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RESEARCH REPORT

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Abstract

Current-carring plasma channels have been proposed for transporting intense ion beams from diodes to a target in a LIB-fusion device. In this papaer, the growth rate of the most dangerous surface mode, that is, axisymmetric sausage instability is examined for the plasma channel. The growth rate is shown to be smaller than that of the plasma channel with no fluid motion in a sharp boundary. It is concluded that the stable plasma channel can be formed.

§1. Introduction

In light ion beam (LIB) fusion, it is necessary to transport intense ion beams from diodes to a target.[1,2] As one method for transporting ion beams, a plasma channel has been proposed. The plasma channel must be kept stable while the ion beam propagates through in it. However the most dangerous surface mode of macro=instability, that is, sausage instability[3,4] appearing in the channel has a possibility to obstruct the beam propagation.

In this paper, the growth rate of the axisymmetric sausage instability appearing in the plasma channel is examined to point out the fact that the growth rate reduces in one or two orders of magnitude from that of the plasma channel with no fluid motion in a sharp or diffuse boundary. It can be shown that the maximum growth rate γ_{max} of the instability in our channel multiplyed by the duration time τ of the channel formation is much smaller than unity. Therefore it is plausible to expect that the plasma channel is kept stable for the most dangerous surface mode of macro=instability, that is, sausage instability during the process of channel formation.

§2. Growth Rate of Axisymmetric Sausage Instability of Plasma Channel

The basic equations for the analysis of the axisymmetric sausage instability in the plasma channel are written down as follows:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\, \rho \mathbf{v} = 0, \tag{1}$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\ \mathbf{v}\right] = -\nabla p + \frac{1}{4\pi}(\nabla \times \mathbf{B})\times \mathbf{B}, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{3}$$

Here t is the time, ρ the density, $\mathbf{B}$ the magnetic field, $\mathbf{v}$ the velocity of the channel plasma, and the pressure p is written by $p = (z + 1)k\rho T/m$, where z is the charge number, k Boltzmann's constant and m the mass of the plasma ion and the temperature T is assumed to be constant. In these basic equations, the energy dissipations by viscosity and thermal and electrical conductivities are completely neglected. The linearized equations of eqs.(1)-(3) in the axisymmetric forms are obtained here under the assumptions: $\partial/\partial\theta = 0$, $v_{z0} = 0$, $v_{\theta 1} = 0$, $B_{\theta 0} = B_0$, $B_{r0} = B_{z0} = 0$, $B_{r1} = B_{z1} = 0$ and perturbed quantities $\propto \exp(i\int k_r dr + ik_z z + \gamma t)$. Here the suffix 0 denotes quantities in equilibrium and the suffix 1 denotes perturbed quantities. The wave numbers k_r and k_z are for the radial and axial directions, respectively. By using non-dimensional quantities, $R = r/r_p$ ($r_p = p_0 c^2/\pi j_0^2$, where p_0 the pressure, c the velocity of the light and j_0 the current density), $K_r = k_r r_p$, $K_z = k_z r_p$, $\Gamma = \gamma r_p/v_s$ ($v_s = \sqrt{kT/m}$), $U = v_{r0}/v_s$ (v_{r0} is the velocity in the radial direction) and $B = B_0 / \sqrt{8\pi\rho_0 v_s^2}$, the dispersion relation is written down as follows:

$$\Gamma^4 + \left(\frac{U}{R} + 3\frac{\partial U}{\partial R}\right)\Gamma^3 + \left\{- 2\left(\frac{2B}{R} + \frac{\partial B}{\partial R}\right)\frac{\partial B}{\partial R} + \frac{\partial U}{\partial R}\left(\frac{U}{R} + 3\frac{\partial U}{\partial R} - U\frac{1}{\rho}\frac{\partial \rho}{\partial R}\right)\right.$$

$$\left. + \left[- 6U^2 + (2B^2 + 1)\right] K_r^2 + (2B^2 + 1)K_z^2 \right\} \Gamma^2$$

$$+ \left\{\left(\frac{\partial U}{\partial R}\right)^2\left(\frac{\partial U}{\partial R} - U\frac{1}{\rho}\frac{\partial \rho}{\partial R}\right) - 2\left(\frac{U}{R} + \frac{\partial U}{\partial R}\right)\left(2\frac{B}{R} + \frac{\partial B}{\partial R}\right)\frac{\partial B}{\partial R} + \left[2\frac{U}{R} + \right.\right.$$

$$2\frac{\partial U}{\partial R}(2B^2+1)]K_z{}^2 + [- U^2(3\frac{U}{R} + 7\frac{\partial \rho}{\partial R}) + 2U(\frac{1}{R} + \frac{1}{\rho}\frac{\partial \rho}{\partial R}) +$$

$$2UB(5\frac{B}{R} + 4\frac{\partial B}{\partial R}) + \frac{\partial U}{\partial R}(2B^2+1)] K_r^2 \}\Gamma$$

$$+ U^2(U^2 - 2B^2 - 1) K_r^4$$

$$+ \{ U^3\frac{\partial U}{\partial R}(\frac{1}{\rho}\frac{\partial \rho}{\partial R} - \frac{1}{R}) - 2U^2(\frac{\partial U}{\partial R})^2 + U\frac{\partial U}{\partial R}[\frac{1}{R} + \frac{1}{\rho}\frac{\partial \rho}{\partial R}+4B(\frac{B}{R} + \frac{\partial B}{\partial R})]$$

$$+ 2U^2[\frac{\partial B}{\partial R}(\frac{2B}{R} + \frac{\partial B}{\partial R}) + \frac{2B}{R}(\frac{B}{R} +\frac{\partial B}{\partial R})] -U^2(2B^2+1)K_z^2 \}K_r^2$$

$$- 2(\frac{2B}{R} + \frac{\partial B}{\partial R})[\frac{\partial B}{\partial R} - B(\frac{1}{R} + \frac{1}{\rho}\frac{\partial \rho}{\partial R})]K_z^2$$

$$+[(\frac{\partial U}{\partial R})^2(2B^2+1) + 2UB\frac{\partial U}{\partial R}(\frac{\partial B}{\partial R} - B\frac{1}{\rho}\frac{\partial \rho}{\partial R})]K_z^2 = 0. \qquad (4)$$

The nondimensional wave number K_r is expressed in the following equation.

$$K_r = \frac{\sqrt{- F_1 + \sqrt{F_1^2 - 4F_3F_2}}}{\sqrt{2F_3}}, \qquad (5)$$

where

$$F_1 = G_1 + G_2K_z^2 + G_3\Gamma + G_4\Gamma^2, \qquad (6)$$

$$F_2 = H_1K_z^2 + H_2\Gamma + H_3K_z^2\Gamma + H_4\Gamma^2 + H_5K_z^2\Gamma^2, \qquad (7)$$

$$F_3 = U^2(2B^2 + 1 - U^2), \qquad (8)$$

$$G_1 = U\frac{\partial U}{\partial R} [- U^2(\frac{1}{\rho}\frac{\partial \rho}{\partial R} - \frac{1}{R}) + 2U\frac{\partial U}{\partial R} - (\frac{1}{R} +\frac{1}{\rho}\frac{\partial \rho}{\partial R})$$

$$- 4B\left(\frac{B}{R} + \frac{\partial B}{\partial R} \right)] - 2U^2\left[\frac{\partial B}{\partial R}\left(\frac{2B}{R} + \frac{\partial B}{\partial R} \right) + \frac{2B}{R}\left(\frac{B}{R} + \frac{\partial B}{\partial R} \right)\right],$$

$$G_2 = U^2(2B^2+1), \tag{9}$$

$$G_3 = - \frac{\partial U}{\partial R}(2B^2+1 -7U^2) + \frac{3U^3}{R} -2U\left(\frac{1}{R} +\frac{1}{\rho}\frac{\partial \rho}{\partial R} +5\frac{B^2}{R} +4B\frac{\partial B}{\partial R} \right),$$

$$G_4 = - (2B^2+1) + 6U^2,$$

$$H_1 = 2\left(\frac{2B}{R}+\frac{\partial B}{\partial R}\right)\left[\frac{\partial B}{\partial R} -B\left(\frac{1}{R}+ \frac{1}{\rho} \frac{\partial \rho}{\partial R}\right)\right] - \left(\frac{\partial U}{\partial R}\right)^2(2B^2+1)$$

$$-2BU\frac{\partial U}{\partial R}\left(\frac{\partial B}{\partial R} - B\frac{1}{\rho}\frac{\partial \rho}{\partial R} \right),$$

$$H_2 = -\left(\frac{\partial U}{\partial R}\right)^2\left(\frac{\partial U}{\partial R} -U\frac{1}{\rho} \frac{\partial \rho}{\partial R} \right) + 2\left(\frac{U}{R} +\frac{\partial U}{\partial R}\right)\left(\frac{2B}{R} +\frac{\partial B}{\partial R} \right)\frac{\partial B}{\partial R}, \tag{10}$$

$$H_3 = -2\left[\frac{\partial U}{\partial R}(2B^2+1) + \frac{U}{R}B^2\right],$$

$$H_4 = 2\left(\frac{2B}{R} + \frac{\partial B}{\partial R}\right)\frac{\partial B}{\partial R} - 3\left(\frac{\partial U}{\partial R}\right)^2 -U\left(\frac{\partial U}{\partial R}\right)^2\left(\frac{1}{R} - \frac{1}{\rho} \frac{\partial \rho}{\partial R}\right),$$

$$H_5 = -2B^2- 1.$$

The growth rate Γ is calculated through eqs.(4) and (5), based on the data of the plasma channel obtained by the computer simulation.[5] Figure 1 shows the growth rate Γ versus the time t. The nondimensional wave numbers K_z=3.1416, 2.0944 and 1.5708 correspond to the wave lengths $\lambda=2r_0, 3r_0$ and $4r_0$, respectively. Figure 2 shows the maximum growth rate Γ_{max} versus the wave number K_z. The growth rate Γ is calculated at each instant. The real growth rate at t must be integrated over the duration time of the wave. In this reason the real value of the growth rate Γ may be smaller than those shown in Figs.1 and 2.

The product of the nondimensional maximum growth rate γ_{max} by the duration time τ of the channel formation versus the nondimensional wave rimber k_z is given in Fig.3.

Here we summarize the growth rates of sausage instability in the plasma channels with no fluid motion.

2.1 channel with sharp boundary

The growth rate γ_1 of the plasma channel with the profiles of the constant magnetic field B_z in the z-direction and the constant density ρ_0 within the plasma column $r = r_0$ and the magnetic field $B_{\theta 0}$ ($B_{\theta 0} = B_0 r_0/r$) outside the column is given by[6)]

$$\gamma_1 = \frac{B_{\theta 0}\, k_z}{\sqrt{4\pi\rho_0}} \sqrt{\frac{I_0'\,(k_z r_0)}{(k_z r_0)\; I_0(k_z r_0)} - \frac{B_{z0}^2}{B_{\theta 0}^2}}\,, \tag{11}$$

where I_0 is the modefided Bessel function.

2.2 channel with finite radius

The growth rate Γ_2 or γ_2 of the plasma channel with the following profiles of the magnetic field B_0, the density ρ_0 and the temterature T_0 (shown in Fig.4)

$$B_0 = \sqrt{4\pi\tilde{\rho}_0}\;\frac{r}{r_p}\,, \tag{12}$$

$$\rho = \tilde{\rho}_0\,(1 - \frac{r^2}{r_p^2})^{2/3}, \tag{13}$$

$$T_0 = \tilde{T}_0(1 - \frac{r^2}{r_p^2})^{1/3}, \tag{14}$$

is given by[3,4)]

$$\Gamma_2 = \frac{A_1 K_z}{\pi(n'+1/2)} \quad \text{or} \quad \gamma_2 = \frac{A_1 \tilde{v}_s k_z}{\pi(n'+1/2)}, \quad (n'\text{: integer}) \tag{15}$$

where $A_1 = \int_{R_1}^{R_2} R\sqrt{\frac{18}{5(1-R^2)+3R^2}}\, dR$ $(R_1 = 0,\ R_2 = 1)$, $\tilde{v}_s = \sqrt{k\tilde{T}_0/m}$, n' is the mode number, r_p the channel radius, and $\tilde{p}_0$, $\tilde{\rho}_0$ and $\tilde{T}_0$ are the equilibrium values of the pressure, density and temperature on the axis.

2.3 channel with infinite radius

For the plasma channel with the profiles (shown in Fig.5) of

$$B_0 = \frac{\sqrt{8\pi\tilde{p}_0}\ r/r_c}{1 + r^2/r_c^2}, \tag{16}$$

$$\rho_0 = \frac{\tilde{\rho}_0}{(1 + r^2/r_c^2)^{2/3}}, \tag{17}$$

$$T_0 = \frac{\tilde{T}_0}{(1 + r^2/r_c^2)^{4/3}}, \tag{18}$$

the growth rate Γ_3 or γ_3 of the sausage instability is written down as follows:[3,4)]

$$\Gamma_3 = \frac{A_2 K_z}{\pi(n'+1/2)} \quad \text{or} \quad \gamma_3 = \frac{A_2 \tilde{v}_s k_z}{\pi(n'+1/2)}, \tag{19}$$

where $A_2 = \int_{R_3}^{R_4} \frac{2R\sqrt{1/3(3+R^2)}}{\sqrt{(5/3+2R^2)(1+R^2)^3}}\, dR$ $(R_3 = 0,\ R_4 = \infty)$.

The product of the maximum growth rate γ_{1max} by the duration time τ of the channel formation is given by eq.(20) for the density $\rho_0 = 3.35 \times 10^{-6}$ g/cm^3, the channel radius $r_0 = 0.4$ cm, the duration time $\tau = 350$ ns and the magnetic field within the plasma $B_{z0} = 0$.

$$\gamma_{1max}\tau = 0.0936\ k_z\ B'_\theta\ \text{(kgauss)}. \tag{20}$$

The soloid lines in Fig.6 show $\gamma_{1max}\tau$ versus k_z for $B'_\theta = 10$ kgauss and 20 kgauss. The growth rates Γ_{2d} and Γ_{3d} of the most dangerous mode ($n' = 1$) of Γ_2 (in eq.(12)) and Γ_3 (in eq.(19)) are given respectively by

$$\Gamma_{2d} = 0.2269\ K_z, \tag{21}$$

$$\Gamma_{3d} = 0.2277\ K_z. \tag{22}$$

The alternate long and short dash line in Fig.6 presents the product of the dimensional maximum growth rates γ_{2dmax} and γ_{3dmax} by the duration time τ versus the dimensional wave number k_z on the same line.

Comparing our growth rate γ_{max} with γ_{1max}, γ_{2dmax} and γ_{3dmax}, we choose the sausage mode of the wave length $\lambda = 2\ r_0$.

$$\frac{\gamma_{max}}{\gamma_{1max}} = 0.0085, \tag{23}$$

$$\frac{\gamma_{max}}{\gamma_{2dmax}} \simeq \frac{\gamma_{max}}{\gamma_{3dmax}} = 0.055. \tag{24}$$

According to these results, we can conclude that the growth rate γ of the channel with moving plasma is smaller than those of γ_1, γ_{2d} and γ_{3d} of the channel with no plasma motion. The plasma channel which has a radial velocity remains stable regarding the sausage instability during the time duration of the channel formation.

§3. Conclusion

The main results obtained in this paper are summarized as follows:

(1) The growth rate of the axisymmetric sausage instability of the plasma channel with radial motions is smaller by one or two order of magnitude than those of the plasma channels with no radial motion. The growth rate of the instability for the channel plasma moving radially is suppressed because the disturbance accompanied with the instability is carried away with the expanding plasma.

(2) The product of the maximum growth rate γ_{max} by the duration time τ of the channel formation is much smaller than 1. Therefore the channel remains stable before the beam propagation.

In this paper, we discuss the stability of the plasma channel of the LIB-fusion device. The stability of the channel, however, must be examined when the ion beam is passing through in it. The time dependent electric and magnetic fields caused by the strong beam current have a possibility to disturb the channel. We try to investigate the effect of the beam propagation on the channel stability in the subsequent paper.

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Figure Captions

Fig.1. The nondimensional growth rate Γ versus the time t. The nondimensional wave numbers K_z=3.1416,2.0944 and 1.5708 correspond to the wave length $\lambda=2r_0, 3r_0$ and $4r_0$, respectively.

Fig.2. The maximum growth rate Γ_{max} versus the wave number K_z.

Fig.3. The product of the maximum growth rate γ_{max} by the duration time τ of the channel formation versus the wave number k_z.

Fig.4. The profiles of the magnetic field B_0, the density ρ_0 and the temperature T_0 of the plasma channel with finite radius.

Fig.5. The profiles of the magnetic field B_0, the density ρ_0 and the temperature T_0 of the plasma channel with infinite radius

Fig.6. The product of the growth rate γ_{max} by the duration time τ of the channel formation. The solid lines for the plasma channel with a sharp boundary and the dotted line for the channel with a finite or infinite radius.

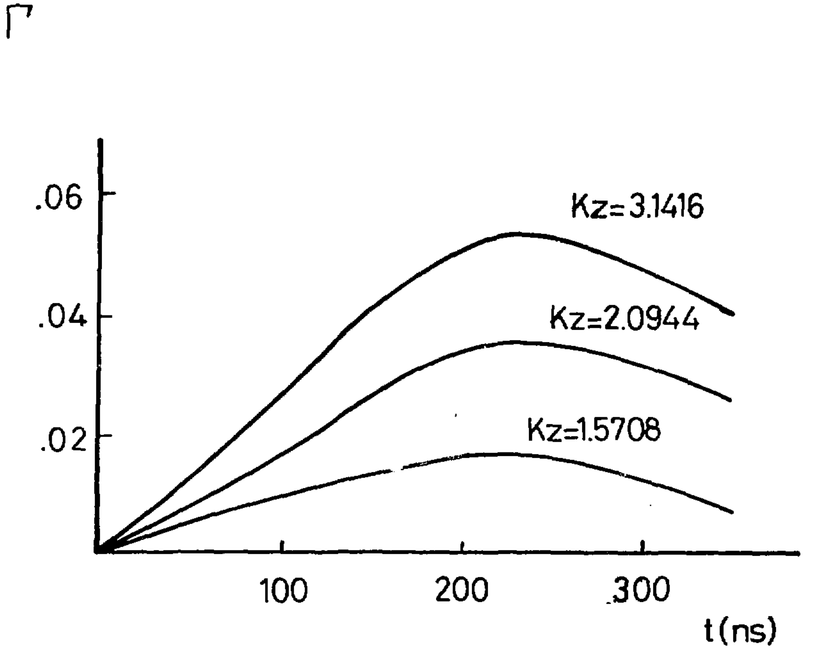


Fig.1

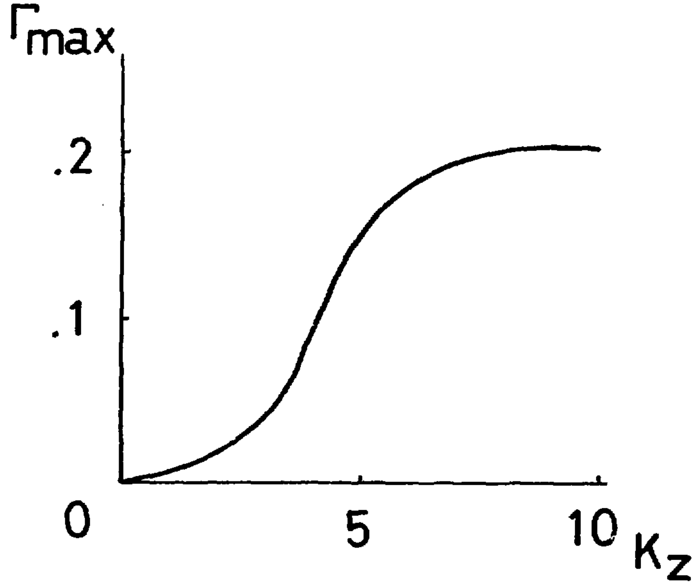


Fig.2

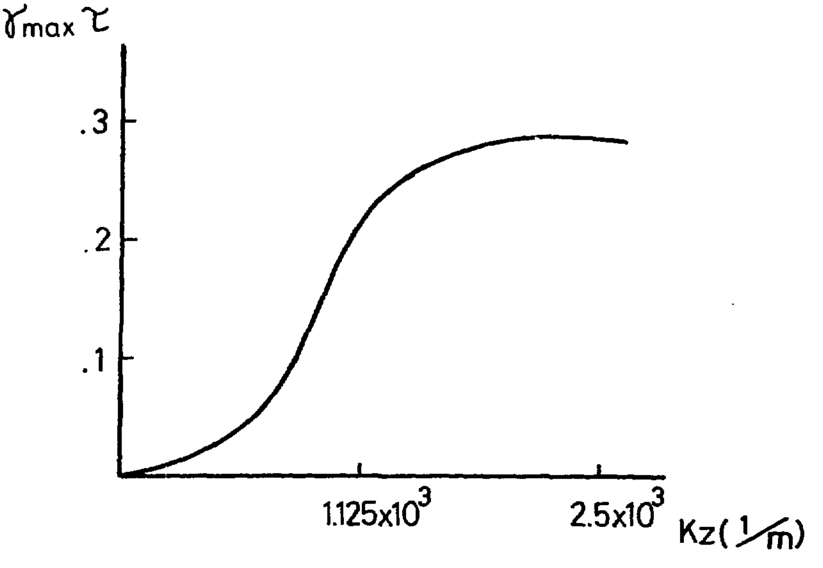


Fig.3

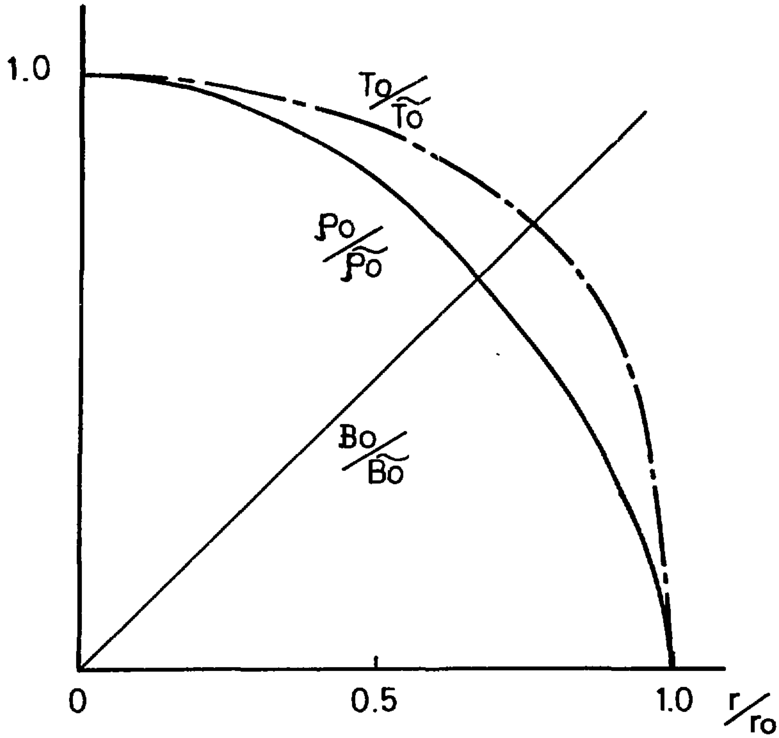


Fig.4

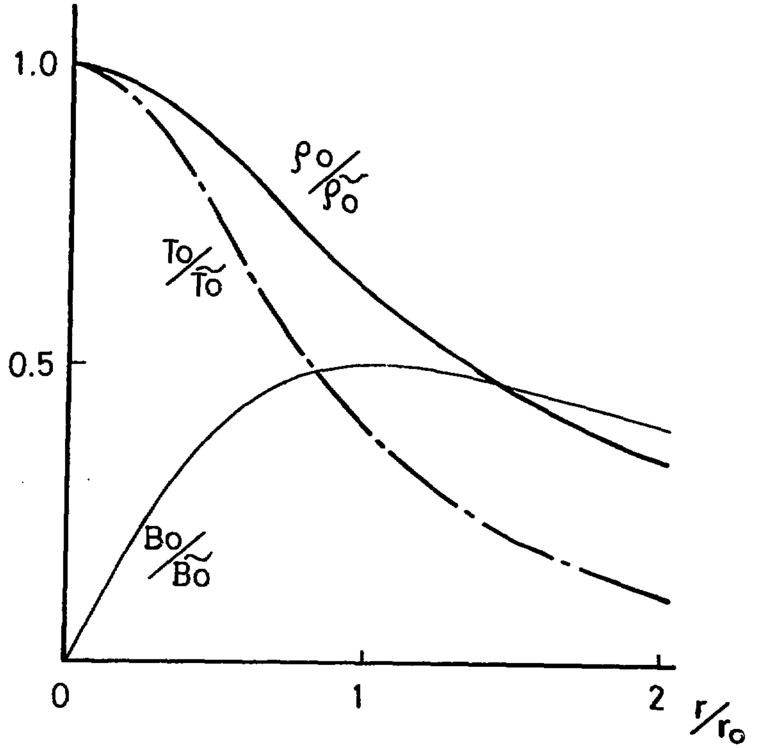
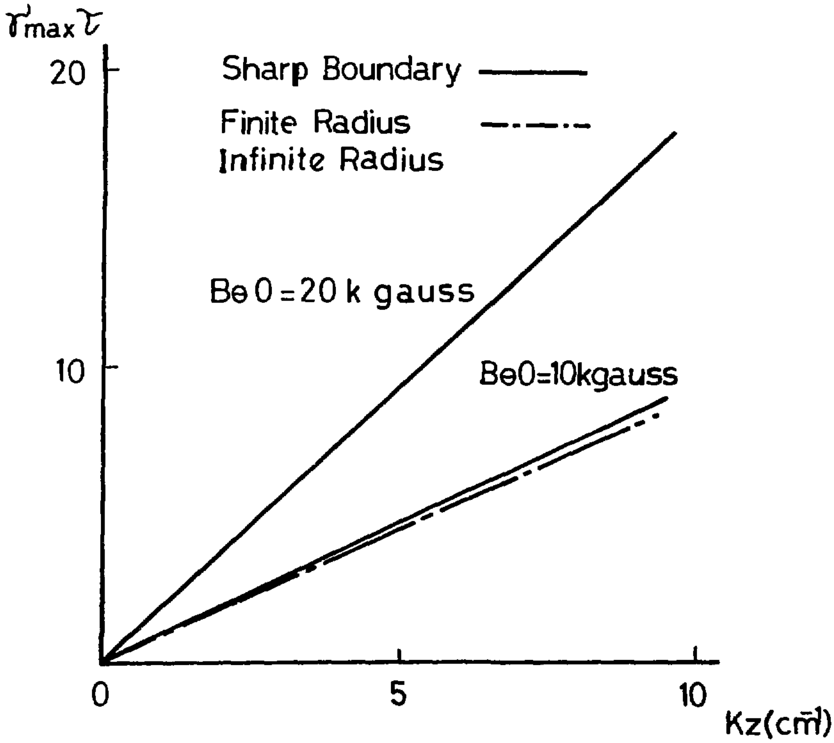


Fig. 5



γmax τ
20
10
Sharp Boundary
Finite Radius
Infinite Radius
Bθ0 = 20 k gauss
Bθ0=10kgauss
0
5
10
Kz(cm⁻¹)

Fig. 6