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TWO-STAGE DECISION APPROACH TO MATERIAL ACCOUNTING

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ABSTRACT

The validity of the alarm threshold 4σ has been checked for hypothetical large and small facilities using a two-stage decision model in which the diverter's strategic variable is the quantity diverted, and the defender's strategic variables are the alarm threshold and the effectiveness of the physical security and material control systems in the possible presence of a diverter. For large facilities, the material accounting system inherently appears not to be a particularly useful system for the deterrence of diversions, and essentially no improvement can be made by lowering the alarm threshold below 4σ. For small facilities, reduction of the threshold to 2σ or 3σ is a cost effective change for the accounting system, but is probably less cost effective than making improvements in the material control and physical security systems.

INTRODUCTION

Large-scale analytical studies of the material accounting system for strategic nuclear materials (SNM) have been performed since 1976. The purpose of these studies has been (a) to directly examine the implications of deliberate diversion on nuclear material accounting, (b) consider the applicability of inventory difference data to statements about unauthorized diversion of SNM, and (c) to provide tools for assessing licensee material accounting safeguards performance standards.

These studies have ranged from advances in statistical testing techniques to simulation techniques to dynamic programming to game theory.

In a review of game theory, a peer review panel noted that the adversarial nature of the relationship between the defender (society, the public utility, and the U.S. government) and the diverter is worthy of consideration in modeling safeguards problems. In this study, a two-stage decision algorithm is presented that maintains the adversarial nature (perhaps even more realistically than some theory) of the problem. The defender is concerned with minimizing total societal cost, whereas the diverter is concerned with maximizing the amount stolen, two entirely different objectives. Both the defender and diverter functions are explicitly modeled in the two-stage model presented here and, therefore, the behaviors of both the defender and diverter are found. Unlike previous studies known to the authors, this algorithm allows for the direct incorporation of the interaction of the physical security and material control systems.

This study, like the game theoretic studies, is concerned with decision criteria to be used in taking action in response to the recorded inventory difference, which is the difference between book inventory and physical inventory performed at the end of an accounting period. If there were no measurement errors, process errors, or human errors, the inventory difference would be zero unless diversion had occurred. Due to various random errors inherent in the system, the inventory difference is not, in general, zero and diversions can conceivably be masked in the inherent errors. Thus, the decision making problem is, given an inventory difference reading, what if any action should be taken to verify theft and/or recover material that may have been diverted.

The present practice in the licensed domestic nuclear industry follows regulations of the Nuclear Regulatory Commission that establish inventory periods and limits on measurement accuracy. The Nuclear Regulatory Commission has also established guidelines and operating suggestions for appropriate action limits. The operational procedure at present is to establish fixed alarm thresholds and to take action when the inventory difference exceeds these thresholds. When the inventory difference exceeds 4.0 times the standard deviation of random errors for the facility, the facility is shut down, a clean-out inventory is conducted, and an investigation of the cause of the inventory difference is initiated. This study models the clean-out inventory action.

THE MODEL

Defining the problem after the adversarial fashion of game theory, the move of the defender is to (a) preset the value of the alarm threshold α, (b) determine the likelihood that a diverter is present, and (c) invest in suitable physical security and material control measures. The likelihood of a diverter being present and successfully defeating the physical security and material control systems is represented by another defender parameter, π. In this two-stage decision model, α is assumed to be under the control of the defender but known to the diverter. The move of the diverter is to divert a quantity, x, between zero and the minimum total quantity of SNM that could pose a severe threat to society, K. K . assumed to be 5 kg in this study. The "move" of nature is a value ε for the value of the inventory difference due to all other causes than a potential diversion. The diverter and defender functions associated with these moves are as follows:

For $y < \alpha$,

$$M_1(x,y,\alpha) = M_2(x,y,\alpha) = C_3 \cdot x$$

For $y \geq \alpha$,

$$M_1(x,y,\alpha) = C_3 \cdot \max\left\{x - \gamma \cdot \min\left[\max(y, y-\alpha),\ \frac{L-C_1}{C_2}\right], 0\right\}$$

$$M_2(x,y,\alpha) = C_1 + C_2 \cdot \min\left\{\max(y, y-\alpha),\ \frac{L-C_1}{C_2}\right\} + M_1(x,y,\alpha)$$

Where:

- M_1 = the benefit to the diverter of SNM successfully diverted by the diverter;
- M_2 = overall cost to the defender, including societal considerations, of SNM successfully diverted;
- x = quantity of SNM diverted by the diverter; $0 \leq x \leq K$, where K is the quantity of SNM that could pose a severe threat to society; K assumed to be 5 kg;
- y = quantity of SNM determined by the defender to be the inventory difference;
- α = alarm threshold preset by the defender and known to the diverter;
- L = maximum total expenditure possible for shutdown inventory and search and recovery;
- γ = fraction of SNM searched for that is successfully recovered;
- C_1 = fixed shutdown inventory cost;
- C_2 = unit search and recovery cost;
- C_3 = unit cost to defender and society of unretrieved SNM.

In this study, γ has been taken to be unity, which is representative of full recovery. In reality, $(L-C_1)/C_2$ has a large value, which corresponds to the defender having a large sum of money to spend on search and recovery of diverted SNM. For the purpose of this model, the realization of the large sum of money available is accomplished by setting $(L-C_1)/C_2$ infinite.

When $y < \alpha$, there is no shutdown or special inventory, the amount x is successfully diverted, and the defender's loss is the diverter's gain (i.e. $M_1 = M_2$). When $y \geq \alpha$, a special shutdown inventory occurs, and search and recovery of diverted SNM is attempted.

Because y is the sum of random errors plus any diversion that may have occurred, it can be expressed as $y = x + \varepsilon$, where ε is drawn from $\eta(0,\sigma)$, the normal distribution centered around zero with the standard deviation of the inventory difference, σ. The expected values, $M_1(x,\alpha)$ and $M_2(x,\alpha)$, to the diverter and defender, respectively, are developed using standard statistical techniques and are represented as follows:

$$M_1(x,\alpha) = C_3 \cdot x \int_{-\infty}^{\alpha-x} \rho(\varepsilon)d\varepsilon + C_3 \begin{cases} 0 & ; \ \alpha > x \geq 0 \\ \int_0^{\alpha-x} \varepsilon\rho(\varepsilon)d\varepsilon & ; \ 0 \leq \alpha \leq x \\ \alpha\int_{\alpha-x}^{\alpha} \rho(\varepsilon)d\varepsilon - \int_{\alpha-x}^{\alpha} \varepsilon\rho(\varepsilon)d\varepsilon & ; \ \alpha < 0 \end{cases}$$

$$M_2(x,\alpha) = C_1\int_{\alpha-x}^{\infty} \rho(\varepsilon)d\varepsilon + C_2\left\{\max\ (x, x-\alpha) \cdot \int_{\alpha-x}^{\infty} \rho(\varepsilon)d\varepsilon\right\} + C_2\int_{\alpha-x}^{\infty} \varepsilon\rho(\varepsilon)d\varepsilon + M_1(x,\alpha)$$

If no credit is given to the physical security or material control systems, and the diverter is assumed always present, then the two-stage algorithm is applied to the functions $M_1(x,\alpha)$ and $M_2(x,\alpha)$. First, for each α fixed and known to the diverter, the diverter determines $x = x_{max}(\alpha)$, which maximizes $M_1(x,\alpha)$, and selects $x_{max}(\alpha)$ or K, whichever is smaller. In the second stage, the defender function $M_2[\min(x_{max}(\alpha),K),\alpha]$ is minimized with respect to α. As a proxy for the probability of the presence of a diverter who successfully defeats the physical security and material control systems, a parameter π, the probability of successful diversion, is introduced. The parameter π is in reality a function of x, the amount diverted. For the purpose of this study, π was chosen to be constant, which is a reasonable approximation, at least for an insider with extensive knowledge of the facility weaknesses. The safeguards system diverter function, Φ_1, is simply the diverter function, M_1, decreased by the factor π:

$$\Phi_1(x,\alpha,\pi) = \pi \cdot M_1(x,\alpha)$$

The safeguards system defender function, Φ_2, is represented by the π-weighted sum of two diverter possibilities, $x = 0$, and $x = x_{max}(\alpha)$ or $x = K$:

$$\Phi_2(x,\alpha,\pi) = \pi \cdot M_2[\min(x_{max}(\alpha),K),\alpha] + (1-\pi)M_2(0,\alpha)$$

Using Φ_1 and Φ_2, the two-stage decision model proceeds as discussed above.

The largest value of π allowed in this study is 0.1, because of the questionable presence of a diverter and the existence of physical security and material control. In all likelihood, the actual value of π is much smaller than the low value chosen in this study, namely 0.001. Enough values of π have been evaluated to indicate trends.

The values of C_1, C_2, and C_3 must now be specified. Although the value of C_1 is facility dependent, both the values of C_2 and C_3 are taken to be constant and independent of the facility in question. The value of C_2 can be assumed constant due to the fact that the desire for recovery will result in the same effort per unit mass, regardless of the size of the facility. The cost to the defender and society of successfully diverted material (C_3) will also be independent of the facility from which it was diverted.

Inasmuch as $M_1(x,\alpha)$ and $M_2(x,\alpha)$ are specified in units of cost, C_1 will be in dollar units, and C_2 and C_3 will be in units of cost per unit mass. Because masses are specified in σ units, C_2 and C_3 will be in units of cost per σ unit. Only the ratios C_1/C_3 and C_2/C_3 must be specified to obtain the unnormalized variation of the functions $\Phi_1(x,\alpha,\pi)$ and $\Phi_2(x,\alpha,\pi)$.

The ratio C_2/C_3 is not easily specified because of the subjectivity of both terms, especially C_3. It is clear that C_3 must have a value larger than C_2, because otherwise it would never be advantageous to attempt recovery. If C_3 becomes too large compared with C_2, the functions of the defender and diverter become almost the same, the economics of the defender is in effect removed from the model, and a game theoretic approach is probably the most effective analytical tool to utilize the problem. For these reasons, in this study, the ratio C_2/C_3 was assumed to take on the values 0.1 and 0.01.

The ratio C_1/C_3 is dependent on the size of the facility, in σ units. Two facility sizes were considered, a small facility for which 10σ is 5 kg, and a large facility for which 0.4σ is 5 kg. The values of C_1 for the small and large facilities are taken to be \$5,000 and \$500,000, respectively. If C_2 is assumed to have a value of \$10,000/kg, then the ratio C_1/C_3 will range from 0.01 to 0.1 when x and α are measured in σ units. Thus, for the small facility, the ratios considered are $C_1:C_2:C_3 = 0.01:0.01:1$ and $0.1:0.1:1$.

Similarly, for the larger facility, the ratio C_1/C_3 will range from 0.04 to 0.4 when x and α are measured in σ units. Thus, the ratios chosen for the large facility are $C_1:C_2:C_3 = 0.04:0.01:1$ and $0.4:0.1:1$.

RESULTS

The graph in Figure 1 shows $M_1(x,\alpha)$ versus x for several values of α between $-\sigma$ and $+6\sigma$. It is apparent from the figure that the diverter's selection of the amount to divert, $x_{max}(\alpha)$, can be highly dependent on the value of α and that it is monotone increasing with positive values of the alarm threshold. In the case of the small facility, $x_{max}(\alpha)$ never exceeds K and the diverter strategy is given by enumerative iteration of the function $M_1(x,\alpha)$ in as fine a mesh as 0.01σ intervals between -10σ and $+10\sigma$. For the large facility, K always exceeds $x_{max}(\alpha)$.

The graphs in Figures 2 and 3 present the defender function $\Phi_2[\min(x_{max}(\alpha),K),\alpha,\pi]$ for the small facility for the cost ratios 0.01:0.01:1 and 0.1:0.1:1, respectively. The graphs in Figures 4 and 5 present the defender function for the large facility for the cost ratios 0.04:0.01:1 and 0.4:0.1:1, respectively. The values presented are unnormalized and present the variation of the defender function with respect to the choice of the alarm threshold α and the value of the interface parameter π. The optimal defender strategy is determined by enumerative iteration of the function $\Phi_2[\min(x_{max}(\alpha),K),\alpha,\pi]$ in as fine a mesh as 0.01σ intervals.

The function $\Phi_2(0,\alpha,\pi)$ is a monotone decreasing function of α, whereas the function $\Phi_2[\min(x_{max},K),\alpha,\pi]$ has a minimum dependent on the size of the facility and the ratio C_2/C_3. This minimum is responsible for the minimums in the small facility curves in Figures 2 and 3 and the large facility curve for $\pi = 0.1$ in Figure 4.

Table 1 presents the expected percentage change in reduction of the safeguards system defender function if the defender selects the optimal α when a two-stage algorithm determines that a value of α less than 4σ is desirable.

CONCLUSIONS

For both large and small facilities, physical security and material controls, represented by the variable π, are seen to be effective minimizers of the defender's objective function. As expected, the value of the defender function is about 100 times smaller when π is reduced from 0.1 to 0.001.

For the large facility, the improvement realized by reducing the alarm threshold from 4σ to the level determined by the two-stage algorithm is at best 4%, in which case failure of the material control and physical security systems is highly likely, i.e. $\pi = 0.1$, and society considers the material valuable, i.e. $C_2/C_3 = 0.01$. This insensitivity to the choice of the alarm threshold is in part a result of the large fixed shutdown inventory cost (C_1) at a large facility. It is also due to the fact that the diverter will attempt to divert an amount that is a threat to society (K), regardless of reductions in the value of the alarm threshold.

The currently utilized standard of 4σ should possibly be reduced to 2σ or 3σ for small facilities. As seen in Table 1, the expected cost to the defender could be reduced by about 37% if the current technology can force the value of π to no smaller than 0.001, and the value of the diverted material to society is 10 times as great as the cost of recovery.

One of the members of the game theory review panel expressed the sentiment that material accounting was inherently not a particularly useful system for deterrence of a diversion. At least for large facilities, this study seems to support that sentiment. No matter how the accounting system is designed, i.e. how the alarm threshold α is chosen, the diverter will attempt to divert an amount that is a threat to society (K). For large facilities, improvements in physical security and material control (which effectively reduce the value of π) are clearly more cost effective than lowering the threshold of the accounting system, because there is essentially no improvement in the safeguards system defender function by reducing the value of the alarm threshold. For small facilities, the material accounting system appears to be a deterrent. The diverter's selection of diversion quantity, x, is monotone increasing with α, for $\alpha > 0$. There is a noticeable change in the safeguards system defender function as the alarm threshold α is lowered at a small facility. However, reduction of the material accounting alarm threshold to 2σ or 3σ is probably less cost effective than making improvements in the physical security and material control systems. To prove this statement about cost effectiveness, an in-depth cost/benefit analysis of changes in the alarm threshold (same π curve, varying α) versus changes in physical security or material controls (same α, varying the π parameter) must be performed. This comparison is beyond the scope of this study.

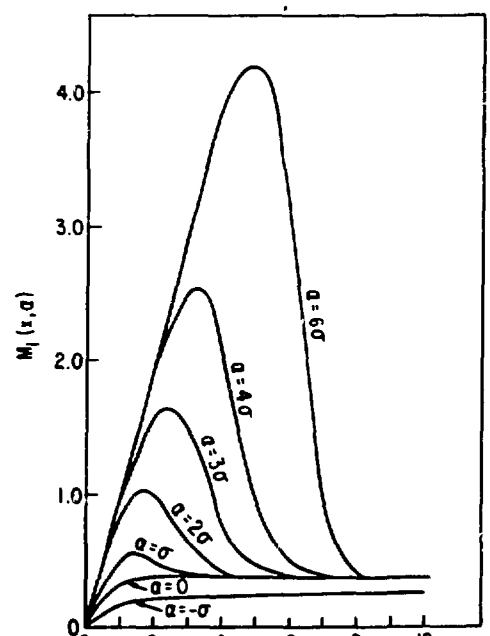

Figure 1. Functional Dependence of the Diverter Function.

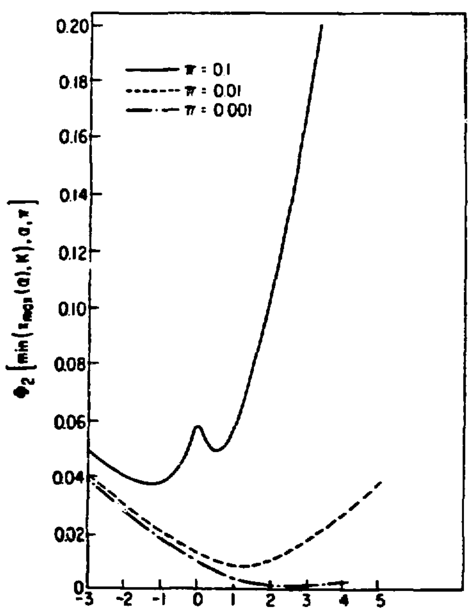

Figure 2. Functional Dependence of the Safeguards System Defender Function for a Small Facility with C_1 = \$5,000, K = 10σ, and $C_1:C_2:C_3 = 0.01:0.01:1$.

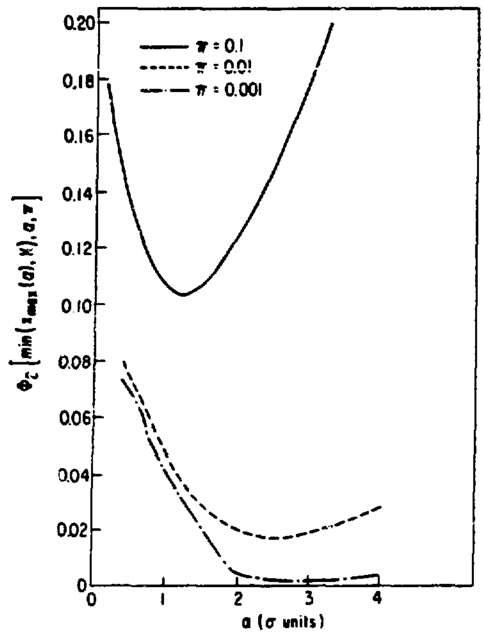

Figure 3. Functional Dependence of the Safeguards System Defender Function for a Small Facility with C_1 = \$5,000, K = 10σ, and $C_1:C_2:C_3$ = 0.1:0.1:1.

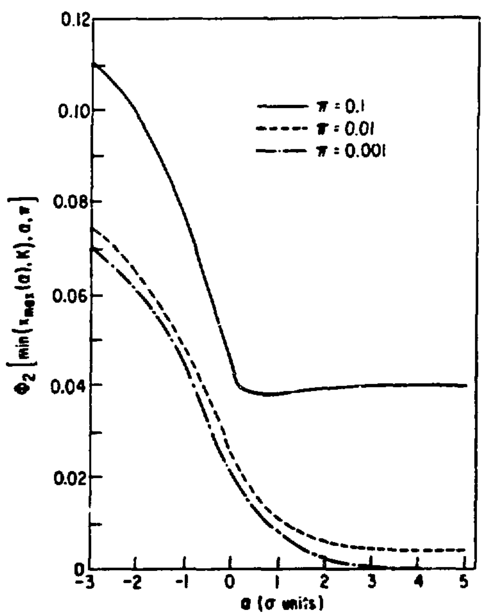

Figure 4. Functional Dependence of the Safeguards System Defender Function for a Large Facility with C_1 = \$500,000, K = 0.4σ, and $C_1:C_2:C_3$ = 0.04:0.01:1.

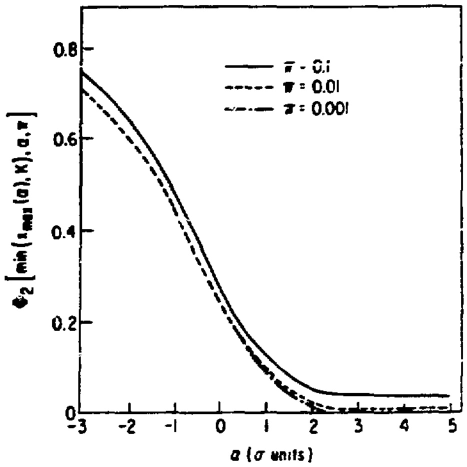

Figure 5. Functional Dependence of the Safeguards System Defender Function for a Large Facility with C_1 = \$500,000, K = 0.4σ, and $C_1:C_2:C_3$ = 0.4:0.1:1.

Table 1. Percent Change in the Safeguards System Defender Function if an Optimal Threshold of ≦ 4σ is Selected

$C_1:C_2:C_3$	π	Minimum M_2 (≦ 4σ)	M_2 (= 4σ)	Change (%)
Small Plant	C_1 = \$5,000, K = 10σ, σ = 0.5 kg			
0.01:0.01:1	0.10	0.050	0.25	98
	0.010	0.0092	0.025	64
	0.0010	0.0016	0.0025	37
0.1:0.1:1	0.10	0.10	0.27	61
	0.010	0.017	0.027	35
	0.0010	0.0023	0.0027	14
Large Plant	C_1 = \$500,000, K = 0.4σ, σ = 12.5 kg			
0.04:0.01:1	0.10	0.038	0.040	4.3
	0.010	0.0020	0.0020	0.0
	0.0010	0.00020	0.00020	0.0
0.4:0.1:1	0.10	0.040	0.040	0.0
	0.010	0.040	0.040	0.0
	0.0010	0.040	0.040	0.0

DISCLAIMER