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AUGUST 1983

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PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

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ABSTRACT

The response of a finite conductivity plasma to resonant magnetic perturbations is studied. The equations, which are derived for the time development of magnetic islands, help one interpret the singular currents which occur under the assumption of perfect plasma conductivity. The relation to the Rutherford regime of resistive instabilities is given.

I. INTRODUCTION

Many plasma confinement concepts are based on the existence of magnetic surfaces. These surfaces are strictly defined only when the trajectory of a field line remains in a topologically toroidal surface forever. In vacuum magnetic fields, a small magnetic perturbation, which resonates spatially with the field lines, can destroy the simple nested topology of the magnetic surfaces by producing magnetic islands.[1] The width of these islands is proportional to the square root of the amplitude of a resonant perturbation.[2] At larger, but nonetheless small, amplitude the magnetic islands on different magnetic surfaces can interact causing the trajectory of an individual field line to cover a finite volume rather than just a surface.[3] The loss of surfaces is quite deleterious to plasma confinement.

The situation is more complicated in the presence of a finite conductivity plasma. When a resonant perturbation is applied, a current is induced in the vicinity of the resonant surface which modifies the resonant perturbation, not only in the vicinity of the rational surface, but also in a large fraction of the plasma volume. The currents in the vicinity of the rational surface are destroyed by the finite resistivity of the plasma, so that over a period of time the island broadens to the width that it would have had in vacuum.

In a slowly evolving plasma, one expects pressure equilibrium

$$\vec{\nabla} P = \frac{1}{c} \vec{j} \times \vec{B} \tag{1}$$

with P the pressure, c the speed of light, $\vec{j}$ the current, and $\vec{B}$ the magnetic field. Suppose the toroidal magnetic flux $2\pi\psi$, contained in each constant pressure surface is given, or equivalently the function $P(\psi)$ is given. The current perpendicular to the magnetic field is then

$$\vec{j}_{\perp} = \frac{c}{B^2} (\vec{B} \times \vec{\nabla}\psi) \frac{dP}{d\psi} \quad . \tag{2}$$

Except for highly symmetric situations, the perpendicular current is not divergence-free. To make the current divergence-free, the parallel current must satisfy

$$\vec{B} \cdot \vec{\nabla} \frac{j_{\parallel}}{B} = \vec{\nabla} \cdot \vec{j}_{\perp} \quad . \tag{3}$$

The current $j_{\parallel}$ is the sum of two physically different parts, the Pfirsch-Schlüter current and the force-free current. The Pfirsch-Schlüter current j_{PS} is the special solution to the inhomogeneous differential equation for $j_{\parallel}$. It is made unique by the condition that there be no net Pfirsch-Schlüter current inside a pressure surface. One can express the Pfirsch-Schlüter current in terms of the variation in the magnetic field strength on a constant pressure surface.[4] In the absence of perfect symmetry, this expression is singular on the rational surfaces. On these special surfaces, the field lines close exactly on themselves. However, the subtleties of the Pfirsch-Schlüter current are not the subject of this paper. Instead, we will be concerned with the subtleties of the force-free current, j_{ff}, which is the general homogeneous solution of the differential equation for $j_{\parallel}$. Therefore, we will assume the plasma is locally force-free around the rational surface of interest.

The differential equation satisfied by the force-free current

$$\vec{B} \cdot \vec{\nabla} (j_{ff}/B) = 0 \tag{4}$$

is more subtle than it may first appear. On irrational magnetic surfaces, the equation has the general solution

$$\frac{j_{ff}}{B} = \frac{c}{4\pi}\,\mu(\psi) \quad . \tag{5}$$

On rational magnetic surfaces, a more general solution for the force-free current is possible since it can be different on each field line of the rational surface.

To give this expression, the rotational transform $\iota(\psi)$ must be defined. If one follows a field line many times around the torus so that the toroidal angle ϕ (see Fig. 1) advances by a large multiple 2π, then the poloidal angle θ will advance by the same multiple of $2\pi\iota$. If n and m are integers, then a rational surface can be defined by $\iota = n/m$. The general solution for the force-free current is

$$\frac{4\pi}{c}\frac{j_{ff}}{B} = \mu(\psi) + \sum_{n,m} K_{nm}|\vec{\nabla}\iota|\,\delta(\iota - \frac{n}{m})\,\exp[i(n\phi - m\theta)] \tag{6}$$

with δ the Dirac delta function and θ and ϕ magnetic poloidal and toroidal angles. By magnetic angles we mean θ and ϕ are defined so that the change in $\Delta\theta$ as a field line moves in toroidal angle by $\Delta\phi$ is $\Delta\theta = \iota\Delta\phi$ for all $\Delta\phi$. Ordinary, nonmagnetic angles only satisfy the weaker relation

$$\lim_{\Delta\phi\to\infty} \frac{\Delta\theta}{\Delta\phi} = \iota \quad . \tag{7}$$

The delta function part of the force-free current is a surface current on each rational surface. Let a rational surface be given by $\iota = n_o/m_o$ with n_o

and m_o mutually prime. The surface current $K_s\vec{B}$ on that rational surface is

$$K_s = \sum_j K_j \exp[ij(n_o\phi - m_o\theta)] \tag{8}$$

with $K_j = K_{n,m}$ for $n = jn_o$ and $m = jm_o$. If $\hat{n}$ is the outward normal to the surface, then the magnetic field obeys the well-known relations for a surface current

$$[\hat{n} \times \vec{B}] = K_s\vec{B} \tag{9}$$

$$[\hat{n} \cdot \vec{B}] = 0 \tag{10}$$

with [...] bein, the jump across the surface.

Although the existence of a surface current on each rational surface may seem unphysical, it arises naturally as the response of the plasma to a symmetry changing perturbation, that is, a perturbation which tends to form a magnetic island. In a finite conductivity plasma, the magnetic islands have finite radial extent. However, a surface current is an accurate approximation to the plasma response as long as the island is thin compared to its length.

There are two different types of resonant perturbations. In the first type, the external driving perturbation may reach a finite amplitude even if the plasma is so highly conductive that an island cannot open. Of course, the magnetic field due to the surface current is equal and opposite to that of the external perturbation over a volume comparable to the regime over which the external perturbation changes. In this case, the island width broadens as $(\eta_\parallel t)^{1/3}$ with t time and $\eta_\parallel$ the parallel resistivity. The island width at any given time during the broadening is proportional to the cube root of the perturbing external magnetic field. Examples of this type of perturbation are an external resonant winding, which is suddenly turned on, or a sudden increase in the plasma pressure. One can show that the Pfirsch-Schlüter currents due to a finite pressure gradient, even well away from a rational surface, cause resonant perturbations except in cases of high symmetry. In this paper we assume the perturbation is applied sufficiently slowly that the plasma can be assumed to be in equilibrium force balance. The opposite assumption is generally made in classical magnetic reconnection theory.[5,6]

A second type of resonant external perturbation depends upon a finite island width for its existence. That is, the magnitude of the driving external perturbation is closely tied to the island width. This is the situation with so-called resistive instabilities in the Rutherford regime. In this case, one obtains the well-known result that the island grows linearly with $(\eta_{\parallel} t)$ and that the growth rate is proportional to Δ', the strength of the instability.

II. STRUCTURE OF VACUUM ISLANDS

When a resonant magnetic perturbation is applied to a vacuum magnetic field, the topology of the field changes by the creation of a magnetic island. The assumption that the magnetic island is thin, compared to its length, allows one to solve analytically for its structure. In particular, one can evaluate the poloidal flux ψ_p outside a surface that contains a given amount of toroidal flux ψ_t as a function of the island half-width Δ. Strictly speaking, the fluxes are $2\pi\psi_p$ and $2\pi\psi_t$, and ψ_p and ψ_t are flux functions. The change in ψ_p with Δ is closely related to the inductive electric field. In a finite conductivity plasma, the inductive electric field would lead to a parallel current. An important feature of the function $\psi_p(\psi_t)$ is that it is independent of Δ except in a region within a few island widths about the resonant surface. Consequently, the induced current in a finite conductivity plasma has a surface current structure. A surface current has the property of changing the component of the magnetic field orthogonal to the surface only on a distance scale comparable to the length of the surface current. Consequently, even in the presence of a finite conductivity plasma, one can assume the radial field which dominates the structure of a magnetic island is essentially constant across the width of the island. In other words, the structure of an island is essentially the same in a finite conductivity plasma and in a vacuum field provided the effect of the plasma current on the magnitude of the resonant radial perturbation is taken into account.

A magnetic field with perfect surfaces can be written[7-9]

$$\vec{B} = \vec{\nabla}\bar{\psi} \times \vec{\nabla}\bar{\theta} + \vec{\nabla}\phi \times \vec{\nabla}\bar{\psi}_p(\bar{\psi}) \tag{11}$$

with $\bar{\theta}$ a poloidal and ϕ a toroidal angle (see Fig. 1). A perturbation can resonate with a surface on which the transform $\bar{\iota} = d\bar{\psi}_p/d\bar{\psi}$ equals a rational number $\bar{\iota} = N/M$. By a simple transformation, one can always make both the resonant transform and the radial coordinate ψ zero on the resonant surface $\bar{\psi} = \bar{\psi}_r$. Let

$$\psi_p = \bar{\psi}_p - (N/M)\bar{\psi} \tag{12}$$

$$\theta = M\bar{\theta} - N\phi \tag{13}$$

$$\psi = (\bar{\psi} - \bar{\psi}_r)/M \tag{14}$$

then one finds

$$\vec{B} = \vec{\nabla}\psi \times \vec{\nabla}\theta + \vec{\nabla}\phi \times \vec{\nabla}\psi_p(\psi) \tag{15}$$

with the transform $\iota = M\bar{\iota} - N$. Near the rational surface, one can then write $\iota(\psi) = \iota'(o)\psi$.

In the ψ,θ,ϕ coordinate system the equations for a field line in the field $\vec{B}$ plus a *small perturbing* field $\vec{b}$ are

$$\frac{d\psi}{d\phi} = \frac{(\vec{B}+\vec{b})\cdot\vec{\nabla}\psi}{(\vec{B}+\vec{b})\cdot\vec{\nabla}\phi} \tag{16}$$

$$\frac{d\theta}{d\phi} = \frac{(\vec{B}+\vec{b})\cdot\vec{\nabla}\theta}{(\vec{B}+\vec{b})\cdot\vec{\nabla}\phi} \quad . \tag{17}$$

If $|\vec{b}| << |\vec{B}|$, then the field line equations are approximated by

$$\frac{d\psi}{d\phi} = \frac{\vec{b}\cdot\vec{\nabla}\psi}{\vec{B}\cdot\vec{\nabla}\phi} \quad , \quad \frac{d\theta}{d\phi} = \iota(\psi) \quad . \tag{18}$$

Using $\iota = \iota'\psi$, one has near the rational surface

$$\frac{d\psi^2}{d\theta} \simeq \frac{2}{\iota'}\,\frac{\vec{b}\cdot\vec{\nabla}\psi}{\vec{B}\cdot\vec{\nabla}\phi} \quad . \tag{19}$$

If $\vec{b}\cdot\vec{\nabla}\psi \,/\, \vec{B}\cdot\vec{\nabla}\phi$ is Fourier decomposed, only the resonant terms, $\sin(\theta)$, $\sin(2\theta)$, etc., affect the island structure. To make the structure problem solvable in terms of tabulated mathematical functions only the $\sin(\theta)$ term is retained. Additional terms would not make a qualitative difference in the solution, but they would complicate the algebraic expressions especially when there are maxima or minima of distinct magnitudes. Consequently, we let

$$\frac{2}{\tau'}\frac{\vec{b}\cdot\vec{\nabla}\psi}{B\cdot\vec{\nabla}\phi} = \frac{\Delta^2}{2}\sin(\theta) \tag{20}$$

with Δ a positive constant. The solution to Eq. (19) for ψ is

$$\psi = \pm\,\Delta[s^2 - \sin^2(\theta/2)]^{1/2} \tag{21}$$

with s^2 a constant of integration. Actually s labels the magnetic surfaces of the perturbed system (see Fig. 2). To make the sign of the square root definite, we take the sign to be that of s. For $|s| < 1$, the two values of s with opposite signs but the same magnitude represent two parts of the same magnetic surface. Despite the apparent artificiality, the double counting turns out to simplify the statement of the solution. For $|s| > 1$, the two signs of s represent topologically unchanged surfaces inside, that is $\bar{\psi} < \bar{\psi}_r$, and outside, $\bar{\psi} > \bar{\psi}_r$, the island chain.

To find the poloidal flux as a function of the toroidal flux, we need to evaluate the poloidal and toroidal fluxes as functions of s after the creation of the island. To within an irrelevant additive constant, the poloidal flux in the absence of the island is just

$$\psi_p(\psi_t) = \frac{1}{2}\tau'\ \psi_t^2 \tag{22}$$

since $\psi_t = \psi$ in the unperturbed system. The poloidal flux in the perturbed system Ψ_p can be simply evaluated by doing an area integral in the $\theta = 0$ plane

$$\Psi_p = \frac{1}{2\pi} \int \vec{B} \cdot \vec{\nabla}\theta \frac{d\psi d\phi}{(\vec{\nabla}\psi \times \vec{\nabla}\theta) \cdot \vec{\nabla}\phi}$$

$$= \frac{1}{2\pi} \int \iota \, d\psi d\phi \tag{23}$$

$$= \frac{1}{2} \iota' \psi^2(s, \theta = 0) + \text{const.}$$

where we have included an integration constant. Using $\psi(s, \theta = 0)$ from Eq. (21) one finds

$$\Psi_p = \frac{1}{2} \iota' \Delta^2 (s^2 + c) \tag{24}$$

with c a constant of integration which we will determine by making $\Psi_p = \psi_p$ for $s^2 >> 1$. The toroidal flux in the perturbed system is also evaluated from area integrals

$$\psi_t(s) = \frac{1}{2\pi} \int \vec{B} \cdot \vec{\nabla} \phi \frac{d\psi d\theta}{(\vec{\nabla}\psi \times \vec{\nabla}\theta) \cdot \vec{\nabla}\phi}$$

$$= \frac{1}{2\pi} \int \psi (s, \theta) \, d\theta \tag{25}$$

with $\psi(s,\theta)$ given by Eq. (21). Of course one must be careful about the range of the θ integration inside the island since $|\sin\theta/2|$ must be less than $|s|$. Performing the necessary elliptic integrals, one finds that inside the magnetic island, $|s| < 1$,

$$\psi_t = \frac{2}{\pi} \Delta \frac{s}{|s|} [E(|s|) - (1-s^2) K(|s|)] \tag{26}$$

while outside the island, $|s| > 1$,

$$\psi_t = \frac{2}{\pi} \Delta \, s \, E(|\frac{1}{s}|) \quad . \tag{27}$$

Again one finds there is a subtlety inside the island. The total toroidal flux inside an island surface labeled by $|s|$ is $2\psi_t(|s|)$ with equal contributions to the toroidal flux being made inside the resonant surface, $-1 < s < 0$, and outside the resonant surface, $0 < s < 1$. The constant c in Eq. (24) can be evaluated by considering $s >> 1$. In this limit

$$\frac{2}{\pi} E(\frac{1}{s}) = 1 - \frac{1}{4} \frac{1}{s^2} + 0(\frac{1}{s^4}) \tag{28}$$

so

$$\psi_t^2 \, (s \rightarrow \infty) = \Delta^2 [s^2 - \frac{1}{2} + 0(\frac{1}{s^2})] \quad . \tag{29}$$

Comparing Eqs. (22) and (24), one finds Ψ_p and ψ_p are equal for $s >> 1$ if $c = -1/2$. Therefore

$$\Psi_p = + \Delta^2 (s^2 - 1/2) \quad . \tag{30}$$

The expression we have derived for the perturbed poloidal flux as a function of the toroidal flux, Eqs. (26), (27), and (30), can be compared with known expressions for the rotational transform inside and outside[10,11] a magnetic island. The transform inside the island, $|s| < 1$, is

$$\iota_i = \frac{1}{2}\frac{d\Psi_p}{d\psi_t}$$

$$= \frac{\iota' \Delta}{2}\,\frac{\pi}{2K(|s|)} \quad . \tag{31}$$

Outside the island, $|s| > 1$, the transform is

$$\iota_o = \frac{d\Psi_p}{d\psi_t}$$

$$= \iota' \;\Delta\;\frac{\pi s}{2K(1/|s|)} \quad . \tag{32}$$

For $s >> 1$, $\iota_o = \iota' \psi_t$ as one would expect.

The quantity of primary interest in the next section will be the change in the poloidal flux at a given value of toroidal flux as Δ changes, that is, $\partial\Psi_p/\partial\Delta$. This change in poloidal flux consists of two parts. The first part comes from the explicit Δ^2 dependence of Ψ_p in Eq. (30). The second part comes from a change in the value of s associated with a given ψ_t as Δ changes. Explicitly,

$$\left(\frac{\partial s}{\partial\Delta}\right)_{\psi_t} = -\frac{\psi_t}{\partial\psi_t/\partial s}\,\frac{1}{\Delta} \quad . \tag{33}$$

Combining the two parts one finds

$$\left(\frac{\partial\Psi_p}{\partial\Delta}\right)_{\psi_t} = \iota' \;V(s)\Delta \tag{34}$$

with

$$V(s) = \begin{cases} \left[1 - \frac{E(1/|s|)}{K(1/|s|)}\right] s^2 - \frac{1}{2}, & |s| > 1 \\ \frac{1}{2} - \frac{E(|s|)}{K(|s|)}, & |s| < 1 \end{cases} \quad . \tag{35}$$

The quantity V is plotted versus the toroidal flux in Fig. 3.

III. EFFECT OF PLASMA

In a finite conductivity plasma, a change in the functional dependence of the poloidal flux on the toroidal flux induces a current. The magnitude of the current is proportional to the rate at which the change is made and the sign of the current is such as to resist the change. One might suppose that the induced current would fundamentally change the structure of the island. Actually, the basic structure is unchanged provided the island is long compared to its width. The effect of the plasma current is to reduce $|\vec{b} \cdot \vec{\nabla}\psi|$ from its vacuum value.

In the magnetic island problem, the poloidal flux Ψ_p is a function of the toroidal flux ψ_t and the island half-width Δ. The time dependence of Ψ_p is determined by its dependence on $\Delta(t)$. The time rate of change in Ψ_p can be related to an electric field $\vec{E}$ by Faraday's Law expressed in magnetic coordinates[4]

$$\left(\frac{\partial \Psi_p}{\partial t}\right)_{\psi_t} = \frac{\partial}{\partial \psi_t}\left(\frac{c}{(2\pi)^2} \int \vec{E} \cdot \vec{B}\, d^3x\right) \tag{36}$$

with c the speed of light. A different derivation of this equation is given in Appendix A. The volume of integration is the interior of the ψ_t surface. For simplicity, in this paper we assume that the plasma is locally force-free around the rational surface of interest. The plasma current in a force-free plasma is

$$\vec{j} = \frac{c}{4\pi}\,\mu(\psi_t)\vec{B}\ . \tag{37}$$

The condition that μ depend on only the variable ψ_t comes from $\vec{j}$ being divergence-free. If we assume the conventional Ohm's law,

$$\vec{E} + \frac{\vec{v}}{c} \times \vec{B} = \eta_\perp \vec{j}_\perp + \eta_\parallel \vec{j}_\parallel\ , \tag{38}$$

then

$$\vec{E} \cdot B = \frac{\eta_\parallel c}{4\pi}\,\mu B^2 \quad . \tag{39}$$

To evaluate $\mu(\psi_t)$, we insert $\vec{E} \cdot \vec{B}$, Eq. (39), into Faraday's law, Eq. (36), and perform the volume integral.

To carry out the volume integral in Eq. (36) t'^ Jacobian of the magnetic coordinates is required. The most convenient expression is derived if we assume the *original* $\bar{\psi}$, $\bar{\theta}$, ϕ *magnetic coordinates* have the *covariant* representation for $\vec{B}$

$$\vec{B} = \bar{g}(\bar{\psi})\vec{\nabla}\phi + \bar{I}(\bar{\psi})\vec{\nabla}\theta \quad . \tag{40}$$

One can show such magnetic coordinates exist if the plasma is locally force free.[4,12] A physical interpretation of $\bar{g}$ and $\bar{I}$ in terms of currents is given in Fig. 1. Transforming to ψ, θ, ϕ coordinates using Eqs. (12 - 14),

$$\vec{B} = g(\psi)\vec{\nabla}\phi + \frac{\bar{I}}{M}\,\nabla\theta \tag{41}$$

with $g(\psi) = \bar{g} + N\bar{I}/M$. The dot product of the contravariant, Eq. (15), and the covariant, Eq. (41), representation of $\vec{B}$ implies

$$B^2 = g(\vec{\nabla}\psi \times \vec{\nabla}\theta) \cdot \vec{\nabla}\phi \tag{42}$$

near the rational surface. Consequently, the Jacobian of the magnetic coordinates is just

$$J = \frac{g}{B^2} \quad . \tag{43}$$

Equations (25, 36, 39, and 43) then imply

$$\mu(\psi_t, t) = \frac{4\pi}{\eta_\parallel c^2} \frac{1}{g} \left(\frac{\partial \Psi_p}{\partial t}\right)_{\psi_t} \quad . \tag{44}$$

In the last section we evaluated the change in the poloidal flux Ψ_p as the island width changes, Eq. (34). Using this equation the force-free current can be written as

$$\mu(s) = \frac{4\pi}{\eta_\parallel c^2} \frac{\iota'}{2g} V(s) \frac{d\Delta^2}{dt} \quad . \tag{45}$$

For large $|\psi_t/\Delta| >> 1$ one can easily show

$$V = \frac{1}{16} \left(\frac{\Delta}{\psi_t}\right)^2 ; \tag{46}$$

so the current is induced only in the vicinity of the magnetic island (see Fig. 3).

When the island is thin, it is a good approximation to represent the induced current as a surface current. The divergence-free nature of a magnetic field implies $\vec{b} \cdot \vec{\nabla}\psi$ is a slowly varying function of ψ, and it is only this component of the perturbation which determines the structure of the island. The equations for the magnetic field in the presence of a surface current were given in the introduction, Eqs. (9) and (10). The outward normal $\hat{n} = \vec{\nabla}\psi/|\vec{\nabla}\psi|$. The surface current $K_s\vec{B}$ is given by

$$K_s(\theta) = \int \mu \frac{d\psi}{|\vec{\nabla}\psi|} \quad . \tag{47}$$

In words, the function $\mu(s)$ can be considered a function of ψ and θ using Eq. (21). This function, divided by $|\vec{\nabla}\psi|$, is integrated over all ψ holding θ constant. Of course, $|\vec{\nabla}\psi|$ can be considered a constant. One then has

$$K_s = \frac{4\pi}{n_\parallel c^2} \frac{\iota'}{2q} \frac{\Delta}{|\vec{\nabla}\psi|} \frac{d\Delta^2}{dt} k(\theta) \tag{48}$$

with

$$k(\theta) = -\frac{1}{\Delta} \int V(s) d\psi \quad . \tag{49}$$

A numerical evaluation of $k(\theta)$ gives

$$k(\theta) = k_o[\cos\theta + 0.07 \cos 2\theta + \ldots] \tag{50}$$

with $k_o = 0.39$ and the higher harmonics, $\cos(3\theta)$ etc., having a less than 1% amplitude. The fact that the average of $k(\theta)$ over θ is zero is a statement that $\vec{A} \cdot \vec{B}$ integrated over the volume is unchanged by the perturbation.[13] The surface current K_s is simply related to Δ' of resistive instability theory.

This relation is evaluated in Appendix B.

Until this point we have made no assumptions about the exact geometric configuration of the unperturbed magnetic field. To complete the self-consistent calculation, we must make some simplyfying assumptions. First, we assume

$$K_s = -2K_o \cos\theta \quad . \tag{51}$$

By comparison with Eqs. (48 - 50), we have

$$K_o = \frac{k_o}{6} \frac{4\pi}{\eta_\| c^2} \frac{\iota'}{g|\vec{\nabla}\psi|} \frac{d\Delta^3}{dt} \; . \tag{52}$$

The most important features of K_o are exact. They are that K_o is inversely proportional to the resistivity $\eta_\|$ and proportional to the rate of change of Δ^3. This is equivalent to the strength of the surface current being proportional to the time rate of change of three halves power of the perturbation field.

Second, we assume simple cylindrical geometry while calculating the magnetic field due to the plasma current $\vec{b}_p$. Actually, we assume $|\vec{\nabla}\psi \cdot \vec{\nabla}\theta| << |\vec{\nabla}\psi||\vec{\nabla}\theta|$ and B almost constant on the rational surface. Letting $d\psi/dr = Br$, one finds that Eqs. (9),(10), and (51) imply

$$\vec{b}_p = \vec{\nabla}(f \sin\theta) \tag{53}$$

with

$$f = \begin{cases} K_o Br & r<r_o \\ -K_o Br_o/r & r>r_o \end{cases} \quad . \tag{54}$$

The resonant surface radius is r_o. One then finds

$$\vec{b}\cdot\vec{\nabla}\psi = K_o B|\vec{\nabla}\psi|\sin\theta + \vec{b}_o\cdot\vec{\nabla}\psi \tag{55}$$

with $\vec{b}_o$ the perturbation in the absence of a surface current. Using Eq. (20) in the obvious way to define $\Delta_o{}^2$, one finds using Eq. (52) for K_o and $\vec{B}\cdot\vec{\nabla}\phi = B^2/q$ that

$$\frac{d\Delta^3}{dt} = \left(\frac{3B}{2k_o}\,\frac{\eta_\parallel c^2}{4\pi}\right)(\Delta_o^2 - \Delta^2) \quad . \tag{56}$$

This is the fundamental equation for the growth of magnetic islands.

IV. ENERGETICS

It is clearly physically important to know the energy required to open a magnetic island in a finite conductivity plasma. The power required is just

$$P = \int\vec{J}\cdot\vec{E}d^3x \quad . \tag{57}$$

The electric field is given by

$$-\frac{1}{c}\frac{\partial\vec{b}}{\partial t} = \vec{\nabla}\times\vec{E} \tag{58}$$

and can be represented in the form

$$\vec{E} = U\vec{\nabla}\phi + E_\psi\vec{\nabla}\psi - \vec{\nabla}\Phi \quad . \tag{59}$$

One then has

$$\frac{\partial U}{\partial \theta} = \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\vec{b}\cdot\vec{\nabla}\psi}{\vec{B}\cdot\vec{\nabla}\phi}\right) \quad . \tag{60}$$

The current is approximated by a surface current

$$\vec{j} = \frac{c}{4\pi} K_s \vec{B} \; |\vec{\nabla}\psi| \, \delta(\psi) \tag{61}$$

with $\vec{B}\cdot\vec{\nabla}\psi = 0$. Therefore,

$$P = -\frac{c}{4\pi} \int_{\psi=0} K_s U \; |\vec{\nabla}\psi| \; d\theta d\phi \quad . \tag{62}$$

Using Eq. (48) for K_s and Eq. (20) for $\vec{b}\cdot\vec{\nabla}\psi/B\cdot\vec{\nabla}\phi$, one finds

$$\begin{aligned} P &= \left(\frac{4\pi}{\eta_\parallel c^2}\right) \frac{\iota'^2}{4g} \Delta^3 \left(\frac{d\Delta}{dt}\right)^2 \int k(\theta)\cos\theta \; d\theta \\ &= \left(\frac{4\pi}{\eta_\parallel c^2}\right) \frac{\pi k_o}{4g} \iota'^2 \Delta^3 \left(\frac{d\Delta}{dt}\right)^2 \end{aligned} \tag{63}$$

using Eq. (50) for $k(\theta)$. If we use Eq. (56) for the time rate of change of Δ, we obtain

$$P = \frac{\pi}{16}\frac{B}{g} \iota'^2 \left(\Delta_o^2 - \Delta^2\right) \frac{d\Delta^2}{dt} \quad . \tag{64}$$

If we let

$$\vec{b}\cdot\vec{\nabla}\psi = - b|\vec{\nabla}\psi| \; \sin(\theta) \tag{65}$$

then Eq. (20) and Eqs. (42) and (43) imply

$$P = \pi \frac{g|\vec{\nabla}\psi|^2}{B^3} (b_o - b) \frac{db}{dt} \quad . \tag{66}$$

The peculiar factor $g|\vec{\nabla}\psi|^2/B^3$ is essentially the volume of the rational surface. In simple cylindrical symmetry it is the volume divided by $2\pi^2|B_\phi/B|$. There is actually a coordinate transformation subtlety. One should integrate θ to $2\pi M$ in volume integrals and not just 2π. The actual power is therefore M times that given above.

V. DISCUSSION

The response of a finite conductivity plasma to a resonant perturbation has been studied in general geometry. To make the results more accessible, we will assume in this section that the unperturbed equilibrium is cylindrically symmetric. A magnetic perturbation can only resonate with a rational magnetic surface, that is, a surface on which the transform ι equals N/M with N and M integers. The transform in a cylinder is

$$\iota = \frac{RB_\theta}{rB_z} \tag{67}$$

with $2\pi R$ the length of the cylinder. The radial component of the perturbation, which is resonant on the surface defined by $\iota = N/M$, is assumed to be of the form

$$b_r = - b\sin(N\phi - M\theta) \ .$$

A magnetic island, Fig. 2, is then formed about the resonant rational surface. The radial half-width of the island δ is related to the perturbation by

$$\frac{b}{B_z} = M \frac{d\imath}{dr} \frac{\delta^2}{4R} \quad . \tag{68}$$

This formula implies even a small magnetic perturbation can give a significant magnetic island. For example a half-Gauss, $M = 2$, $N = 1$, perturbation will give an island with a half-width of 1% of the plasma radius using typical tokamak parameters.

If currents away from a rational surface produce a resonant radial magnetic perturbation b_o, then a plasma with resistivity $\eta_\parallel$ will have a surface current on that rational surface. This current tends to cancel the external perturbation. If δ_o is related to the field b_o by Eq. 68, then the island half-width δ is

$$\frac{d\delta^3}{dt} = \frac{3}{2k_o} \frac{\eta_\parallel c^2}{4\pi} \frac{M}{r_o} \left|\frac{B}{B_z}\right| (\delta_o{}^2 - \delta^2) \quad . \tag{69}$$

The constant $k_o \simeq 0.39$, c is the speed of light, r_o is the radius of the resonant rational surface, and $|B/B_z|$ is the ratio of the total to the axial magnetic field strength. As the resistivity goes to zero, an external perturbation cannot change the width of the magnetic islands. The convergence to the zero resistivity limit is slow. It depends on $(\eta_\parallel t)^{1/3}$ with t the observation time.

Let V_o be the volume enclosed by the resonant rational surface, the power, which is being dissipated by the surface current on the resonant rational surface, is

$$P = \frac{V_o}{2\pi M} \left|\frac{B_z}{B}\right| (b_o - b) \frac{db}{dt} \quad . \tag{70}$$

The energy dissipation required to open a magnetic island of given width can be zero if the island is opened slowly. However, it can be as large as

$$\frac{V_o}{2\pi M} \left|\frac{B_z}{B}\right| b^2 \tag{71}$$

if it is opened quickly.

An interesting application of these results is to calculate the rate an island broadens as a result of a tearing mode.[14] This problem was first studied by Rutherford.[15] There has been analytic and numerical confirmation of the characteristic features of his solution. In a tearing mode the plasma regions away from the resonant rational surface respond to a magnetic perturbation at the resonant surface without dissipation but with an energy change[16]

$$\delta W = - V_o \frac{r_o \Delta'}{M^2} \frac{b^2}{8\pi} \quad . \tag{72}$$

The only quantity which has not been previously defined is Δ'. The radial component of the radial perturbation cannot change across the resonant surface but its derivative can and

$$\Delta' = \frac{1}{b} \left[\frac{db}{dr}\right] \tag{73}$$

with [...] meaning the change across the resonant surface. In a typical unstable tearing mode, $r_o \Delta' \approx 10$. If one equates the energy released away from the resonant surface as b increases with the power dissipated in the island, Eq. (70), one finds

$$b_o - b = \left|\frac{B}{B_z}\right| \frac{r_o\Delta'}{2M} b \quad . \tag{74}$$

Substituting this expression into Eq. (69) and using $b \propto \delta^2$, one finds

$$\frac{d\delta}{dt} = \frac{1}{4k_o} \frac{\eta_\parallel c^2}{4\pi} \Delta' \tag{75}$$

with $1/(4\ k_o) \simeq 0.64$. This relation, which is derived in Appendix B by a different method, is essentially Rutherford's well-known result. The best previous calculation gave a rate which was about 20% faster. If the exact results of Sec. III are used, one has the interesting result that the $n = 1$, $m = 2$ tearing mode is coupled with a 7% amplitude to the $n = 2$, $m = 4$ mode.

Away from the magnetic island, magnetic perturbations which change topology do not modify the relation between the poloidal and the toroidal flux. Consequently, the current $J_\parallel/B$ is the same on a magnetic surface enclosing a given amount of toroidal flux. However, the shape of the magnetic surface is modified by the perturbation so there is a change in the current at a given point in space. If $\mu(\psi_t)$ is the force-free current with the perturbation and $\mu_o(\psi)$ the force-free current of the unperturbed field $\vec{B}$, then the perturbation field $\vec{b}$ obeys

$$\vec{\nabla} \times \vec{b} = [\mu(\psi_t) - \mu_o(\psi)]\vec{B} + \mu(\psi_t)\vec{b} \quad . \tag{76}$$

Away from the island μ and μ_o have the same functional form so that

$$\mu(\psi_t) - \mu_o(\psi) = (\psi_t - \psi)\mu_o' \quad . \tag{77}$$

One can easily show that away from the island

$$\psi_t - \psi \propto \frac{\vec{b}\cdot\vec{\nabla}\psi}{n - \iota m} \quad . \tag{78}$$

Equation (76) leads to the well-known ideal region equation for the tearing mode. If $\mu(\psi_t)$ is augmented by the induced current Eq. (45) near the island, then Eq. (76) applies globally. One can view the reduction of the induced current to a surface current K_s in Sec. III and the relation between K_s and Δ' which was derived in Appendix B, as a demonstration.

APPENDIX A Faraday's Law

A magnetic coordinate representation of Faraday's law is required. To evaluate this expression some magnetic coordinate relations are required. In a region in which the magnetic field has perfect surfaces, it can be written

$$\vec{B} = \vec{\nabla}\psi \times \vec{\nabla}\theta + \vec{\nabla}\phi \times \vec{\nabla}\psi_p(\psi,t) \tag{A1}$$

with ψ, θ, ϕ the magnetic coordinates and t time. Let $\vec{X}(\psi,\theta,\phi,t)$ be the transformation equations from the magnetic coordinates to a fixed set of Cartesian coordinates. The covariant basis vectors are

$$\vec{e}_\alpha \equiv \frac{\partial\vec{X}}{\partial\alpha} \tag{A2}$$

with α equal to ψ, θ, or ϕ. The Jacobian is

$$J = \vec{e}_\psi \cdot (\vec{e}_\theta \times \vec{e}_\phi) \quad . \tag{A3}$$

The theorems of partial differentiation imply

$$\vec{\nabla}\psi = J(\vec{e}_\theta \times \vec{e}_\phi) \quad . \tag{A4}$$

Relations for $\vec{\nabla}\theta$ and $\vec{\nabla}\phi$ are given by permutations of the ψ, θ, ϕ labels. Equation (A1) can be rewritten as

$$\vec{B} = \frac{1}{J}\left(\vec{e}_\phi + \frac{d\psi_p}{d\psi}\vec{e}_\theta\right) \quad . \tag{A5}$$

Since the Cartesian coordinates $\vec{x}$ are assumed fixed, $d\vec{x}/dt = 0$ and the velocity of the magnetic coordinates, $\vec{V}_B = \partial\vec{x}/\partial t$ is equal to

$$\vec{V}_B = -\sum_\alpha \left(\frac{\partial\alpha}{\partial t}\right)_{\vec{x}} \vec{e}_\alpha \, . \tag{A6}$$

Using Eqs. (A4) - (A6), one finds

$$\vec{V}_B \times \vec{B} = \left(\frac{\partial\psi}{\partial t}\right)_{\vec{x}} \left(\vec{\nabla}\theta - \frac{\partial\psi_p}{\partial\psi}\vec{\nabla}\phi\right) - \left(\frac{\partial\theta}{\partial t}\right)_{\vec{x}} \vec{\nabla}\psi + \left(\frac{\partial\phi}{\partial t}\right)_{\vec{x}} \vec{\nabla}\psi_p \quad . \tag{A7}$$

Faraday's law relates the time rate of change of the magnetic field to the curl of the electric field. From Eq. (A1),

$$\left(\frac{\partial B}{\partial t}\right)_{\vec{x}} = \vec{\nabla} \times \left[\left(\frac{\partial\psi}{\partial t}\right)_{\vec{x}} \vec{\nabla}\theta - \left(\frac{\partial\theta}{\partial t}\right)_{\vec{x}} \vec{\nabla}\psi - \left(\frac{\partial\psi_p}{\partial t}\right)_{\vec{x}} \vec{\nabla}\phi + \left(\frac{\partial\phi}{\partial t}\right)_{\vec{x}} \vec{\nabla}\psi_p\right] \quad . \tag{A8}$$

Using the relation

$$\left(\frac{\partial\psi_p}{\partial t}\right)_{\vec{x}} = \left(\frac{\partial\psi_p}{\partial t}\right)_{\psi} + \frac{\partial\psi_p}{\partial\psi}\left(\frac{\partial\psi}{\partial t}\right)_{\vec{x}} \tag{A9}$$

we find

$$\left(\frac{\partial\vec{B}}{\partial t}\right)_{\vec{x}} = \vec{\nabla} \times \left[\vec{V}_B \times \vec{B} - \left(\frac{\partial\psi_p}{\partial t}\right)_{\psi} \vec{\nabla}\phi\right] \quad . \tag{A10}$$

Faraday's Law,

$$\left(\frac{\partial\vec{B}}{\partial t}\right)_{\vec{x}} = -c\,\vec{\nabla} \times \vec{E} \tag{A11}$$

then implies

$$\vec{E} + \frac{\vec{v}_B}{c} \times \vec{B} = \frac{1}{c}\left(\frac{\partial\psi_p}{\partial t}\right)_\psi \vec{\nabla}\phi - \vec{\nabla}\Phi \quad . \tag{A12}$$

Finally, using the relation, $\vec{B}\cdot\vec{\nabla}\phi = 1/J$, one obtains

$$\left(\frac{\partial\psi_p}{\partial t}\right)_\psi = \frac{c}{(2\pi)^2}\frac{\partial}{\partial\psi}\left(\int_\psi \vec{E}\cdot\vec{B}\, d^3x\right) \tag{A13}$$

which is the desired expression.

APPENDIX B Delta Prime Evaluation

In this appendix, the relation between the logarithmic derivative of the radial component of the magnetic perturbation, Δ', and the surface current K_s will be derived. First, the perturbation is written as

$$\vec{b} = b^\psi\, \nabla\theta \times \vec{\nabla}\phi + b^\theta\vec{\nabla}\phi \times \vec{\nabla}\psi + b^\phi\vec{\nabla}\psi \times \vec{\nabla}\theta \tag{B1}$$

so that

$$b^\psi = \frac{\vec{b}\cdot\vec{\nabla}\psi}{\vec{B}\cdot\vec{\nabla}\phi} \quad . \tag{B2}$$

The equation for the surface current Eq. (9) plus the covariant representation for $\vec{B}$, Eq. (4) yields three relations with $I = \bar{I}/M$ and $n = \vec{\nabla}\psi/|\vec{\nabla}\psi|$.

$$\frac{1}{|\vec{\nabla}\psi|}\left([b^\phi]\vec{\nabla}\psi \cdot \vec{\nabla}\theta - [b^\theta]\ \vec{\nabla}\psi \cdot \vec{\nabla}\theta\right)\vec{\nabla}\psi = 0 \tag{B3}$$

$$\frac{1}{|\vec{\nabla}\psi|}\left([b^\psi]\vec{\nabla}\psi \cdot \vec{\nabla}\phi - [b^\psi]\,|\vec{\nabla}\psi|^2\right)\vec{\nabla}\theta = I\, K_s\, \vec{\nabla}\theta \tag{B4}$$

$$\frac{1}{|\nabla\psi|}\left([b^{\theta}]|\vec{\nabla}\psi|^{2} - [b^{\psi}]\vec{\nabla}\psi \cdot \vec{\nabla}\theta\right)\vec{\nabla}\phi = q\, K_{s}\, \vec{\nabla}\phi \quad . \tag{B5}$$

These relations plus $\vec{B} \cdot \vec{\nabla}\psi = 0$ imply

$$[b^{\psi}] = 0 \quad , \quad [b^{\theta}] = \frac{q\, K_{s}}{|\vec{\nabla}\psi|} \quad , \quad [b^{\phi}] = -\frac{I\, K_{s}}{|\vec{\nabla}\psi|} \quad . \tag{B6}$$

The null divergence of $\vec{B}$ is equivalent to

$$\frac{\partial}{\partial\psi} b^{\psi} + \frac{\partial}{\partial\theta} b^{\theta} + \frac{\partial}{\partial\phi} b^{\phi} = 0 \quad . \tag{B7}$$

$$\left[\frac{\partial}{\partial\psi} b^{\psi}\right] = -q \frac{\partial}{\partial\theta}\frac{K_{s}}{|\vec{\nabla}\psi|} + I \frac{\partial}{\partial\phi}\frac{K_{s}}{|\vec{\nabla}\psi|} \quad . \tag{B8}$$

If we ignore the θ and ϕ dependence of $|\vec{\nabla}\psi|$, use Eq. (48) for K_{s}, approximate $k(\theta)$ by $k_{o} \cos\theta$, and use Eq. (20) for Δ^{2}, then

$$\frac{1}{b^{\psi}}\left[\frac{\partial}{\partial\psi} b^{\psi}\right] = \frac{4\, k_{o}}{|\vec{\nabla}\psi|^{2}} \frac{4\pi}{\eta_{\parallel} c^{2}} \dot{\nabla} \quad . \tag{B9}$$

The radial width of the island in centimeters is $\delta = \Delta/|\vec{\nabla}\psi|$ and

$$\Delta' \equiv \frac{1}{b^{\psi}}\left[\frac{\partial}{\partial\psi} b^{\psi}\right]|\vec{\nabla}\psi| \tag{B10}$$

so that

$$\Delta' = 4k_{o} \frac{4\pi}{\eta_{\parallel} c^{2}} \dot{\delta} \quad . \tag{B11}$$

This is just the Rutherford rate of island growth derived by another method in Sec. V.

The neglect of the θ and ϕ dependence of $|\vec{\nabla}\psi|$ as well as complex form $k(\theta)$ are nontrivial approximations. Actually, one should take a more complex dependence of $b\cdot\vec{\nabla}\psi$ on θ so that when K_s is evaluated Eq. (B8) has the same angular dependence on the two sides. This determines the mode-mode coupling of the island region.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

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FIGURE CAPTIONS

FIG. 1. Magnetic Coordinates The magnetic coordinates are the toroidal flux $2\pi\psi$, a poloidal angle θ, and a toroidal angle ϕ. There are two area elements, a toroidal area element $d\vec{a}_t$ and a poloidal area element $d\vec{a}_p$. The functions $I(\psi)$ and $g(\psi)$ are obviously related to the total toroidal and poloidal current. The poloidal flux is $2\pi\ \psi_p(\psi)$.

FIG. 2. Magnetic Island A magnetic island is illustrated with surfaces labeled with the appropriated value of the parameter s. Note that different parts of the surfaces inside the island labeled by a positive and a negative value of s. The sign of s is determined by whether that part of the perturbed surface lies inside or outside of the original resonant magnetic surface.

FIG. 3. Induce Voltage The function $V(\psi_0)$ is proportional to the induced voltage when the island half-width Δ is changed. The induce force-free current $j_\parallel/B$ is also proportional to V. The spike with amplitude 0.5 is at the separatrix. Note the position of zero on the figure.

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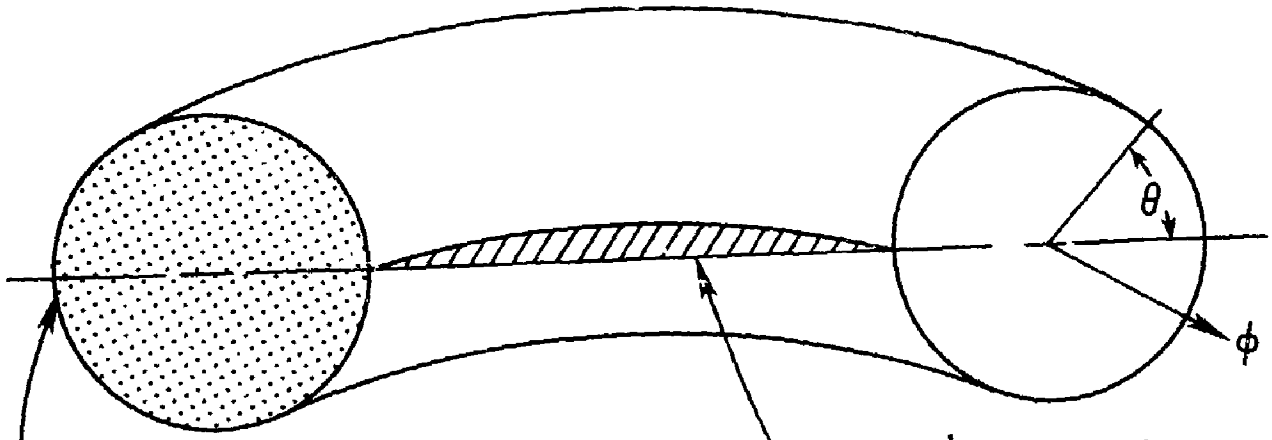

$$\psi = \frac{1}{2\pi} \int \vec{B} \cdot d\vec{a}_t$$

$$I = \frac{1}{2\pi} \int \vec{\nabla} \times \vec{B} \cdot d\vec{a}_t$$

$$\psi_p = \frac{1}{2\pi} \int \vec{B} \cdot d\vec{a}_p$$

$$g = \frac{1}{2\pi} \int \vec{\nabla} \times \vec{B} \cdot d\vec{a}_p$$

Fig. 1

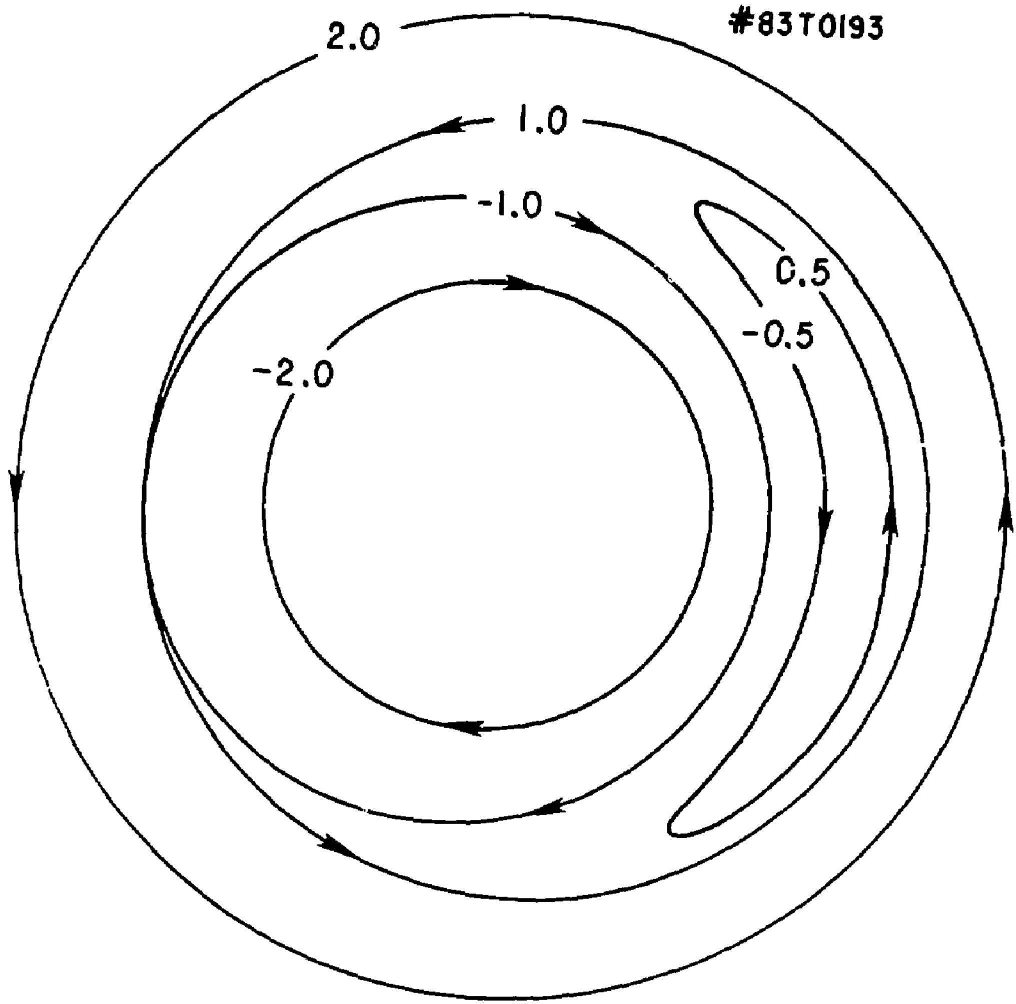

Fig. 2

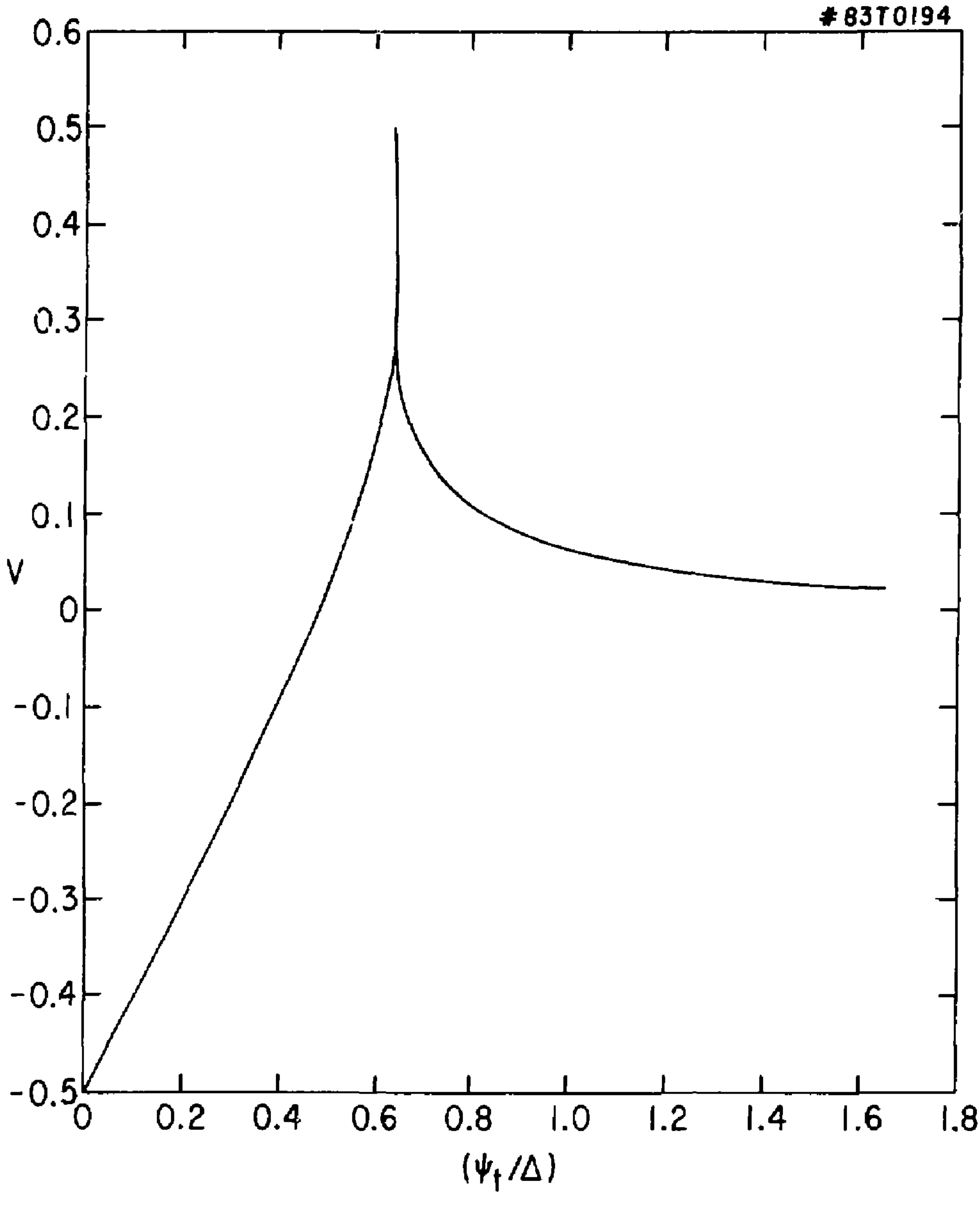
#83T0194
0.6
0.5
0.4
0.3
0.2
0.1
V
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
(ψf/Δ)

Fig. 3

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