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# ON THE PROTON DECAY *

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## ABSTRACT

The problem of the proton decay is considered taking into account that in actual experiments there is an interaction of the proton with its environment which could imply an increase of its theoretical lifetime. It is seen that, by application of the time-energy uncertainty relation, no prolongement of the lifetime is obtained in this case.

The recognition that in actual experiments aiming to the determination of the lifetime of an unstable quantum system, the system itself cannot be considered as evolving undisturbed, but it is repeatedly subjected to random measurements ascertaining whether it is decayed or not, has led to a reconsideration of the theoretical description of such processes. The proper formulation of this theoretical description as well as a detailed analysis of the various interesting consequences of this approach have been developed in a series of papers of our group (1-4). The most important advantages of the approach can be briefly summarized as follows.

1. The description takes explicitly into account the specific characteristics of the actual physical situation under which the experiments are performed.
2. The theoretical framework implies, in a rigorous way, that for almost all times the non-decay probability is given by an exponential, yielding therefore a unification of the classical and quantum description of the decay processes, a fact which is quite relevant from a conceptual point of view.
3. The approach shows that a decay process, when properly described, can be associated to a representation of the time translation semigroup. This fact also constitutes an important conceptual achievement, since it relates naturally to the irreversible character of actual decay processes and allows a group theoretical basic approach to the study of such processes.
4. The description makes two important facts very clear, i.e. a) that the basic feature in decay processes is given by the destruction of coherence between the wave function of the decay products and the one of the unstable system and b) that this loss of coherence is induced essentially by processes which can be identified with localization measurements on the decay products.

The two above-mentioned points are characteristic of every decay experiment, so that a unified picture of these processes is obtained.

5. The description has made very clear the problem of the unstable state formation by spatial localization, a fact which had not been properly appreciated before.

For the discussion which follows there is specifically a point which has particular relevance and deserves to be stressed. Within the theoretical scheme defined above to describe decay processes, the non-exponential character of the quantum non-decay probability $P(t)$ could lead to a dependence of the observed lifetime on the specific characteristics of the actual experiment.

Let us be more explicit on this feature. As is well known, one can prove that the time derivative of the quantum non-decay probability $P(t)$, defined as

$$P(t) = \left| \langle u | e^{-\frac{i}{\hbar}Ht} | u \rangle \right|^2 , \qquad (1)$$

$|u\rangle$ being any state vector for the system, vanishes for $t = 0$:

$$\left.\frac{dP}{dt}\right|_{t=0} = 0 .$$

It follows that for small times the theoretical quantum non-decay probability is larger than the classical one, which is of the exponential type.

By writing (1) for small times as a power series of t

$$P(t) = 1 - \Delta E^2 t^2/\hbar^2 + O(t^4), \qquad (2)$$

where

$$\Delta E^2 = \langle u|H^2|u\rangle - \langle u|H|u\rangle^2, \qquad (3)$$

and matching (2) with the power expansion of the exponential exp $(-\gamma t/\hbar)$, where $\gamma$ is the width of the resonance, we obtain an evaluation of the small-time region $0 \leq t \leq t_M$ where deviations occur from the exponential:

$$t_M = \frac{\gamma\hbar}{\Delta E^2 + \gamma^2/2} , \qquad (4)$$

which, if $\gamma \ll \Delta E$, reads:

$$t_M \simeq \frac{\gamma\hbar}{\Delta E^2} . \qquad (5)$$

We see from (4) or (5) that the larger the energy spread $\Delta E$ of the unstable state $|u\rangle$, the smaller is the time region where the deviations are relevant. We would like to point out that

i) The parabolic behaviour given by Eq. (2) is for sure established in the region $(0, t_M)$ due to the rigorous bound given by Fleming(5).

ii) General estimates(2) and model calculations(3) show that, when $\gamma/\Delta E$ is very small, as in our case, the non decay probability $P(t)$ is practically $\exp(-\gamma t/\hbar)$ for $t > t_M$. Therefore, in order that in a measurement the system be found undecayed with a larger probability than the one given by $\exp(-\gamma t/\hbar)$, one has to perform measurements at times $t < t_M$.

We can conclude that, if the measurement processes take place at times $t < t_M$, the measurements themselves tend to make the quantum system more stable, so that the observed lifetime would result larger than the one associated with the Breit-Wigner width of the resonance. For more and more frequent reductions, this argument leads to the conclusion (which in this context has first been pointed out in ref. (4)) that in the limit for continuous measurement processes the system will never be found decayed. This feature has been referred in the literature as Zeno's paradox.

Following this line of thought, two papers have recently appeared(6,7) investigating whether the above arguments can be relevant for the case of proton decay, a problem which is now calling the attention of the scientific community.

*If* this decay process takes place, the estimated lifetime of the system should be so long ($\tau \simeq 10^{34}$ years) that measurement processes on the system would certainly take place at a very small fraction of its lifetime. In view of the departure of the decay law from exponential at times very small compared with the lifetime of the decaying system, an uncritical use of Zeno's paradox could bring one to conclusions such as the suppression of the proton decay.

The two mentioned papers by Chiu, Misra, Sudarshan(6) and by Horwitz, Katznelson(7) take into account the processes of measurement from two different points of view*. As outlined in ref. (1) there are in general two types of measurements: those performed by externally operated apparatuses, and those induced by the interaction with the environment. Chiu et al.(6) take as a lower limit for measurement times the present shortest experimental resolution of time measuring devices, which they evaluate to be $10^{-12}$ sec. Horwitz et al.(7) instead stipulate that in actual experiments the proton decays in bulk matter, so that reductions are induced by the environment itself, i.e. by the nucleons of the nucleus in which the decay occurs; on the basis of observed relaxation times, they estimate the reduction time to be of the order of $10^{-23}$ sec.

Let us briefly summarize the results of refs. (6) and (7) by using the very simple procedure of ref. (3).

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* The problem of the consequences of the non-exponentiality of the decay law on the proton nonstability has first been raised by Khalfin(8), who however does not give any consideration to the repeated measurement processes which are by all means present in this case.

By simply cutting-off the Breit-Wigner form factor for $|E-E_R|>\mu$, $E_R$ being the resonance energy, we get*

$$\Delta E = \frac{\gamma}{2}\left(\frac{2\mu}{\gamma\,\mathrm{arctg}\left(\frac{2\mu}{\gamma}\right)}-1\right)^{1/2} \simeq \frac{1}{2}(\mu\gamma)^{1/2} . \qquad (6)$$

Combining (5) with (6) one gets $t_M \simeq 4\hbar/\mu$ . Since, in order to see an enhancement of the lifetime, one must have that the time interval between reductions $t_R$ satisfies $t_R < t_M$, one finally gets

$$\mu < \frac{4\hbar}{t_R} . \qquad (7)$$

From Eq. (7) the following estimates of $\mu$ are obtained:

$\mu < 2\cdot 10^{-3}$ eV when $t_R = 10^{-12}$ sec, as chosen by Chiu, Misra, Sudarshan; (8a)

$\mu < 264$ MeV when $t_R = 10^{-23}$ sec, as chosen by Horwitz, Katznelson . (8b)

Chiu et al. consider the value of $\mu$ given by (8a) completely unrealistic, from which they infer that one cannot reach the small time deviation region in actual experiments. Horwitz et al. obtain instead the reasonable value (8b) for the cutoff energy, from which they conclude that proton decay is tested at times where deviations are present, so that the experimental lifetime could be appreciably longer than the one predicted theoretically.

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* This type of dependence of $\Delta E$ on $\mu$ is obtained also for other cut-off functions(3).

We are inclined to agree with Horwitz and Katznelson on the fact that the environment is the entity providing the reductions. It is clear that the estimates given for the reduction time are based on what one thinks the reduction mechanism should be. Each choice can be subjected to criticisms and debate. For example, the choice, made in ref. (7), that the nucleons in the nucleus are responsible for the reduction (leading to $10^{-23}$ sec as the relevant reduction time) has been recently criticized in ref. (9). Another possibility would be to assume that the ionization of individual atoms is the entity responsible for the reduction (in which case one gets times of the order of $10^{-16} - 10^{-17}$ sec.). As we shall see now, whichever the reduction mechanism at work for the proton decay in bulk matter, one reaches the conclusion that no prolongation of its lifetime is obtained.

In fact, as pointed out in ref. (3) (see also the discussion of ref. (10)), for the natural interaction of the environment with the physical system one must apply to the process the Heisenberg time-energy uncertainty principle $\Delta E \Delta t \geq \hbar$, where $\Delta t$ is the time necessary to complete the measurement, and therefore is smaller or equal to the time interval $t_R$ between two successive reduction processes.

Combining the uncertainty principle with Eq. (5), one gets

$$t_M = \frac{\gamma (\Delta t)^2}{\hbar} \quad . \qquad (9)$$

Since $\Delta t$ is extremely small compared to the life-time $\tau_p = \frac{\hbar}{\gamma}$ of the proton, from (9) one gets

$$t_M = \left(\frac{\gamma}{\hbar} \Delta t\right) \Delta t \ll \left(\frac{\gamma}{\hbar} \tau_p\right) \Delta t = \Delta t \; . \qquad (10)$$

Eq. (10) shows that, when the time-energy uncertainty principle works, the time between reductions is much larger* than the small time deviation region $(0, t_M)$ and there is consequently no possibility of increasing the lifetime by reductions. Therefore the proton experimental lifetime equals the theoretical one. In ref. (3) it is argued that this holds in all practical cases**.

Our conclusion agrees therefore with that of Chiu et al., but in order to obtain it one must resort to the time-energy uncertainty principle since proton decays in bulk matter. Had it not been for this principle the lifetime of the proton could have turned out to be prolonged by the interactions with the environment, as in the hopes of Horwitz and Katznelson.

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* If one takes $\Delta t = 10^{-23}$ sec, one gets from (9) $t_M = 3.10^{-85}$ sec; if $\Delta t = 10^{-12}$ sec, one obtains $t_M = 3.10^{-63}$ sec.

**There remains the possibility of measuring the decay of the proton avoiding any interaction with the environment, and using an externally monitored apparatus which violates the $\Delta E \Delta t \geq \hbar$ uncertainty relation. However, firstly this is not the case for actual experiments where the proton decays in bulk matter, and secondly such apparatuses are in practice very difficult to project(11) when such short times, as those requested to test the small time deviation region, are involved.

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