IC/83/9
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ON THE EQUIVALENCE OF TWO APPROACHES IN THE EXCITON-POLARITON THEORY *

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ABSTRACT

The polariton effect in the optical processes involving photons with energies near that of an exciton is investigated by the Bogolubov diagonalization and the Green function approaches in a simple model of the direct band gap semiconductor with the electrical dipole allowed transition. To take into account the non-resonant terms of the interaction Hamiltonian of the photon-exciton system the Green function approach derived by Nguyen Van Hieu is presented with the use of Green's function matrix technique analogous to that suggested by Nambu in the theory of superconductivity. It is shown that with the suitable choice of the phase factors the renormalization constants are equal to the diagonalization coefficients. The dispersion of polaritons and the matrix elements of processes with the participation of polaritons are identically calculated by both methods. However the Green function approach has an advantage in including the damping effect of polaritons.

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February 1983

* Submitted for publication.

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I. INTRODUCTION

In recent years many optical processes in semiconductors have been intensively studied in the resonant region: resonant Raman scattering [1)-3)], resonant scattering of light by light [4),5)], resonant electronic Raman scattering [6),7)] and high order harmonic generation at resonance frequencies [8)-10)].

Due to the photon-exciton interaction in semiconductors there arises new elementary excitation, which is a mixture of the photon and the exciton: excitonic polaritons, or briefly the polaritons. Polariton effect plays a very important role at the resonance because in this region the dispersion of the polaritons is rather different from those of the photon and the exciton.

The polariton theory was developed by many authors: method of Bogolubov diagonalization [12)-16)], Green's function approach [17),18)] and other microscopic approaches (for example, see Ref.19). Recently a new Green function approach to the scattering of the polaritons was derived by Nguyen Van Hieu [20)]. By the use of the renormalization recipe, this method allows us to construct the matrix element of all optical processes involving photon at the resonant energies. Moreover in this Green function approach to the theory of polaritons the effect of the damping of polaritons can be included in a rather elegant manner.

The purpose of this work is to compare two approaches on the polariton problem: the Bogolubov diagonalization approach and the Green function approach [20)]. We show that these approaches lead to the same results, but the second has an advantage in including the damping effect of polaritons. To this purpose we take a simple model of the direct band gap semiconductor with the electrical dipole allowed transition and with on-degenerate valence and conduction bands whose extrema are located at the centre of the Brillouin zone. We suppose that the polarization state of all photons are given. Here only the $n = 1$ exciton state is taken into account.

In Sec.II we give a review of the Bogolubov diagonalization method. In Sec.III the Green function approach derived by Nguyen Van Hieu is presented with the use of the Green function matrix technique analogous to that suggested by Nambu in the theory of superconductivity [21)]. Sec.IV is devoted to the study of the amplitudes of some polariton interaction processes. We shall use the unit system with $\hbar = c = 1$.

II. BOGOLUBOV DIAGONALIZATION APPROACH

In Ref.20 it was shown that the exciton-photon transitions can be described by the following effective interaction Hamiltonian:

$$\mathcal{H}^{\gamma E_x}(t) = g \int A(\vec{r},t)\, B(\vec{r},t)\, d^3r \qquad (1)$$

with the effective coupling constant

$$g = \frac{e}{m} \left(2E_0\right)^{1/2} \varphi(0)\, \Pi \quad , \qquad (2)$$

where e and m are the (free) electron charge and mass, $E_0 = E(\vec{p} = 0)$ is the energy of the exciton with the total momentum $\vec{p} = 0$, $\varphi(0) \equiv \varphi(\vec{r} = 0)$ is the external wave function of the exciton in point $\vec{r} = 0$. Π denotes the matrix element of the dipole transition between the valence and conduction bands

$$\Pi = i \langle c | (\vec{\xi}\vec{P}) | v \rangle \equiv \frac{1}{\Omega} \int_{\Omega} u_0^*(\vec{r}) (\vec{\xi}\vec{\nabla})\, v_0(\vec{r})\, d^3r \qquad (3)$$

Ω is the elementary cell volume, $\vec{\xi}$ is the unit vector characterizing the polarization of the photon, $A(\vec{r},t)$ and $B(\vec{r},t)$ are the effective scalar quantum fields for describing the photons and excitons in the given polarization state. We write

$$A(\vec{r},t) = A^{(+)}(\vec{r},t) + A^{(-)}(\vec{r},t)$$

$$B(\vec{r},t) = B^{(+)}(\vec{r},t) + B^{(-)}(\vec{r},t) \quad , \qquad (4)$$

where $A^{(+)}(\vec{r},t) = A^{(-)}(\vec{r},t)^+ \quad ; \quad B^{(+)}(\vec{r},t) = B^{(-)}(\vec{r},t)^+$

$$A^{(+)}(\vec{r},t) = \frac{1}{(2\pi)^{3/2}} \int \frac{\hat{a}(\vec{k})\, e^{i[\vec{k}\vec{r} - \omega(k)t]}}{[2\epsilon_0\, \omega(k)]^{1/2}}$$

$$B^{(+)}(\vec{r},t) = \frac{1}{(2\pi)^{3/2}} \int \frac{\hat{b}(\vec{p})\, e^{i[\vec{p}\vec{r} - E(p)t]}}{[2E(p)]^{1/2}} \qquad (5)$$

$\hat{a}(\vec{k})$ and $\hat{b}(\vec{p})$ are the annihilation operators of the photons and the excitons, respectively, ϵ_0 is the background dielectric constant, $\omega(k)$ is the energy of the photon with momentum $\vec{k}$.

To apply the Bogolubov diagonalization method we write the Hamiltonian of the photon-exciton system as:

$$H = H_0 + H^{\gamma - E_x} \qquad (6)$$

$$H_0 = \sum_{\vec{k}} \omega(k)\, a^+(\vec{k})\, a(\vec{k}) + \sum_{\vec{p}} E(p)\, b^+(\vec{p})\, b(\vec{p}) \quad . \qquad (7)$$

According to (1) $H^{\gamma - E_x}$ is written in the form

$$H^{\gamma - E_x} = \sum_{k} g_0(k) \left[a^+(\vec{k})\, b(\vec{k}) + a(\vec{k})\, b^+(\vec{k}) + a^+(\vec{k})\, b^+(-\vec{k}) + a(\vec{k})\, b(-\vec{k}) \right] \qquad (8)$$

where

$$g_0(k) = \frac{g}{2\sqrt{\epsilon_0\, E(k)\, \omega(k)}} \qquad (9)$$

By the Bogolubov diagonalization method we can rewrite the Hamiltonian (6)

$$H = \sum_{\nu, \vec{k}} \mathcal{E}_\nu(k)\, C^+_\nu(\vec{k})\, C_\nu(\vec{k}) \quad . \qquad (10)$$

In this formula $C_\nu(\vec{k})$ and $C^+_\nu(\vec{k})$ are the annihilation and creation operators, respectively, of the polariton with momentum $\vec{k}$ in the branch ν , $\varepsilon_\nu(k)$ is the polariton energy and can be determined by the algebraic equation

$$\frac{\omega^2(k)}{\mathcal{E}_\nu^2(k)} = 1 + \frac{4 g_0^2(k)\, \omega(k)\, E(k)}{\mathcal{E}_\nu^2(k) \left[E^2(k) - \mathcal{E}_\nu^2(k) \right]} \qquad (11)$$

It is easy to obtain the precise expression for $\varepsilon_\nu(k)$

$$2\mathcal{E}^2_{1,2}(k) = E^2(k) + \omega^2(k) \pm \left\{ \left[E^2(k) - \omega^2(k) \right]^2 + 4 g_0^2(k)\, E(k)\, \omega(k) \right\}^{1/2} . \qquad (12)$$

The annihilation and creation operators of polaritons are connected with those of photons and excitons as follows:

$$C_\nu(\vec{k}) = u_\nu^\gamma(k)\, a(\vec{k}) - v_\nu^\gamma(k)\, a^+(-\vec{k}) + u_\nu^{Ex}(k)^*\, b(\vec{k}) - v_\nu^{Ex}(k)^*\, b^+(-\vec{k}) \tag{13}$$

with the transformation coefficients

$$u_\nu^\gamma(k) = \frac{\omega(k) + \mathcal{E}_\nu(k)}{2\sqrt{\omega(k)\mathcal{E}_\nu(k)}}\, \beta_\nu^\gamma(k)$$
$$v_\nu^\gamma(k) = \frac{\omega(k) - \mathcal{E}_\nu(k)}{2\sqrt{\omega(k)\mathcal{E}_\nu(k)}}\, \beta_\nu^\gamma(k)$$
$$u_\nu^{Ex}(k) = \frac{g_0(k)}{\mathcal{E}_\nu(k) - E(k)} \sqrt{\frac{\omega(k)}{\mathcal{E}_\nu(k)}}\, \beta_\nu^\gamma(k)$$
$$v_\nu^{Ex}(k) = \frac{-g_0(k)}{\mathcal{E}_\nu(k) + E(k)} \sqrt{\frac{\omega(k)}{\mathcal{E}_\nu(k)}}\, \beta_\nu^\gamma(k), \tag{14}$$

where

$$\beta_\nu^\gamma(k) = \frac{|\mathcal{E}_\nu^2(k) - E^2(k)|}{\left\{[\mathcal{E}_\nu^2(k) - E^2(k)]^2 + 4g_0^2(k)E(k)\omega(k)\right\}^{1/2}} . \tag{15}$$

The annihilation operators of photons and excitons are expressed in terms of the annihilation and creation operators of polaritons by the inverse transformation:

$$a(\vec{k}) = \sum_\nu \left[u_\nu^\gamma(k) C_\nu(\vec{k}) + v_\nu^\gamma(k)^* C_\nu^+(-\vec{k}) \right]$$
$$b(\vec{k}) = \sum_\nu \left[u_\nu^{Ex}(k) C_\nu(\vec{k}) + v_\nu^{Ex}(k)^* C_\nu^+(-\vec{k}) \right] . \tag{16}$$

Using (16) one can construct matrix elements of each interaction process with the participation of polariton from the corresponding processes involving photons or excitons. This method was applied to the study of resonant Raman scattering [14], absorption of light [16], scattering of light by light [4),5] and non-linear optical effect [15].

III. GREEN'S FUNCTION APPROACH

It is essential to notice that due to the presence of the terms

$$\int A^{(+)}(\vec{r},t)\, B^{(+)}(\vec{r},t)\, d^3r \qquad \text{and} \qquad \int A^{(-)}(\vec{r},t)\, B^{(-)}(\vec{r},t)\, d^3r$$

in the Hamiltonian (1) corresponding to the non-resonant interaction between exciton and photon we must apply the below matrices of Green's functions analogous to those used by Nambu [21] in the theory of superconductivity.

First we define the matrices of free field operators of photon and exciton

$$\tilde{A}(\vec{r},t) = \begin{pmatrix} A^{(+)}(\vec{r},t) \\ A^{(-)}(\vec{r},t) \end{pmatrix} , \quad \tilde{A}^+(\vec{r},t) = \left(A^{(-)}(\vec{r},t) ,\ A^{(+)}(\vec{r},t) \right)$$
$$\tilde{B}(\vec{r},t) = \begin{pmatrix} B^{(+)}(\vec{r},t) \\ B^{(-)}(\vec{r},t) \end{pmatrix} , \quad \tilde{B}^+(\vec{r},t) = \left(B^{(-)}(\vec{r},t) ,\ B^{(+)}(\vec{r},t) \right) . \tag{17}$$

We denote by $\hat{\tau}$ the following matrix:

$$\hat{\tau} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} . \tag{18}$$

Then we can rewrite Hamiltonian (1) in the form

$$\mathcal{H}^{\gamma\text{-}Ex}(t) = \frac{1}{2} g \int \tilde{A}^+(\vec{r},t)\, \hat{\tau}\, \tilde{B}(\vec{r},t)\, d^3r + \frac{1}{2} g \int \tilde{B}^+(\vec{r},t)\, \hat{\tau}\, \tilde{A}(\vec{r},t)\, d^3r . \tag{19}$$

Introduce the matrix of Green's functions of free fields

$$\tilde{D}(\vec{r}_1-\vec{r}_2, t_1-t_2) \equiv i \langle 0 | T \left\{ \tilde{A}(\vec{r}_1,t_1)\, \tilde{A}^+(\vec{r}_2,t_2) \right\} | 0 \rangle =$$
$$= \begin{pmatrix} D_{11}(\vec{r}_1-\vec{r}_2, t_1-t_2) & D_{12}(\vec{r}_1-\vec{r}_2, t_1-t_2) \\ D_{21}(\vec{r}_1-\vec{r}_2, t_1-t_2) & D_{22}(\vec{r}_1-\vec{r}_2, t_1-t_2) \end{pmatrix} \tag{20}$$

where

$$D_{11}(\vec{r}_1-\vec{r}_2, t_1-t_2) \equiv i\langle 0|T\left\{A^{(+)}(\vec{r}_1,t_1),\ A^{(-)}(\vec{r}_2,t_2)\right\}|0\rangle$$

$$D_{12}(\vec{r}_1-\vec{r}_2, t_1-t_2) \equiv i\langle 0|T\left\{A^{(+)}(\vec{r}_1,t_1)\ A^{(+)}(\vec{r}_2,t_2)\right\}|0\rangle$$

$$D_{21}(\vec{r}_1-\vec{r}_2, t_1-t_2) \equiv i\langle 0|T\left\{A^{(-)}(\vec{r}_1,t_1)\,A^{(-)}(\vec{r}_2,t_2)\right\}|0\rangle$$

$$D_{22}(\vec{r}_1-\vec{r}_2, t_1-t_2) \equiv i\langle 0|T\left\{A^{(-)}(\vec{r}_1,t_1)\,A^{(+)}(\vec{r}_2,t_2)\right\}|0\rangle \quad . \qquad (20)$$

Analogously we define

$$\widetilde{G}(\vec{r}_1-\vec{r}_2,\ t_1-t_2) \equiv i\langle 0|T\left\{\widetilde{B}(\vec{r}_1,t_1)\,\widetilde{B}^{+}(\vec{r}_2,t_2)\right\}|0\rangle \quad . \qquad (21)$$

Here we denote by $|0\rangle$ the vacuum state of non-interacting photon-exciton system. We also denote by $|\gamma(\vec{k})\rangle$ or $|E_{ex}(\vec{p})\rangle$ the one-photon or one-exciton state with definite energies $\omega(k)$ or $E(p)$ and momenta $\vec{k}$ or $\vec{p}$, respectively, in the interaction representation. In the Heisenberg representation we have the interacting field operators determined as follows:

$$A_H^{(\pm)}(\vec{r},t) = S(0,t)\ A^{(\pm)}(\vec{r},t)\,S(t,0)$$

$$B_H^{(\pm)}(\vec{r},t) = S(0,t)\ B^{(\pm)}(\vec{r},t)\,S(t,0) \quad , \qquad (22)$$

where

$$S(t,t_0) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{t_0}^{t}\cdots\int_{t_0}^{t} T\left\{\mathcal{H}^{\gamma-E_x}(t_1)\cdots\mathcal{H}^{\gamma-E_x}(t_n)\right\}dt_1\cdots dt_n \qquad (23)$$

and the quantum interacting (dressed) field states

$$|\gamma^H(\vec{k})\rangle = S(0,-\infty)\,|\gamma(\vec{k})\rangle$$

$$|E_x^H(\vec{p})\rangle = S(0,-\infty)\,|E_x(\vec{p})\rangle \quad . \qquad (24)$$

In a similar manner we define the matrices of the interacting field operators of photon and exciton:

$$\widetilde{A}_H(\vec{r},t) = \begin{pmatrix} A_H^{(+)}(\vec{r},t) \\ A_H^{(-)}(\vec{r},t)\end{pmatrix} \ ; \quad \widetilde{A}_H^{+}(\vec{r},t) = \left(A_H^{(-)}(\vec{r},t)\ ,\ A_H^{(+)}(\vec{r},t)\right)$$

$$\widetilde{B}_H(\vec{r},t) = \begin{pmatrix} B_H^{(+)}(\vec{r},t) \\ B_H^{(-)}(\vec{r},t)\end{pmatrix} \ ; \quad \widetilde{B}_H^{+}(\vec{r},t) = \left(B_H^{(-)}(\vec{r},t)\ ,\ B_H^{(+)}(\vec{r},t)\right) \qquad (25)$$

and the matrices of the Green functions of interacting fields

$$\widetilde{D}_H(\vec{r}_1-\vec{r}_2, t_1-t_2) \equiv i\langle\phi_0|T\left\{\widetilde{A}_H(\vec{r}_1,t_1)\,\widetilde{A}_H^{+}(\vec{r}_2,t_2)\right\}|\phi_0\rangle$$

$$\widetilde{G}_H(\vec{r}_1-\vec{r}_2, t_1-t_2) \equiv i\langle\phi_0|T\left\{\widetilde{B}_H(\vec{r}_1,t_1)\,\widetilde{B}_H^{+}(\vec{r}_2,t_2)\right\}|\phi_0\rangle \ . \qquad (26)$$

Here $|\phi_0\rangle$ denotes the vacuum state of photon-exciton system in the Heisenberg representation.

From (20) and (21) we have the expression for the Fourier transforms of each element of the matrix of Green's function of free fields

$$D_{11}(\vec{k},\omega) = \frac{1}{2\epsilon_0\,\omega(k)[\omega(k)-\omega]}$$

$$D_{22}(\vec{k},\omega) = \frac{1}{2\epsilon_0\,\omega(k)[\omega(k)+\omega]} \qquad (27)$$

$$D_{12}(\vec{k},\omega) = D_{21}(\vec{k},\omega) = 0 \quad ,$$

$$G_{11}(\vec{k},\omega) = \frac{1}{2E(k)[E(k)-\omega]}$$

$$G_{22}(\vec{k},\omega) = \frac{1}{2E(k)[E(k)+\omega]} \qquad (28)$$

$$G_{12}(\vec{k},\omega) = G_{21}(\vec{k},\omega) = 0$$

We denote

$$D(\vec{k},\omega) \equiv \sum_{i,j=1,2} D_{ij}(\vec{k},\omega) = \frac{1}{\epsilon_0[\omega^2(k)-\omega^2]}$$

$$G(\vec{k},\omega) \equiv \sum_{i,j=1,2} G_{ij}(\vec{k},\omega) = \frac{1}{E^2(k)-\omega^2} \quad . \tag{29}$$

From (19), (22), (23) and the perturbative expansions of the Green functions of the interacting fields it is easy to verify that the Fourier transforms of these matrices of Green's functions must satisfy the linear matrix equation:

$$\widetilde{D}_H(\vec{k},\omega) = \widetilde{D}(\vec{k},\omega) + g^2\,\widetilde{D}(\vec{k},\omega)\,\hat{\tau}\,\widetilde{G}(\vec{k},\omega)\,\hat{\tau}\,\widetilde{D}_H(\vec{k},\omega)$$

$$\widetilde{G}_H(\vec{k},\omega) = \widetilde{G}(\vec{k},\omega) + g^2\,\widetilde{G}(\vec{k},\omega)\,\hat{\tau}\,\widetilde{D}(\vec{k},\omega)\,\hat{\tau}\,\widetilde{G}_H(\vec{k},\omega) \,. \tag{30}$$

From (30) we obtain

$$D^H_{12}(\vec{k},\omega) = D^H_{21}(\vec{k},\omega) = \frac{g^2 G(\vec{k},\omega)\,D(\vec{k},\omega)}{4\epsilon_0\omega^2(k)[1-g^2G(\vec{k},\omega)D(\vec{k},\omega)]}$$

$$D^H_{11}(\vec{k},\omega) = \frac{4\epsilon_0\omega^2(k)D_{11}(\vec{k},\omega) - g^2G(\vec{k},\omega)D(\vec{k},\omega)}{4\epsilon_0\omega^2(k)\,[1-g^2G(\vec{k},\omega)D(\vec{k},\omega)]}$$

$$D^H_{22}(\vec{k},\omega) = \frac{4\epsilon_0\omega^2(k)D_{22}(\vec{k},\omega) - g^2G(\vec{k},\omega)D(\vec{k},\omega)}{4\epsilon_0\omega^2(k)[1-g^2G(\vec{k},\omega)D(\vec{k},\omega)]}$$

$$D^H(\vec{k},\omega) = \sum_{i,j=1,2} D^H_{ij}(\vec{k},\omega) = \frac{D(\vec{k},\omega)}{1-g^2G(\vec{k},\omega)D(\vec{k},\omega)} \tag{31}$$

and similarly

$$G^H_{12}(\vec{k},\omega) = G^H_{21}(\vec{k},\omega) = \frac{g^2D(\vec{k},\omega)G(\vec{k},\omega)}{4E^2(k)[1-g^2G(\vec{k},\omega)D(\vec{k},\omega)]}$$

$$G^H_{11}(\vec{k},\omega) = \frac{4E^2(k)G_{11}(\vec{k},\omega) - g^2D(\vec{k},\omega)G(\vec{k},\omega)}{4E^2(k)[1-g^2G(\vec{k},\omega)D(\vec{k},\omega)]}$$

$$G^H_{22}(\vec{k},\omega) = \frac{4E^2(k)G_{22}(\vec{k},\omega) - g^2D(\vec{k},\omega)G(\vec{k},\omega)}{4E^2(k)[1-g^2G(\vec{k},\omega)D(\vec{k},\omega)]}$$

$$G^H(\vec{k},\omega) = \sum_{i,j=1,2} G^H_{ij}(\vec{k},\omega) = \frac{G(\vec{k},\omega)}{1-g^2D(\vec{k},\omega)G(\vec{k},\omega)} \,. \tag{32}$$

Each polariton corresponds to a pole of the Green function $G^H_{ij}(\vec{k},\omega)$ or $D^H_{ij}(\vec{k},\omega)$. From (31) and (32) we can see that the pole of the Green functions satisfies the equation

$$1 - g^2D(\vec{k},\omega)\,G(\vec{k},\omega) = 0 \,. \tag{33}$$

Its solutions are

$$2\mathcal{E}^2_\nu(k) = E^2(k) + \omega^2(k) \pm \sqrt{[E^2(k)-\omega^2(k)]^2 + \frac{g^2}{\epsilon_0}} \,. \tag{34}$$

By a comparison of expressions (12) and (34) we conclude that the dispersions of polaritons derived by Green function approach coincide with the result obtained in the Bogolubov diagonalization approach.

It is easy to see that

$$D_{11}(\vec{k},-\omega) = D_{22}(\vec{k},\omega) \quad ; \quad G_{11}(\vec{k},-\omega) = G_{22}(\vec{k},\omega)$$

$$D^H_{11}(\vec{k},-\omega) = D^H_{22}(\vec{k},\omega) \quad ; \quad G^H_{11}(\vec{k},-\omega) = G^H_{22}(\vec{k},\omega) \,. \tag{35}$$

From (31), (32) and (34) we have

$$D^H_{11}(\vec{k},\omega) = \sum_\nu \frac{|\beta^\gamma_\nu(k)|^2}{8\epsilon_0\omega^2(k)\mathcal{E}_\nu(k)}\left[\frac{\mathcal{E}_\nu(k)+\omega(k)}{\mathcal{E}_\nu(k)-\omega} + \frac{\mathcal{E}_\nu(k)-\omega(k)}{\mathcal{E}_\nu(k)+\omega}\right]$$

$$G^H_{11}(\vec{k},\omega) = \sum_\nu \frac{|\beta^{Ex}_\nu(k)|^2}{8E^2(k)\mathcal{E}_\nu(k)}\left[\frac{\mathcal{E}_\nu(k)+E(k)}{\mathcal{E}_\nu(k)-\omega} + \frac{\mathcal{E}_\nu(k)-E(k)}{\mathcal{E}_\nu(k)+\omega}\right] \tag{36}$$

where $\beta^\gamma_\nu(k)$ is determined as in (15) and

$$|\beta^{Ex}_\nu(k)|^2 = \frac{[\mathcal{E}^2_\nu(k)-\omega^2(k)]^2}{[\mathcal{E}^2_\nu(k)-\omega^2(k)]^2 + \frac{g^2}{\epsilon_0}} \,. \tag{37}$$

From (37) and (15) we have the identity

$$|\beta^{Ex}_\nu(k)|^2 = \frac{g^2}{\epsilon_0}\,|G(\vec{k},\mathcal{E}_\nu(k))|^2\,|\beta^\gamma_\nu(k)|^2 \,. \tag{38}$$

In order to calculate the matrix elements of the interaction processes with the participation of polaritons we define the matrix wave functions of bare (non-interacting) photons and excitons as the matrix elements:

$$\langle 0|\tilde{A}(\vec{r},t)|\gamma\rangle = \tilde{\mathcal{U}}(\vec{r},t) = \frac{1}{2\pi^4}\int e^{i(\vec{q}\vec{r}-\varepsilon t)}\tilde{u}(\vec{q},\varepsilon)\,d^3q\,d\varepsilon$$

$$\langle 0|\tilde{B}(\vec{r},t)|E_x\rangle = \tilde{\mathcal{V}}(\vec{r},t) = \frac{1}{2\pi^4}\int e^{i(\vec{q}\vec{r}-\varepsilon t)}\tilde{v}(\vec{q},\varepsilon)\,d^3q\,d\varepsilon\,. \qquad (39)$$

The matrix wave function of the dressed (interacting) photons and excitons are determined in a similar manner

$$\langle\Phi_0|\tilde{A}_H^H(\vec{r},t)|\gamma^H\rangle = \tilde{\mathcal{U}}^H(\vec{r},t) = \frac{1}{2\pi^4}\int e^{i(\vec{q}\vec{r}-\varepsilon t)}\tilde{u}^H(\vec{q},\varepsilon)\,d^3q\,d\varepsilon$$

$$\langle\Phi_0|\tilde{B}_H(\vec{r},t)|E_x^H\rangle = \tilde{\mathcal{V}}^H(r,t) = \frac{1}{2\pi^4}\int e^{i(\vec{q}\vec{r}-\varepsilon t)}\tilde{v}^H(\vec{q},\varepsilon)\,d^3q\,d\varepsilon \qquad (40)$$

The matrix wave functions $\tilde{u}^H(\vec{q},\varepsilon)$ and $\tilde{v}^H(\vec{q},\varepsilon)$ satisfy the equations:

$$\tilde{u}^H(\vec{q},\varepsilon) = \tilde{u}(\vec{q},\varepsilon) + g^2\tilde{D}(\vec{q},\varepsilon)\,\hat{\tau}\,\tilde{G}(\vec{q},\varepsilon)\,\hat{\tau}\,\tilde{u}^H(\vec{q},\varepsilon)$$

$$\tilde{v}^H(\vec{q},\varepsilon) = \tilde{v}(\vec{q},\varepsilon) + g^2\tilde{G}(\vec{q},\varepsilon)\,\hat{\tau}\,\tilde{D}(\vec{q},\varepsilon)\,\hat{\tau}\,\tilde{v}^H(\vec{q},\varepsilon)\,. \qquad (41)$$

Expressions (41) show again that the energies and momenta of the dressed photon and exciton must obey the dispersion equations of the polaritons, i.e. they are the polaritons.

From expansions (5) it follows that for the one-photon or one-exciton state with the definite energy $\omega(k)$ or $E(p)$ and momentum $\vec{k}$ or $\vec{p}$ we have

$$\tilde{u}(\vec{q},\varepsilon) = \frac{\delta[\varepsilon-\omega(k)]\,\delta^3(\vec{k}-\vec{q})}{\sqrt{2\epsilon_0\,\omega(k)}}\begin{pmatrix}1\\0\end{pmatrix}$$

$$\tilde{v}(\vec{q},\varepsilon) = \frac{\delta[\varepsilon-E(k)]\,\delta^3(\vec{k}-\vec{q})}{\sqrt{2E(k)}}\begin{pmatrix}1\\0\end{pmatrix}\,. \qquad (42)$$

Similarly for the polariton branch

$$\tilde{u}_\nu^H(\vec{q},\varepsilon) = \frac{\delta[\varepsilon-\mathcal{E}_\nu(k)]\,\delta^3(\vec{k}-\vec{q})}{\sqrt{2\epsilon_0\,\omega(k)}}\begin{pmatrix}Z_\nu^\gamma(k)\\W_\nu^\gamma(-k)\end{pmatrix}$$

$$\tilde{v}_\nu^H(\vec{q},\varepsilon) = \frac{\delta[\varepsilon-\mathcal{E}_\nu(k)]\,\delta^3(\vec{k}-\vec{q})}{\sqrt{2E(k)}}\,. \qquad (43)$$

With the renormalized constants $Z_\nu^\gamma(k)$, $W_\nu^\gamma(-k)$, $Z_\nu^{Ex}(k)$, $W_\nu^{Ex}(-k)$.

From the spectral representation of the Green function we have

$$D_{11}^H(\vec{k},\omega) = \frac{1}{\epsilon_0}\sum_{\nu=1,2}\left\{\frac{|Z_\nu^\gamma(k)|^2}{2\omega(k)[\mathcal{E}_\nu(k)-\omega]} + \frac{|W_\nu^\gamma(-k)|^2}{2\omega(k)[\mathcal{E}_\nu(k)+\omega]}\right\}$$

$$G_{11}^H(\vec{k},\omega) = \sum_{\nu=1,2}\left\{\frac{|Z_\nu^{Ex}(k)|^2}{2E(k)[\mathcal{E}_\nu(k)-\omega]} + \frac{|W_\nu^{Ex}(-k)|^2}{2E(k)[\mathcal{E}_\nu(k)+\omega]}\right\} \qquad (44)$$

By a comparison of Eqs.(44) and (36) we obtain

$$|Z_\nu^\gamma(k)|^2 = \frac{[\mathcal{E}_\nu(k)+\omega(k)]^2}{4\omega(k)\mathcal{E}_\nu(k)}|\beta_\nu^\gamma(k)|^2$$

$$|W_\nu^\gamma(-k)|^2 = \frac{[\mathcal{E}_\nu(k)-\omega(k)]^2}{4\omega(k)\mathcal{E}_\nu(k)}|\beta_\nu^\gamma(k)|^2$$

$$|Z_\nu^{Ex}(k)|^2 = \frac{[\mathcal{E}_\nu(k)+E(k)]}{4E(k)\mathcal{E}_\nu(k)}|\beta_\nu^{Ex}(k)|^2$$

$$|W_\nu^{Ex}(k)|^2 = \frac{[\mathcal{E}_\nu(k)-E(k)]}{4E(k)\mathcal{E}_\nu(k)}|\beta_\nu^{Ex}(k)|^2\,. \qquad (45)$$

From (14), (15), (37) and (45) it follows that with a suitable choice of the phase factors of the renormalization constants

$$Z_\nu^\gamma(k),\ W_\nu^\gamma(-k),\ Z_\nu^{E_x}(k),\ W_\nu^{E_x}(-k)$$

they exactly coincide with the Bogolubov transformation coefficients

$$u_\nu^\gamma(k),\ v_\nu^\gamma(k),\ u_\nu^{E_x}(k),\ v_\nu^{E_x}(k)\ .$$

If we use the phase factor as in Eqs.(14) we have the following identities

$$\frac{1}{\sqrt{\epsilon_0\omega(k)}}\begin{pmatrix} Z_\nu^\gamma(k) \\ W_\nu^\gamma(-k)\end{pmatrix} = g\,\tilde{D}(\vec{k},\mathcal{E}_\nu(k))\,\hat{\tau}\,\frac{1}{\sqrt{E(k)}}\begin{pmatrix} Z_\nu^{E_x}(k) \\ W_\nu^{E_x}(-k)\end{pmatrix} \tag{46a}$$

$$\frac{1}{\sqrt{E(k)}}\begin{pmatrix} Z_\nu^{E_x}(k) \\ W_\nu^{E_x}(-k)\end{pmatrix} = g\,\tilde{G}(\vec{k},\mathcal{E}_\nu(k))\,\hat{\tau}\,\frac{1}{\sqrt{\epsilon_0\omega(k)}}\begin{pmatrix} Z_\nu^{\gamma}(k) \\ W_\nu^{\gamma}(-k)\end{pmatrix} . \tag{46b}$$

They will be used in Sec.IV.

IV. APPLICATION AND DISCUSSION

Let us apply the above results to the study of interaction processes with the participation of the polaritons. To explain the method we shall consider two simple examples. As a first example we consider the process

$$P_\nu(\vec{k}) + a \rightarrow b + c + \cdots \tag{I}$$

where a,b,c denote some elementary excitations different from polaritons, $P_\nu(\vec{k})$ is the polariton in the branch ν with the momentum $\vec{k}$ this process has been considered in Ref.20, and we will not go into details here. We just want to recall that besides the processes

$$\gamma(\vec{k}) + a \rightarrow b + c + \cdots \tag{Ia}$$

$$E_x(\vec{k}) + a \rightarrow b + c + \cdots \tag{Ib}$$

which give the main contribution to the matrix element of the process (I) there are two other processes which can also give some contribution

$$a \rightarrow \gamma(-\vec{k}) + b + c + \cdots \tag{Ic}$$

$$a \rightarrow E_x(-\vec{k}) + b + c + \cdots \tag{Id}$$

This contribution is negligible in comparison with that of (Ia) and (Ib) in the resonant domain and is often negelected in the resonant approximation. Let the matrix elements of these processes in the absence of the photon-exciton interaction be T_{Ia}, T_{Ib}, T_{Ic}, T_{Id} , respectively.

From expression (46) we have

$$\begin{aligned}
\frac{Z_\nu^\gamma(k)}{\sqrt{\epsilon_0\omega(k)}} &= \frac{g}{\sqrt{E(k)}}\,D_{11}[\vec{k},\mathcal{E}_\nu(k)]\,[\,Z_\nu^{E_x}(k) + W_\nu^{E_x}(-k)\,] \\
\frac{W_\nu^\gamma(-k)}{\sqrt{\epsilon_0\omega(k)}} &= \frac{g}{\sqrt{E(k)}}\,D_{22}[\vec{k},\mathcal{E}_\nu(k)]\,[\,Z_\nu^{E_x}(k) + W_\nu^{E_x}(-k)\,] \\
\frac{Z_\nu^{E_x}(k)}{\sqrt{E(k)}} &= \frac{g}{\sqrt{\epsilon_0\omega(k)}}\,G_{11}[\vec{k},\mathcal{E}_\nu(k)]\,[\,Z_\nu^{\gamma}(k) + W_\nu^{\gamma}(-k)\,] \\
\frac{W_\nu^{E_x}(-k)}{\sqrt{E(k)}} &= \frac{g}{\sqrt{\epsilon_0\omega(k)}}\,G_{22}[\vec{k},\mathcal{E}_\nu(k)]\,[\,Z_\nu^{\gamma}(k) + W_\nu^{\gamma}(-k)\,] .
\end{aligned} \tag{47}$$

By a manner analogous with that in Ref.20 we obtain the following exact matrix element of process (I):

$$\begin{aligned}
T_\nu^{pol}(\vec{k}) = {} & Z_\nu^\gamma(k)\,\Big[\,T_{Ia} + g\sqrt{\frac{E(k)}{\epsilon_0\omega(k)}}\,G_{11}[\vec{k},\mathcal{E}_\nu(k)]\,T_{Ib} \\
& + g\sqrt{\frac{E(k)}{\epsilon_0\omega(k)}}\,G_{22}[\vec{k},\mathcal{E}_\nu(k)]\,T_{Id}\,\Big] + \\
& + W_\nu^\gamma(-k)\,\Big[\,T_{Ic} + g\sqrt{\frac{E(k)}{\epsilon_0\omega(k)}}\,G_{11}[\vec{k},\mathcal{E}_\nu(k)]\,T_{Ib} \\
& + g\sqrt{\frac{E(k)}{\epsilon_0\omega(k)}}\,G_{22}[\vec{k},\mathcal{E}_\nu(k)]\,T_{Id}\,\Big] .
\end{aligned} \tag{48}$$

According to (47) $T_{\nu}^{pol}(\vec{k})$ can be rewritten in an equivalent form

$$T_{\nu}^{pol}(\vec{k}) = Z_{\nu}^{\gamma}(k)\, T_{Ia} + Z_{\nu}^{Ex}(k)\, T_{Ib} + W_{\nu}^{\gamma}(-k)\, T_{Ic} + W_{\nu}^{Ex}(-k)\, T_{Id} \quad (49)$$

Before considering the second example we formulate the recipe for the construction of the matrix element of each interaction process with the participation of polaritons in the initial and final states. From the above line of reasoning we see that for this purpose it is enough to write down all the lowest order matrix elements of this interaction process with the direct absorption and (or) emission of photon or exciton without exciton-photon transition. The sum of these lowest order matrix elements which are multiplied by the corresponding renormalization constants $Z_{\nu}^{\gamma}(k)$ for an external photon and $Z_{\nu}^{Ex}(k)$ for an external exciton will be the main contribution to the total matrix element of the polariton interaction process. To obtain the non-resonant terms of the matrix element of this process we must write down all those lowest order matrix elements but instead of the absorption (or emission) of one photon (or exciton) we have the emission (or absorption) of this photon (or this exciton) with the momentum in the inverse direction. Each matrix element is then multiplied by the corresponding renormalization constants: $W_{\nu}^{\gamma}(-k)$ for a substituted external photon and $W_{\nu}^{Ex}(-k)$ for a substituted external exciton. The sum of these matrix elements will be the non-resonant contribution to the total matrix element of this polariton interaction process.

Now we consider the following process:

$$P_{\nu_1}(\vec{k}_1) + a \rightarrow P_{\nu_2}(\vec{k}_2) + b + c + \cdots \quad (II)$$

Corresponding to the above recipe we have the following lowest-order processes which give the main contribution to the total matrix element:

$$\gamma(\vec{k}_1) + a \rightarrow \gamma(\vec{k}_2) + b + c + \cdots \quad (IIa)$$

$$E_x(\vec{k}_1) + a \rightarrow \gamma(\vec{k}_2) + b + c + \cdots \quad (IIb)$$

$$\gamma(\vec{k}_1) + a \rightarrow E_x(\vec{k}_2) + b + c + \cdots \quad (IIc)$$

$$E_x(\vec{k}_1) + a \rightarrow E_x(\vec{k}_2) + b + c + \cdots \quad (IId)$$

and the other lowest-order processes which give the non-resonant contribution

$$\gamma(-\vec{k}_2) + a \rightarrow \gamma(-\vec{k}_1) + b + c + \cdots \quad (IIe)$$

$$\gamma(-\vec{k}_2) + a \rightarrow E_x(-\vec{k}_1) + b + c + \cdots \quad (IIf)$$

$$E_x(-\vec{k}_2) + a \rightarrow \gamma(-\vec{k}_1) + b + c + \cdots \quad (IIg)$$

$$E_x(-\vec{k}_2) + a \rightarrow E_x(-\vec{k}_1) + b + c + \cdots \quad (IIh)$$

$$\gamma(\vec{k}_1) + \gamma(-\vec{k}_2) + a \rightarrow b + c + \cdots \quad (IIi)$$

$$a \rightarrow \gamma(-\vec{k}_1) + \gamma(\vec{k}_2) + b + c + \cdots \quad (IIj)$$

$$\gamma(-\vec{k}_2) + E_x(\vec{k}_1) + a \rightarrow b + c + \cdots \quad (IIk)$$

$$a \rightarrow \gamma(\vec{k}_2) + E_x(-\vec{k}_1) + b + c + \cdots \quad (IIm)$$

$$E_x(-\vec{k}_2) + \gamma(\vec{k}_1) + a \rightarrow b + c + \cdots \quad (IIn)$$

$$a \rightarrow E_x(\vec{k}_2) + \gamma(-\vec{k}_1) + b + c + \cdots \quad (IIo)$$

$$E_x(\vec{k}_1) + E_x(-\vec{k}_2) + a \rightarrow b + c + \cdots \quad (IIp)$$

$$a \rightarrow E_x(\vec{k}_2) + E_x(-\vec{k}_1) + b + c + \cdots \quad (IIq)$$

Then the total matrix element can be written as

$$T^{pol\ pol}_{\nu_1(\vec{k}_1)\ \nu_2(\vec{k}_2)} =$$

$$= Z_{\nu_1}^{\gamma}(k_1) Z_{\nu_2}^{\gamma}(k_2)\, T_{IIa} + Z_{\nu_1}^{Ex}(k_1) Z_{\nu_2}^{\gamma}(k_2)\, T_{IIb} +$$

$$+ Z_{\nu_1}^{\gamma}(k_1) Z_{\nu_2}^{Ex}(k_2)\, T_{IIc} + Z_{\nu_1}^{Ex}(k_1) Z_{\nu_2}^{Ex}(k_2)\, T_{IId} +$$

$$
\begin{aligned}
&+ W_{\nu_2}^{\gamma}(-k_2)\, W_{\nu_1}^{\gamma}(-k_1)\, T_{II e} + W_{\nu_2}^{\gamma}(-k_2)\, W_{\nu_1}^{Ex}(-k_1)\, T_{II f} + \\
&+ W_{\nu_2}^{Ex}(-k_2)\, W_{\nu_1}^{\gamma}(-k_1)\, T_{II g} + W_{\nu_2}^{Ex}(-k_2)\, W_{\nu_1}^{Ex}(-k_1)\, T_{II h} + \\
&+ Z_{\nu_1}^{\gamma}(k_1)\, W_{\nu_2}^{\gamma}(-k_2)\, T_{II i} + Z_{\nu_2}^{\gamma}(k_2)\, W_{\nu_1}^{\gamma}(-k_1)\, T_{II j} + \\
&+ Z_{\nu_1}^{Ex}(k_1)\, W_{\nu_2}^{\gamma}(-k_2)\, T_{II k} + Z_{\nu_2}^{\gamma}(k_2)\, W_{\nu_1}^{Ex}(-k_1)\, T_{II m} + \\
&+ Z_{\nu_1}^{\gamma}(k_1)\, W_{\nu_2}^{Ex}(-k_2)\, T_{II n} + Z_{\nu_2}^{Ex}(k_2)\, W_{\nu_1}^{\gamma}(-k_1)\, T_{II o} + \\
&+ Z_{\nu_1}^{Ex}(k_1)\, W_{\nu_2}^{Ex}(-k_2)\, T_{II p} + Z_{\nu_2}^{Ex}(k_2)\, W_{\nu_1}^{Ex}(-k_1)\, T_{II q} .
\end{aligned}
$$

The matrix elements of processes (I) and (II) can also be calculated by the Bogolubov diagonalization method. From the expression (16) we can transform the interaction Hamiltonian of photons and excitons with other elementary excitations into the interaction Hamiltonian of polaritons with them. Then by use of this interaction Hamiltonian we easily obtain the same expressions (49) and (50) for the $T_{\nu}^{pol}(\vec{k})$ and $T^{pol\ pol}_{\nu_1(\vec{k}_1)\nu_2(\vec{k}_2)}$ but instead of the renormalization constants $Z_{\nu}^{\gamma}(k)$, $Z_{\nu}^{Ex}(k)$, $W_{\nu}^{\gamma}(-k)$, $W_{\nu}^{Ex}(-k)$ we have the diagonalization constants $U_{\nu}^{\gamma}(k)$, $U_{\nu}^{Ex}(k)$, $V_{\nu}^{\gamma}(k)$, $V_{\nu}^{Ex}(k)$. Due to the suitable choice of the phase factors it is shown above that these constants exactly coincide. The dispersions of polariton derived by the two approaches also coincide. We conclude that these two approaches are equivalent for the simple model of direct band gap semiconductor. Our result can be generalized to more complicated cases.

In conclusion we emphasize once more that the Green function approach which was derived in Ref.20 can include the effect of the damping of polaritons. The results of the treatment of the damping effect of polaritons will be published elsewhere.

ACKNOWLEDGMENTS

The authors would like to thank Professor Nguyen Van Hieu for suggesting the problem and fruitful discussions. One of them (Ha Vinh Tan) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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