

APR 27 1983

DESIGN GUIDANCE FOR ELASTIC FOLLOWUP

CONF-830607--25

DE83 016085

Frank V. Naugle  
Rockwell International  
Energy Systems Group  
Canoga Park, California

ABSTRACT

The basic mechanism of elastic followup is discussed in relation to piping design. It is shown how mechanistic insight gained from solutions for a two-bar problem can be used to identify dominant design parameters and to determine appropriate modifications where elastic followup is a potential problem. It is generally recognized that quantitative criteria are needed for elastic followup in the creep range where badly unbalanced lines can pose potential problems. Approaches for criteria development are discussed.

INTRODUCTION

In a 1955 paper, (1) E. L. Robinson addressed the problem of potential large creep strain concentrations in steam piping at regions of maximum stress and coined the phrase, "elastic followup." Since that time, the ASME Boiler and Pressure Vessel Code has repeatedly cautioned the piping designer about potential strain concentration due to elastic followup. The elevated-temperature Code (Case (2) is more specific in stating that, "Included in this category . . ." (Load-Controlled Quantities) are . . . secondary stresses with a large amount of elastic followup . . ." (-3213(a) Terms Relating to Analysis). However, a specific definition of elastic followup is not provided nor are the critical parameters identified. The design guidance in the Code tends to be inadequate and, in some instances, is misleading. The Code alternative of reclassifying almost all secondary stresses as primary stresses virtually eliminates elastic analysis as a means for qualifying elevated-temperature piping. In the absence of a specific definition of elastic followup or of an acceptance criterion, the piping designer is left with the options either of performing expensive and time-consuming inelastic analysis or of justifying his secondary stresses as producing no significant elastic followup.

The objectives of this paper are: (1) to illustrate the basic characteristics of elastic followup through discussion of simple examples, (2) to propose specific relationships to quantify elastic followup, and (3) to provide design guidance through application of the basic principles to design problems. For clarity of description, this paper will concentrate on plasticity response under elastic followup conditions; creep response to elastic followup under non-cyclic loadings can be evaluated in an analogous manner using isochronous stress-strain curves.

BASIC PHENOMENON IN A NONHARDENING MATERIAL

The characteristics of elastic followup are easiest to understand by considering the two-bar problem shown in Figure 1. This example illustrates the basic approach for series-loaded conditions in the load-deformation regime. The load-deformation curves for the two regions of the two-bar problem are easily determined from elementary principles:

$k = \text{tensile stiffness} = AE/l \quad (1)$

where

A = cross-sectional area

E = modulus of elasticity

l = length.

For the example considered, Region 1 has an area half that of Region 2. However, Region 1 is four times as stiff since its length is one-eighth that of Region 2. For simplification, a nonhardening (elastic-perfectly plastic) material is assumed in this example. For the nonhardening material assumption, all deformation subsequent to yielding is concentrated in the region which is first to yield (Region 1 in this case).

**NOTICE**  
**PORTIONS OF THIS REPORT ARE ILLEGIBLE.**

It has been reproduced from the best available copy to permit the broadest possible availability.

**MASTER**

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

*EAB*

For a series-loaded system, the load-deformation curve for the system is determined by summing the deformations of the component regions. The system and component inelastic response to a deformation-controlled load is then determined by the total applied system deformation ( $e_T$ ) as denoted by the arrows in Figure 1. The corresponding elastically calculated responses are denoted by dashed lines. The system yield deformation ( $e_y$ ) is noted in Figure 1. Elastic followup is apparent in Region 1 by the inelastically calculated concentration of deformation at C compared to the elastically calculated

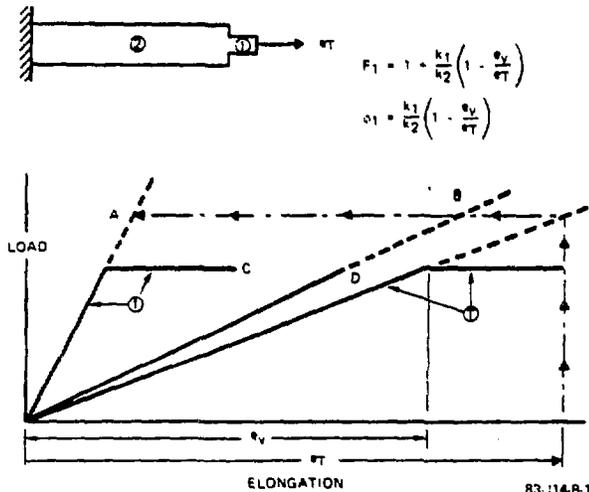


Figure 1. Elastic Followup Response in Two-Bar Problem for Nonhardening Material

deformation at A (see Figure 1) and in Region 2 by the inelastically calculated deformation reduction of D from the elastically calculated deformation at B. The ratio (F) of the inelastic deformation to the elastically calculated deformation can be taken as a measure of elastic followup. F is equal to one when there is no elastic followup, as in the fully elastic range, and increases with increased concentration of deformation. Another measure of elastic followup is the incremental deformation in excess of the elastically calculated value ( $\phi$ ) which is zero when there is no elastic followup.

Consideration of the two-bar model reveals some very significant aspects of elastic followup. First, it is essential that the system be loaded inelastically (plasticity or creep) for any elastic followup to occur. The nonhardening material assumption accentuates the second essential characteristic, unequal stress distribution between regions of the system. Without hardening, the region which is first to yield will absorb all subsequent deformation; even though balancing the stiffness between regions will tend to minimize elastic followup, it cannot eliminate elastic followup in a structure having a nonuniform stress distribution. The parenthetical quantity in the F and  $\phi$  equations (see Figure 1) represents the fraction of the total system deformation that is

inelastic and can range from zero to a value which approaches one for large inelastic deformation. The degree of imbalance between region stiffnesses as represented by the stiffness ratio ( $k_1/k_2$ ) is obviously the most significant parameter with respect to the magnitude of elastic followup.

It must be emphasized that the equations in Figure 1 are based on Region 1 being the high-stressed region; thus, large amounts of elastic followup will occur when the high-stressed region is the stiffer region, ( $k_1 > k_2$ ). Contrary guidance is found in Paragraph -3138 of Code Case N-47, and revision of the code case is under consideration. Although elastic followup will occur when the high-stressed region is more flexible, the elastic followup will be much greater when the high-stressed region is stiffer. Since large deformations (or strains) can occur with little or no elastic followup (as in a conventional tensile or creep test), the magnitude of deformations in an inelastic system analysis is not a sufficient basis to assess the significance (or insignificance) of elastic followup.

#### BASIC PHENOMENON IN A BILINEAR HARDENING MATERIAL

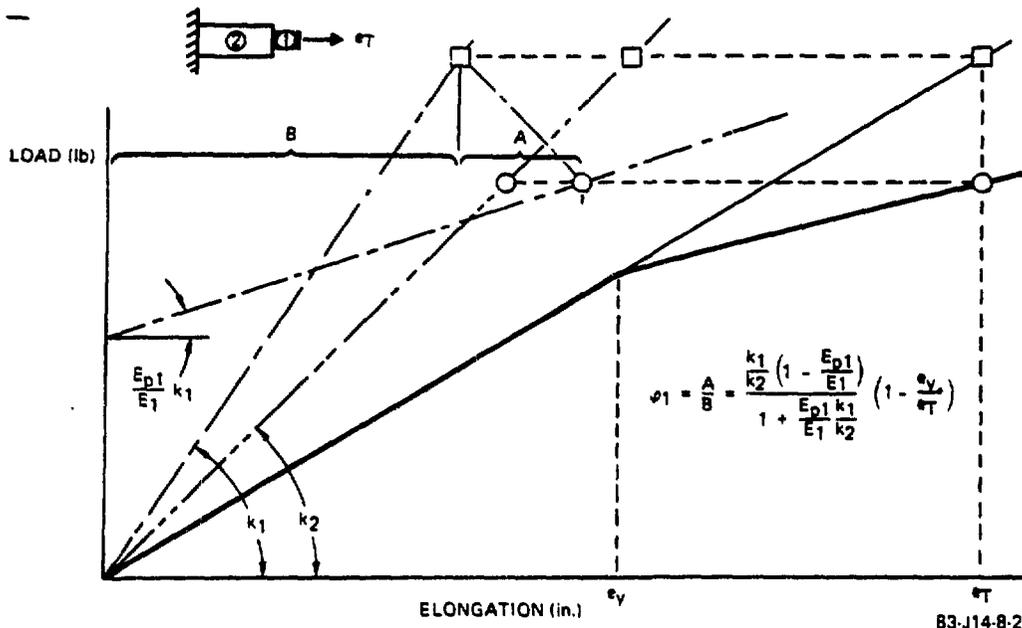
The preceding two-bar analysis can be extended\* to include material hardening. A bilinear hardening assumption will be presented for its mathematical simplicity, however, the analysis can be readily extended to address multilinear hardening assumptions if desired.

While there are several loading regimes for the two-bar problem, the case of one region loaded inelastically while the other region remains elastic is of most interest.

The inelastic/elastic loading regime for the two-bar problem with hardening is diagrammed in Figure 2. The corresponding equation for  $\phi$  is also shown in that figure. The equation for  $\phi$  with hardening is seen to include the additional parameter ( $E_{p1}/E_1$ ) which is the ratio of the plastic modulus to the elastic modulus;  $E_{p1}/E_1 = 0$  corresponds to nonhardening material and  $E_{p1}/E_1 = 1$  corresponds to the elastic solution.

The sensitivity of elastic followup to the various parameters becomes more apparent when  $\phi$  is plotted as a function of  $e_T/e_y$  and  $k_1/k_2$  for specific values of  $E_{p1}/E_1$  as in Figure 3. The effect of the system deformation ratio ( $e_T/e_y$ ), which is the driving force, is to increase elastic followup rapidly at first and then to saturate at a level determined by the stiffness and modulus ratios. It can be seen from Figure 3 that the potential for elastic followup is greatest when the high-stressed region is much stiffer than the lower stressed regions (i.e.,  $k_1/k_2$  is large), and the potential for elastic followup is diminished by increasing the flexibility of the high-stressed region (i.e., reduce  $k_1/k_2$ ). It is apparent that reduced hardening also enhances the potential for elastic followup.

A study of Figure 3 shows that any system in which one region exceeds yield will have some degree of elastic followup. The critical question then is what degree of elastic followup is acceptable. One criterion would be to limit the deformation in the



83-J14-8-2

Figure 2. Elastic Followup in a Series-Loaded, Two-Bar Problem with Hardening

high-stressed region so that the maximum strain would not exceed some acceptable magnitude. The effect of limiting the maximum strain to  $2\%$  when the stress in the critical region is two times the stress in the lower stressed region is indicated by the dotted lines in Figure 3.

It is apparent that the maximum strain criterion restricts the total system deformation ( $e_T$ ) to a smaller multiple of system yield deformation ( $e_y$ ) where the stiffness ratio ( $k_1/k_2$ ) is large (i.e., when the high-stressed region is stiffer than the remaining system) than when the high-stressed region is more flexible than the remaining system. There are two important points to be drawn from this observation; first, that the guidance provided by the Code (see Figure 4) is misleading, and second, that it can be dangerous to extrapolate conclusions drawn from the behavior of "relatively well balanced" systems to determine criteria for "badly unbalanced" systems.

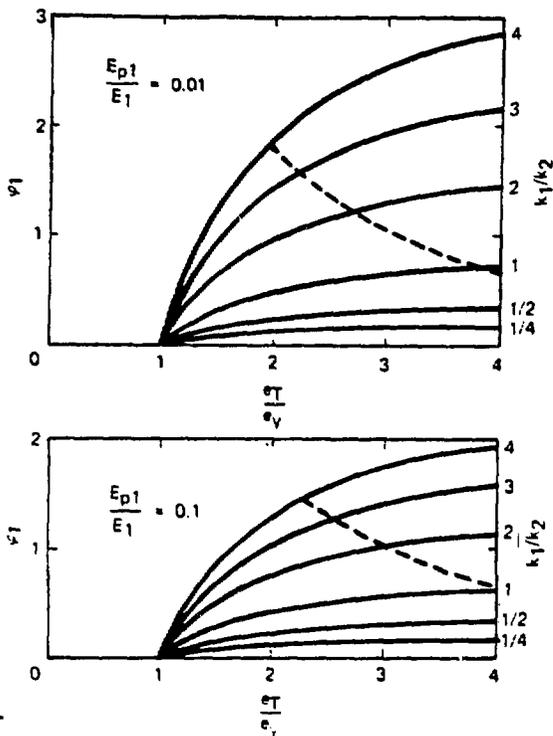
Although the relationships developed are directly applicable only to the two-bar problem, they show promise for extension to prototypic piping configuration, and work is continuing toward this goal. However, the simple two-bar model provides considerable insight into the basic phenomena of elastic followup and yields valuable design guidance.

#### DESIGN GUIDANCE

Consider the heat exchanger piping configuration shown in Figure 5 as an example of a design with enhanced potential for elastic followup. In this example, elastic followup has been made more severe

by a reduction in pipe size at the closure welds due to a requirement for an automatic weld. The thermal expansion of the heating coil must be absorbed by the coil and its end connections. Due to the large bend radius and torsional loading of the heating coil, its stiffness and stress level are low relative to the cantilever end connection which is most highly stressed at the connection to the chamber nozzles. The cantilever is several orders of magnitude stiffer than the coil. The stress in the closure weld section is approximately proportional to the inverse of the diameter squared, and the stiffness is approximately proportional to the cube of the diameter-to-length ratio. Because of its short length, this means the closure weld section is much more highly stressed and orders of magnitude stiffer than the cantilever.

There are three courses of action open to the designer: (1) to reduce the applied deformation so the piping is virtually elastic, (2) to reduce the stress in the stiffer region below that of the more flexible region, and (3) to reduce the stiffness of the high-stressed region. One means to minimize the applied deformation is to locate the coil inlet and outlet close to one another, however, this could have a major impact on either the dimensional envelope or the heat transfer performance of the coil. The obvious means to reduce the stress in the stiff closure weld region is to increase the diameter at the closure weld, however, this has been ruled out by the automated weld design requirement. Another means of reducing the stress level in the closure weld region would be to restrain the rotation of the coil end of the cantilever. Although the opposing restraint



83-J14-8:3

Figure 3. Elastic Followup in Two-Bar Model for Two Levels of Hardening

moment could nearly double the maximum stress in the cantilever, the moment distribution would reverse sign in the stiff closure region near the mid-length of the cantilever. This approach would shift the potential elastic followup from the closure weld region to nearly equal levels at both ends of the cantilever resulting in a substantial net improvement. However, the support required to restrain the coil end rotation could become complex. The third course of action of reducing the stiffness of the high-stressed closure weld region is perhaps easiest to implement by extending the length of the reduced diameter closure weld region as far as is practical. Since bending stiffness varies inversely with the cube of the length for an end load and with the square of the length for an end moment, substantial reductions in stiffness can be obtained. This example shows how the principles of the two-bar problem can guide the designer towards minimizing the potential for elastic followup.

As another example, the backup pump connection for maintenance as shown in Figure 6 includes two features which are relatively common in plant designs; one is the bypass circuit or standby flow path which is shut off and cold during normal operation, and the second is the reduction and expansion in line size to accommodate a smaller, cheaper valve. If the bypass loop is coupled close to the main line and the bypass loop has been sized for

equal flow capacity, there can be large initial strains even if there is little elastic followup. Where the intake and discharge lines for the backup pump are long and relatively flexible, there is potential for significant elastic followup in the connections to the reduced-size valves.

In this case, the best solution is to replace the reduced-size valves with full-size valves. Reducing the line size of the intake and discharge to the backup pump has the beneficial effect of increasing the system deformation to initial yield. However, since the weld joints at the small ends of the reducers are the highest stressed regions, reducing the intake and discharge line sizes has the detrimental effect of reducing the stiffness of the lower stressed region. The overall effect tends to be beneficial but is unlikely to produce a major improvement. Relocating the valve to the point where the moment changes sign has the potential for a substantial reduction in the maximum stress, but must consider the possibility of in-situ repair or removal of the main pump for maintenance.

As a final example, Figure 7 shows a sketch of the uppermost portion of a containment spray line. A typical configuration consists of three to five ring sections, where line size decreases with increased containment elevation. The header tee connections are usually offset for successive ring sections to provide flexibility to accommodate thermal expansion in the meridional direction during an accident condition. In addition, the ring sections require numerous seismic supports due to potentially large accelerations at the high containment elevations, and hard supports are preferred because of access difficulties. The sketch shows a support configuration with considerable potential for elastic followup. The 25.4-cm (10-in.) header, which is long and flexible, is restrained at the upper end by supports on the 10.2-cm (4-in.) ring sections close to the reducers. If the reducers were spread apart to accommodate the initial upper ring restraints on headersize piping, the potential for elastic followup would be alleviated considerably.

#### SUMMARY AND CONCLUSIONS

The two-bar model has been used to illustrate the essential characteristics of elastic followup and to define a quantitative measure for elastic followup. The significance of a maximum strain criterion relative to elastic followup has been investigated for a monotonic deformation loading. While the two-bar model is a gross simplification of a prototype piping system, the two-bar relationships provide valuable design guidance concerning how to recognize and avoid or improve a design with potential for excessive elastic followup. Although this presentation has been limited to plastic response predictions to elastic followup, isochronous stress-strain curves can be used in an analogous manner to predict creep response to elastic followup under noncyclic loadings.

Generalization of the two-bar relationship for application to prototypic piping systems requires consideration of a number of technical questions. Appropriate interaction relations between in- and out-of-plane bending and torsional moments is one of these questions. While the conservation of the vector summation of moments has been established for the

Below The Creep Range

NC-3672.6 Stresses. Calculations . . .

(b) *Local Overstrain.* All the commonly used methods of piping flexibility analysis assume elastic behavior of the entire piping system. This assumption is sufficiently accurate for systems in which plastic straining occurs at many points or over relatively wide regions but fails to reflect the actual strain distribution in unbalanced systems in which only a small portion of the piping undergoes plastic strain or in which, for piping operating in the creep range, the strain distribution is very uneven. In these cases, the weaker or higher stressed portions will be subjected to strain concentrations due to elastic follow-up of the stiffer or lower stressed portions. Unbalance can be produced:

(1) by use of small pipe runs in series with larger or stiffer pipe, with the small lines relatively highly stressed;

(2) by local reduction in size or cross section, or local use of a weaker material;

(3) in a system of uniform size, by use of a line configuration for which the neutral axis or thrust line is situated close to the major portion of the line itself, with only a very small offset portion of the line absorbing most of the expansion strain.

(c) Conditions of this type shall be avoided where materials of relatively low ductility are used; if unavoidable, they shall be mitigated by the judicious application of cold spring.

(d) It is recommended that the design of piping systems of austenitic materials be approached with greater overall care as to general elimination of local stress raisers, examination, material selection, fabrication quality, and erection.

For Elevated Temperature Applications (N47-17)

-3138 Elastic Follow-up

(a) When only a small portion of the structure undergoes inelastic strains while the major portion of the structural system behaves in an elastic manner, the calculations of load forces, stresses and strains shall consider the behavior of the entire structural system. In these cases, certain areas may be subjected to strain concentrations due to the elastic follow-up of the rest of the connected structure. These abnormally large strain concentrations may result when structural parts of different flexibility are in series and the flexible portions are highly stressed. Examples include:

(1) Local reduction in size of a cross section or local use of a weaker material.

(2) In a piping system of uniform size, a configuration for which most of the system lies near the hypothetical straight line connecting the two anchors, (stiffeners, flanges, or other stiff members), and with only a small portion departing from this line. Then the small portion absorbs most of the expansion strain.

(b) If possible, the above conditions should be avoided in design. Where such conditions cannot be avoided, the analysis required in -3250 will determine the acceptability of the design to guard against harmful consequences of elastic follow-up.

-3213 Terms Relating to Analysis

In this Case the stress and strain limits for design evaluation are related to the type of structural behavior under loading. The controlled quantities fall into two general categories:<sup>2</sup>

<sup>2</sup> Note that the expansion stress ( $P_e$ ) defined in NB-3227.3 is deleted from this Code Case. Stresses resulting from the constraint of free end displacement and the effects of anchor motion shall be assigned to either primary or secondary stress categories (see -3213 (a), -3213 (b) and -3217).

T-1320 Satisfaction of Strain Limits Using Elastic Analysis

T-1324 Test No. 3

(a) For axisymmetric structures . . .

(d) Unless otherwise justified, any stress with elastic follow-up (e.g., secondary stresses except those caused by radial through-wall temperature variations) should be included as primary stresses for purposes of this evaluation. Alternatively, the strains due to such stresses may be calculated and added to the strains due to  $\sigma_c$ , the sum being limited to the values of T-1310. If the latter is done, stresses with elastic follow-up should be treated as secondary.

(e) As an alternate to the limit of subparagraph (b), the total inelastic strains due to any number of selected operational cycles may be evaluated separately using an inelastic analysis. The sum of these strains plus the strains due to elastic follow-up (evaluated using either method described above), plus the strains due to  $\sigma_c$  must not exceed 1 percent for parent metal and 1/2 percent for weld metal.

T-1507 BUCKLING AND INSTABILITY

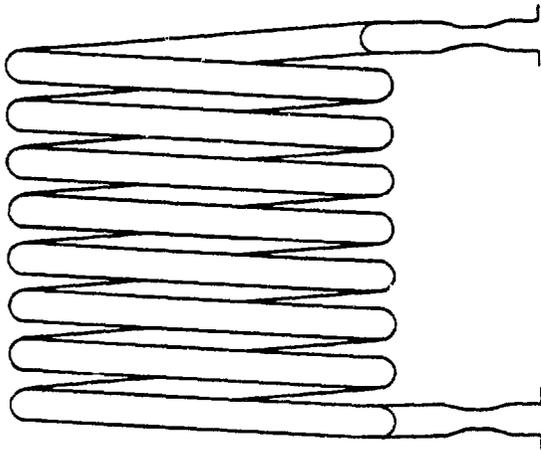
T-1510 General Requirements

(a) The stability limits in

(d) For conditions where significant elastic follow-up may occur, the Load

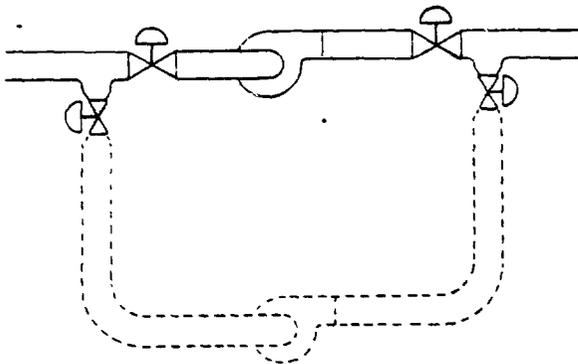
Factors applicable to load-controlled buckling shall also be used for strain-controlled buckling.

Figure 4. Code Guidance Concerning Elastic Followup



83-J14-8-6

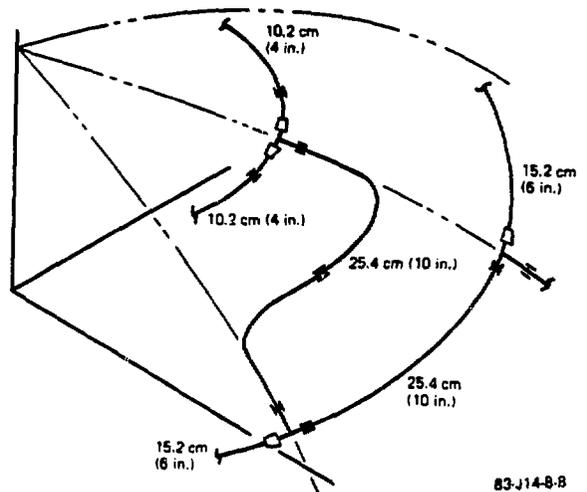
Figure 5. Heat Exchanger Piping



83-J14-8-7

Figure 6. Bypass Loop for Backup Pump

elastic range, the sensitivity of inelastic deformation to peak stress levels could distort the combined effect of in- and out-of-plane loading. This same concern must be resolved for pseudoinelastic methods which use a common value of reduced modulus for in- and out-of-plane loading. In addition, internal program limitations on the response of a specific finite element combined with the modeling of the structure by the analyst may implicitly restrict or exclude elastic followup. For example, the response of an elbow may be distorted if it is restricted to uniform deformation between its end nodes. Since the stresses vary widely within an elbow, inelastic response calculations may be subject to significant distortion. Work is continuing towards resolution of these questions and on the extension of the two-bar



83-J14-8-8

Figure 7. Partial Isometric of Containment Spray Line

approach to allow its application to prototypic piping configurations.

The two-bar model has been used to derive the incremental deformation parameter ( $\phi$ ) as a quantitative measure of elastic followup. This measure was used as the basis for recognizing design situations with the potential for significant elastic followup, and examples have been provided to show how knowledge of the underlying parameters can be used to identify appropriate design modifications.

#### ACKNOWLEDGEMENTS

This work was funded in part by the Department of Energy under the High-Temperature Structural Design Program.

#### REFERENCES

1. Robinson, E. L., "Steam-Piping Design to Minimize Creep Concentrations," Transactions of the ASME, October 1955, pp 1147-1162
2. "Class 1 Components in Elevated Temperature Service," ASME Boiler and Pressure Vessel Code Case N-47, published by the American Society of Mechanical Engineers, New York

## **DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.