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B. LUKÁCS  
L.P. CSERNAI

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VIA ENERGETIC NUCLEAR COLLISIONS

*Hungarian Academy of Sciences*

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**BUDAPEST**

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**B. LUKÁCS, L.P. CSERNAI**

**Central Research Institute for Physics  
H-1525 Budapest 114, P.O.B. 49, Hungary**

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## ABSTRACT

The possible effects of the nucleon-quark phase transition on the dynamics of heavy ion collisions are discussed. It is shown that the formation of the quark phase can be expected at recent experiments. Nevertheless, the compressibility of the two-phase mixture remains relatively low, thus the quark phase remains limited in both space and time, and the observables are not strongly affected.

## АННОТАЦИЯ

Рассматривается возможное влияние фазового перехода нуклонов в кварки на динамику столкновения тяжелых ионов. Показывается, что при настоящих энергиях ожидается формирование кварковой фазы. Однако, смесь двух фаз является сравнительно твердой, поэтому кварковая фаза остается ограниченной и в пространстве и во времени, и наблюдаемые величины значительно не изменяются.

## KIVONAT

Megvizsgáljuk a kvark-nukleon fázisátmenet lehetséges hatásait a nehéz-ionütközések dinamikájára. Megmutatjuk, hogy a kvarkfázis kialakulása várható a jelenleg elérhető energiákon. Mindazonáltal a két fázis keverékének kompresszibilitása viszonylag alacsony marad, így a kvarkfázis térben is, időben is korlátozott marad, és a megfigyelhető mennyiségek nem változnak lényegesen.

## 1. INTRODUCTION

Although the quark hypothesis has been proven very efficient for explaining various experimental facts about the elementary particles and their interactions, it is some kind of frustration that free quarks have not been observed, so one has to devoid the most direct proof. Of course, the total confinement is concordant with the Quantum Chromodynamics, nevertheless it would be slightly strange to tell that the absence of observed free quarks is an evidence for the QCD.

Nevertheless, if the confinement is full, one simply cannot obtain the most direct evidence. However, there is the next best possibility: to observe a (confined) quark plasma. If it contains a sufficient number of quarks, then these quarks are approximately free within, and the distinctive features of the quarks can clearly manifest themselves. Thus it is promising to look for situations in which a quark plasma can exist. However, as far as we know, such a plasma is not stable under usual circumstances; high densities and/or temperatures are needed.

The opinions in the literature are not fully concordant about the necessary circumstances. For example, Gyulassy estimates that some  $2 \text{ GeV}/\text{fm}^3$  energy density would be needed, and he thinks that for U+U collisions cca. 100 GeV/nucleon beam energy would be sufficient [1]. On the other hand, thermodynamical calculations yield some 5-7  $n_0$  for the (lower) transition density at  $T = 0$  [2], [3], with a serious decrease at higher temperatures. This density is high enough, but not inaccessible;  $\therefore$  two-fluid calculation of Amsden et al. 234 MeV maximal temperature and  $n_0$  maximal density was obtained for 2.1 GeV/nucleon beam energy [4]. According to Ref. 3 at  $T = 200 \text{ MeV}$  the transition begins at 3-4  $n_0$ , while in one version of Chin's calculation [2] the lower transition density is even 0 above  $T = 190 \text{ MeV}$ .

While the first estimation is too high for the present heavy ion experiments, and then only cosmic ray observations would be hopeful (with poorer statistics), in the second case we would be near to the production of the plasma in the recent experiments, in fact, combining the results of Refs. 4 and 2 or 3, one gets that the transition should start in the 2.1 GeV experiments in the very center of the compressed nuclear matter, and should continue for at least  $1.10^{-23} \text{ s}$ .

Here the differences between the predictions have been slightly exaggerated, because it is explicitly stated in Ref. 1 that the transition may

happen below  $2 \text{ GeV}/\text{fm}^3$  energy density, and this value belongs to temperatures where the plasma is already similar to an ideal gas. Nevertheless, there remains the fact that it would be difficult to decide if one can expect phase transition in recent experiments or not, and in the best case these energies would be only just enough.

There is no clear evidence for phase transition in the experiments (see Ref. 5), and there are only awkward signals in cosmic ray observations [1], [6] at 4-5 TeV/nucleon. It is not obvious that there would be any contradiction between these negative facts and the more optimistic estimations. E.g. the strange particle production from the plasma has to be preceded by formation of  $s\bar{s}$  pairs, and this needs some  $2 \cdot 10^{-23}$  s [7]. Another dubious point is whether the phase transition can be finished. Namely, it seems that the phase transition is of first order [3], thus there is a density gap between the phases (some 9-10  $n_0$  at  $T = 0$ ). Between the lower and upper transition densities the matter is a mixture of the two phases, so the system may spend a considerable part of the whole reaction time in the mixture stage where the thermodynamical (and thus also the hydrodynamical) behaviour of the matter is more complicated than before or after the transition. Thus this stage of the evolution needs special attention in the present situation, when the possible beam energies are in a delicate region.

In this paper we want to discuss the thermodynamical behaviour in the two-phase region. In Sect. 2 we present the conditions for a stable equilibrium, in Sect. 3 an explicit way is given for calculating the thermodynamical data during the transition. Sect. 4 discusses some consequences of the finite characteristic time of the interaction, while in Sect. 5 we investigate the possibility of some hydrodynamical instabilities. Finally, in Sect. 6 the compressibility of the mixture is calculated.

## 2. PHASE TRANSITION AND PHASE EQUILIBRIUM IN HEAVY ION COLLISION

There are at least two phase transitions which may lie in the experimentally accessible range, namely the pion condensation [8], [9], and the transition into free quarks. The questions arise: can we use the continuum approach here, and if so how should we use it?

In principle, the first question cannot be answered within the continuum description. Nevertheless, one can list some essential assumptions which are used in the formulation of this approach. First, the characteristic data should be continuous functions of the space coordinates. This assumption may or may not be true even for one phase. Second, the near-equilibrium thermodynamics should be applicable for the system. Let us detail this condition.

We assume that the full thermodynamical information is given, i.e. we know the actual values of the independent extensive densities (or the conjugate intensives), and the form of the corresponding potential (e.g. we know the function of the energy density versus particle and entropy densities  $n$ ,  $s$ )

$$T(s, n) = \frac{\partial \rho}{\partial s}$$

$$\mu(s, n) = \frac{\partial \rho}{\partial n} \quad (2.1)$$

$$p(s, n) = s \frac{\partial \rho}{\partial s} + n \frac{\partial \rho}{\partial n} - \rho$$

However, these relations are obtained from the assumption that the energy is also an extensive quantity, i.e. it is additive for the subsystems [10]. This is true in equilibrium, if there are no volume forces, but may not be true e.g. far from the equilibrium.

The estimations yield that a collision satisfies the near-equilibrium conditions below 500 MeV/nucleon [11]. Nevertheless, if the equilibrium is unstable, the system cannot remain in it. In this case even the data  $(n, s)$  begin to rapidly vary, or the energy distribution of the particles essentially differs from the thermal one. In both cases the thermodynamical approach has left its grounds.

Generally if the specific data of the two phases differ, i.e. the phase transition is of first order (which is probably not the situation for pion condensation [9]), the equation of state has a region in which the equilibrium is not stable, i.e. where the matrix

$$M_{ik} = \frac{\partial^2 \rho(\xi_i)}{\partial \xi_i \partial \xi_k} \quad (2.2)$$

(where  $\xi_i$  stands for the independent extensive densities) is not positive definite [10]. So generally the thermodynamical description with the original equation of state is not self-consistent during the phase transition. Of course, we do not state that the performed calculations give necessarily wrong results in this region, however the relevance of the results becomes awkward.

There is a process, which, if it happens, eliminates this problem, namely the formation of coexisting phases. For the sake of simplicity, let us first investigate the special case  $s = 0$  ( $T = 0$ ). Then the only stability condition is

$$\rho'_{nn} = \mu'_{nn} > 0. \quad (2.3)$$

If this condition is not fulfilled between some  $n_1$  and  $n_2$

$$\rho'_{nn}(n_1) = \rho'_{nn}(n_2) = 0 > \rho'_{nn}(n_1 < n < n_2) \quad (2.4)$$

then  $\mu$  and  $p$  have local maxima at  $n_1$  and local minima at  $n_2$ . So there are values of  $\mu$  and similarly of  $p$  belonging to two different densities  $n_3 < n_1$ ,  $n_4 > n_2$ ; and there is such a pair  $(n_3, n_4)$  for which both  $p(n_3) = p(n_4)$  and  $\mu(n_3) = \mu(n_4)$ . These two densities represent two phases in equilibrium. If  $n$  is slowly increasing and the fluctuations are sufficiently large, then reaching

$n_3$  the phase with  $n_4$ , existed as a fluctuation, does not vanish, and with increasing density the relative weight of this second phase is increasing. Finally at  $n_4$  the first phase vanishes. The interactions guaranteeing the equilibrium of the phases are the same as those ensuring the equilibrium within one phase. In this process the matter escaped the unstable state between  $n_1$  and  $n_2$ . Similarly, for  $T \neq 0$  during the transition in each volume element the local state is a mixture of two stable phases characterized by  $(n_1, s_1)$  and  $(n_2, s_2)$  so that

$$\begin{aligned} T(n_1, s_1) &= T(n_2, s_2) \\ \mu(n_1, s_1) &= \mu(n_2, s_2) \\ p(n_1, s_1) &= p(n_2, s_2) \end{aligned} \quad (2.5)$$

where the actual forms of the functions  $T, \mu$  and  $p$  are determined by the equation of state. Since (2.5) yields three equations for four quantities, the coexisting phases are point pairs on a line  $\varphi(n, s) = 0$  of the  $(n, s)$  plane. The interior of this curve does not represent really existing state, but the average data of the mixture. The transition starts when the state reaches the boundary line, and ends when it leaves (Fig. 1).

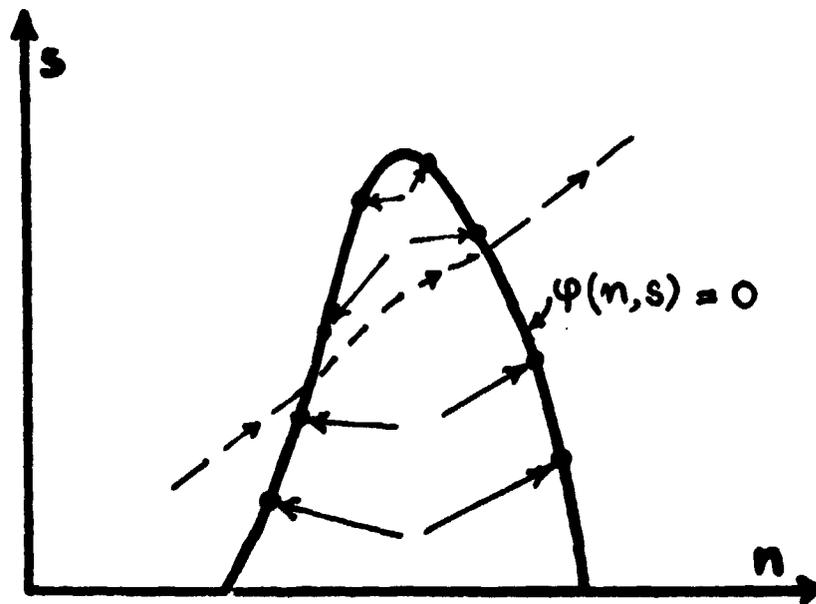


Fig. 1. The two-phase region on the  $(n, s)$  (particle density, entropy density) plane. The coexisting phases are on the full line. The points within the two-phase region represent mixtures of states on the borderline. The dashed line indicates the evolution of a system during compression.

The hydrodynamical equations of motion and the continuity equation describe the behaviour of the average data. For any extensive density the average value has the form

$$\xi = q\xi_1 + (1 - q)\xi_2. \quad (2.6a)$$

Being  $n$  an extensive density, it has the same form, whence  $q$  can be expressed as

$$q = \frac{n_2 - n}{n_2 - n_1} \quad (2.6b)$$

Of course, the (2.1) relations are not valid for the averages. One way to calculate a hydrodynamical process through phase transition is to evaluate all thermodynamical quantities in advance for all  $(n, s)$  pairs inside the phase transition region [12].

Now we show, how a phase transition can be directly incorporated into the hydrodynamical calculations.

### 3. AN EXPLICIT WAY TO CALCULATE THE DYNAMICS OF A FIRST ORDER PHASE TRANSITION

Consider a non-perfect (e.g. viscous) fluid as a model for the nuclear matter during and after the collision. There are five hydrodynamical equations for  $n, \rho$  and  $u^i$  [4], [13], [14] of which three are the equations of motion for the three independent components of the velocity; these three will not be discussed here. The remaining two yield:

$$\dot{n} + nu^x_{;x} = 0 \quad (3.1)$$

$$\dot{\rho} + (\rho + p)u^x_{;x} = \Sigma = \eta \left(\frac{\dot{n}}{n}\right)^2 \quad (3.2)$$

where  $\dot{\rho} \equiv u^x \partial_x \rho$ ,  $\Sigma$  denotes the energy production of the irreversible processes, and  $u^i$  is the four-velocity.

For one single phase,  $p$  is to be taken from eq. (2.1) and then (3.2) gives

$$T(\dot{s} + su^x_{;x}) = \Sigma \quad (3.3)$$

(see Appendix).

For two phases, the equations are the simplest by using  $n$  and  $T$  as variables. Then the proper thermodynamical potential is the free energy density  $f$ , and, instead of eqs. (2.1) we have [13]

$$\begin{aligned}
 s &= - \frac{\partial f}{\partial T} \\
 u &= \frac{\partial f}{\partial n} \\
 p &= n \frac{\partial f}{\partial n} - f \\
 \rho &= f - T \frac{\partial f}{\partial T}
 \end{aligned}
 \tag{3.4}$$

Now the energy density of the mixture is composed as

$$\rho = \frac{1}{n_1 - n_2} [(n_2 - n)\rho(n_1, T) + (n - n_1)\rho(n_2, T)]
 \tag{3.5}$$

being T the same for both phases. Substituting this form into eq. (3.2) one gets

$$\dot{\rho}(n, n_1, n_2, T) - (\rho + p) \frac{\dot{n}}{n} = \Sigma
 \tag{3.6}$$

where p, being the same for both phases, can be calculated from eq. (3.4) for any phase.

Then eqs. (2.5), (3.1) and (3.6) yield four equations for the four quantities n, n<sub>1</sub>, n<sub>2</sub> and T. In order to get explicit equations for  $\dot{n}_A$  and  $\dot{T}$  we take the time derivatives of eqs. (2.5) and perform the derivation in (3.6). The result is as follows:

$$\underline{K} \dot{\underline{\xi}} = \underline{w}; \quad \underline{\xi} \equiv (n_1, n_2, T)
 \tag{3.7}$$

where

$$\underline{K} = \begin{bmatrix} (n_2 - n)\delta_1 & (n - n_1)\delta_2 & -(n_2 - n_1)[(n_2 - n)\gamma_1 + (n - n_1)\gamma_2] \\ \alpha_1 & -\alpha_2 & \beta_1 - \beta_2 \\ n_1\alpha_1 & -n_2\alpha_2 & s_1 - s_2 + n_1\beta_1 - n_2\beta_2 \end{bmatrix}$$

$$\underline{w} = \begin{bmatrix} (n_2 - n_1)^2 T^{-1} \dot{\Sigma} + n^{-1} (n_2 - n_1) (n_2 s_1 - n_1 s_2) \dot{n} \\ 0 \\ 0 \end{bmatrix}
 \tag{3.8}$$

$$\alpha = f,_{nn}$$

$$\beta = f,_{nT}$$

$$\gamma = f,_{TT}$$

$$\delta_A = s_1 - s_2 + (n_1 - n_2)\beta_A$$

Thus the evolution of the constituents of the mixture is determined by  $\dot{n}$ , which is given by the velocity field through eq. (3.1). If we wanted to calculate the hydrodynamical behaviour too, the velocity could be calculated from the ignored components of the energy-momentum balance equation.

As we have seen, the transition starts at a curve

$$\phi(n,T) = 0 \quad (3.9)$$

Thus one should start with the one-phase equations (3.1), (3.3); at the curve (3.9) switch over to (3.7-8), and when  $n=n_2$ , take the one-phase equations again.

#### 4. ALTERNATIVE PATHS OF THE PHASE TRANSITION

In the preceding section we showed that the phase transition through coexisting phases removes the thermodynamical instability from the path of the system. The only necessary condition is that the "nuclei of condensation" of the second phase be present due to fluctuations when the system arrives at the region of coexisting phases. This condition is fulfilled if the fluctuations are sufficiently large or if the thermodynamical state changes sufficiently slowly.

The estimated fluctuation of an arbitrary thermodynamical quantity of a one component matter is given by the well known thermodynamical formula [10] in the variables  $n,s$  as

$$\frac{(\overline{\Delta X})^2}{X^2} = \frac{n\rho_s}{X^2 \Delta N} \frac{1}{\rho_{ss}} (\text{Det}^{-1}(\underline{M})(X_{,n\rho_{ss}} - X_{,s\rho_{sn}})^2 + X^2_{,s}) \quad (4.1)$$

where  $\Delta N$  is the number of the particles in the investigated cell and  $\underline{M}$  is the matrix defined by eq. (2.2). It can be seen that the fluctuations become infinitely large if either  $\rho_{ss}$  or  $\text{Det}(\underline{M})$  vanishes, i.e. when the matrix  $\underline{M}$  loses its positive definiteness, at the boundary of the unstable region. Of course, there the first order formula breaks down. Nevertheless, the formula qualitatively shows that there the fluctuations are great in the system.

Consider an equation of state with an unstable region. First one can calculate  $(\overline{\Delta X})^2$  ( $X = n, s$  or  $\rho$ ) on the phase transition line  $\phi(s,n) = 0$ . From the details of the interactions the condition can be determined that the fluctuations be sufficiently large for the formation of the second phase, so there will be intervals on the phase transition line where this criterion is fulfilled (at high temperatures or entropies, near to the top of the curve if it has a top) and there will be sections where it is not fulfilled (at low temperatures). If the system reaches the curve in the first interval, the phase transition will take place in the way described in the preceding sections. Otherwise the system may cross the phase transition line  $\phi$  in a one phase state. Can it follow an unstable path?

In order to achieve a reduction ad absurdum consider the case  $T=s=0$ . Then the fluctuations vanish, and the system can reach an arbitrarily close proximity of the unstable region. Let us assume that the system just has arrived at the boundary of the unstable region, i.e.  $\mu_{,n} = 0$ . Then eq. (4.1) yield  $\frac{0}{0}$  thus now there may be fluctuations. What is the situation when the system enters the unstable region,  $\mu_{,n}$  becomes negative?

Assume that there are fluctuations in a fixed volume  $V$  containing matter of average density  $n$ . The simplest fluctuation is the formation of two subsystems with  $(V_1, n_1)$  and  $(V_2, n_2)$ .

Then

$$\begin{aligned} E &= \rho(n_1)V_1 + \rho(n_2)V_2 \\ V_1 + V_2 &= V \\ n_1V_1 + n_2V_2 &= nV \end{aligned} \tag{4.2}$$

where  $V_1, V_2, n_1$  and  $n_2$  may change due to the fluctuations. Expanding  $E$  into Taylor series around  $n$  one obtains

$$\begin{aligned} E &= V\left(\rho(n) + \frac{1}{8} \mu_{,n}(n) \left(1 - \frac{W^2}{V^2}\right) \delta^2 + o(\delta^3)\right) \\ \delta &= n_2 - n_1 \\ W &= V_2 - V_1 \end{aligned} \tag{4.3}$$

Thus if  $\mu_{,n} < 0$ , then the energy in the fixed volume decreases for arbitrary (small) fluctuations producing inhomogeneities, i.e. the local homogeneity cannot be ensured after  $\mu_{,n} = 0$ . Unless the hydrodynamical change of  $n$  and  $s$  is very rapid compared to the local interactions (which is not probable below 500 MeV/nucleon bombarding energy [11]), the unstable region is not realised, there is a spontaneous separation into stable subsystems.

The most probable fluctuation is the one giving minimal energy. Minimizing  $E/V$ , the result is that

$$\begin{aligned} \mu_1 &= \mu_2 \\ P_1 &= P_2 \end{aligned} \tag{4.4}$$

that is, we have obtained the conditions for phase coexistence. So the cold matter can reach the boundary of the unstable region, but there it dissociates into two coexisting phase, and thence the phase transition continues regularly.

Mutatis mutandis, the whole argument can be repeated for  $T > 0$ . The two thermodynamical variables governed by the balance equations are  $\rho$  and  $n$ . Consider the entropic equation of state  $s = s(\rho, n)$ . Then the fluctuations can make the entropy increase if the matrix

$$M'_{ik} = \frac{\partial^2 s}{\partial \xi_i \partial \xi_k} ; \quad \xi_i = (\rho, n) \quad (4.5)$$

is not negative definite. Evaluating this condition, we arrive at the non-definiteness of  $M$ . Thus reaching the unstable region, the local homogeneity breaks down. Then maximizing the entropy:

$$\frac{\partial}{\partial \phi_A} \{s_1 V_1 + s_2 V_2 + \lambda (\rho_1 V_1 + \rho_2 V_2 - \rho V) + \lambda' (n_1 V_1 + n_2 V_2 - nV) + \lambda'' (V_1 + V_2 - V)\} = 0 \quad (4.6)$$

$$\phi_A \equiv (\rho_A, n_A)$$

whence we obtain Conds. (2.5), i.e. the system dissociates into coexisting phases again. Nevertheless, for  $T > 0$  this dissociation happens before the instability: eq. (4.1) shows that going to the unstable region the fluctuations go to infinity, thus there will be some point where they are sufficiently large to produce the second phase.

Although we have seen that a low temperature system reaches the coexisting phases only somewhere between the phase transition line  $\phi(s, n) = 0$  and the boundary of the unstable region, thence the most probable state already fulfils Conds. (2.5), thus the ordinary phase transition seems to be a modest approximation for the process. At the separation of phases there may be a jump in the evolution, but the form of the variation principle (4.6) shows that there is not a jump in either  $n$  or  $\rho$ .

If one assumes that the motion of the system on the phase plane is very fast compared to the velocities of the structural changes in the matter, then there is a further possibility similar to the classical undercooled fluids: a sudden change carrying the whole matter into the second phase, jumping over the unstable region (see path c on Fig. 2). In the hydrodynamical flow this transition appears as a sharp front, since the continuity of all parameters cannot be ensured.

The theoretical description of this process is complicated because the Rankine-Hugoniot relations and the hydrodynamical equations have to be simultaneously used. In Ref. 8, due to the special equation of state, the situation is more fortunate than it could be in general. The unstable region is compressed into one line and no energy and density changes occur. Then, due to the continuity equation, the velocity is continuous too. The pressure has a jump in the front, but its effect to the energy change can be ignored because of the nonrelativistic treatment. These facts together gave the possibility to describe the process in nonrelativistic continuum hydrodynamics without using the Rankine-Hugoniot equation. Such a treatment is equivalent with the assumption that there is no enough time for regular phase transition.

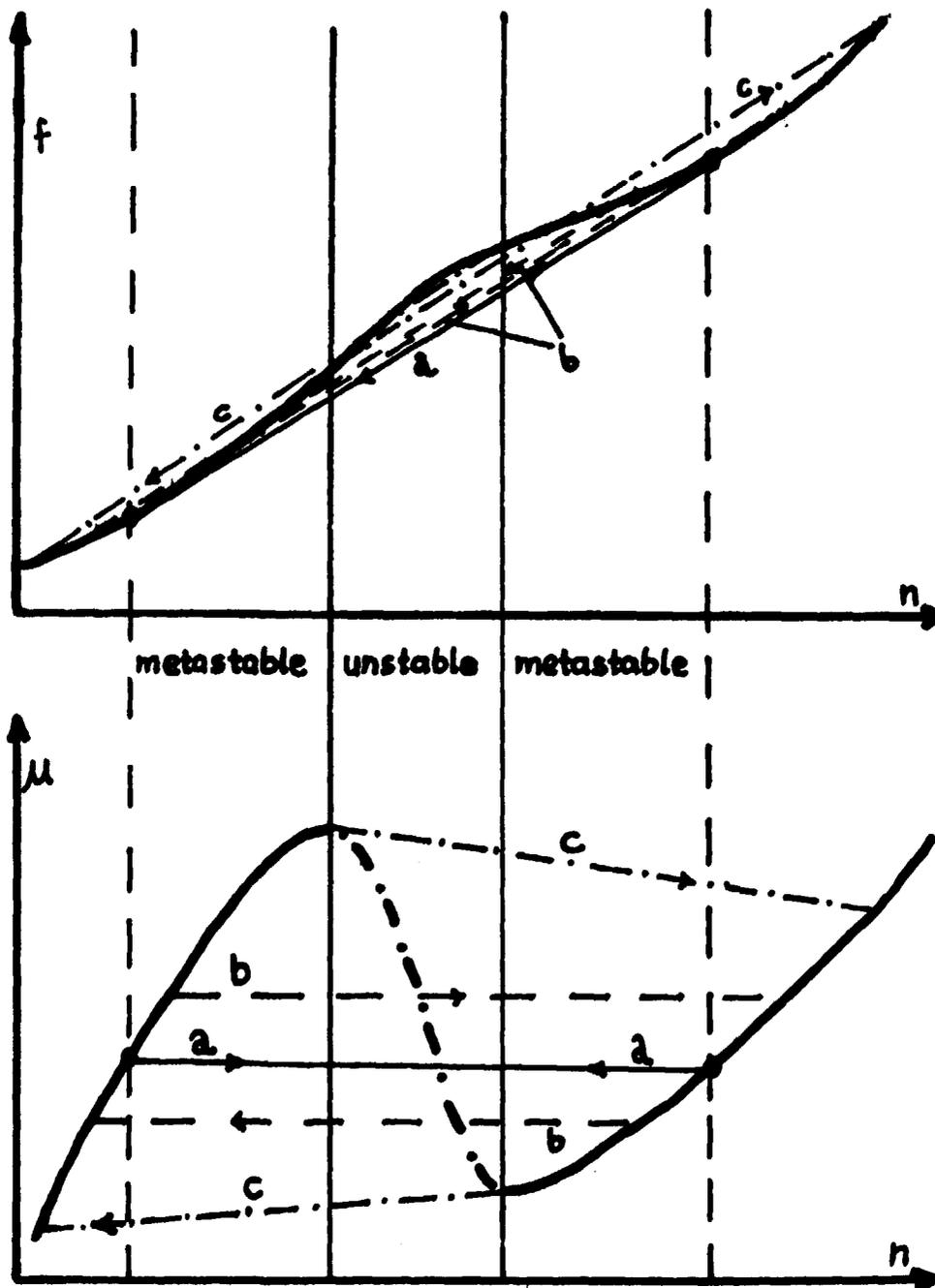


Fig. 2. Free energy density  $f$  and chemical potential  $\mu$  versus the baryon density  $n$  at the phase transition. The path labelled by  $a$  (full line) corresponds to the equilibrium phase transition satisfying the Maxwell Rule. Small supercompression or superexpansion is represented by curves  $b$ , while curves  $c$  show the transition starting from the last stable state. The latter two curves ( $b, c$ ) do not represent complete equilibrium.

## 5. MICROSCOPIC COLLAPSE OR THE NEGATIVE PRESSURE

Some equations of state contain regions where  $p < 0$ . (See e.g. Refs. 8, 12.) Danielewicz, who discussed this problem [12], concluded that such a state is awkward but not contradictory. In fact such a state may be thermodynamically stable in the sense of eq. (2.2). However, in this region the continuum is unstable against droplet formation. If  $p < 0$ , the matter collapses in each element (microscopic collapse [15]) till reaching a state  $p > 0$ . If a piece of matter is collapsing faster than its neighbourhood, it becomes a droplet, and since the pressure acts through a surface, becomes decoupled from the matter. Thus the probable final state is a system of drops of positive pressures. This process (and its final state) cannot be calculated by means of continuum equations. If the system reaches the  $p < 0$  region, we can calculate but cannot expect that the calculated states are realized in the nature.

Generally the  $p < 0$  regions are preceded by thermodynamical instabilities. E.g. for van der Waals gases the isotherms of low temperature have such sections but they are removed by the Maxwell Rule. Danielewicz also states that for his equation of state the first part of the  $p < 0$  interval of the isotherms is unstable and there is a phase transition. The case in Ref. 8 is slightly more complicated since the equation of state does not contain any unstable region except a singular line where the derivatives of  $\rho$  do not exist. However, if  $\rho(s,n)$  were smoothed, the state would become unstable in a region. If we assume that the system escapes this instability by phase transition through coexisting phases, it escapes the  $p < 0$  states too, because in the stable phase before the instability  $p > 0$ , and thus in the coexisting second phase the pressure must be positive too.

## 6. THE COMPRESSIBILITY OF THE MIXTURE

The compressibility of the matter in a real process can be measured by means of the inverse of the ratio  $\dot{p}/\dot{h}$  (which is a generalization of the isothermal or adiabatic compressibility). For a single phase one can calculate  $\dot{h}$  and  $\dot{T}$  from eqs. (3.1-2) and then the result for  $\dot{p}$  is

$$\dot{p} = \frac{n}{\gamma} \left( [\alpha\gamma - (\beta + \frac{\epsilon}{n})^2] \dot{h} - (\beta + \frac{\epsilon}{n}) \frac{\dot{\Sigma}}{T} \right), \quad (6.1)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are defined in (3.8). For two phases  $\dot{h}_1$ ,  $\dot{h}_2$  and  $\dot{T}$  can be obtained from eqs. (3.7-8) and then

$$\dot{p} = \frac{\frac{1}{n} \frac{(n_2 s_1 - n_1 s_2)^2}{n_2 - n_1} \dot{n} + (n_2 s_1 - n_1 s_2) \frac{\Sigma}{T}}{\frac{n_2 - n}{a_1} (\Delta + \beta_1)^2 + \frac{n - n_1}{a_2} (\Delta + \beta_2)^2 - (n_2 - n) \gamma_1 - (n - n_1) \gamma_2}, \quad (6.2)$$

$$\Delta = \frac{s_2 - s_1}{n_2 - n_1}.$$

If the irreversible processes can be neglected, then  $\dot{p}$  is linear in  $\dot{n}$ , so the inverse effective compressibility  $\dot{p}/\dot{n}$  is fully determined by the momentary local thermodynamical data.

Naturally the compressibility of the mixture depends on the equations of state of both phases, and in order to calculate it first it is necessary to calculate the location of the phase transition on the phase plane. Nevertheless even without detailed calculations one can see the low temperature behaviour of the compressibility. Namely, for most cases the free energy starts as  $a(n) + b(n)T^2$ , and then, if  $\Sigma = 0$ , the effective compressibility  $\dot{n}/\dot{p}$  is proportional to  $T^{-2}$  (and it is infinite at  $T = 0$ ). On the other hand, eq. (6.2) indicates that at high temperatures the matter may show quite substantial resistance against compression in spite of the phase transition.

In order to get a more quantitative estimation for the experimental situation  $E/A = 2.1$  GeV beam energy, let us take the temperature according to Ref. 4 (cca. 200 MeV). Then the phase transition starts at  $n_1 = 3n_0$  and ends at  $n_2 = 6n_0$  [3], [5], while the chemical potential is cca. 1 GeV. The entropy of the quark plasma can be calculated from these data. For an estimation one can use a quadratic equation of state

$$f = \frac{K}{18 n_0} (n - n_0)^2 + n W_0 + F(n, T) \quad (6.3)$$

in the nucleon phase, where  $W_0 = -15.96$  MeV and  $K = 200$  MeV [16], while  $F$  is taken from the  $T^2$  approximation of nonrelativistic Fermi gases [17]. Then, comparing the compressibilities just below and above  $n_1$ , the result is that the first is  $300 \text{ erg}^{-1}$ , while for the mixture the compressibility is the double of this value. One can see that the mixture is still more compressible, but the difference is not too great.

Such a result can be expected at this temperature because here the three different characteristic energy parameters, i.e. the temperature  $T$ , the Fermi energy, and the parameter  $K$  more or less coincide. A similar study of the sensitivity of the compressibility on the cold nuclear equation of state led to the conclusion that at the present accuracy the experimental determination of the compressional part of the equation of state is not promising [18]. There the thermal and compressional pressures were compared to each other and the total pressure was insensitive on the details of the equation of

state. Here we see that a (quasi) equilibrium phase transition does not cause an essential change in the picture, at least at the discussed temperature.

## 7. CONCLUSION

We have seen that one cannot expect a high compressibility for the two phase mixture at 2.1 GeV/nucleon beam energy, it remains almost the same after the formation of the second phase. But then the one phase calculation of Ref. 4 remains more or less relevant even during the phase transition, thus the maximal density accessible in the collision does not essentially increase. Comparing the calculated maximal density of Ref. 4 to the phase diagram of the n-q system [3] one can conclude that the phase transition can just be completed in the very center, at the moment of the maximal compression. But then the plasma stage is limited in both space and time, and thus one cannot expect very definite signals of its existence at this energy.

Also we can conclude that full 3 dimensional dynamical calculations including viscosity, heat conduction and final fragment formation effects [19], and theoretical evaluation of more sensitive observables as two and more particle correlations [20] with their accurate measurements are necessary in order to gain definite information on the equation of state. This is obviously a formidable task, but the prize is worth the effort.

APPENDIX:  
THE IRREVERSIBLE PROCESSES

Consider a one-phase fluid with the independent extensive densities  $n$  and  $s$ . The balance equations yield:

$$T^{ir}_{,r} = 0 \quad (A.1)$$

$$(nu^r)_{,r} = 0 \quad (A.2)$$

The energy-momentum tensor can be written as

$$T^{ik} = (\rho+p)u^i u^k + pg^{ik} + q^i u^k + q^k u^i + \tau^{ik} \quad (A.3)$$

$$u^r u_r = -1, \quad q^r u_r = \tau^{ir} u_r = 0$$

where  $q$  is the heat flux and  $\tau^{ik}$  stands for the viscous stresses. Both extra terms come from irreversible processes. Three components of (A.1) determine the three independent components of the four-velocity  $u^i$ ; the fourth one and (A.2) govern the extensive densities:

$$\dot{\rho} + (\rho+p)u^r_{,r} - q^r_{(r;s)} u^r u^s + \tau^{rs} u_{(r;s)} = 0 \quad (A.4)$$

$$\dot{n} + n_{,r} u^r = 0 \quad (A.5)$$

Since  $\rho$  is a function of  $n$  and  $s$ , its derivative is

$$\dot{\rho} = \rho_{,s} \dot{s} + \rho_{,n} \dot{n} = T\dot{s} + \mu\dot{n} \quad (A.6)$$

and then (A.4) gives

$$T(\dot{s} + su^r_{,r}) = -q^r_{,r} + q^r_{(r;s)} u^r u^s - \tau^{rs} u_{r;s} \quad (A.7)$$

In the simplest approximation

$$q^i = 0$$

$$\tau^{ik} = -2\eta(u^{(i;k)} + u^r u^i u^k)_{,r} - \eta'(g^{ik} + u^i u^k) u^r_{,r} \quad (A.8)$$

and thus

$$T(\dot{s} + su^r_{,r}) = \eta(u^{r;s} u_{r;s} + u^r_{;s} u_{s;r} + u^r u^s u^t_{,r} u_{t;s}) + \eta' (u^r_{,r})^2 \quad (A.9)$$

Neglecting the shear viscosity or restricting ourselves to one dimensional flow only the last term remains:

$$T(\dot{s} + su^r_{;r}) = \eta' \left(\frac{\dot{n}}{n}\right)^2 \quad (\text{A.10})$$

If the heat flux does not vanish, then the entropy flux has the form [10]

$$s^i = su^i + \frac{1}{T} q^i \quad (\text{A.11})$$

with the source

$$s^r_{;r} = -\frac{1}{T} \{q^r (T_{,r} + u^s u_{r;s}) + \tau^{rs} u_{r;s}\} \quad (\text{A.12})$$

Thus the bracketed term has to be negative semidefinite, which is a thermodynamical constraint on the transport laws.

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Felelős kiadó: Szegő Károly  
Szakmai lektor: Borbély Imre  
Nyelvi lektor: Dolinsky Tamás  
Gépelte: Balczér Györgyné  
Példányszám: 415 Törzsszám: 83-471  
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