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RPA METHOD BASED ON THE SELF-CONSISTENT CRANKING MODEL FOR ^{168}Er AND ^{158}Dy

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Recently, we have seen papers[1,2]. where attempts have been made to interpret some low-lying positive parity rotational bands in ^{168}Er as bands based on two-gamma phonon vibrational states. The question of existence of two-phonon states in the low-lying spectrum has been discussed extensively in recent years (see, e.g., ref.[3]). In the present note we study this question in the framework of RPA method based on the SCCM (see refs. [4-6]). The second aim of this paper is to test the applicability of the RPA + SCCM method, originally introduced for high spins, in the low-spin region of nuclear spectrum.

We have started with the cranking Hamiltonian[4,5]:

$$H' = H - \sum_{\tau} \lambda_{\tau} \hat{N}_{\tau} - \Omega \hat{J}_{x}. \tag{1}$$

The total nuclear Hamiltonian H consists of three parts[4,5]:

$$H = H_{AV} - \frac{1}{4} \sum_{\tau} G_{\tau} P_{\tau}^{+} P_{\tau} - \frac{1}{2} \sum_{\mu=-2}^{2} \kappa_{\mu} \mathcal{Q}_{2\mu}^{+} \mathcal{Q}_{2\mu}. \tag{2}$$

where H_{AV} describes the average nuclear field, the second term represents pairing correlations and the third one stands for the residual interactions which are assumed in a separable quadrupole-quadrupole form, as we restrict ourselves to positive parity states only. To get the agreement with experimental data we introduce, in contrast to ref.[4]. the dependence of strength constants of the quadrupole-quadrupole term on projection μ. The total Hamiltonian H has to satisfy the following symmetry conditions[4-6]:

$$[H, \hat{J}_{i}] = [H, \hat{N}_{\tau}] = 0 \rightarrow \begin{array}{ll} [H', \hat{J}_{x}] = 0, & [H', \hat{J}_{z}] = -i\Omega \hat{J}_{y} \\ [H', \hat{J}_{y}] = i\Omega \hat{J}_{z}, & [H', \hat{N}_{\tau}] = 0. \end{array} \tag{3}$$

The SCCM + RPA method involves two separate steps:

i) In the first step, the SCCM solution is found. For this purpose we have followed the method described in ref.[7], where axially deformed Woods-Saxon Hamiltonian was used for the average field H_{AV} in solving the Hartree-Fock-Bogolubov problem. Its parameters for the nuclei considered here have been taken from ref.[8]. The deformation parameters have been obtained using the Strutinski method and they are: $\beta_2 = 0.265$, $\beta_4 = 0.044$

for ^{158}Dy and $\beta_2 = 0.284$, $\beta_4 = -0.001$ for ^{168}Er. The pairing strength constants G_τ have been determined from the experimental values of neutron and proton pairing energies for nonrotating nuclei and kept constant for all spins, J. The pairing gap begins to decrease at $J = 8$ for ^{168}Er and $J = 6$ for ^{158}Dy and therefore we have restricted the calculations to $J \leq 8$ and $J \leq 6$, respectively.

ii) In the second step, the vibrations about the SCCM solutions (i.e., about the yrast line states) are described by the RPA method. Introducing the two-quasiparticle bosons as in refs. /4,5/ $b^{+}_{k\bar\ell} = \alpha^{+}_{k}\alpha^{+}_{\bar\ell}$, $b^{+}_{k\ell} = i\alpha^{+}_{k}\alpha^{+}_{\ell}$, $b^{+}_{\bar k\bar\ell} = i\alpha^{+}_{\bar k}\alpha^{+}_{\bar\ell}$, the Hamiltonian H' can be divided, up to the second order in the boson expansion, into two parts:

$$H' = \langle\Omega\,|H'|\,\Omega\rangle + H_{(+)} + H_{(-)}, \tag{4}$$

where $\langle\Omega\,|H'|\,\Omega\rangle$ is the mean value in the quasiparticle vacuum (yrast state with given Ω). Since H' is invariant with respect to the rotation by angle π around the x-axis, $R_\pi(\pi)$, the RPA *equations for the positive* $(H_{(+)})$ *and negative* $(H_{(-)})$ *signature* parts of the Hamiltonian H' separate, and one can write the phonon creation operators as a linear combination of bosons $b^{+}_{k\bar\ell}$, $b_{k\bar\ell}$ (for $H_{(+)}$), and $b^{+}_{k\ell}$, $b_{k\ell}$, $b^{+}_{\bar k\bar\ell}$, $b_{\bar k\bar\ell}$ (for $H_{(-)}$). The RPA equations of motion provide a system of linear homogeneous equations for coefficients in these combinations and the secular equation for energy $\omega_\lambda(\Omega)$ of the one-phonon states, $Q^{+}_{\lambda}(\pm)|\Omega\rangle$ (see refs. /4-8/).

From $R_x(\pi)$ symmetry it follows that the positive signature one-phonon states $Q^{+}_{\lambda}(+)|\Omega\rangle$ correspond to even values of the total spin, J, and the negative signature ones, $Q^{+}_{\lambda}(-)|\Omega\rangle$, to odd values, respectively (see refs. /4,5/). From the comparison of the RPA equations of motion with the symmetry conditions (3) one can see that $\hat J_x$ and $\hat N_\tau$, with their conjugated angles θ_x and θ_τ, represent the zero-energy Goldstone modes of $H_{(+)}$, and that one can construct the normal modes of $H_{(-)}$ with energy $\omega = \Omega$ from $\hat J_y$ and $\hat J_z$ (see refs. /4,5/). The orthogonality of all normal modes of the RPA solutions to the Goldstone mode (θ_x, J_x) and the fact that in SCCM one has $\langle\Omega|J_x|\Omega\rangle \approx \sqrt{J(J+1)}$ justify the interpretation of the RPA solutions as is schematically shown in fig.1.

The energies to be compared with the experimental ones are then given by:

$$E_\lambda(J) = E_{yr}(J_0) + \frac{h^2}{2\mathcal{J}(J_0)}[J(J+1) - J_0(J_0+1)] + h\omega_\lambda(J_0), \tag{5}$$

where $J = J_0$ and $J = J_0 \pm 1$ *for even and odd* spins J, respectively, $\mathcal{J}(J_0)$ represents the moment of inertia of a nuclear with spin J_0,

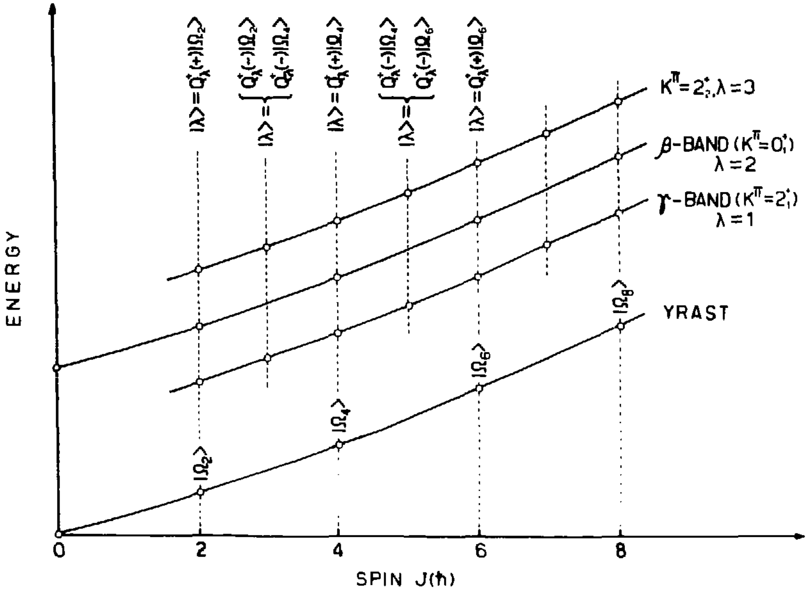

Fig.1. Scheme of the interpretation of the one-phonon states near the yrast line in RPA + SCCM method for low-spin region.

and can be obtained from the symmetry condition $[H', \hat{J}_x] = 0$. For $\Omega = 0$ it is given by the simple Belyaev formula (see refs. /6,10/). In the same way the condition $[H', \hat{N}_\tau] = 0$ determines the values of λ_τ in (1).

Since our Woods-Saxon deformed average field violates the symmetries given by Eq.(3), the strength constants κ_μ in Eq.(2) have been chosen appropriately to restore the required symmetry. Analogously, the experimental values of G_τ in Eq.(2) have to be slightly renormalized to satisfy $[H', \hat{N}_\tau] = 0$. The symmetry relations (3) provide only the condition for κ_1. The remaining constants, κ_0 and κ_2, have been determined from the experimental values of energies of the lowest levels in β- and γ-band, respectively. The values of the strength constants G_τ, κ_μ for both nuclei are shown in table 1.

In table 1 the column for constants G_τ denoted by HFB contains values obtained from experimental pairing energy and the column denoted by RPA contains the values obtained from the condition $[H', \hat{N}_\tau] = 0$. Only one number (for spin $J = 2$) is given because the values of κ_1 and $(G_\tau)_{RPA}$ appear spin-independent within 0.5 per cent.

The spin dependence of the moment of inertia is presented in table 2. For comparison, the moments of inertia calculated from the Belyaev formula are given together with our results.

Table 1

Strength constants of the residual interaction of the Hamiltonian H

	$\varkappa_0$ $\left[10^{-3}\frac{keV}{fm^4}\right]$	$\varkappa_1=\varkappa_{-1}$ $\left[10^{-3}\frac{keV}{fm^4}\right]$	$\varkappa_2=\varkappa_{-2}$ $\left[10^{-3}\frac{keV}{fm^4}\right]$	G_N [MeV]		G_p [MeV]	
				HFB	RPA	HFB	RPA
^{168}Er	0.816	0.774	1.514	0.104	0.085	0.134	0.119
^{158}Dy	0.808	0.833	1.602	0.111	0.093	0.135	0.123

Table 2

The dependence of moment of inertia in spin

Spin		0	2	3	4	5	6	7	8
^{168}Er	$\mathcal{J}_x$ [$\hbar^2$/Mev]	27.97	29.35	/	32.97	/	41.80	/	62.09
	$\mathcal{J}_x$ or $\mathcal{J}_y$ Belyaev [$\hbar^2$/Mev]	27.97	29.02	28.55	31.63	32.19	37.42	36.07	56.30
^{158}Dy	$\mathcal{J}_x$ [$\hbar^2$/Mev]	26.97	29.24	/	36.56	/	65.41	-	-
	$\mathcal{J}_x$ or $\mathcal{J}_y$ Belyaev [$\hbar^2$/Mev]	26.97	28.74	30.04	34.10	34.63	52.45	-	-

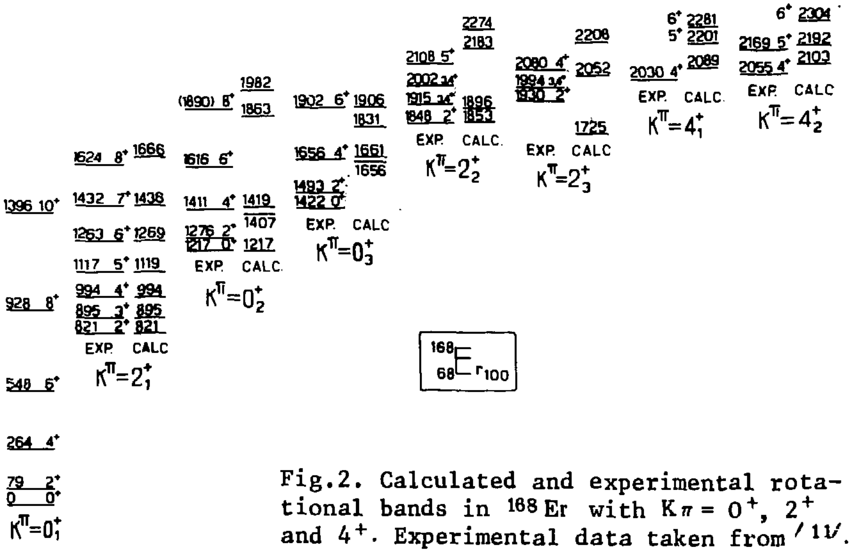

Fig.2. Calculated and experimental rotational bands in ^{168}Er with $K\pi = 0^+$, 2^+ and 4^+. Experimental data taken from /11/.

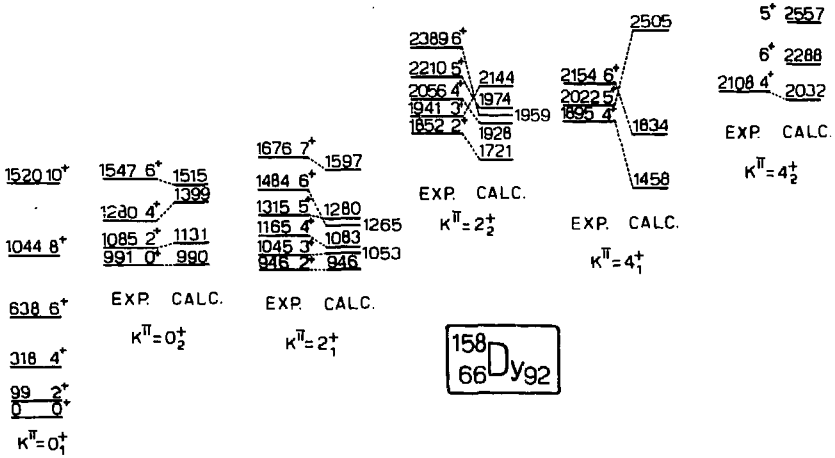

Fig.3. Calculated and experimental rotational positive parity bands in ^{158}Dy. Experimental data taken from /12/.

The calculated spectra for both nuclei are shown in figs.2 and 3, where the experimental values are taken from refs./11,12/. Since the results for $(G_\tau)_{HFB}$ and $(G_\tau)_{RPA}$ practically coincide, only the values obtained with $(G_\tau)_{RPA}$ are presented. To establigh the correspondence between the experimental and our calculated points, it has been required that the structure of phonons building up one rotational band should change slowly with spin (adiabatic condition).

Figures 2 and 3 show that all solutions of RPA equations can be interpreted experimentally, and that RPA + SCCM method provides quite good agreement with experiment in the low-energy region, without introducing two phonon states. The only exception is the K = 3_1^+ band in ^{168}Er (see ref./12/) which has no experimental partner in RPA + SCCM and therefore is not involved in fig.2.

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Квасил Я. и др. E4-83-314
Применение ПСФ + ССКМ в ядрах ^{168}Er и ^{158}Dy

Анализируются низколежащие состояния ядер ^{168}Er и ^{158}Dy в приближении случайной фазы /ПСФ/, основанные на самосогласованной кренкинг-модели/ССКМ/. Момент инерции, значение химического потенциала и силовая константа κ_1 определяются из условия симметрии. Константы κ_0, κ_2 определяются из экспериментальных значений энергий низколежащих уровней в β- и γ-полосах. Константы парного взаимодействия G_τ определены из экспериментальных значений нейтронных и протонных парных энергий для неротационных ядер. Получены спектры энергий состояний с положительной четностью, которые хорошо согласуются с экспериментальными данными без введения двухфононных вибрационных состояний.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Kvasil J. et al. E4-83-314
RPA Method Based on the Self-Consistent Cranking Model
for ^{168}Er and ^{158}Dy

The low-lying nuclear states in ^{168}Er and ^{158}Dy are analysed within the Random Phase Approximation (RPA) method based on the Self-Consistent Cranking Model (SCCM). The moment of inertia, the value of chemical potential, and the strength constant κ_1 have been obtained from the symmetry condition. The constants κ_0, κ_2 have been determined from the experimental values of energies of the lowest levels in β- and γ-band, respectively. The pairing strength constants G_τ have been determined from the experimenral values of neutron and proton pairing energies for nonrotating nuclei. A quite good agreement with experimental energies of states with positive parity was obtained without introducing the two-phonon vibrational states.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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