (12.71 MeV) and isovector (15.11 MeV) $1^+$ states in $^{12}$C.

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Table 3: Proton Spectroscopic amplitudes for the $2^+$ state excitations in $^{12}$C. Three models results for the $T=0$, 4.43 MeV state excitation are compared with one for the $T=1$, 16.11 MeV state excitation.

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<th>PHM</th>
<th>PHM(T=1)</th>
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Figure Captions.

Fig.1: The momentum components of the singlet odd central force t-matrices at 400 MeV incident proton energy. The curves are identified in the text.

Fig.2: The momentum components of the singlet even central force t-matrices at 400 MeV incident proton energy.

Fig.3: The momentum components of the triplet even central force t-matrices at 400 MeV incident proton energy.

Fig.4: The momentum components of the triplet odd central force t-matrices at 400 MeV incident proton energy. There is no real part in the Picklesimer-Walker interaction.

Fig.5: The momentum components of the triplet even tensor force t-matrices at 400 MeV incident energy. There is no imaginary component in the Picklesimer-Walker interaction.

Fig.6: The momentum components of the triplet odd tensor force t-matrices at 400 MeV incident energy.

Fig.7: The momentum components of the triplet even spin-orbit force t-matrices at 400 MeV incident energy.

Fig.8: The momentum components of the triplet odd spin-orbit force t-matrices at 400 MeV incident energy. There is no imaginary component in the Picklesimer-Walker interaction.

Fig.9: The momentum components of the singlet (central) forces in the L-F matrix at 800 MeV incident energy. The real and imaginary components are identified by the labels R and I respectively.

Fig.10: The momentum components of the triplet forces in the L-F t-matrix at 800 MeV incident energy. The central tensor and spin-orbit components are identified by the solid, long dash and short dash curves respectively.
Fig. 11: A comparison of the ADWA predictions using the various two nucleon t matrices with data from the inelastic scattering of 402 MeV protons exciting the isoscalar and isovector $1^+$ states in $^{12}\text{C}$. The curves are identified in text and all were obtained using the shell model spectroscopy.

Fig. 12: ADWA predictions obtained using the Love-Franey two-nucleon t-matrix and compared with the inelastic scattering data from 800 MeV proton excitation of the isoscalar and isovector $1^+$ states in $^{12}\text{C}$. The solid curve depicts the results when the shell model spectroscopy was used whilst the broken curve gives those from use of the PHM spectroscopy.

Fig. 13: The comparison with 800 MeV data of an ADWA calculation for (a) the excitation of the $2^-$ state in $^{16}\text{O}$ (b) the excitation of the isovector $2^+$ state in $^{12}\text{C}$. In both cases the Love-Franey interaction was used. The $^{16}\text{O}$ spectroscopic amplitudes were obtained from a Shell model calculation whilst those for $^{12}\text{C}$ was obtained from the PHM evaluations. The dashed curve gives the result of an (unnormalised) collective model calculation.

Fig. 14: A comparison of ADWA calculations with data from the excitation of the $2_1^+$ state in $^{12}\text{C}$ initiated by 800 MeV protons. On the left are given the results of (microscopic) model calculations in which the Love-Franey two nucleon t-matrix was used and with shell model (dash-dot), PHM (dash) and PHFBA (solid) spectroscopic amplitudes.
SINGLET ODD CENTRAL

REAL

IMAGINARY

\( g_{\ell fm} \)

\( q (\text{fm}^{-1}) \)
TRIPLET EVEN
CENTRAL

REAL

IMAGINARY

$q [fm^{-1}]$
TRIPLET EVEN TENSOR

REAL

IMAGINARY

$|q|_{fm^{-1}}$
TRIPLET EVEN
SPIN-ORBIT

REAL

IMAGINARY

\[ T^2 \]
\[ T^3 \]
\[ \tilde{r} \]
\[ a \] (fm)

\[ q \ (fm^{-1}) \]
SINGLET ODD (central)

SINGLET EVEN (central)

$q_\parallel (fm^{-1})$
$^{12}\text{C} (p,p')$

800 MeV

$1^+ (12.71 \text{ MeV})$

$1^+ (15.11 \text{ MeV})$

$d\sigma/d\Omega$ (mb/sr)

$\theta_{\text{c.m.}}$ (degrees)
800 MeV (p,p')

\[ \sigma/4\pi \text{ (mb/sr)} \]

\[ \theta \text{ c.m. (degrees)} \]

\( \text{^{16}O} \ 2^{-} (8.88) \)

\( \text{^{12}C} \ 2^{+} (16.11) \)

Graphs showing angular distributions for different isotopes and spin states.
$^{12}$C $^{(p,p')}$ $2^+$ (4.44 MeV) $E_p = 800$ MeV

- microscopic
- collective

$d\sigma/d\Omega$ (mb/sr)

$\theta$ cm. (degrees)