PPPL-2025

UC20-B,D,G

205
9-9-83 JS ①

PPPL-2025
Dr. 1747-1

I-11083

PPPL--2025

DE83 017716

HELICAL-AXIS STELLARATORS WITH NONINTERLOCKING PLANAR COILS

By

A. Reiman and A. Boozer

AUGUST 1983

PLASMA
PHYSICS
LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

MASTER

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

HELICAL-AXIS STELLARATORS WITH NONINTERLOCKING PLANAR COILS

A. Reiman and A. Boozer

Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08544

ABSTRACT

The properties of helical-axis stellarator fields generated by unlinked, planar coils are described. It is shown that such fields can have a magnetic well and large rotational transform, implying large equilibrium and stability beta limits.

Peg

There has been a renewed interest in stellarators in recent years.[1] This has been due largely to the possibility of achieving high β, steady-state, disruption free operation, and to promising experimental[2,3] and theoretical[4,7] results concerning transport. A drawback of stellarators has been the complicated coil structure required. In this paper we show that stellarator fields having favorable properties can be generated by a very simple coil system consisting only of unlinked planar coils.

We require that our stellarator fields have a magnetic well and large rotational transform. Numerical stability results for stellarators indicate that the presence of a well provides good stability properties.[8,9] For equilibria having low shear (which we will be considering), low β Mercier stability is determined by the sign of V''. The stability β limit for ballooning modes scales like ι^2/A, where A is the aspect ratio and ι is the rotational transform.

The equilibrium beta limit is determined by the Shafranov shift. Three-dimensional equilibrium codes show that for a field with a magnetic well the toroidal shift at small β relative to the minor radius scales like $\beta A/\iota^2$, where A is the aspect ratio and ι is the rotational transform.[10-12] A large rotational transform gives a large equilibrium beta limit.

At large rotational transform, the toroidal corrections to the magnetic well and rotational transform are relatively small. This means that we cannot rely on toroidal effects to give us a well. It also means that it is reasonable to neglect toroidal effects in a first analysis. We do so in this paper, specializing to straight helical systems. By ignoring those parameters on which the transform and well depend only very weakly, we obtain a simple general characterization of those configurations which give a magnetic well and large transform. By assuming helical symmetry, we guarantee that the drift orbits are identical to those in an axisymmetric torus.[13] The question of how best to bend our system into a (nonaxisymmetric) torus, so as to minimize the transport, can be dealt with separately. We do not consider that question here.

Figure 1 shows a typical coil configuration. In assuming helical symmetry, we are ignoring the ripple due to the discreteness of the coils.

The centers of our coils trace out a helix, which we call the coil axis. The location of this helical coil axis is given by

$$\underset{\sim}{X}(z) = z\hat{z} + \hat{x}r_o\cos kz + \hat{y}r_o\sin kz \quad ,$$

where r_o is the coil displacement and $2\pi/k$ is the periodicity length, and where we have used a Cartesian coordinate system (x,y,z). Our method of solving for the field is an analytical expansion about this coil axis. The expansion parameter is

$$\frac{a_c}{r_h} = \frac{k^2 r_o a_c}{1+k^2 r_o^2} ,$$

where a_c is the coil radius and r_h is the helical radius of curvature. We will see that the smaller this quantity is, the easier it is to get a magnetic well, so that our expansion is reasonable in the regime of interest. Note that if this expansion parameter approaches one, coils perpendicular to the axis collide.

We work in a helical coordinate system. Let $\underset{\sim}{n}$ be the unit normal vector to the coil axis, and let $\underset{\sim}{b}$ be the binormal vector. We express a general vector as

$$\underset{\sim}{r} = \underset{\sim}{X}(s) + \underset{\sim}{n}(s)\ \rho\cos\theta + \underset{\sim}{b}(s)\rho\sin\theta\ ,$$

where $\underset{\sim}{X}(s)$ defines the coil axis, s measures the distance along the coil axis, and ρ is the distance from this axis. In helical symmetry, $\underset{\sim}{B}$ is a function only of θ and ρ. The shape of the helical solenoid is taken to be

$$\rho = a_c + \delta_2\cos2\theta + \delta_3\cos3\theta. \tag{1}$$

Of course, a $\cos\theta$ term is precluded from appearing in this expression because it would shift the coil centers. The displacement of the coils has already been specified by $\underset{\sim}{X}(z)$. In Eq. (1), δ_2 determines the ellipticity of the coils, while δ_3 gives the triangularity. As δ_3 increases from 0, the coils first become increasingly D shaped, and then somewhat bean shaped. The addition of higher harmonics of $\rho(\theta)$ does not affect either ι or V" at the magnetic axis. The variation of ι and V" across the coil radius relative to their values at the magnetic axis is of the order of a_c/r_h. We take $|\delta_2/a_c| \ll 1$ and $|\delta_3/a_c| \ll 1$ in the following.

Using the helical symmetry and $\nabla \cdot \underset{\sim}{B} = 0$, we can express $\underset{\sim}{B}$ in terms of a helical flux function $\psi(\rho,\theta)$,

$$\frac{\partial\psi}{\partial\rho} = \kappa\rho B_s - h_s B_\theta\ ,\quad \frac{\partial\psi}{\partial\theta} = \rho h_s B_\rho\ , \tag{2a}$$

where

$$\kappa = k/(1+ k^2 r_o^{\ 2}) \tag{2b}$$

is the helical torsion, and

$$h_s = 1 - \frac{\rho}{r_h}\cos\theta$$

is the scale factor for the s coordinate. For the vacuum field, $\nabla \times \underset{\sim}{B} = 0$ gives the condition

$$h_s B_s + \kappa\rho B_\theta = I = \text{constant}, \tag{3}$$

and the equation for ψ,

$$\rho \frac{\partial}{\partial\rho}\left[\rho \frac{h_s \partial\psi/\partial\rho - \kappa\rho I}{h_s^2 + \kappa^2\rho^2}\right] + \frac{\partial}{\partial\theta}\left(\frac{1}{h_s}\frac{\partial\psi}{\partial\theta}\right) = 0 \quad . \tag{4}$$

We take our coils to be perpendicular to the coil axis (see Fig. 1). This is equivalent to the requirement that the s component of the coil current vanish. It gives a jump condition for ψ at the coils

$$h_s \left[\frac{\partial\psi}{\partial\rho}\right] = \kappa\rho[I] \quad , \tag{5}$$

where [ ] denotes the jump in value across the coil location given by Eq. (1). In addition to Eq. (5), we take ψ to be continuous across the coils, and $|\underset{\sim}{B}| \to 0$ for ρ large.

To lowest order in a_c/r_h, Eq. (4) for ψ reduces to that for a straight solenoid. Neglecting δ_2/a_c and δ_3/a_c in this order, we get $\psi_0 = (1/2)\kappa I\rho^2$ inside the coils, and $\psi_0 = (1/2)\kappa I a_c^2$ outside the coils. To this solution we add the lowest order corrections due to finite a_c/r_h, δ_2/a_c, and δ_3/a_c. We get

$$\psi \approx \frac{1}{2}\,\kappa I\Big[\rho^2 + \frac{1}{2}\,\frac{k^2 r_o}{1+k^2 r_o^2}\Big(\frac{3}{2}\,\rho^3 - a_c^2\rho\Big)\cos\theta - \frac{\delta_2}{a_c}\,\rho^2\cos 2\theta - \frac{\delta_3}{a_c^2}\,\rho^3\cos 3\theta\Big] \quad . \tag{6}$$

With our solution for ψ, we obtain explicit expressions for ι and V''.[14] To lowest order in a_c/r_h, both ι and V'' are constant across the plasma. Note that this low shear limit is precisely the limit of interest for helical axis stellarators because it allows low order rational surfaces to be avoided. The rotational transform is

$$\iota/m \approx 1 - \Big[\frac{1-4(\delta_2/a_c)^2}{1+k^2 r_o^2}\Big]^{1/2} \quad , \tag{7}$$

where m is the number of periods. Estimating the change in V' across the plasma by $\Delta V' \approx V''(1/\iota_h)\Delta\psi$, we determine the well depth,

$$\frac{\Delta V'}{V'} \approx \frac{k^2 a_c^2}{(1+k^2 r_o^2)}\Big[k^2 r_o^2 + \Big(\frac{\delta_2}{a_c}\Big)^2 + \frac{5}{4}\,\frac{\delta_2}{a_c}\,k^2 r_o^2 - 3\,\frac{\delta_2}{a_c}\,\frac{\delta_3}{a_c}\,\frac{r_o}{a_c}\,(1+k^2 r_o^2)\Big] \quad . \tag{8}$$

Requiring that the flux surfaces (and coils) not be too elongated, we can see from Eqs. (6) and (7) that most of the transform must come from the finite value of kr_o for ι relatively large. For ι/m of order 1/2 we must have kr_o of order 1.

To determine the conditions for the presence of a magnetic well, we solve $V'' = 0$ for δ_3/a_c,

$$\frac{\delta_3}{a_c} = \frac{(\delta_2/a_c)^2 + k^2 r_o^2\,(1+ 5/4\ \delta_2/a_c)}{3(\delta_2/a_c)\ (1+k^2 r_o^2)}\,\frac{a_c}{r_o} \quad . \tag{9}$$

Equation (9) gives the coil triangularity required to produce a magnetic well for a given coil ellipticity and a given helical pitch of the coil axis. A nonzero ellipticity and triangularity are both required in order to get a magnetic well from a reasonable coil deformation. Note that the required value of δ_3/a_c is proportional to a_c/r_o, which can be small. Figure 2 is a *contour plot of the required* value of $(r_o/a_c)(|\delta_3|/a_c)$, as given by Eq. (9).

Taking $kr_o \sim 1$, we scan across Fig. 2 to see that we can in fact obtain a magnetic well and large transform with reasonable coil deformation. As an example, Fig. 3 shows a coil cross section and corresponding flux surfaces for $kr_o = 1.0$, $\delta_2/a_c = -0.3$, and $\delta_3/a_c = -0.1$. This configuration has $\iota/m \approx 0.43$ and a marginal magnetic well. Larger values of a_c/r_o require an increasingly bean-shaped coil (i.e., increasing triangularity) to maintain the well. Conversely, a D shape is sufficient if the helical aspect ratio is somewhat larger. Note from Eq. (8) that while it becomes easier to obtain a well as our expansion parameter gets smaller, the wells obtained also get shallower. Within the domain of validity of our expansion, the well depths are limited to the order of 1%.

We conclude that a configuration of noninterlocking plasma coils can generate stellarator fields with large transform and a magnetic well, for very reasonably shaped coils. Our configuration is not optimal. It could be improved somewhat, for example, by including higher harmonics in the coil shape, and by allowing the coils to be slightly nonplanar. The configuration does demonstrate that very attractive *stellarator fields can be generated by* extremely simple, modular coil systems.

ACKNOWLEDGMENTS

We are grateful to J. Johnson for an illuminating discussion of the Mercier criterion in helical axis stellarators.

This work supported by U.S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

REFERENCES

[1] For a review of recent work see J.L. Johnson, Nucl. Technol./Fusion 2, 340 (1982); in Proceedings of the Fifth Topical Meeting on the Technology of Fusion Energy, Knoxville, Tennessee, 1983, Nucl. Technol. Fusion (to be published).

[2] T.V. Bartlett, G. Cannici, G. Cattanei, D. Dorst, G. Grieger, H.H. Hacker, J. How, H. Jackel, R. Jaenicke, P. Javel, J. Junker, M. Kick, R. Lathe, J. Meyer, C. Mahn, S. Marlier, G. Müller, W. Ohlendorf, F. Rau, H. Renner, H. Ringler, J. Sapper, P. Smeulders, M. Tutter, B. Ulrich, A. Weller, E. Würsching, H. Wobig, M. Zippe, D. Cooper, K. Freudenberger, G. Lister, W. Ott, and E. Speth, in Proceedings of the Eighth International Conference/ Plasma Physics and Controlled Nuclear Fusion Research, Brussels, Belgium, (International Atomic Energy Agency, Vienna, 1981) Vol. 1, p. 185.

[3] A. Iiyoshi, M. Sato, O. Motojima, T. Mutoh, S. Sudo, M. Iima, S. Kinoshita, H. Kaneko, H. Zushi, S. Besshou, K. Kondo, T. Mizuuchi, S. Morimoto, and K. Uo, Phys. Rev. Lett. 48, 745 (1982).

[4] R. E. Potok, P. A. Politzer, and L. M. Lidsky, Phys. Rev. Lett. 45, 1328 (1980).

[5] A. H. Boozer and G. Kuo-Petravic, Phys. Fluids 24, 851 (1981).

[6] H. E. Mynick, Phys. Fluids 25, 325 (1982).

[7] H. E. Mynick, T. K. Chu, and A. H. Boozer, Phys. Rev. Lett. 48, 322 (1982).

[8] D. Monticello, R. Dewar, and J. Manickam, Phys. Fluids (to be published).

[9] R. Gruber, W. Kerner, P. Merkel, J. Nührenberg, W. Schneider, and F. Troyon, Phys. Fluids (in press).

[10] R. Chodura and A. Schlüter, J. Comput. Phys. 41, 68 (1981).

[11] F. Bauer, O. Betancourt, and P. Garabedian, in Proceedings of the Third Stellarator Study Workshop, Courant Institute Mathematical Sciences, New York University, N.Y., 1982 (unpublished).

[12] F. Hernegger, Z. Naturforsch. 37a, 879 (1982).

[13] A. Boozer, Phys. Fluids 2, 496 (1983).

[14] N.M. Zueva and L.S. Solov'ev, A. Energ. 20, 396 (1966) [Sov. J. At. Energy, 20(5), 444 (1966)].

FIGURE CAPTIONS

FIG. 1 Three-dimensional helically symmetric array of noninterlocking planar coils. The coil parameters are the same as those in Fig. 3.

FIG. 2 Coil triangularity, $-(r_0/a_c)\delta_3/a_c$, required to produce a magnetic well for a given coil ellipticity and a given helical pitch. Here δ_2 and δ_3 are both taken negative.

FIG. 3 Outline of a noninterlocked planar coil and the magnetic surfaces a set of such coils would produce. The illustrated case has a marginal magnetic well and a transform per period of 0.43.

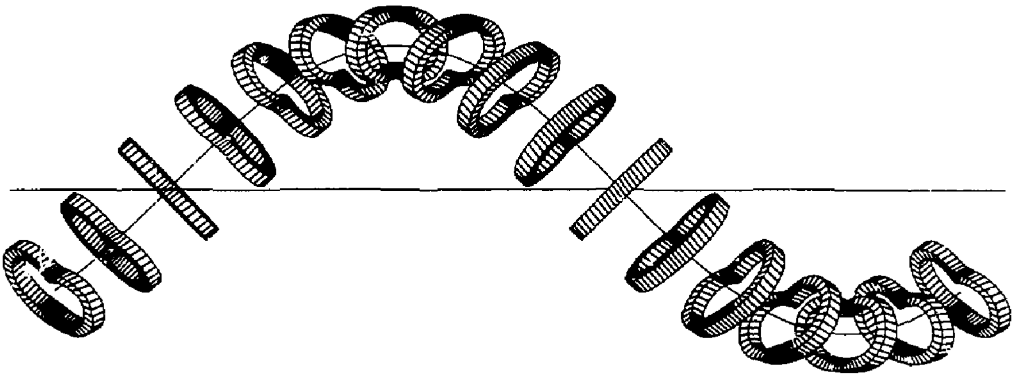

Fig. 1

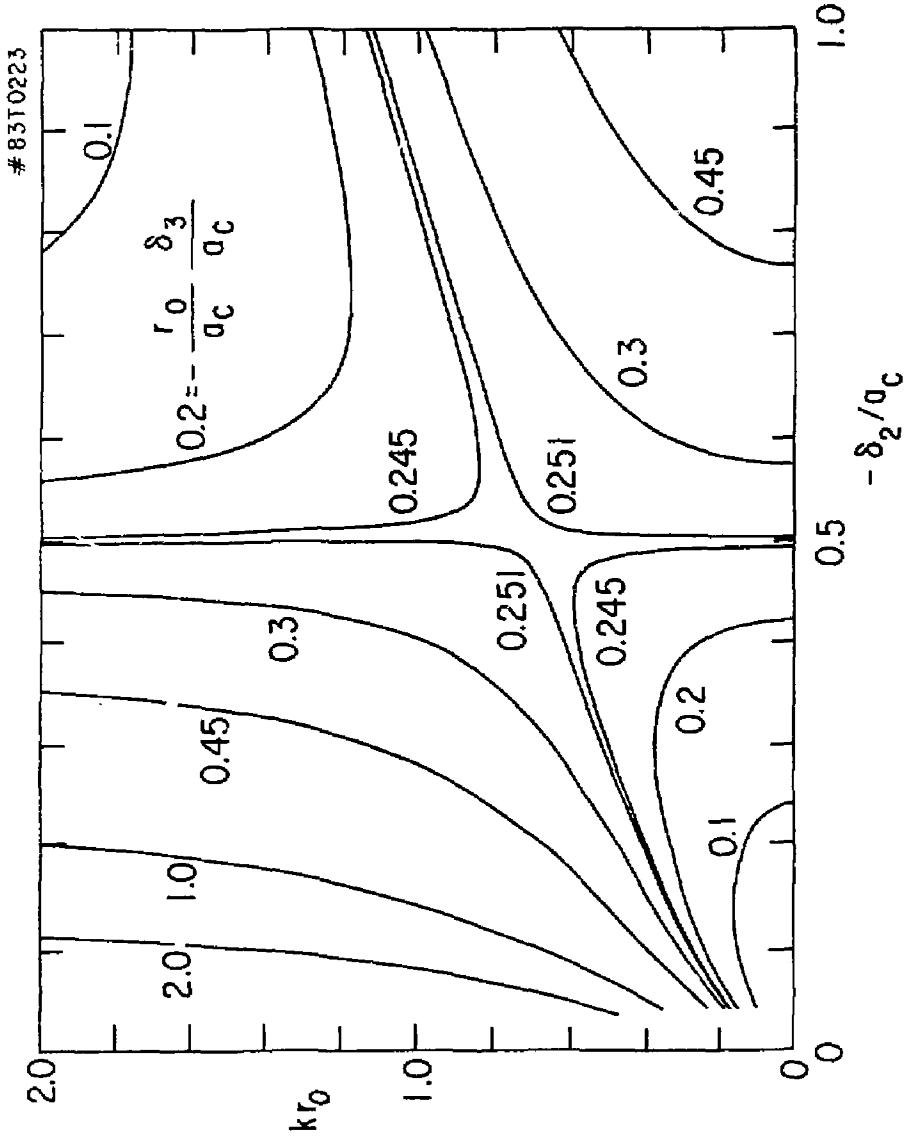
#83T0223
kr_0
2.0
1.0
0
0
0.5
1.0
$-\delta_2/a_c$
$0.2 = -\frac{r_0}{a_c}\frac{\delta_3}{a_c}$
0.1
0.245
0.251
0.3
0.45
0.251
0.245
0.3
0.2
0.45
0.1
1.0
2.0

Fig. 2

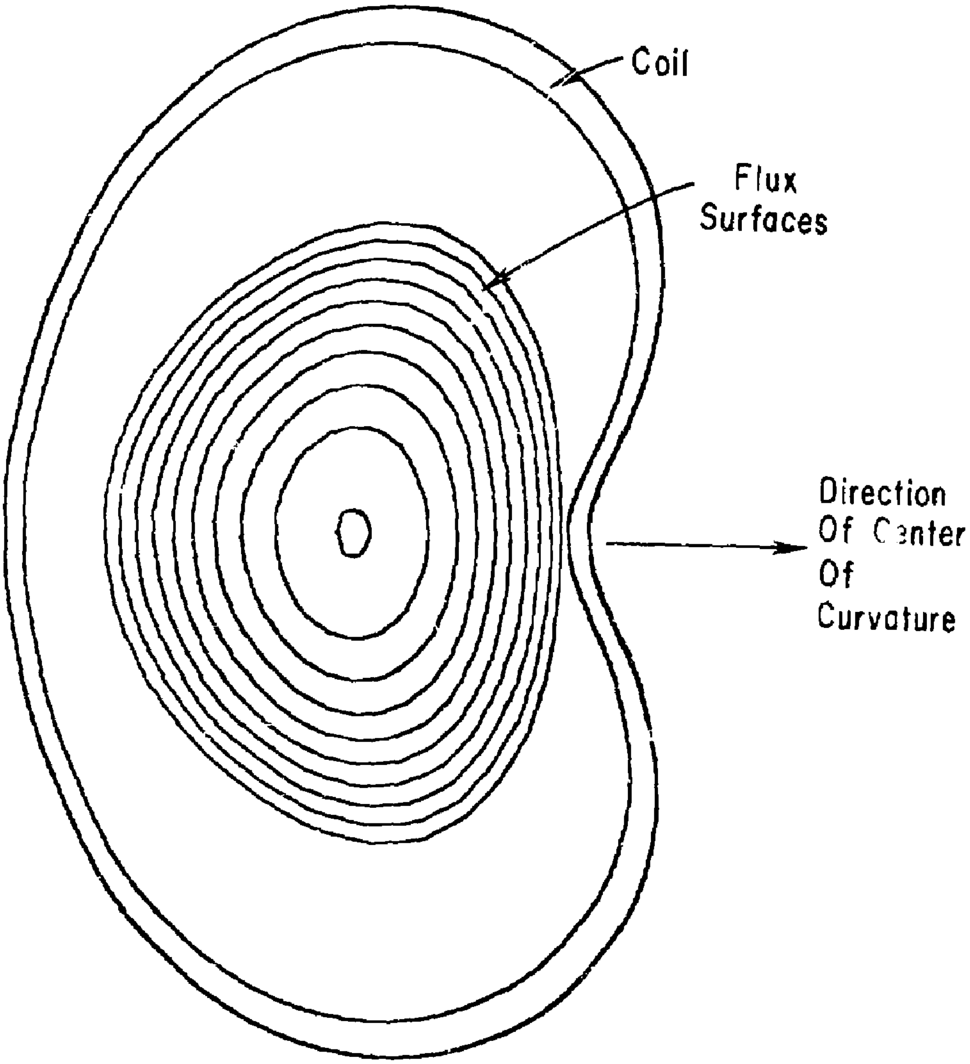

Fig. 3

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

Plasma Res Lab, Austra Nat'l Univ, AUSTRALIA
Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
Prof. I.R. Jones, Flinders Univ., AUSTRALIA
Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
Prof. F. Cap, Inst Theo Phys, AUSTRIA
Prof. Frank Verheest, Inst theoretische, BELGIUM
Dr. D. Palumbo, Dg XII Fusion Prog, BELGIUM
Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
Dr. P.H. Sakanaka, Univ Estadual, BRAZIL
Dr. C.R. James, Univ of Alberta, CANADA
Prof. J. Teichmann, Univ of Montreal, CANADA
Dr. H.M. Skarsoerd, Univ of Saskatchewan, CANADA
Prof. S.R. Sreenivasan, University of Calgary, CANADA
Prof. Tudor W. Johnston, INRS-Energie, CANADA
Dr. Hannes Barnard, Univ British Columbia, CANADA
Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
Zhengwu Li, SW Inst Physics, CHINA
Library, Tsing Hua University, CHINA
Librarian, Institute of Physics, CHINA
Inst Plasma Phys, SW Inst Physics, CHINA
Dr. Peter Lukac, Komenskeho Univ, CZECHOSLOVAKIA
The Librarian, Culham Laboratory, ENGLAND
Prof. Schatzman, Observatoire de Nice, FRANCE
J. Radet, CEN-BP6, FRANCE
AM Dupas Library, AM Dupas Library, FRANCE
Dr. Tom Mual, Academy Bibliographic, HONG KONG
Preprint Library, Cent Res Inst Phys, HUNGARY
Dr. A.K. Sundaram, Physical Research Lab, INDIA
Dr. S.K. Trehan, Panjab University, INDIA
Dr. Indra, Mohan Lal Das, Banaras Hindu Univ, INDIA
Dr. L.K. Chavda, South Gujarat Univ, INDIA
Dr. R.K. Chhajlani, Var Ruchi Marg, INDIA
P. Kaw, Physical Research Lab, INDIA
Dr. Phillip Rosenau, Israel Inst Tech, ISRAEL
Prof. S. Cuperman, Tel Aviv University, ISRAEL
Prof. G. Rostagni, Univ Di Padova, ITALY
Librarian, Int'l Ctr Theo Phys, ITALY
Miss Clelia De Palo, Assoc EURATOM-CNEN, ITALY
Biblioteca, del CNR EURATOM, ITALY
Dr. H. Yamato, Toshiba Res & Dev, JAPAN
Prof. M. Yoshikawa, JAERI, Tokai Res Est, JAPAN
Prof. T. Uchida, University of Tokyo, JAPAN
Research Info Center, Nagoya University, JAPAN
Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
Prof. Sigeru Mori, JAERI, JAPAN
Library, Kyoto University, JAPAN
Prof. Ichiro Kawakami, Nihon Univ, JAPAN
Prof. Satoshi Itoh, Kyushu University, JAPAN
Tech Info Division, Korea Atomic Energy, KOREA
Dr. R. England, Civdad Universitaria, MEXICO
Bibliotheek, [illegible] Inst Voor Plasma, NETHERLANDS
Prof. B.S. Liley, University of Waikato, NEW ZEALAND

Dr. Suresh C. Sharma, Univ of Calabar, NIGERIA
Prof. J.A.C. Cabral, Inst Superior Tech, PORTUGAL
Dr. Octavian Petrus, ALI CUZA University, ROMANIA
Dr. R. Jones, Nat'l Univ Singapore, SINGAPORE
Prof. M.A. Hellberg, University of Natal, SO AFRICA
Dr. Johan de Villiers, Atomic Energy Bd, SO AFRICA
Dr. J.A. Tagle, JEN, SPAIN
Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
Dr. Lennart Stenflo, University of UMEA, SWEDEN
Library, Royal Inst Tech, SWEDEN
Dr. Erik T. Karlson, Uppsala Universitet, SWEDEN
Centre de Recherchesen, Ecole Polytech Fed, SWITZERLAND
Dr. W.L. Weise, Nat'l Bur Stand, USA
Dr. W.M. Stacey, Georg Inst Tech, USA
Dr. S.T. Wu, Univ Alabama, USA
Prof. Norman L. Oleson, Univ S Florida, USA
Dr. Benjamin Ma, Iowa State Univ, USA
Prof. Magne Kristiansen, Texas Tech Univ, USA
Dr. Raymond Askew, Auburn Univ, USA
Dr. V.T. Tolok, Kharkov Phys Tech Ins, USSR
Dr. D.D. Ryutov, Siberian Acad Sci, USSR
Dr. M.S. Rabinovich, Lebedev Physical Inst, USSR
Dr. G.A. Eliseev, Kurchatov Institute, USSR
Dr. V.A. Glukhikh, Inst Electro-Physical, USSR
Prof. T.J. Boyd, Univ College N Wales, WALES
Dr. K. Schindler, Ruhr Universitat, W. GERMANY
Nuclear Res Estab, Julich Ltd, W. GERMANY
Librarian, Max-Planck Institut, W. GERMANY
Dr. H.J. Kaeppler, University Stuttgart, W. GERMANY
Bibliothek, Inst Plasmaforschung, W. GERMANY