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ARE HIGGS PARTICLES STRONGLY INTERACTING?

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Abstract

The order of magnitude of Yukawa couplings in some theories with flavour violating Higgs particles is estimated. Based on these couplings, mass bounds for flavour violating Higgs particles are derived from the K_L-K_S mass difference. The Higgs particles have to be very heavy, implying that the Higgs sector quartic couplings are very large. Thus, these theories seem to require a strongly interacting Higgs sector unless one adjusts the Higgs-fermion Yukawa couplings to within two orders of magnitude, so as to suppress the coupling of Higgs particles to the flavour-violating $\bar{s}d$ current. Most models with flavour violating Higgs particles have the same general features, so the conclusions are likely to hold for a wide class of models with flavour violating Higgs particles.

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1. Introduction

In many extensions of the standard weak interaction model one has flavour-violating Higgs particles. These must be made very heavy if they are not to violate the experimental bounds on flavour-changing neutral currents (in particular, the bound from the K_L-K_S mass difference). This requires making the Higgs quartic couplings very large, and Higgs loop effects then become comparable to the tree effects. There is a screeming theorem due to Veltman [1,2] which suggests that the effect of strongly interacting Higgs particles is difficult to detect. However, it is interesting to see if theories with flavour changing Higgs particles require a strongly interacting Higgs sector. In this paper we will investigate this aspect for some models. The main features of the argument are quite general, and a strongly interacting Higgs sector is likely to occur in many models which have flavour violating Higgs scalars, unless the Higgs-fermion Yukawa couplings are properly adjusted.

The organisation of this paper is as follows: In Sect. 2 we will study a simple scalar theory, to illustrate the well-known fact that the Higgs sector becomes strongly interacting when $\frac{\lambda}{4\pi^2} \approx O(1)$ [3], where λ is a typical Higgs quartic coupling. In Sect. 3 we discuss the standard model with two Higgs doublets and spontaneous CP violation. We will discuss the order of magnitude of flavour violating Higgs-fermion couplings, and derive a lower bound on the masses of flavour violating neutral Higgs particles. We will then present the neutral Higgs particle mass matrix in terms of Higgs quartic couplings and Higgs vacuum expectation values (VEV's), and show that the quartic couplings must be much larger than $4\pi^2$ if the flavour violating neutral currents are to be suppressed sufficiently. In Sect. 4 we will discuss a left-right symmetric theory. The discussion is similar to Sect. 3. In Sect. 5 we present our conclusions.

2. Higgs loops in a scalar theory

In this section we will discuss a scalar theory with a global SU(2) symmetry, with the Higgs particles transforming as a complex doublet. We will study the Higgs four point function, to illustrate that loop effects become comparable to tree effects unless $\lambda/4\pi^2 \ll 1$. The complex doublet can be written as $(H_1 + iH_2,\ H_3 + iH_4)$ where the H's represent hermitean scalar fields. The Lagrangian we study is

$$L = (\partial^\mu\phi)^+\partial_\mu\phi - \mu^2\phi^+\phi - \lambda(\phi^+\phi)^2 \ .$$

We study the case where $\mu^2 > 0$. When $\mu^2 < 0$ the Higgs particles develop a VEV. The calculation of the loop corrections becomes more complicated because a trilinear coupling is induced by the symmetry breaking. This case is now under study, and the results will be presented elsewhere. For $\mu^2 > 0$ the Higgs particles have a mass $m_H = \sqrt{2}\mu$. The leading contribution to the four-point function with four H_3 fields is shown in Fig. 1. To order λ the four point function is

$$\Gamma^4(P_1,P_2,P_3,P_4) = -24i\lambda \ . \qquad (2.1)$$

The λ^2 contribution can be divided into three classes, Fig. 2 with the H_3 field in the loops, Fig. 2 with the other fields in the loops, and the counter-term contribution Fig. 3. We will use a Pauli-Villars regularisation procedure, introducing a propagator with a large mass Λ to absorb the divergences. When the H_3 particle occurs in the loop Fig.2(a) makes the contribution

$$\frac{18i\lambda^2}{\pi^2}\left(\log\left(\frac{\Lambda^2}{m_H^2}\right) - 1 - \int_0^1 dx\, \log\left(\frac{m_H^2 - x(1-x)s}{m_H^2}\right)\right.$$

$$= \frac{18i\lambda^2}{\pi^2}\left(a + \sqrt{\frac{s-4m_H^2}{s}}\,\log\left(\frac{\sqrt{s}-\sqrt{s-4m_H^2}}{\sqrt{s}+\sqrt{s-4m_H^2}}\right) + i\pi\sqrt{\frac{s-4m_H^2}{s}}\right) \text{ for } s > 4m_H^2 ,$$

$$= \frac{18i\lambda^2}{\pi^2}\left(a - 2\sqrt{\frac{4m_H^2-s}{s}}\,\tan^{-1}\sqrt{\frac{s}{4m_H^2-s}}\right) \text{ for } 0 < s < 4m_H^2 ,$$

$$= \frac{18i\lambda^2}{\pi^2}\left(a + \sqrt{\frac{4m_H^2-s}{|s|}}\,\log\left(\frac{\sqrt{4m_H^2-s}-\sqrt{|s|}}{\sqrt{4m_H^2-s}+\sqrt{|s|}}\right)\right) \text{ for } s < 0 , \tag{2.2}$$

where $s = (P_1+P_2)^2$ and $a = \log(\Lambda^2/m_H 2)^{-1}$. The real part of the diagram for $s > 4m_H^2$ can be related to the $O(\lambda)$ contribution by the unitarity of the S-matrix. Figures 2(b) and 2(c) make similar contributions with s replaced by t and u respectively, where $t = (P_1-P_3)^2$ and $u = (P_1-P_4)^2$. When the fields H_1, H_2 and H_4 occur in the loops the sum over these fields gives the same contribution as Eq. (2.2), with the factor 18 replaced by 6. The counterterm contribution (Fig. 3) is defined by the renormalisation condition: $\Gamma^{(4)} = -24i\lambda_R$ when $s = t = u = 4m_H^2/3$ and $P_i^2 = m_H^2$. This renormalisation condition leads to the counterterm

$$-\frac{72i\lambda^2}{\pi^2}\left(\log\frac{\Lambda^2}{m_H^2} + 1 - 2\sqrt{2}\,\tan^{-1}\frac{1}{\sqrt{2}}\right) . \tag{2.3}$$

We will compare the $O(\lambda^2)$ effects to the $O(\lambda)$ contribution at two physical points: for $s = 45m_H^2$, $t = u = -m_H^2/2$, and for $s = 8m_H^2$, $t = u = -2m_H^2$. For the former point the $O(\lambda^2)$ contribution is

$$-\frac{47i\lambda^2}{\pi^2} + i\pi\left(\frac{8\, i\lambda^2}{\pi^2}\right) . \tag{2.4}$$

This becomes comparable to the $O(\lambda)$ contribution $-24i\lambda$ when $\lambda/\pi^2 \approx 0.5$. For the second physical point the $O(\lambda^2)$ contribution is

$$-\frac{85i\lambda^2}{\pi^2} + i\pi\left(\frac{12\sqrt{2}\, i\lambda^2}{\pi^2}\right) . \tag{2.5}$$

This becomes comparable to the leading contribution when $\lambda/\pi^2 \approx 0.25$.

In both cases the loop effects become an order of magnitude larger than the tree effects when $\lambda/(4\pi^2)\approx O(1)$. Thus, we expect that $\lambda/(4\pi^2)$ should be much less than 1 if the Higgs particles are not to become strongly interacting.

3. Standard model with two doublets

In this section we will show that the Higgs quartic couplings should be much larger than $4\pi^2$ in the standard model with two Higgs doublets and spontaneous CP violation, if the contribution of the neutral flavour changing Higgs particles to the K_L-K_S mass difference is to be suppressed sufficiently. Thus, it seems likely that the Higgs particles will be strongly interacting in this model. The lower bound on the Higgs masses to be derived in Sect. 3.1 is valid when an arbitrary number of Higgs doublets are present, provided no discrete symmetry is imposed to get natural flavour conservation (NFC). The neutral Higgs particle mass matrix becomes more complicated than the case discussed in Sect. 3.2. However, it seems difficult to evade strongly interacting Higgs particles even with an arbitrary number of doublets, without imposing NFC or adjusting the Higgs-fermion couplings.

3.1 MASS BOUND ON HIGGS PARTICLES

When an arbitrary number of Higgs doublets are introduced into the standard model, the Higgs-fermion Yukawa couplings in the charge-1/3 quark sector can be written as

$$\sum_j \overline{U}_L F_j D_R \phi_j^+ + \overline{D}_2 F_j D_R \frac{e^{i\theta_j}}{\sqrt{2}} (\rho_j + R_j + iI_j) + \text{h.c.} , \qquad (3.1)$$

where U and D stand for the gauge eigenstate fermions (u,c,t) and (d,s,b) respectively, $\rho_j e^{i\theta_j}/\sqrt{2}$ are the VEV's of the n Higgs doublets ($j = 1$ to n), the ϕ_j^+ are the charged Higgs fields, the R_j and I_j are the neutral Higgs fields. The F_j are 3×3 matrices containing the Yukawa couplings. They are real if CP violation occurs spontaneously. h.c. stands for hermitean conjugate. The diagonalisation of the fermion mass matrix leads to the relation

$$\sum_j \rho_j e^{i\theta_j} F_j = C_L M_d C_R^+ , \tag{3.2}$$

where C_L and C_R are unitary 3 × 3 fermion mixing matrices and M_d is a real diagonal matrix containing the charge -1/3 quark masses. We thus see that the entries in the matrices F_j are of order m_b/ρ times mixing angles*, where m_b is the bottom quark mass and $\rho^2 = \sum_j \rho_j^2$. ρ is related to the weak interaction Fermi coupling constant by $\rho^2 = 1/(\sqrt{2}\, G_F)$, and thus ρ = 250 GeV. The coupling of the Higgs particles to the flavour violating $\bar{s}d$ current is thus of order $\sqrt{G_F}\, m_b$ times mixing angles. The real part of the K_L-K_S mass difference then imposes a lower bound of 150 TeV on the Higgs boson masses [4], assuming that the mixing angles are of order one. The imaginary part of the K_L-K_S mass difference is 10^{-3} times smaller. Thus, even if we allow for a mixing angle suppression for the amplitude of order 10^{-3}, we find that the flavour changing neutral Higgs particle masses must be greater than about 150 TeV.

3.2 HIGGS MASS MATRIX

With two Higgs doublets there are three neutral Higgs particles. The mass^2 matrix M takes the form [5]:

$$2\begin{pmatrix} \left\{A\rho_1^2 + \frac{\cos\theta}{2}(D\rho_2^2\cos\theta + E\rho_1\rho_2)\right\} & \left\{\frac{\rho_1\rho_2}{2}[C - D(1+\cos^2\theta)]\right\} & \left\{-\rho\rho_2\left(E\frac{\rho_1}{\rho_2} - F\frac{\rho_2}{\rho_1}\right)\frac{\sin\theta}{8}\right\} \\ \left\{\frac{\rho_1\rho_2}{2}[C - D(1+\cos^2\theta)]\right\} & \left\{B\rho_2^2 + \frac{\cos\theta}{2}(D\rho_1^2\cos\theta + F\rho_1\rho_2)\right\} & \left\{\rho\rho_1\left(E\frac{\rho_1}{\rho_2} - F\frac{\rho_2}{\rho_1}\right)\frac{\sin\theta}{8}\right\} \\ \left\{-\rho\rho_2\left(E\frac{\rho_1}{\rho_2} - F\frac{\rho_2}{\rho_1}\right)\frac{\sin\theta}{8}\right\} & \left\{\rho\rho_1\left(E\frac{\rho_1}{\rho_2} - F\frac{\rho_2}{\rho_1}\right)\frac{\sin\theta}{8}\right\} & \left\{\frac{D\rho^2}{2}\sin^2\theta\right\} \end{pmatrix} . \tag{3.3}$$

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*We are assuming that the different Yukawa couplings are of the same order of magnitude.

ρ_1, ρ_2 and θ occur in the usual parametrisation of the Higgs vacuum expectation values. ρ_1 and ρ_2 are positive. The constants A-F are Higgs particle quartic couplings and are real (C is the sum of two quartic couplings). ρ is defined as $\sqrt{\rho_1^2+\rho_2^2}$ and is equal to 250 GeV. Thus, ρ_1 and ρ_2 are both less than 250 GeV. Also [5],

$$-4D\cos\theta = \left(E\frac{\rho_1}{\rho_2} + F\frac{\rho_2}{\rho_1}\right) . \tag{3.4}$$

The sum of the squares of the Higgs masses is equal to the trace of M. If λ_{max} is the maximum of the absolute values of the quartic couplings occurring in the diagonal entries and m_H is the mass of the heaviest neutral Higgs particle†, then

$$m_H^2 < 2\left(A\rho_1^2 + B\rho_2^2 + \frac{D\rho^2}{2} + (E+F)\rho_1\rho_2\frac{\cos\theta}{2}\right)$$

$$< 2\lambda_{max}\,\rho^2\left(1.5 + \frac{\rho_1\rho_2}{\rho^2}\cos\theta\right) . \tag{3.5}$$

Since $\rho_1^2+\rho_2^2 = \rho^2$, the maximum value of $\rho_1\rho_2/\rho^2$ is 1/2. Therefore m_H^2 is less than $4\lambda_{max}\,\rho^2$. If m_H is to be larger than 150 TeV, then λ_{max} has to be larger than 10^5. With such a large value for λ_{max} the Higgs particles will be strongly interacting. Since the Higgs-fermion Yukawa couplings are not completely determined by the fermion masses and mixing angles, one could adjust the Yukawa couplings so that the Higgs particle couplings to the $\bar{s}d$ current are small. Barring such arbitrary adjustment of parameters, the model requires strongly interacting Higgs particles. If more than two Higgs doublets are present the bound on m_H^2 is again of the form $\lambda_{max}\,\rho^2$ times a number of order one. Thus even for the general case the Higgs sector would be strongly interacting,

†The idea of using the trace to find an upper bound on the Higgs mass was suggested by Ling-Fong Li.

unless the Yukawa couplings are adjusted to suppress the flavour-changing $\bar{s}d$ coupling.

The above remarks hold if NFC is not imposed on the theory. If a discrete symmetry is imposed to get NFC, Branco [6] has shown that CP violation must be solely due to Higgs particle exchanges if the violation is due to spontaneous symmetry breaking. In this case the theory may run into problems with the experimental value of the ε' parameter [7,8] and the neutron dipole moment [9]. The problem arises because CP violation occurs in flavour changing processes, and in a theory with NFC the Higgs particles have to be very light to overcome the flavour change suppression effect.

If we demand that $\lambda_{max} << 4\pi^2$, the heaviest Higgs particle mass must be less than 3 TeV. If m_1 is the mass of the lightest Higgs particle, then the trace of M is greater than $3m_1^2$, which means that $m_1 <$ 1.7 TeV. An upper bound of 1 TeV for the Higgs particle mass has been found for the standard model with one Higgs doublet [10,11]. The quartic couplings in Eq. (3.3) cannot be arbitrarily small because they get contributions from loops involving the exchange of two gauge bosons. This provides lower limits for the Higgs particle masses of order 7 GeV [12].

3.3 GERSCHGORIN'S THEOREM

In this section we derive the conclusions of Sect. 3.2 using a slightly different approach. We will use Gerschgorin's theorem [13] to answer the question: if all Higgs quartic couplings are much less than $4\pi^2$, what is the largest Higgs mass that is allowed? Gerschgorin's theorem gives certain regions in the complex plane within which the eigenvalues of a matrix must lie. Let A be an

$n \times n$ matrix and x an eigenvector, i.e., $Ax = \lambda x$. Let the entry in x which has the largest absolute value be x_i. One can divide all entries in x by x_i. Thus, x has the properties: $x_i = 1$ and $|x_j| \leq 1$. (In this paragraph j is supposed to take all values $1 \leq j \leq n$ except that $j \neq i$.) The i^{th} row of the equation $Ax = \lambda x$ can be written as $(a_{ii} - \lambda) = - \sum_j a_{ij} x_j$. Taking the absolute values of both sides and using $|x_j| \leq 1$, we get

$$|a_{ii} - \lambda| \leq \sum_j |a_{ij}| \tag{3.6}$$

Thus, for any matrix A the eigenvalues lie inside the set of n circles with centres a_{ii} and radius $\sum_j |a_{ij}|$. The Higgs mass matrix is a real symmetric matrix with real eigenvalues. Thus Gerschgorin's theorem locates the eigenvalues of this matrix to lie within certain intervals on the real axis. Our strategy in what follows is to find the limits on the absolute values of the entries in the Higgs mass matrix, and then use Eq. (3.6) to find the bounds on the neutral Higgs particle masses.

If the quartic couplings A-F occurring in Eq. (3.3) are all less than $4\pi^2 \approx 40$ in absolute value, it is straightforward to set limits on the absolute values of all entries in M except for the non-diagonal entries of the third row and third column. For example, $|M_{33}|$ is less than $2 * [40*(250\ \text{GeV})^2/2]$. For the other entries in the third column the occurrence of ρ_1 and ρ_2 in the denominator necessitates a more careful treatment. It is sufficient to find the upper limit of

$$y = \left(E \frac{\rho_1}{\rho_2} - F \frac{\rho_2}{\rho_1}\right) \frac{\sin\theta}{8} \tag{3.7}$$

with the constraint Eq. (3.4). Let us denote $-4D \cos\theta$ by k. Since $|D| < 40$, $|k| < 160$. Let ρ_1/ρ_2 be x. Then Eq. (3.4) gives

$x = (k \pm \sqrt{k^2 - 4EF})/(2E)$. Since x is real, we find that $4EF < k^2$. Also, $|EF| < (40)^2$ if perturbation theory is not to break down. Equation (3.7) can now be written as

$$y = \left(\frac{k \pm \sqrt{k^2-4EF}}{2} - \frac{2EF}{k \pm \sqrt{k^2-4EF}}\right)\frac{\sin\theta}{8}$$
$$= \frac{\pm\sqrt{4D^2\sin^2(2\theta)-4EF\sin^2(\theta)}}{8} \tag{3.8}$$

This implies that $|y| < 14$. We can now exhibit the upper limits on the absolute values of the entries in Eq. (3.3):

$$2(250\ \text{GeV})^2\begin{pmatrix}70 & 30 & 14\\ 30 & 70 & 14\\ 14 & 14 & 20\end{pmatrix}. \tag{3.9}$$

Gerschgorin's theorem tells us that the maximum eigenvalue of M [Eq.(3.3)] is less than $(70 + 44) \times 2 * (250\ \text{GeV})^2$. Thus, the Higgs particle masses must all be less than 4 TeV if perturbation theory is not to break down. This upper bound is much lower than the lower bound of 150 TeV from the K^0-$\overline{K}^0$ transition. Gerschgorin's theorem can also be used to find a lower bound on the eigenvalues of M. This requires setting a lower limit on the diagonal entries. From an inspection of Eq.(3.3) it is clear that one of the eigenvalues goes to zero when $\sin\theta = 0$. Since the eigenvalues of M must be non-negative on physical grounds, this means that the lowest Higgs mass allowed is zero. Note that $\sin\theta = 0$ is not allowed phenomenologically because it implies CP conservation. However, there seem to be too many parameters to set a meaningful lower limit on $\sin\theta$. Gerschgorin's theorem gives a slightly less stringent limit than the trace argument. The reason is that we found the maximum value of each entry in M without considering the other entries. The bounds can be improved if we take into account the fact that all entries cannot simultaneously achieve their maximum values.

4. Left-right symmetric model

In this section we will study the $SU(2)_L \times SU(2)_R \times U(1)$ theory with the Higgs transforming as $\chi_L(1/2,0,1)$, $\chi_R(0,1/2,1)$ and $\phi(1/2,1/2^*,0)$ where the numbers in brackets represent $SU(2)_L$ isospin, $SU(2)_R$ isospin and $U(1)$ hypercharge respectively. The quarks transform as $Q_{iL} = (1/2,0,1/3)$ and $Q_{iR} = (0,1/2,1/3)$ where the index i represents the generation index. For example,

$$Q_{1L} = \begin{pmatrix} u \\ d \end{pmatrix}_L \; ,$$
$$Q_{1R} = \begin{pmatrix} u \\ d \end{pmatrix}_R \qquad (4.1)$$

and the left-handed doublet fields are singlets under $SU(2)_R$ while the right-handed fields are singlets under $SU(2)_L$. The Higgs-fermion couplings which generate masses for the quarks may be written as

$$\sum_{i,j} \overline{Q}_{iL}(a_{ij}\,\phi + b_{ij}\,\tilde{\phi})Q_{jR} + \overline{Q}_{jR}(a^*_{ij}\,\phi^+ + b^*_{ij}\,\tilde{\phi}^+)Q_{iL} \qquad (4.2)$$

where ϕ is written as a 2×2 matrix and $\tilde{\phi}$ is $\tau_2\,\phi^*\tau_2$ [14]. The a_{ij} and b_{ij} are Yukawa coupling constants. These couplings lead in general to flavour violating Higgs interactions. This is a very general feature of left-right symmetric models [15]. ϕ develops the VEV:

$$<\phi> = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} . \qquad (4.3)$$

4.1 MASS BOUND ON HIGGS PARTICLES

The quark mass matrices in terms of gauge eigenstate quarks can be written as

$$M_{1ij} = a_{ij}\,k + b_{ij}\,k'^*, \quad M_{2ij} = a_{ij}\,k' + b_{ij}\,k^* \qquad (4.4)$$

where the subscript 1 stands for charge 2/3 quarks and 2 stands for charge -1/3 quarks. The VEV's k and k′ contribute to the W_L gauge boson mass. Therefore the weak interaction Fermi coupling constant constrains them to be less than 250 GeV in magnitude. The a_{ij} and b_{ij} are then of order $m_t/(250\ \text{GeV})$ times mixing angles, where m_t is the mass of the top quark. The flavour violating Higgs coupling to the $\bar{s}d$ current is of order $\sqrt{G_F}\, m_t$ times mixing angles. The imaginary part of the K_L-K_S mass difference then constrains the masses of flavour-violating Higgs particles to be greater than 600 TeV, assuming a mixing angle suppression for the K^0-$\bar{K}^0$ transition amplitude of order 10^{-3}. Here we have used the fact that the lower limit on the top quark mass is of order 20 GeV. The new feature of L-R symmetric models relative to the standard model is that the Yukawa couplings contributing to the K^0-$\bar{K}^0$ transition are constrained by the quark mass matrices of both charge 2/3 and charge -1/3 quarks, whereas in the standard model the relevant Yukawa couplings were constrained only by the charge -1/3 quarks. That is why the mass bound on the Higgs particles is 4 times more stringent than for the standard model with many Higgs doublets.

4.2 HIGGS MASS MATRIX

The lower bound for the Higgs mass derived in Sect. 4.1 implies that the Higgs quartic couplings must be very large. We discuss this aspect in two cases: (1) for the case where the right-handed gauge bosons get very heavy masses and (2) for the case where the right-handed currents have phenomenological consequences (i.e., when the right-handed gauge bosons have masses close to present experimental limits).

When the right-handed gauge bosons get very heavy masses, most of the neutral Higgs bosons get masses of the order of λv^2, where λ is a typical Higgs quartic coupling and v is the large vacuum expectation value (VEV) which gives a heavy mass to the W_R boson. Because of this large VEV these Higgs masses can be made heavy enough to sufficiently suppress their contribution to the K_L-K_S mass difference. There may exist one light Higgs boson [14] which in general would have flavour-violating couplings. Ref. [14] gives the following mass^2 matrix for some of the neutral Higgs particles in the theory:

$$\begin{pmatrix} A' & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 4k^2(\lambda_1+\lambda_2) & 2vk(\alpha_1+\alpha_2') \\ 0 & 0 & 2vk(\alpha_1+\alpha_2') & 4\rho_1 v^2 \end{pmatrix}$$

where

$$A' = (4\lambda_3 + \lambda_5 + \lambda_6 - \lambda_2)2k^2 + v^2(\alpha_2 - \alpha_2')$$

$$B = (\rho_2 - 2\rho_1)v^2 \ . \tag{4.5}$$

The λ, α and ρ parameters are Higgs sector quartic couplings. k is the VEV contributing to the usual left-handed weak gauge boson mass. It is of order $(\sqrt{2}G_F)^{-1/2} = 250$ GeV. From Eq. (4.5) we find the mass of the light Higgs boson to be

$$m_H^2 = 2k^2(\lambda_1 + \lambda_2) + 2\rho_1 v^2 - \sqrt{4(\rho_1 v^2 - k^2(\lambda_1+\lambda_2))^2 + k^2v^2(\alpha_1+\alpha_2')^2}$$

$$\approx 4k^2(\lambda_1 + \lambda_2) - \frac{k^2(\alpha_1 + \alpha_2')^2}{4\rho_1} \ . \tag{4.6}$$

From the condition that the Higgs mass matrix is positive one finds that $\rho_1 > 0$ and $4\rho_1(\lambda_1 + \lambda_2) > (\alpha_1 + \alpha_2')^2$. Thus, the mass of the Higgs particle is less than $\sqrt{4k^2(\lambda_1 + \lambda_2)}$. The quartic couplings must be of order 10^6 if this Higgs particle has to have a mass of order 600 TeV.

Such a large value for λ would imply strongly interacting Higgs particles. If λ_1 and λ_2 are not to exceed $4\pi^2$ the mass of this Higgs particle should be less than 4 TeV. It should be noted that Senjanovic has derived his results imposing a discrete symmetry on the Lagrangian. This discrete symmetry has to be relaxed on phenomenological grounds (it implies that quarks of one particular charge are massless). It would be interesting to see if the occurrence of the light Higgs boson is predicted even when the discrete symmetry is removed. The terms which break the discrete symmetry cannot be too large, because they lead to mixing between the right and left-handed gauge bosons. One has stringent experimental bounds on this L-R mixing.

We will now discuss the case where the W_R has a phenomenologically interesting mass, $M_{W_R} \approx 3M_{W_L}$. For this case also there is a problem with the light Higgs boson. In addition the other Higgs bosons will also have problems, because now $v^2 \approx 8k^2$. For example, one finds that the particle with $m^2 = A'$ has an upper bound on m of order $\sqrt{\lambda_{max}(14k^2 + 2v^2)}$ where λ_{max} is the magnitude of the largest quartic coupling occurring in A′ [Eq. (4.5)]. λ_{max} must be larger than 10^5 if the mass of this Higgs particle is to be greater than 600 TeV. If we demand that $\lambda_{max} < 4\pi^2$ and $m_H > 600$ TeV, we find that v must be at least 100 times larger than k, i.e., the mass of the W_R gauge boson must be at least 100 times the mass of the W_L gauge boson. Thus, it seems likely that the Higgs particles in a left-right symmetric theory would be strongly interacting if the right-handed currents have detectable low energy effects, unless the Higgs-fermion Yukawa couplings are adjusted to suppress the Higgs contribution to the K_L-K_S mass difference.

5. Conclusions

We have studied the order of magnitude of Yukawa couplings in some theories which have flavour violating Higgs particles. Based on these couplings we derived lower bounds on Higgs particle masses from the K_L-K_S mass difference. These lower bounds are of the order of 100 TeV unless one adjusts the Higgs-fermion Yukawa couplings to within two orders of magnitude, so as to suppress the coupling of Higgs particles to the flavour-violating $\bar{s}d$ current. To get such heavy Higgs particles the Higgs quartic couplings must be of order 10^5. This implies that the Higgs particles are strongly interacting. Most models with flavour violating Higgs particles have the same general features that were used in the present analysis. Thus, similar conclusions should hold for these models.

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Figure captions

Fig. 1. Tree level contribution to Higgs particle four-point function.

Fig. 2. *One-loop corrections.*

Fig. 3. Counter-term contribution.

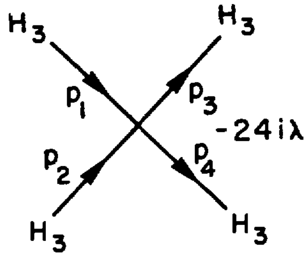
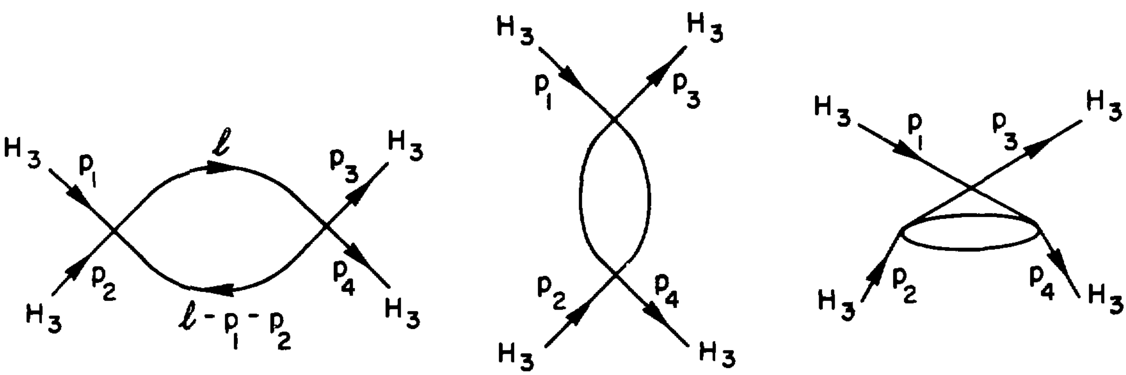


Fig. 1



(a) (b) (c)

Fig. 2

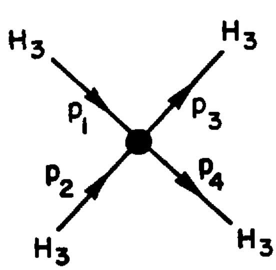


Fig. 3