Conf-820180--1

CONF-820180--1

DE83 017182

A REVIEW OF THE BETA SITUATION*

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January 18, 1982

*Research sponsored by the Office of Fusion Energy, U.S. Department of Energy, under contract W-7405-eng-26 with the Union Carbide Corporation.

A Review of the Beta Situation

INTRODUCTION

This note lists some of the possible causes of beta limitation in tokamaks and discusses what is known and what is involved in investigating them. The motivation for preparing this note is the observed degradation of confinement with increasing beta poloidal (β_p) and beam power (P_b) in ISX-B. It was shown in ISX-B that approximately $\beta_p I^{1/2} \propto P_b^{1/3}$

$$\text{and since } \beta_p \simeq \frac{0.2a^2\, nT}{I^2}$$

$$\text{and } \tau_E = \frac{0.1a^2R\, nT}{P_b}, \text{ we have}$$

$$\tau_E \propto \frac{I^{1/2}}{\beta_p^2} \propto \frac{I^{3/2}}{P_b^{2/3}}$$

a, R(m), n (x 10^{19} m^{-3}), T (keV), P_b (MW), I (MA), τ_E(S)

It is not clear, however, whether the effects are a result of the increase in beta poloidal (β_p) or the increase in beam power or power density. It is observed, however, that τ_E is independent of beta $\langle\beta\rangle$ at fixed β_p.

In Appendix A a discussion is given of the implications of a relationship of the form $\beta_p^x \; I^y \propto P_b^w$ for transport, using the analysis of Connor and Taylor (Nuclear Fusion 17, 5, 1977) to determine the form of the transport consistent with kinetic and fluid phenomena.

In Table 1 a summary is given of issues which are important to understanding the effects which are observed on ISX-B. Apparently, similar effects may be occurring in other experiments (T-11, JFT-2, PLT) since in none of them are $\langle\beta\rangle$ and β_p substantially different.

Table 1

Issues	Theory	Scale Length	Diagnostics	Status
(1) Equilibrium $\varepsilon\beta_p \to 1$	Exists	$L \sim a$	Magnetic Loops	No problem on ISX-B[(a)] since no separatrix at $R_o - a$
(2) Low m,n MHD (1, 2, 3,...)	Ideal and Resistive code calculations exist	$L \sim a$	Magnetic Loops PIN arrays	Resistive modes clearly[(b)] seen on ISX-B, but they are not the only possible cause of confinement degradation
(3) Medium→High m,n MHD ($\sim$ 4→20?)	Ideal and Resistive code calculations exist	$\sim$cm$<L<a$	Cross correlation of PIN signals. IR, FIR interferometer and polarimeter Ion Beam Probe	Difficult to measure.[(c)] Resolution experimentally and theoretically is a problem
(4) Current Profile	Bootstrap effects Beam driven effects	Resolution needed $\ll a$	Faraday rotation Thomson scattering $T_e(r)$ + continuum for $Z_{eff}(r)$ + PINs for q rational	Being installed[(d)] on ISX-B
(5) Kinetic a. Drift waves b. $v_b > c_A$ (beams)	Electrostatic $\omega = c_A k_{11}$ $\omega = c_A k$ $\omega = w_{ci}$ Alfven m=2,3,4	$k_\perp \rho_{it} \sim 1$ $\beta \neq 0$ $L \lesssim$ cm	Microwave, FIR scattering. Cyclotron noise	Difficult to analyze[(e)] theory and experiment
(6) Rotation	Modification of transport being studied Kelvin-Helmholtz instability?	$\sim a$	Need $n_e(r)$, $T_e(r)$, $T_i(r)$, $n_z(r)$, $V_{11}(r)$ $\phi(r)$ co- + counter injection	First tests, JFT-2, PLT.[(f)] Tests with full diagnostics to be undertaken on ISX-B

Table 1 - continued

Issues	Theory	Scale Length	Diagnostics	Status
(7) Heating Effects	Theory exists for NB and rf heating, but some rotation and electric field effects are not included	Both $\sim a$ and microscopic $\lesssim$cm	As above	$\|\|$co + counter NB PLT,[g] ISX-B, JFT-2 $\sim\perp$PDX, DIII ICRH in PLT
(8) Miscellaneous	Ion orbit calculations, non-uniform n_n calculations, electric field calculations. Generally, profile effects.	As above	As above + neutron measurements and $f_i(r,\theta,\phi,t)$ Detailed profile measurements.	Need more extensive[h] theoretical calculations including electric fields and non-uniformity and n_n measurements. Profile control is needed.

DISCUSSION OF POSSIBLE EFFECTS

a. Equilibrium. For certain profiles, the separatrix moves into the inner side of the plasma as β_p increases to of order the aspect ratio R/a. Careful measurements on ISX-B with multiple magnetic coils show clearly that the poloidal field on the inside of the plasma is not canceled by the increasing vertical field required to support the increasing β_p.

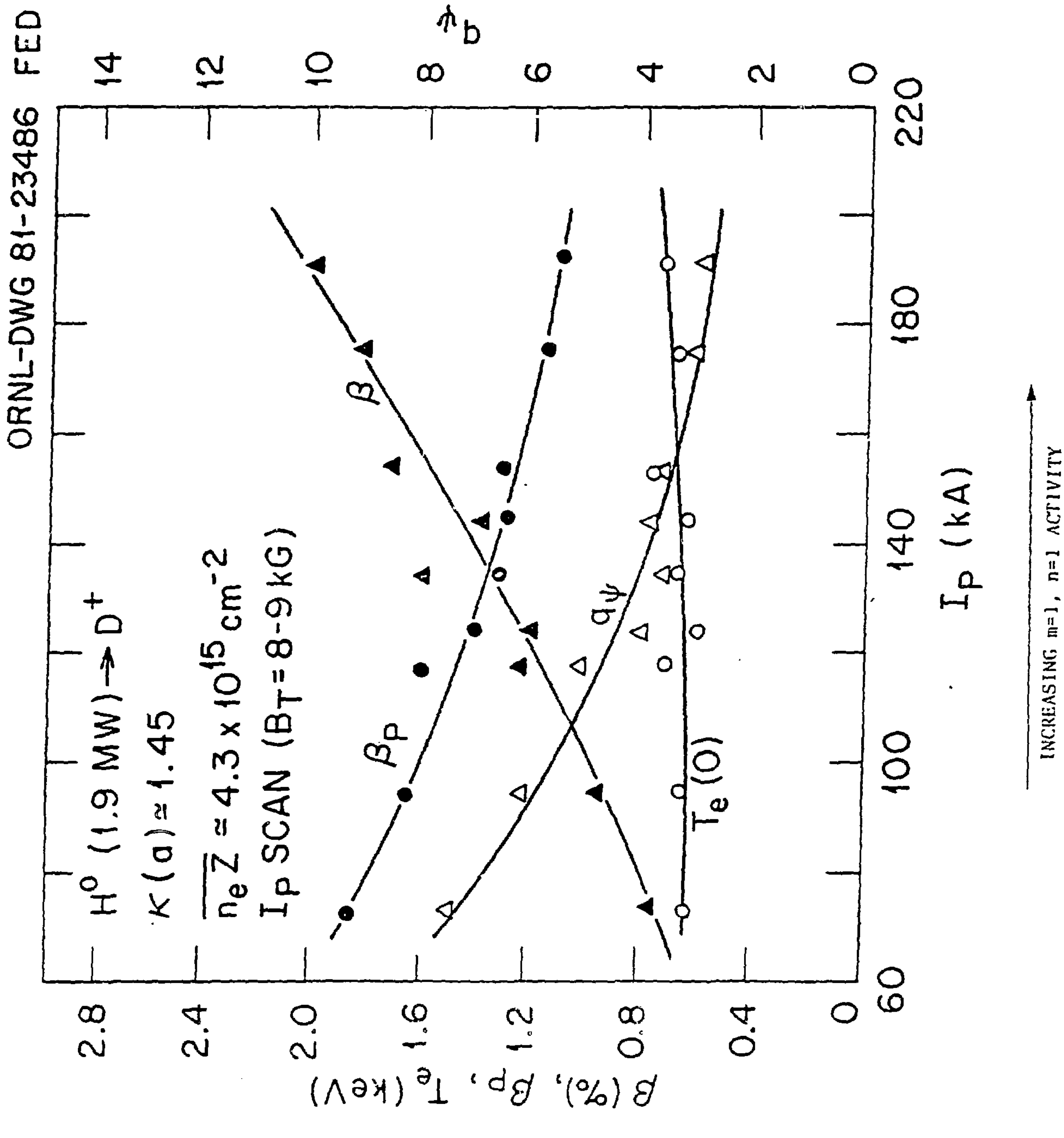
b. Low m,n MHD modes. There is remarkably good agreement between the mode structure observed on ISX-B and the theoretical predictions of Carreras, et al, of the high β_p distortion of m=1/n=1 mode, (see Figure 1). This is convincing evidence that the resistive mode effects are predictable. However, the results of changing toroidal field while maintaining constant, n_e, I, and P_b show that degradation of confinement occurs both in the presence or absence of these low m and n resistive modes. Two basic interpretations are possible.

(1) The m=1/n=1 resistive mode is not the main cause of confinement degradation.

(2) The m=1/n=1 mode degrades confinement at low q_a, and something else degrades confinement at high q_a; and the two mechanisms trade off systematically to maintain constant τ_E.

There is some evidence from theoretical calculations (Carreras) that the presence of low m and n modes suppresses the high m and n modes. Thus, the effect could be a trade off between the low and high m and n modes. However, as shown in Figure 1, with changing I_p at constant n and P_b the m=1/n=1 activity was increasing while τ_E was increasing suggesting that the first interpretation is correct. This will be investigated further on ISX-B.

Figure 1. The variation of β, β_p, q_ψ, T_e with plasma current I_p is shown. Since the toroidal field is approximately constant then $\tau_E \propto \beta$ is increasing with I_p even though the m=1, n=1 activity is also necessary.



c. <u>High m,n MHD modes</u>. It is difficult to resolve the high m and n modes, both from a computational point of view (grid size) and from an experimental point of view (resolution). In ISX-B the radius is $\simeq$ 25 cm, and practically it is "difficult" to attain a resolution of order a centimeter with fluctuation detection devices such as PIN arrays and magnetic loops. Nevertheless, attempts will be made to use cross-correlation techniques on the PIN diode signals on ISX-B.

d. <u>Current profile</u>. There is considerable evidence, mainly circumstantial, that shows a marked dependence of confinement on the current profile. The form of MHD activity depends sensitively on the profile and q_a level, and for a fixed current at $q_a \sim 3$ confinement can vary from nominal ($\sim$ Alcator) to disastrous (disruption), depending on the profile. It is difficult to determine the resistive equilibrium current profile at high β from the conductivity, since in general, $Z_{eff}(r)$ is not known accurately and neo-classical effects, micro- and macro-instabilities, and beam-driven currents could modify this profile.

In order to understand the importance of the current profile to beta and to understand beam effects it is important to measure it, even though the measurement may be poor.

With an N-channel Faraday rotation scheme it is in principle possible to resolve j(r) to $\sim$ a/N. In addition the magnetic center of the plasma may be detected by monitoring where $\int_{-z}^{+z} n\, B_z\, dz$ changes sign. This measurement coupled with detection of rational q surfaces, $T_e(r)$, $Z_{eff}(r)$ helps consolidate the experimental determination of j(r). On ISX-B a vertical 5-channel Faraday rotation will be operational in 1982, and this could be extended later to a 9-channel device.

e. Kinetic. Micro-instabilities driven by subtle variations of the distribution functions, and having scale lengths such that, cm > $L \sim \lambda_D \rightarrow \rho_{it}$, may be the cause of confinement degradation. Typically, the scale lengths have the dimensions, Debye length $\lambda_D \sim 10\ \text{cm}^{-3}$, thermal ion gyro radius $\rho_{it} \sim 10^{-1}$ cm. There is reasonable evidence that drift waves of some form or other are responsible for the normal degraded electron confinement (Alcator Scaling). However, after approximately 10 years of research, involving numerous non-linear theoretical calculations and a variety of experimental measurements (generally by the coherent scattering of electromagnetic radiation), there is no clear cut model which explains quantitatively Alcator scaling or the experimental data. In this circumstance it is optimistic to assume that a higher level of understanding could be obtained in the beta area. Nevertheless it is worthwhile to make scattering measurements, and to look at cyclotron noise to obtain qualitative information on whether kinetic effects are candidates to explain the beta effects, and this will be done on a number of experiments.

f. Rotation and g. Heating Effects. The effects of rotation on both MHD and transport are being studied theoretically. Calculations suggest that rotation could not modify the MHD behavior substantially enough to cause the ISX-B beta effects at the measured level of rotation velocity.

1. There are, however, effects associated with electric fields set up by rotation and these should be included in energy balance analyses. Basically rotation should lead to a radial electric field. The flux of electrons and ions moving radially must, respectively, either do work against or have work done by the electric field. If, for example,

$v_\phi = v_{\phi o} = 1.4 \times 10^5$ m.s^{-1} and $B_\Theta = 0.2T$, then $E_r \sim 2.8 \times 10^4$ V.m^{-1}. On ISX-B assuming a parabolic current profile, a triangular toroidal velocity profile, and that $E_r = v_\phi B_\Theta$, then the above numbers give $\phi(a) = 760$ V for coinjection. As far as modeling is concerned care must be taken in using the energy equations to differentiate between random and directed energy, since in most codes the calculations of the beam-plasma energy transfer and subsequent energy balance are done in the ion rest frame. If a significant amount of energy is transferred to directed ion motion (rotation) then terms must be included to transfer this energy in the correct fashion. Note that for deuterons, $v = 1.4 \times 10^5$ m.s^{-1} corresponds to an energy of 200 eV. This effect could be important to the understanding of whether either electron or ion confinement or both is degraded when beta is raised using unidirectional beam heating.

2. A second confusion caused by rotation, or more generally by momentum input, is the observed effect on the transport of impurities. It is conceivable that some of the ISX-B effects are a result of such transport changes and it is important to vary rotation at a variety of beta poloidal values all the way from $+v_{\phi o}$ (max) through $v_\phi = o$ to as far into the $-v_{\phi o}$ (counter-injection) direction as reasonable plasmas can be obtained. In this way it may be possible to unravel the conceivably separate effects of impurity flow reversal, density clamping, rotation (E-fields) and beta poloidal effects. Tests on JFT-2, PLT and ISX-B are important in this area.

It has been argued that if beam power density is the problem, then in the FED the problem will be very small. The basis for this is a comparison with ISX-B in which the FED electron transport is not degraded below Alcator scaling. In fact, the power densities in ISX-B are comparable with those in the FED, possibly a factor of 2 greater.

ISX-B ($a = 27$, $b = 43$, $R = 93$, $P = 1$ MW) $p = 0.24$ W/cm^3 [For 2 MW confinement is substantially degraded]

FED ($a = 130$, $b = 208$, $R = 500$, $P = 72$ MW) $p = 0.13$ W/cm^3

Finally, the use of near perpendicular injection on PDX and DIII, and of ICRH on PLT will help confirm models of the plasma behavior.

h. Miscellaneous. The effects of finite ion orbits on transport and MHD should be studied. The MHD effects on fast ion orbits could change deposition profile. The effects of axial non-uniformity in particular non-uniformity of the neutral density (n_n) may be a factor contributing to fast beam ion losses. The effects of fluctuating electric fields and electric or magnetic convective cells should be studied. There is some evidence from ISX-B that beta, for a given q_ψ is relatively higher when impurities are present. Unfortunately, the presence of impurities also limits q_ψ to higher values than can be obtained in a clean plasma. Nevertheless, this result may indicate the importance of profile effects, in that the peaked profiles typical of modestly impure plasmas may, when heated, lead to more stable pressure profiles. Thus, the accurate measurement and control of profiles is an important avenue for increased effort.

2. Measurement Accuracy of Beta and Beta Poloidal

It is important in understanding and comparing experimental beta results to take into account the type of measurement and overall accuracy of the beta and beta poloidal values.

a. In the evaluation of $<\beta>$ and β_p from plasma measurements it is necessary strictly to obtain values for $n_e(r)$, $n_z(r)$, $T_e(r)$, $T_i(r)$, $T_z(r)$, f_b $(\varepsilon_b, r, \Theta, \phi)$, B and $j_\phi(r)$

$$\text{since} \quad <\beta> \propto \int_0^a 2\pi r\, dr \left[n_e T_e + \left(n_e - \sum_z Z\, n_z(r) - n_b \right) T_i + \sum_z n_z T_z + \frac{2}{3} \overline{n_b \varepsilon_b} \right] \Big/ B^2$$

$$\text{and} \qquad \beta_p \propto <\beta> \frac{B^2}{B_p^2} \; .$$

In practice, if Z_{eff} is small in the bulk of the plasma, the impurity terms may be neglected; and if f_b (the non-Maxwellian part of the ion distribution) is not measured it may be obtained by computing the beam deposition and slowing down or the rf heating effects. In these circumstances it is informative to assess the accuracy of $<\beta>$ and β_p if all measurements or estimates are accurate to ± 10% and r and B are known exactly.

$\boxed{\text{Then } <\beta>, \sim \pm 14\% \text{ accurate}}$

If very great care is taken, then one might expect n_e (± 5%), T_e (± 5%), T_i (± 5%), $\overline{n_b \varepsilon_b}$ (± 10%), B (± 2%) where the accuracy of position is included in the other errors. Then, if $\langle n_e T_e \rangle \simeq \langle n_i T_i \rangle + \frac{2}{3} \langle n_b \varepsilon_b \rangle$, the error in $\boxed{\langle \beta \rangle \text{ will be } \sim \pm 9\%}$

For the calculation of β_p it will, in addition, be necessary to use magnetics measurements to obtain a value for $\overline{B_p^2}$.

b. What then is the accuracy of β_p obtained from magnetics measurements directly? In the analysis used on ISX-B, the vertical field is measured and related to β_p through an equation of the form

$$B_Z \simeq \frac{\mu_o I}{4\pi R} \left(\ln \frac{8R}{a} + \beta_p + \frac{\ell_i}{2} - 3/2 \right)$$

where strictly $\beta_p \; \alpha \left[\langle p \rangle + mV_\phi^2 + \frac{1}{2} mV_\perp^2 \right]$

where V is directed velocity of the plasma.

A more accurate version of this Shafranov equation was obtained by analyzing a large number of computed examples of free boundary equilibria in ISX-B

$$B_Z \simeq I_p \left[1.95 + 1.05 \left(\beta_p + \frac{\ell_i}{2} \right) - 0.05 \left(\beta_p + \frac{\ell_i}{2} \right)^2 \right]$$

The errors, however, can be estimated from the original formula

$$\beta_p = \frac{4\pi R \; B_z}{\mu_o I} + 3/2 - \frac{\ell_i}{2} - \ln \left(8 \; R/a \right)$$

Consider the case R = 0.93 m, a = 0.27 m, $I = 2 \times 10^5$ A, and $\ell_i = 1.6$; then,

$\beta_p = 1.14$ for $B_z = 0.08$ T

$\beta_p = 1.57$ $\quad B_z = 0.09$ T

$\beta_p = 2.03$ $\quad B_z = 0.10$ T

Now, ℓ_i is typically in the range $1.0 \rightarrow 2.0$ (a totally flat current profile has $\ell_i = 0.5$), and if a very careful comparison is made between magnetics and plasma measurements, $\ell_i/2$ accuracy of $\pm$ 0.05 might be possible. The vertical field is obtained from an equation involving essentially the difference in current between the outer and inner coils, and if calibrations of current shunts are say ± 2%, the accuracy of B_z might be $\leq \pm 5\%$. Note that errors will be mainly systematic though pick up on the current measuring circuits will lead to some additional random errors. In principle the plasma current I could be measured to ± 2%, and the major radius R to say ± 0.5 cm on ISX-B. Nevertheless the total error is then given approximately by

$$\beta_p = 1 = 5 \pm 5\% - 4 \pm 0.05 \ \underset{-}{\sim} \ 1 \pm 0.30$$

$$\beta_p = 2 = 6 \pm 5\% - 4 \pm 0.06 \ \underset{-}{\sim} \ 2 \pm 0.35$$

If we now allow that much of the systematic error in B_z can be eliminated by a comparison of numerous magnetics and plasma measurements then at best we might achieve $\boxed{\beta_p \pm 10\% \text{ accuracy}}$ (particularly at the higher β_p).

To calculate $\langle\beta\rangle$ from β_p we use the formula

$$\langle\beta\rangle = \beta_p \frac{\overline{B_p^2}}{B^2} \propto \beta_p \frac{I^2}{B^2} \ a^2$$

Consider the change if instead of $R = 0.93$ m, $a = 0.27$ m the correct values were $R = 0.935$ m, $a = 0.265$ m with fixed $B = 1.2T$, $I = 2 \times 10^5$ A, $\ell_i = 1.6, B_z = 0.1T$,

then $\beta_p = 2.03$ is the same but $<\beta>$ is increased by 4%.

If $R = 0.925$ m, $a = 0.265$ m.

Then $\beta_p = 2.00$ and $<\beta>$ is increased by 2%.

Conclusions. It is very difficult to obtain ± 10% accuracy for measurements of $<\beta>$ or β_p. After a number of years of studies on ISX-B it is probable that averaged over a set of results such an accuracy may be assigned in studying trends and in setting, globally, beta achievements. On any particular result however, taken alone, an accuracy of ± 20% is more realistic.

Among the measurements which have been refined or reinterpreted over the life of ISX-B are Thomson scattering (calibration), charge exchange T_i (neutral density profile estimate and the effect of rotation), neutron measurements of T_i (effects of isotopic fractions of deuterium in a hydrogen beam fired into a deuterium plasma), magnetics (more accurate analysis of data), and neutral beam input (power to plasma and species mix).

In comparison of data between experiments very great care must be taken so as not to read into differences more than could come under the category of experimental error.

3. The Relation of Scale Length to Experimental Capabilities

In Table 1 a list was given of factors which possibly might contribute to the observed beta effects. Given in the table was a list of characteristic scale lengths. The reason that this list was given is that, in practice, it is the scale length rather than the frequency which impacts experimental capability.

There are three directions of importance — toroidal, azimuthal and radial — and three scale lengths,

$$L \sim a,\ L \sim \rho_{it} \rightarrow \lambda_D \text{ and } a > L >> \rho_{it} \rightarrow \lambda_D$$

Toroidal	Measurements in the toroidal direction are possible but likely to be expensive or suffer from access problems.
Radial/Azimuthal	Measurements at one toroidal position, over the r,θ, plane, are in principle possible but in many experiments access does not exist.
$L \sim a$	Measurements where $L \sim a$ are numerous, and include low m,n fluctuations, and radial dependences (resolution $\lesssim 0.1$ a typically).
$L \sim \rho_{it} \rightarrow \lambda_D$	Measurements where $L \sim \rho_{it}, \rightarrow \lambda_D <<$ cm for example, are possible by coherent scattering, but measurement over all, or a sizeable portion of the r,θ range is difficult (expensive) and quantitative interpretation of results is very difficult.
$a > L >> \rho_{it} \rightarrow \lambda_D$	Measurements in this intermediate range are hell, since the measurement parameter is of order the resolution length. For coherent scattering if D is the size of the probing beam then Fourier analysis of the scattering volume encompasses wavelengths up to $\frac{D}{2\pi}$, e.g. D = 1 cm, $L_{max} \sim 0.15$ cm.

For wavelengths $\sim L_{max} \rightarrow \frac{a}{5}$, for example, coherent scattering is not possible, and in either active or passive probing we must look for correlations between two signals. It is difficult, in practice, to obtain a resolution of even 1%

of the plasma radius (0.3 cm in ISX-B), so to obtain correlations from say line integrating passive signals is a real challenge. Active probing with say multiple sources of radiation might be used, but if in fact the beta problems lie in this range, considerable diagnostic development will be required to unravel the problems.

Conclusions. The experimental army marches on its access and diagnostics, and a lesson for future experiments, not simply in the tokamak area, is that we must not underestimate their importance.

4. Levels of Scientific Understanding

Consider the three following levels of scientific understanding, with examples in the beta area.

Level I Qualitatively an effect is observed — Beta does not rise linearly with injected neutral beam power.

Level II Accurate measurements are made and compared with theoretical models — Both electron and ion confinement are observed to degrade as β_p increases. Qualitatively low m,n resistive tearing modes do not appear to explain all of this behavior.

Level III Accurate measurements are made, and quantitatively all the results agree with a theoretical model over a significant range of all significant parameters — Beta degradation is not a result of an undesirable equilibrium.

By reference back to Sections 2 and 3 it can be seen that our ability to tackle problems in each phase depends upon our measurement accuracy and extent, which in turn depend upon access, phenomena scale length, and diagnostic capability, upon our theoretical ability and upon our ability to coordinate the experiment and theory. To some extent our theoretical conquests will depend also on the experimental data since the problems are surely non-linear, the equations to be solved numerous with an appalling number of terms. and simplications may justifiably be effected based upon experimental observation. Thus solution of the problems requires theory and experiment to advance hand in hand But in all cases it takes considerable time to accomplish a significant objective let alone to cope with a change in direction. It took approximately three years to set up and compare the low m and n number resistive code calculations, and the multiple PIN array, and x-ray tomography results on ISX-B, even though at the start numerous MHD codes existed and PIN arrays and some tomography had been undertaken.

Conclusions. While solution of the beta "problems" may come quickly in the sense that beta goes up to a qualitatively new level - say 5% - in for example, a bigger machine or with a different heating system, it is possible that understanding of transport at high beta may take as many years of dedicated slog as was needed to reach where the tokamak program is now. This is particularly possible if the beta problems lie in fluctuations of the intermediate scale length range where level 3 understanding might require the construction of a special maximum access device and the invention/development of new diagnostics and new theoretical analyses. A view of the time scales to unravel transport and beta issues to different levels is given in Table 2.

Table 2

Suggested time scales to reach a given level of undertanding of various beta-related phenomena.

Issue	Scale Length	Completed	Near Term FY 82-83	Long Term → ∿ FY 85
Equilibrium	$L \sim a$	Level 3	—	—
Low m,n MHD	$L \sim a$	Level 2		(Level 3)
Medium-High m,n modes	$\sim \text{cm} < L < a$	—	Level 1	Level 2
Current Profile	$L \ll a$	(Level 1)	Level 1 → 2	Level 2
Kinetic	$\text{cm} > L \sim \rho_{it} \rightarrow \lambda_D$	—	Level 1	Level 2
Rotation / Heating Effects	$L \sim a$ / $L \lesssim \text{cm}$	Level 1	Level 2	Level 3
Miscellaneous	Finite ρ_i	—	Level 1	Level 2
	$n_n(\phi)$	—	Level 1 → 2	Level 2

Appendix A

J. Sheffield and A. J. Wootton

Taylor and Connor in their Nuclear Fusion paper (17, 5, 1977) make the point that if the basic equations decribing plasma behavior are invariant under a certain group of transformations, then any scaling law derived from these equations must be invariant under the same group of transformations. This analysis may be used to limit, or even determine the parameter dependences of scaling laws. Caution must be exercised in placing weight on the result of applying their constraints, since for example, the neglect of radiation can result in overconstraining the dependences, and the assumption of a simple power law dependence on the parameters may not be valid. Nevertheless, their approach gives insight into the possible physics behind phenomena and with this goal it is used here.

Since the formula $\beta_p I^{1/2} \alpha\ P_b^{1/3}$ is approximate, and, in fact, in the central region a formula $\beta_p I\ \alpha\ \mathrm{fn}\ (P_b)$ is a better fit, we will start by using

$$\beta_p^x\ I^y\ \alpha\ P_b^w \quad (1)$$

now
$$\tau\ \alpha\ \frac{nT}{P_b} \quad (2)$$

consequently
$$\tau\ \alpha\ (nT)^{1 - \frac{x}{w}}\ I^{\frac{2x}{w} - y/w} \quad (3)$$

1. Collisional Vlasov High-β Model

Connor and Taylor give the following power law for energy confinement (with $\beta \rightarrow \frac{I}{a}$),

$$\tau\ \ \alpha\ \ \frac{a}{I}\ \left(na^2\right)^p\ \left(Ta^{1/2}\right)^q\ \left(Ia^{1/4}\right)^r \quad (4)$$

Equating the exponents of n, T and I in (3) and (4) leads to

$$\left.\begin{array}{l} n \rightarrow 1 - \frac{x}{w} = p, \quad T \rightarrow 1 - \frac{x}{w} = q \\ \\ I \rightarrow \frac{2x}{w} - \frac{y}{w} = -1 + r \end{array}\right\} \qquad (5)$$

substituting from (5) into (4) yields

$$\tau \;\alpha\; \frac{a^{15/4 - \frac{2x}{w} - \frac{y}{4w}}\, I^{\frac{2x}{w} - y/w}}{(nT)^{x/w - 1}} \qquad (6)$$

or

$$\tau \;\alpha\; \frac{a^{7/4 - \frac{y}{4w}}\, I^{2 - y/w}}{\beta_p^{\,x/w - 1}} \qquad (7)$$

Equation (6) automatically satisfies the constraints of Table 1 of the reference for the collisional high β model.

(a) For the usually quoted ISX-B case

$x = 1$, $y = 1/2$ and $w = 1/3$ which leads to

$$\boxed{\tau \;\alpha\; \frac{a^{11/8}\, I^{1/2}}{\beta_p^2}} \qquad (8)$$

There is some uncertainty in the exponents and it is informative to look at other possibilities.

(b) $x = 1,\ y = 1/2,\ w = 1/4$

yields $\boxed{\tau \ \alpha \ \dfrac{a^{5/4}}{\beta_p^3}}$ (9)

(c) $x = 1,\ y = 2/3,\ w = 1/3$

yields $\boxed{\tau \ \alpha \ \dfrac{a^{5/4}}{\beta_p^2}}$ (10)

2. <u>Collisionless High-β Vlasov Model</u>

With the above scalings the following dependences are found

(a) $x = 1,\ y = 1/2,\ w = 1/3 \rightarrow \tau \ \alpha \ \dfrac{aI^{1/2}}{\beta_p^2}$ (11)

(b) $x = 1,\ y = 1/2,\ w = 1/4 \rightarrow \tau \ \alpha \ \dfrac{a}{\beta_p^3}$ (12)

(c) $x = 1,\ y = 2/3,\ w = 1/3 \rightarrow \tau \ \alpha \ \dfrac{a}{\beta_p^2}$ (13)

3. <u>Resistive Fluid Model</u>

$$\tau \ \alpha \ \frac{n^{1/2}\ a^2}{I} \left(\frac{na^{3/2}}{I^2}\right)^p \left(Ta^{1/2}\right)^q \qquad (14)$$

Equating (3) and (13) leads to

$p = 1/2 - \frac{x}{w}\ \ q = 1 - \frac{x}{w},\ \ w = y/2$. It seems that the restriction $w = y/2$ stems from the inclusion of the ohmic heating term, therefore this form should be used at low additional heating power levels.

These values yield $\boxed{\tau \,\alpha\, \frac{a^{5/4}}{\beta_p^{x/w\ -1}}}$ (15)

with a restriction to the cases such as,

(b) $x = 1,\ y = 1/2,\ w = 1/4$ $\boxed{\tau \,\alpha\, \frac{a^{5/4}}{\beta_p^{3}}}$ (16) = (9)

(c) $x = 1,\ y = 2/3,\ w = 1/3$ $\boxed{\tau \,\alpha\, \frac{a^{5/4}}{\beta_p^{2}}}$ (17) = (10)

These formulae satisfy the Resistive MHD constraints of Table 1 of the reference.

Conclusions

The implications for future devices, are, not suprisingly, dependent upon the fitted power dependence.

The degradation of confinement with increasing β_p is always present and no assumption has been made as to whether β_p or P_b is a dependent variable. The dependence on size for all of these models is reduced from a^2 - scaling, though the most general use of the collisional Vlasov High-β model (equation 8) comes pretty close to a^2 - scaling. Clearly the new larger experiments, particularly TFTR in the relatively near future, should play an important role in tying down this dependence.

Author: ______________________

Report: ______________________

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	(Eqs.)
	Refs.
	Figs.
	Tables
	Abbreviations
	Hyphenations
	Miscellaneous