A BOLTZMANN EQUATION APPROACH TO THE DAMPING OF GIANT RESONANCES IN NUCLEI+

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ABSTRACT

The Vlasov equation plus collision term (Boltzmann equation) represents an appropriate frame for the treatment of giant resonances (zero sound modes) in nuclei. With no adjustable parameters we obtain correct positions and widths for the giant quadrupole resonances.

It is well known [1] that schematic forces of the multipole-multipole type give a surprisingly good description of giant resonances within the RPA - or equivalently the TDHF-approach; this concerns for instance the position of energies and to less extent transition densities but it is of course impossible within this formalism to account for the (spreading) width of the giant resonances (we treat here intermediate to heavy nuclei for which the decay or escape width should be negligible).

We will be concerned with the giant quadrupole resonance only but our theory is applicable for other multipoles as well [2]. For our model (harmonic oscillator plus quadrupole-quadrupole force) the Vlasov equation becomes identical to the TDHF-equation and moments of the Vlasov equation with respect to powers of the momentum break off after the second. This means that the coupled fluid dynamic equations for density, velocity and pressure tensor are still exact.

To account for damping we include a collision term [3] to the Vlasov equation but still break off the moments after the second, certainly a very good approximation in view of what we said above. For the collision integral we take the form of Uehling-Uhlenbeck [4] which has certain approximative features as for instance energy conservation of the quasiparticle energies during the collision process and its validity for zero excitation only [4] . This however does not seem to be a serious drawback since we are able to explain the width of the giant quadrupole resonance quantitatively.

For the two body collisions we take for convenience only the s-wave part of the Gogny force as an average representation of the scattering in all channels and we neglect exchange. The Gogny force [5] has been chosen because it is a good phenomenological representation of a microscopic G-matrix which should be used in the collision integral [4]. The fact that we use different effective forces in the collision integral and for the mean field is not contradictory since they have to describe very different processes.

In this way we arrive at a model [6] for the damping of the giant quadrupole resonance which is free of parameters (the strength of the quadrupole-quadrupole force is as usual determined from the self-consistency condition of the harmonic oscillator).

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Our basic equation is the Boltzmann equation [4] :

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\cdot\frac{\partial f}{\partial \vec{R}} - \frac{\partial V}{\partial \vec{R}}\cdot\frac{\partial f}{\partial \vec{p}} = I[f] \tag{1}$$

$$I[f] = \frac{2\pi}{\hbar}\int \frac{d^3p'}{(2\pi\hbar)^3}\,\frac{d^3p_1}{(2\pi\hbar)^3}\,\frac{d^3p_1'}{(2\pi\hbar)^3}\, v^2(\vec{p}-\vec{p}_1)\,\delta(\vec{p}+\vec{p}'-\vec{p}_1-\vec{p}_1')$$

$$\delta(\varepsilon_p + \varepsilon_{p'} - \varepsilon_{p_1} - \varepsilon_{p_1'})\quad [\bar{f}\bar{f}'\, f_1 f_{1'} - f f' \bar{f}_1 \bar{f}_{1'}]$$

where $\bar{f} = 1 - f$ and $f' = f(\vec{R},\vec{p}',t)$ etc., $\varepsilon_p = p^2/2m$, and $v(p)$ is the Fourier transform of the two body interaction; the mean field is given by

$$V(\vec{R},t) = \frac{1}{2}\, m\, \omega_0^2 R^2 - \chi q(t)\,\hat{Q} \;=:\; \frac{1}{2}\, m\,(\Omega_x^2(t)\,X^2 + \Omega_y^2(t) Y^2 + \Omega_z^2(t) Z^2)$$

$$q(t) = \mathrm{Tr}(\hat{Q}\rho(t)); \quad \hat{Q} = 2Z^2 - X^2 - Y^2. \tag{2}$$

For this potential the left hand side of (1) is the exact transcription into Wigner space of $i\hbar\,\dot{\rho} = [h,\rho]$ and in general it is the $\hbar = 0$ limit of TDHF. It should be realized that the collision term in (1) represents the $\hbar \to 0$ limit of its quantum mechanical counterpart; this corresponds to treating collisions in the local density approximation, i.e. the nucleus is considered locally as a piece of nuclear matter. In this sense we think that the approximation of energy conservation during the scattering process is less important than in the discrete case. The approximation that I[f] or (1) is strictly only valid for zero excitation [4,7] is certainly less justified since we here are concerned with giant resonances. This point needs further studies in the future but at the moment we take the classical form of the collision term as it can be found in the literature [4]. Finally we want to point out that in the small amplitude limit studied here the collision term (1) can be derived from the sum of 2 particle-2 hole Feynmann graphs shown in Fig. 1 [4].

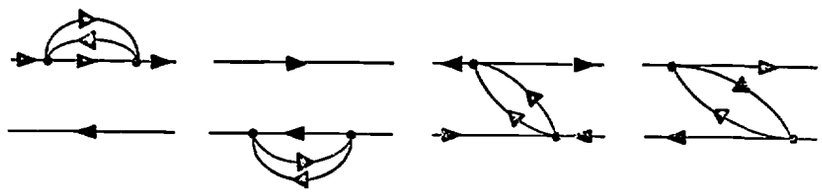

Fig. 1 Feynman graphs contributing to the collision integral in the linearised form.

Without collision term (1) can be solved analytically (up to the solution of classical eqs. of motion). The result is :

$$f(\vec{R},\vec{p},t) = F(\lambda - \tilde{H}) \tag{3}$$

where $F(x)$ is an arbitrary function and

$$\tilde{H} = \frac{1}{2m}\sum_{i=1}^{3}(\bar{p}_i^2 + \frac{m}{2}\,\Omega_i^2\,\bar{X}_i^2)$$

$$\bar{p}_i = \varepsilon_i(p_i - m\,u_i)\;;\quad \bar{X}_i = \frac{X_i}{\varepsilon_i}\;;\; u_i = \dot{\alpha}_i X_i\;;\; \alpha_i = \ln \varepsilon_i\;. \tag{4}$$

The parameters ε_i obey the (coupled) set of classical equations :

$$\ddot{\varepsilon}_i - \frac{\Omega_i^2(0)}{\varepsilon_i^3} + \Omega_i^2(t)\,\varepsilon_i = 0 \qquad \varepsilon_i(0) = 1;\;\; \dot{\varepsilon}_i(0) = 0 \tag{5}$$

In the small amplitude limit, χ being determined from the usual oscillator self-consistency condition, this yields the well known result [1]

$\Omega_{2^+} = \sqrt{2}\,\omega_0 \approx 60\, A^{-1/3}$ MeV in very good agreement with experiment.

For small amplitudes it is possible to develop (3) around equilibrium $F_0(\lambda - H)$ where F_0 is now the Wigner transform of the groundstate density matrix.

For what follows it is very important not to choose for F_0 a step function as would be suggested by lowest order Thomas Fermi theory [8]. This would correspond to an infinite Fermi system at zero temperature (T = 0) and consequently no two body collisions can take place since the collision integral is proportional to T^2 [9]. Nuclei are at zero temperature but even there a smearing out of the Fermi step function is present due to either two body correlations or, most importantly the finiteness of the system. For a harmonic potential one can deduce such a finite diffusivity T from a Strutinsky averaged Wigner transform of the density matrix [10]. One obtains a value constant over the nucleus of T = 4 MeV (A = 224) which is a rather high value indeed. More realistic is a Woods Saxon potential for which we show in Fig. 2 the phase space distribution [11] for A = 184 and in Fig.3 the corresponding diffusivity is extracted in fitting to the curves in Fig.2 a Fermi distribution of the type

$$f(R,p) = \left[1 + \exp \frac{\epsilon_p - \nu(R)}{T(R)}\right]^{-1} \qquad (6)$$

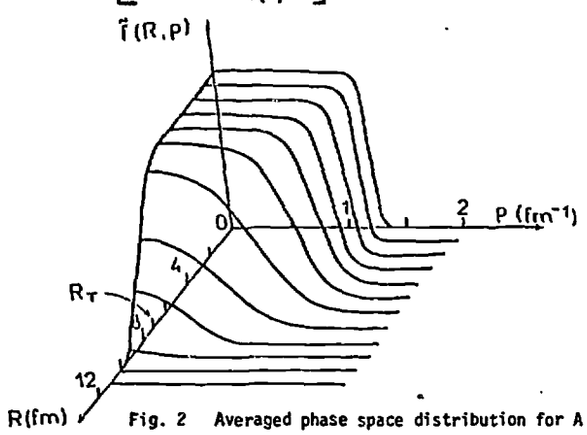

Fig. 2 Averaged phase space distribution for A = 184 in a Woods Saxon potential. For details see ref.[11].

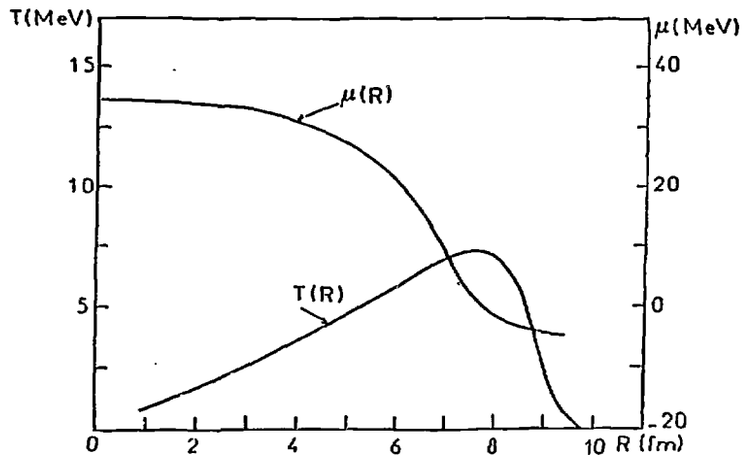

Fig. 3 Local diffusivity T and Fermi energy μ as extracted from the distribution in Fig. 2.

We see that here the diffusivity is strongly varying from small values in the interior to rather high values in the surface. In spite of this fact the Fermi function (6) can be quite effectively represented by the usual Sommerfeld expansion

$$f = \theta(\mu - \epsilon_p) - \frac{\pi^2}{6} T^2 \delta'(\mu - \epsilon_p) \qquad (7)$$

which then helps very much to perform the integrals in (1) . Taking in addition moments of (1) with respect to $\vec{p}$ and truncating after the

second we obtain [12] (we use summation convention) :

$$\dot{\rho} + \mathrm{div}\,(\rho\,\vec{u}) = 0$$

$$m\left(\frac{\partial}{\partial t} + u_k\,\frac{\partial}{\partial X_k}\right) u_i = -\,\frac{\partial V}{\partial X_i} - \frac{1}{\rho}\,\frac{\partial}{\partial X_k}\,P_{ki} \qquad 8(a\text{-}c)$$

$$\left(\frac{\partial}{\partial t} + u_k\,\frac{\partial}{\partial X_k}\right) P_{ij} + P_{ik}\,\frac{\partial}{\partial X_k}\,u_j + (i \leftrightarrow j) = \frac{1}{m}\int d^3p\; p_i p_j I$$

as a coupled system for ρ, $\vec{u}$, $\overleftrightarrow{P}$, the density, velocity field and pressure tensor [8]. As mentioned above these equations are exact in our model for vanishing collision term, which can be verified with (2) and (4).

Evaluating the second moment of the collision integral in (8c) one obtains in the small amplitude limit

$$\int d^3p\; p_i p_j I = \frac{m p^{id}}{\tau}\,\pi_{ij} \qquad (9)$$

where $\overleftrightarrow{\pi}$ is a traceless tensor defined by

$$P_{ij} = p^{id}\,(\delta_{ij} + \pi_{ij}), \qquad p^{id} = \frac{1}{5}\,(3\pi^2)^{2/3}\cdot\rho_o^{5/3} \qquad (10)$$

which describes the deformation of the Fermi surface. In (9) τ is the local collision time, p^{id} is the pressure of an ideal gas and ρ_o the corresponding density. In the same limit, and in first order of the expansion (7), the inverse collision-time can be calculated analytically and turns out to be

$$\frac{1}{\tau(R)} = \frac{m^3}{16\pi\hbar^7}\,T^2\cdot\frac{1}{p_F^5}\int_0^{2p_F} dp\; p^2\,(4p_F^2 - p^2)\,v^2(p)\,, \qquad (11)$$

$$p_F(R) = \left(\frac{3\pi^2}{2}\rho_o\right)^{1/3}.$$

In the case of a superposition of gaussians like the Gogny-force even the last integral in (11) can be performed, and we therefore get an explicit expression for the local collision time.

With (10) we can go over from eq.(8c) for the pressure tensor $\overleftrightarrow{P}$ to an equation for the deformation tensor $\overleftrightarrow{\pi}$. Making further use of the restriction to small amplitudes, we obtain

$$\pi_{ij} = \delta_{ij} - \int_{-\infty}^{t} dt'\; e^{-\frac{t-t'}{\tau}}\left(\frac{\partial u_i}{\partial X_j} + (i \leftrightarrow j) - \frac{2}{3}\,\frac{\partial u_i}{\partial X_i}\,\delta_{ij}\right)_{t'} \qquad (12)$$

In this limit we have in our model (see eq.(4)).

$$\vec{u}(t) = \dot{\alpha}(t)\vec{u}\;; \qquad \vec{u} = (-X, -Y, 2Z) \qquad (13)$$

and therefore we obtain a position independent π_{ij} if we introduce an effective inverse collision time averaged over the pressure which turns out to be the result of the expansion of (8) to first order in $(\Omega_\infty\,\tau)^{-1}$ (see (16)).

$$\frac{1}{\tau_{eff}} = \int d^3R\;\frac{p^{id}}{\tau}\Big/\int d^3R\; p^{id} \qquad (14)$$

From (8(a-c), 12-14) one then obtains [12] :

$$B\,\ddot{\alpha} + C_o\,\alpha + (C_\infty - C_o)\dot{\beta} = 0; \quad ; \quad \dot{\beta} = \int_{-\infty}^{t} dt' e^{-\frac{t-t'}{\tau_{eff}}}\,\dot{\alpha} \qquad (15)$$

with

$$B = m\int d^3r\;\rho|\vec{u}|^2 \qquad C_o = B\,\Omega_o^2 \;;\; C_\infty = B\,\Omega_\infty^2 \qquad (16)$$

where Ω_o and Ω_∞ are the well known [8,9] frequencies of normal sound ($\tau_{eff} = 0$) and zero sound ($\tau_{eff} = \infty$) respectively. Equation (15) is the one of a damped harmonic oscillator with a friction kernel which is non local in time reflecting the fact that an appreciable elapse of time can occur during successive collisions. This memory effect was discussed qualitatively by Nörenberg in a recent work [13] but no determination of τ_{eff} for zero sound modes was attempted. Here we give for the first time a completely self contained theory with no adjustable parameters for giant resonances in nuclei.

In order to discuss more specifically the influence of damping it is convenient to transform (15) to a third order differential equation [12] :

$$B\,\dddot{\beta} + \frac{B}{\tau_{eff}}\,\ddot{\beta} + C_\infty\dot{\beta} + \frac{C_0}{\tau_{eff}}\,\beta = 0 \qquad (17)$$

This equation immediately reproduces the well-known limiting frequencies of ordinary and zero sound but for finite τ_{eff}, as in our case, the characteristic equation of (16) has in general complex frequencies Ω as solution. In the limit $\mathrm{Re}\,\Omega\,\tau_{eff} \to \infty$ one obtains [12] :

$$\Omega = \Omega_\infty\left[1 + \frac{1}{4}\left(-\frac{1}{2} - \left(\frac{\Omega_0}{\Omega_\infty}\right)^2 + \frac{3}{2}\left(\frac{\Omega_0}{\Omega_\infty}\right)^4\right)\left(\frac{\Omega_\infty}{\tau_{eff}}\right)^2\right] - \frac{i}{2\tau_{eff}}\left(1 - \left(\frac{\Omega_0}{\Omega_\infty}\right)^2\right) \qquad (18)$$

From (14) and (9) we can calculate τ_{eff} with the s-wave part of the Gogny force [5] or more conveniently with a one Gaussian equivalent to the Gogny force : $v(r) = v_0\, e^{-r^2/r_0^2}$ with $v_0 = 26.5$ MeV and $r_0 = 2.25$ fm (this force yields the same pairing properties as the original Gogny force [14]). For the model nucleus A = 224 and the corresponding harmonic oscillator diffusivity [10] T = 4 MeV we then obtain a width of

$$\Gamma = \frac{T}{\tau_{eff}} = 3\ \mathrm{MeV} \qquad (19)$$

The only A-dependence of the width in our model comes from the A-dependence of T. As a matter of fact the distribution function $f(\varepsilon)$ for a harmonic oscillator [10] becomes (not unexpectedly) more and more infinite matter (i.e. step function) like the larger the nucleus and a detailed investigation [15] shows that the h.o. - diffusivity can be represented very accurately by the law (for a position dependent diffusivity in a Woods Saxon like potential (Fig. 2) this A-dependence could be changed somewhat)

$$T^2 \sim A^{-1/3} \qquad (20)$$

Together with (19) we thus have the smooth trend of the width for all nuclei which is represented together with the experimental values in Fig. 4. (continuous line).

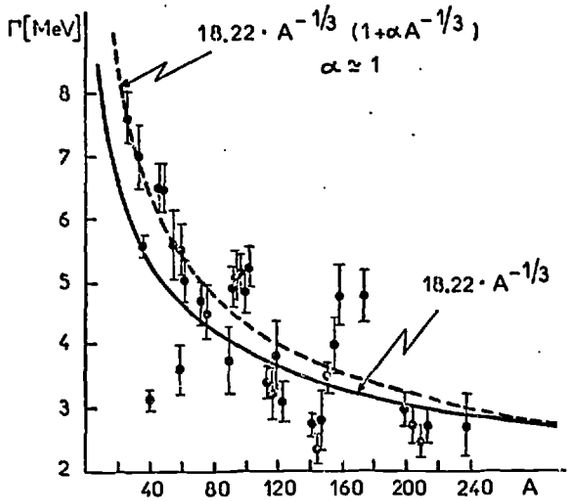

Fig. 4 Comparison of experiment [17] and our results for the width of the giant quadrupole resonance. The broken line includes the Landau correction for finite excitation energy.

Our self-contained model with no adjustable parameters agrees indeed very nicely with the experimental findings. This seems to indicate that our approximations, for instance the evaluation of two body collisions at zero excitation energy, are not too drastic. This can be substantiated somewhat in observing that Landau [7] proposed the following lowest order correction for finite excitation energies to our formula (11) : $T^2 \to T^2 + (\Omega/2\pi)^2$. Inserting numbers shows that $(\Omega/2\pi)^2 = T^2 \cdot A^{-1/3} \simeq 100 \cdot A^{-2/3}$, i.e. for heavy

nuclei this contribution to the width is about five times smaller than the one coming from the finite diffusivity of the phase snace distribution. For lighter nuclei this contribution is becoming more important as is indicated by the broken line in Fig. 4. Further studies have however to be performed to see in detail how excitation energy influences the rate of two body collisions. Possible ways how this can be done are given in Ref.[7]. We also would like to stress at this point that our theory is a linear one. A classical estimate for the quadrupole giant resonance in heavy nuclei yields roughly a 8 % change of nuclear half axes compared to their spherical equilibrium values. This seems to justify the linearization though second and higher order contributions like those shown in Fig.5 [16] cannot be ruled out completely.

In spite of these restrictions it seems evident from this work that the surprisingly strong diffusivity of the Wigner distribution and its A-dependence play an important role in the two-body scattering mechanism.

A more elaborate version of this work will be given elsewhere.

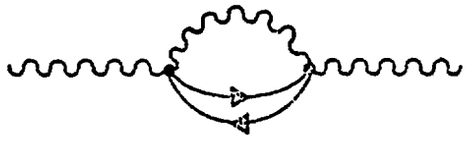

Fig. 5 Second order effect in the change of the density leading to incoherent particle scattering from the surface oscillation.

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