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"ON THE MULTIPHOTON EMISSION DURING U.V. AND X-RAY ABSORPTION BY ATOMS IN INTENSE LASER FIELDS"(*)

by

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ABSTRACT

A discussion of the u.v. and x-ray absorption cross section by a hydrogen atom in the presence of an intense i.r. laser field is presented, taking into account the influence of laser field on the electronic states.

RESUMO

Neste trabalho discute-se a seção de choque de um átomo de hidrogênio para absorção de raios-X e ultra-violeta na presença de um campo intenso de laser, levando-se em conta as modificações dos estados ligados pelo campo do laser.

The x-ray absorption by an atom with the subsequent electron ejection in the presence of an intense non-resonant laser field has attracted the attention of a number of authors [1-5]. According to these authors, the main novelty brought about by the presence of the intense laser field is to change the field-free cross-section in such way that the x-ray absorption now becomes accompanied by laser multiphoton absorption and emission. An even more interesting aspect regarding the changes produced by the laser field has recently been claimed by Ferrante et.al.[4], namely, that in the laser-assisted x-ray absorption tha laser multiphoton emission processes dominate over the absorptions. This means that in the laser-assisted x-ray absorption a transfer of energy takes place, through the ejected electrons, from the x-ray field to the assisting laser field so that this process may, in principle, be used as an amplifier of the coherent radiation[4,5]. Furthermore, as shown in Refs. 4 and 5, this amplification occurs only when the polarizations of the laser and x-ray electric fiels are parallel.

The above findings were obtained as follows[4,5]: the x-ray absorption by the bound electron were calculated considering the electromagnetic interaction between the x-ray photon and the electron as a perturbation responsable for the electron "bound-free" transition. The transition probabilities were then calculated, within first-order perturbation theory, using for the wavefunction of the ejected electron a laser modulated Coulomb wave, whereas for the bound state the wavefunction used was the unperturbed ground-state wavefunction (i.e., 1s - state wavefunction). This kind of description for the electronic states in the presence of an intense laser field is, of course, inadequate and may lead to incorrect results. The solution of these difficulties is the purpose of this letter in which the same calculations as those of Refs. 1-5 are reported using, however, the recently proposed[6] laser-dressed

description of the electronic states in both the Coulomb and the laser fields. To be specific, we consider only hidrogen atoms as absorbers of the high-frequency radiation. The full Hamiltonian for our problem is then written as

$$H = \frac{1}{2m}\left(\hat{p} + \frac{e}{c}\hat{A}_L(t)\right)^2 - \frac{e^2}{|\vec{r}|} + \frac{e}{mc}\vec{A}_x \cdot \hat{p} \qquad (1)$$

The first two terms of Eq.(1) represent the Hamiltonian for the hydrogen atom in the laser field, whereas the last one is the high-frequency photon-electron interaction causing the transition of interest. The laser beam is represented by a righ-hand circularly polarized plane wave, in the dipole approximation, of frequency w propagating in the z-direction, namely, $\vec{A}_L = A(\hat{x}\cos wt + \hat{y}\sin wt)$, and $\vec{A}_x = A_o\vec{\epsilon}\, e^{i(\vec{K}.\vec{r}-\Omega t)}$ is the vector potential of the ionizing high-frequency field of frequency Ω and polarization $\vec{\epsilon}$.

Using first-order perturbation theory the probability amplitude for a transition from the state Ψ_i to the state Ψ_f is given by

$$a(i\to f) = -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt\, \langle\Psi_f|\frac{eA_o}{mc}\, e^{i(\vec{K}.\vec{r}-\Omega t)}\vec{\epsilon}.\hat{p}|\Psi_i\rangle \quad , \qquad (2)$$

where the Ψ's are the exact solutions to the Schrödinger equation

$$i\hbar\dot{\Psi} = \left\{\frac{1}{2m}\left(\hat{p} + \frac{e}{c}\vec{A}_L(t)\right)^2 - \frac{e^2}{|\vec{r}|}\right\}\Psi \qquad (3)$$

Provided one knows the solution to Eq.(3), the above expression for the probability amplitude retains the laser field strength to all orders. The nonperturbative solution to Eq.(3) has recently been discussed in Ref.6. It consists essentially in writing Ψ as

$$\Psi = e^{i\vec{\delta}(t).\hat{p}/\hbar}\, e^{i\eta(t)/\hbar}\, \phi \qquad (4)$$

where

$$\vec{\delta}(t) = -\frac{e}{mc}\int^t dt'\, \vec{A}_L(t') \quad ; \quad \eta(t) = -\frac{e^2}{2mc^2}\int^t dt'\, A_L^2(t') \qquad (5)$$

and Φ obeys the Schrödinger equation for the laser-dressed Coulomb potential[6], namely,

$$i\hbar\dot{\Phi} = \left\{\frac{\hat{p}^2}{2m} - \frac{e^2}{(r^2+a^2)^{\frac{1}{2}}}\right\}\Phi \tag{6}$$

where $a = |\vec{\delta}(t)| = eA/mcw$. Here, we note that for the laser field-free case (i.e., $a = 0$), Eqs.(4) and (6) reduce to the unperturbed hydrogen atom solution. Furthermore, as discussed in Ref.6, the above solution for the hydrogen atom in a laser field has no bearing as to the laser field strength. Hence, in this form, the above procedure is suitable for the description of an atom under either an intense or a weak laser field. The ground-state solution to Eq.(6) was found in Ref.6 by resorting to the variation method. On has:

$$\Phi_0 = (\beta^3/\pi)^{\frac{1}{2}} e^{-\beta r} e^{-iE_0 t/\hbar} \tag{7}$$

where the parameter β and the ground-state energy E_0 are now parametrically dependent upon the laser field strength[6]. In Fig.1 we show the variation of β and E_0 for several laser field strength settings. In our calculations we have introduced the parameters $\lambda = a/a_0$, as a measure of the laser field strength, ϵ_0 as a measure of the ground-state energy E_0 in units of e^2/a (i.e., $E_0 = -e^2\epsilon_0/a$), and $\eta = \beta a$ as a measure of β in units of a^{-1}. In Fig.2 we plot the ground-state energy of the laser-dressed atom, in atomic units (i.e., $E_0 = \frac{e^2}{2a_0}\frac{2\epsilon_0}{\lambda}$) as a function of the laser field. It follows from Figs.1 and 2 that the effect of an intense laser field is to weaken the electron binding to the nuclens in such a way that both β and E_0 are monotonically decreasing functions of the laser field strength. In particular, we note that the ionization threshold (E_0) is now field-dependent. Hence, at large laser field strengths, ionization can now take place for relatively low frequency (say, optical) ionizing radiation instead of

of at soft x-ray frequencies. In other words, the ionization threshold is now tunable by the intense nonresonant ir laser field. As for the final (continuum) state we shall approximate Φ_f, for the sake of simplicity, by a plane wave, namely,

$$\Phi_f = e^{i\vec{k}.\vec{r}}\, e^{-i\hbar k^2 t/2m} \tag{8}$$

Here, we note that instead of representing a crude approximation for the final state, the plane wave approximation now becomes a good representation for the final state the larger the laser field strength is, since, as remarked above, the ionization threshold now decreases on increasing the laser intensity.

Using Eqs.(7) and (8) in the corresponding expressions for the initial and final electronics states (c.f., Eq.(4)), and followings the usual procedures, one gets the ionization probability amplitude involving the simultaneous absorption ($\nu>0$) or emission ($\nu<0$) of $|\nu|$ laser photons:

$$a^{\nu}(i\to f) = -2\pi i \frac{eA_0\hbar}{mc}\left(\frac{\beta^3}{\pi}\right)^{1/2} \frac{8\pi\beta\, \vec{\epsilon}.\vec{k}}{\left[(\vec{k}-\vec{K})^2+\beta^2\right]^2} (-1)^{\nu} J_{\nu}(K_{\perp}a) \cdot$$
$$\cdot\, e^{i\nu\varphi_x}\delta\left(\frac{\hbar^2k^2}{2m} - E_0 - \hbar\Omega - \nu\hbar w\right) \tag{9}$$

where $K_{\perp} = K \sin\theta_x$, with θ_x and φ_x being the polar and azimuthal angles of $\vec{K}$ with the z-axis. Equation (9) tells us that if the x-ray is polarized parallel to the laser field (i.e., $K_{\perp} = 0$) there is no multiphoton processes, since for zero argument the Bessel function vanishes unless $\nu = 0$. This is in contrast with the conclusions drawn in Refs. 4 e 5 in which to have amplification (laser photon emissions larger than absorptions) it is necessary to have the laser electric field parallel to that of the x-ray.

To further understand the role of the multiphoton processes let us consider the total absorption cross section for the specific geometry in which the high-frequency beam propagates along the x-direction and is linearly polarized along the z-axis (direction of propagation of the laser beam). Performing the same calculations as those of Refs. 1-5 the total cross section σ is given by

$$\sigma = \sigma_0 \sum_{\nu=-\nu_1}^{\infty} S_\nu \quad ; \quad S_\nu = J_\nu^2(Ka)\frac{X_\nu^3}{(X_\nu^2+1)^4} \quad ; \quad \sigma_0 = \frac{256\pi e^2}{3mc\Omega} \qquad (10)$$

where

$$X_\nu^2 = \frac{\lambda^2}{\eta^2}\frac{w}{\Omega_b}(\nu+\nu_1) \quad ; \quad \nu_1 = \frac{\Omega_b}{w}\left(\mu - \frac{2\epsilon_0}{\lambda}\right) \; ; \; \mu = \frac{\Omega}{\Omega_b} \quad ; \qquad (11)$$

Here, μ measures the high-frequency photon energy ($\hbar\Omega$) in units of the bare atom ionization energy $\hbar\Omega_b$ (=13,5 eV). Now, since $Ka = 3{,}6\mu\lambda \times 10^{-3}$ is usually much smaller than unit, we see from Eq.(10) that the only significant processes are $\nu = 0, \pm 1$. In Table I, we present the results from our numerical calculations for the ratio $R_1 = S_{-1}/S_1$ of the one-photon emission (S_{-1}) to the one-photon absorption (S_1) cross sections, for several laser field strengths (λ) and ionizing frequencies (μ). It follows from the Table that the laser photon emission process dominates over the absorption process for moderate ionizing frequencies, in agreement with Ferrante et.al.[4] conclusions. However, on increasing the ionizing frequency (μ) these two processes tend to become equally intense thereby producing no net effect.

Summarizing, we have shown in this paper that taking properly into account the influence of the laser field on the electronic states of a hydrogen atom the following results regarding the laser-assisted x-ray absorption obtains: (i) the bound-free transition can now take

place at lower ionizing radiation frequency than that of the unperturbed atom; (ii) the laser multiphoton processes occurs only when the incident high-frequency beam propagates at an angle with the assisting laser field. This result is in contrast with the previous predictions[4,5]; (iii) for the perpendicularly propagation configuration (i.e., $\vec{k}_L \perp \vec{K}$), the relevant laser-assiting process are the single-photon emission and absorption, with one-photon emission being usually larger than the one-photon absorption; (iv) for very high ionizing frequencies (i.e., $\mu \gg 1$) the one-photon emission and absorption processes become of equal strength thereby producing no net effect.

λ	$\frac{2\epsilon_0}{\lambda}$	η	$\mu=1$		$\mu=2$		$\mu=3$	
			ν_1	R_1	ν_1	R_1	ν_1	R_1
1	0.52	0.54	5.5	1.41	17	1.24	28.5	1.16
2	0.38	0.80	7.1	1.61	18.6	1.26	30.1	1.16
3	0.29	0.98	8.2	1.62	19.7	1.26	31.2	1.16
4	0.24	1.12	8.7	1.63	20.2	1.26	31.7	1.16
5	0.21	1.23	9.1	1.63	20.6	1.26	32.1	1.16
6	0.19	1.32	9.3	1.63	20.8	1.26	32.3	1.16
7	0.17	1.42	9.5	1.63	21.0	1.26	32.5	1.16
8	0.15	1.51	9.8	1.61	21.3	1.26	32.8	1.16
9	0.14	1.58	9.9	1.61	21.4	1.26	32.9	1.16
10	0.12	1.65	10.1	1.60	21.6	1.25	33.1	1.16

Table I: Dependence of the ratio R, of the laser-assisted one-photon emission to the one-photon absorption cross sections of a hydrogen atom illuminated by a Nd:glass laser ($\hbar w = 1.17$ eV) as a function of the laser field strength (λ) and the ionizing frequency (μ).

Figure Captions

Figure 1: Variation of the ground-state energy (ϵ_0) and of the parameter $\beta(=\eta/a)$ as a function of the laser field. The parameter λ is related to the laser intensity in W/cm^2 by $\lambda = 6.5 \times 10^{24} w^{-2} I$.

Figura 2: Ground-state energy of the laser-dressed atom, in atomic units ($e^2/2a_0$), as a function of the laser field strength.

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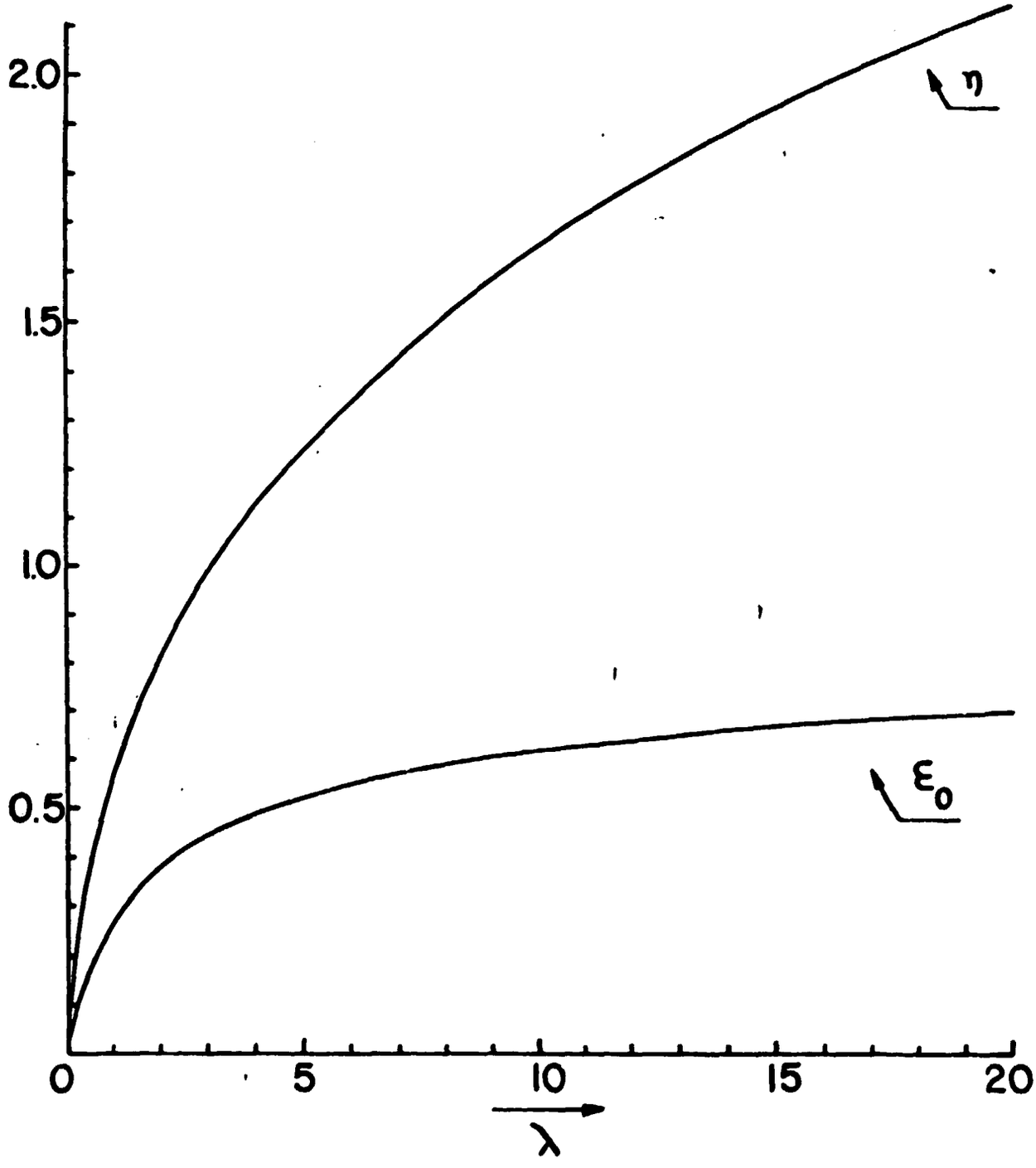
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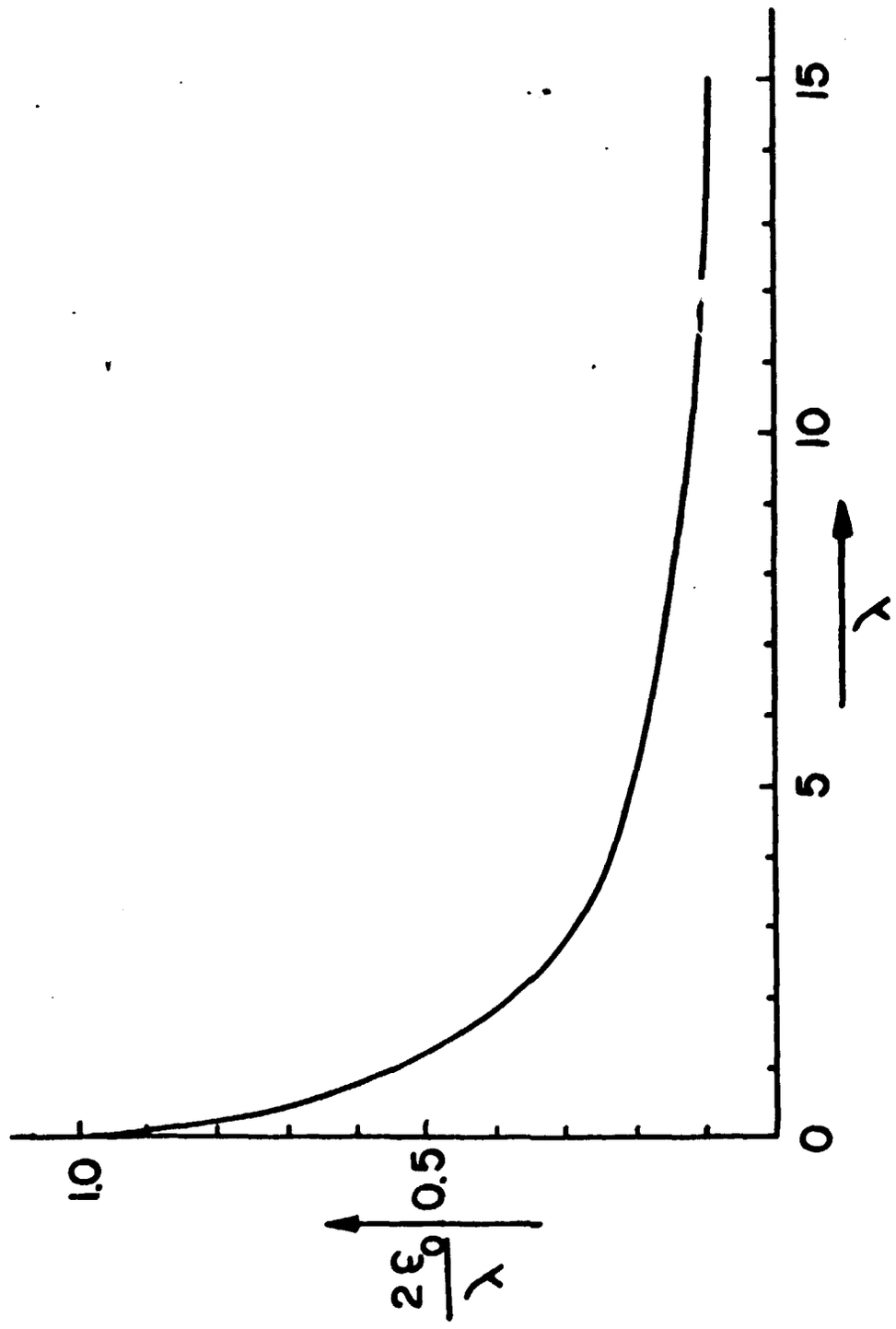
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