POLARIZATION IN HEAVY-ION REACTIONS

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Foreword

Determination of the polarization and spin alignment of reaction products emitted from heavy-ion reactions should provide a sensitive test of reaction mechanisms. Although polarization studies with heavy ions is really in its infancy the usefulness of such data has already been amply demonstrated.\textsuperscript{1} Techniques for producing both polarized beams and polarized targets are advancing rapidly. At the Oak Ridge National Laboratory interest in this field has lead to the design and construction of a laser optically pumped polarized target by illuminating a supersonic gas jet.\textsuperscript{2,3} This target, which is mounted in the scattering chamber of a magnetic spectrometer, will be used to observe effects when deformed polarized targets are bombarded by heavy ions.

Mutual research interests led us to invite Professor Fick, a pioneer in heavy-ion polarization research who recently reviewed\textsuperscript{4} the status of this field, to Oak Ridge. While at ORNL he presented a series of lectures on this subject. Drafts of these talks were compiled and written by B. Shivakumar\textsuperscript{5} and then corrected by Professor Fick. Polarization phenomena are attracting wide interest, therefore, we felt that these lectures should be presented as an ORNL technical report to make them more generally available.

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A Note to the Reader

What follows are lecture notes taken by a member of the audience, B. Shivakumar, and later checked by myself for accuracy in order to eliminate obvious misunderstandings and errors. No attempt has been made to generate material in a form suitable for publication in a journal or as a textbook. We have tried, wherever possible, to keep the text in its original form and to retain a "lecture notes" flavor. As a consequence, there are minor inconsistencies in notation, inadequate discussions of figures and a lack of completeness in several of the ideas presented. These shortcomings will, I hope, be taken care of by the list of references to the original work from where the interested reader can glean all the details that were not presented during the course of these lectures.

I thank all my colleagues for their kind hospitality during my stay and for their patience during my visit.

Dieter Fick
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1. General Considerations

The first lecture is going to be technical in nature. We will start by defining quantities used in the description of polarized particles. This will be followed by a brief discussion of the coordinate systems in use. Next we will define the quantities measured in experiments, namely the analyzing powers. In conclusion, a discussion of the symmetry and dynamical aspects of nucleus-nucleus interactions will tell us about the kind of information we can hope to obtain by measuring the analyzing powers of nuclear reactions.

Polarization nuclear physics is not a special type of nuclear physics. The use of a polarized source to perform an experiment is comparable to using a high resolution spectrometer or a good accelerator. There are certain classes of problems which can be addressed to advantage with the use of polarized beams but unfortunately, there are many more instances where the use of a polarized beam serves only to add to the existing confusion. In general, the attitude "If we do not understand a phenomenon and a polarized source is available, let's turn it on" is the wrong way to go.

Heavy ion polarization phenomena can be treated under two broad classifications, viz: entrance and exit channel phenomena. Figure 1.1a shows the collision of an aligned deformed nucleus with a spherical target nucleus. The effect of the alignment is to produce different overlaps of the interacting nuclei for the same impact parameter. This is an example of an entrance channel phenomenon. Angular momentum
transfer to the projectile in a deep-inelastic collision (as shown in fig. 1.1b), $\beta$-asymmetry, circular polarization of emitted gamma rays, and $m$-substate populations are examples of measurements which focus on exit channel phenomena.

In order to describe polarization phenomena we need to quantify the states of polarization of beams and targets. Our definitions are in accordance with the Madison convention (Mad 71) and the spherical tensors have been defined as by Brink and Satchler (Br Sa 71). For a comprehensive review of the quantities to be defined, we direct you to the article by M. Simonius (Sim 74) and to the introduction of the article by K. Sugimoto et al. (Sug 82). The two bases in general use for the description
of the polarization operators are the Cartesian and the spherical. We will use only the spherical basis extensively, as the usefulness of the Cartesian basis is restricted to the description of particles with spin 1/2 and maybe spin 1.

1.1. Polarization of Beams and Targets

The state of an ensemble of particles, with spin I, e.g. a beam or a target, can be described by a density matrix \( \rho_{mm'} \)

\[
\rho_{mm'} = \langle a_{m} a_{m}^{*} \rangle
\]  

(1.1)

where \( a_{m} \) and \( a_{m}^{*} \) represent the amplitudes for a particle in the ensemble to be in eigenstates \( |I_m\rangle \) and \( |I_{m'}\rangle \) of the system. Thus the elements of the density matrix are ensemble averages of the products of the amplitudes \( a_{m} \) and \( a_{m}^{*} \). The expectation value of any observable \( A \) can then be obtained as

\[
\langle A \rangle = \text{Tr}(\rho A). 
\]

(1.2)

It is convenient to work in a spherical basis with spherical tensor operators \( \tau_{kq} \), and expectation values \( t_{kq} \). The indices \( k \) and \( q \) are restricted to take the following values:

\[
0 \leq k \leq 2I \\
-k \leq q \leq k
\]

(1.3)

The operator \( \tau_{kq} \) is defined as:

\[
\tau_{kq} = \hat{\mathbf{S}} \sum_{m,m'} (-1)^{I-m}(I_{m'I_{m-1}}| kq \rangle | I_{m'}<I_{m}|)
\]

(1.4)
with \( \langle \text{Im'} \mid kq \mid \text{Im} \rangle \) a Clebsch-Gordan coefficient. The matrix elements of this operator are:

\[
\langle \text{Im'} \mid \tau_{kq} \mid \text{Im} \rangle = (\tau_{kq})_{m'm} = \hat{I}(-1)^{I-m}(\text{Im'} \mid l-m \mid kq)
\]

These tensors are normalized such that

\[
\tau_{oo} = E
\]

where \( E \) is the unit matrix. The expectation values of the tensor operators can then be obtained from eq. 1.2

\[
t_{kq} = \text{Tr}(\rho \tau_{kq}) = \sum_{mm'} \rho_{mm'}(\tau_{kq})_{m'm}
\]

whence, from eq. 1.5,

\[
t_{kq}(I) = \hat{I} \sum_{mm'} (-1)^{I-m}(\text{Im'} \mid l-m \mid kq)\rho_{mm'}
\]

with the normalization \( t_{oo}(I) = 1 \). The tensor operators \( \tau_{kq} \) and hence the expectation values \( t_{kq} \) transform under rotation like spherical harmonics:

\[
t_{kq} = \sum_{q'} t_{kq'}^q \delta_{q'q}(R).
\]

\( R \) is a set of Euler's angles of the rotation taking the old, primed axes in which \( t_{kq} \) is described, into the new, unprimed axes which are used to describe \( t_{kq} \). Note the different convention to define the rotation matrices, in particular the sign of \( R \) (Br Sa 71, footnote on p. 21).

In the case of a polarized source or a target, we can always work in a coordinate frame in which the density matrix is diagonal. This is not the case when the polarization is produced in a nuclear reaction. When
the density matrix is diagonal, we have
\[ \rho_{mm'} = \delta_{mm'} N_m \]  
(1.10)
where \( N_m \) are the occupational numbers of the substates \( m (\Sigma N_m = 1) \). For the statistical tensors, this implies \( q = 0 \) and eq. 1.8 then reduces to
\[ t_{k0}(I) = \hat{I} \sum_{m} (-1)^{I-m}(ImI-m \mid k0) N_m \]  
(1.11)
Passing this beam through a beam transport system (electrostatic mirrors, deflection magnets, Wien filters, etc.) the frame of reference changes. The components of the tensors \( t_{kq} \) in the new reference frame can easily be calculated by means of eq. 1.9.

We can now go through a few examples to see how the quantities \( t_{kq} \) are related to the occupation numbers \( N_m \). The spin = 1/2 case is trivial. We obtain from eq. 1.1 for spin = 1 (e.g.: \( \Lambda L_1 \))
\[ \sqrt{2/3} t_{10} = P_Z = (N+1 - N-1) \]  
(1.12a)
\[ \sqrt{2} t_{20} = P_{ZZ} = (N+1 - 2N_0 + N-1) \]  
\[ = 1 - 3N_0 \]  
(1.12b)
and for spin = 3/2 (e.g.: \( \Lambda L_1, \Lambda^n \Lambda \))
\[ \sqrt{579} t_{10} = P_Z = (N_{3/2} - N_{-3/2}) + (N_{+1/2} - N_{-1/2})/3 \]  
(1.13a)
\[ t_{20} = P_{ZZ} = (N_{+3/2} + N_{-3/2}) - (N_{+1/2} + N_{-1/2}) \]  
(1.13b)
\[ \sqrt{579} t_{30} = P_{ZZZ} = (N_{3/2} - N_{-3/2})/3 - (N_{+1/2} - N_{-1/2}) \]  
(1.13c)
In nuclear physics experiments polarized beams are most useful when only one rank $K$ does not vanish and the degree of polarization $t_{K0}$ for this rank reaches a maximum positive (or negative) value. "Inverting" eq. 1.1 one obtains

$$N_m = \frac{1}{2I+1} \sum_{I,I=0} (-1)^{I-I_m} (\ Salman-m|K_0) t_{K0} \tag{1.14}$$

From the conditions $t_{K0} = 0$ for $K \neq 0, K$, and the requirement for $t_{K0}$ to reach a maximum positive value one obtains with $t_{oo} = 1$ the following occupation numbers $N$

$$N_m = \frac{1}{2I+1} (1 + \frac{m}{I}) \text{, for } K = 1 \tag{1.15}$$

$$N_m = \frac{1}{2I+1} (1 + \frac{3m^2-I(I+1)}{I(I+1)}) \text{ for } K = 2 \text{ and } I \text{ integer} \tag{1.15}$$

$$N_m = \frac{1}{2I+1} (1 - \frac{3m^2-I(I+1)}{3/4 - I(I+1)}) \text{ for } K = 2 \text{ and } I \text{ half integer.}$$

The numbers $N_m$ for $K>2$ can be easily calculated if the Clebsch-Gordan coefficients $(\ Salman-m|K_0)$ are known.

Figures 1.2a and 1.2b show the magnetic substate population distributions of beams of pure rank $K = 1$ and 2 respectively with maximum positive polarization for a beam of spin $= 5/2$ (e.g. Eu).

'Conventional' polarized ion sources cannot produce beams of this quality (Ste 81). However recent developments in optical pumping and in high frequency transitions allow the production of beams in almost pure
m-substates (Jre 83). Beams with properties according to eq. 1.15 can be achieved by switching on beams in the various \( m \)-substates according to the probabilities indicated by these equations (for \( I = 5/2 \) according to figs. 1.2a and 1.2b).

1.2. Analyzing Powers, Coordinate Systems, Invariances

ANALYZING POWERS: We now go on to define the quantities we want to measure. We define an analyzing power \( T_{kq}(\theta) \) in terms of measured differential cross sections as:

\[
\sigma_{\text{pol}} = \sigma \cdot \sum_{k, q} t_{kq} T_{kq}^* \tag{1.16}
\]
where the subscript 'pol' indicates that the cross section is measured for a polarized beam. Defined in this manner, with $t_{00} = 1$ we obtain $T_{00} = 1$. The $T_{kq}$ are hermitian

$$T_{kq}^* = (-1)^q T_{k-q}. \quad (1.17)$$

We measure these quantities $T_{k0}$ by preparing a beam in a pure polarization state

$$t_{k0} = \delta_{kk} t_{k0}. \quad (1.18)$$

Substituting eq. 1.18 in eq. 1.16 we obtain

$$\sigma_{\text{pol}} = \sigma (1 + t_{k0} T_{k0}) \quad (1.19)$$

therefore,

$$T_{k0} = \frac{1}{t_{k0}} \left( \frac{\sigma_{\text{pol}}}{\sigma} - 1 \right). \quad (1.20)$$

In the general case when $q \neq 0$, the analyzing powers measured will be a combination of $T_{kq}$'s. We can then obtain the individual $T_{kq}$'s by repeating the experiment with beams polarized in different planes, as will be discussed.

COORDINATE SYSTEMS: The use of a coordinate system should be dictated by the physical problem and not by convention. Several conventions are in use, and three of them are described below. $\hat{a}$ denotes a unit vector in direction of the vector $a$.

A. Madison convention: notation $T_{kq}$:

$$\hat{z} = \hat{k}_1, \quad \hat{y} = \hat{n} = \frac{\hat{k}_1 \times \hat{k}_f}{|\hat{k}_1 \times \hat{k}_f|}, \quad \hat{x} = \hat{y} \times \hat{z} \quad (1.21)$$
B. Barred system: notation: $\mathbf{\bar{T}}_{kq}$

\[
\hat{z} = \frac{\hat{k}_{in} + \hat{k}_{fin}}{|\hat{k}_{in} + \hat{k}_{fin}|}, \quad \hat{y} = \hat{N} = \frac{\hat{k}_{in} \times \hat{k}_{fin}}{|\hat{k}_{in} \times \hat{k}_{fin}|}, \quad \hat{x} = \hat{y} \times \hat{z}
\]

(1.22)

C. Transverse system: notation $\mathbf{T}_{T_{kq}}$

\[
\hat{z} = \hat{N} = \frac{\hat{k}_{in} \times \hat{k}_{fin}}{|\hat{k}_{in} \times \hat{k}_{fin}|}, \quad \hat{y} = \hat{k}_{in}, \quad \hat{x} = \hat{y} \times \hat{z}
\]

(1.23)
Measurement of analyzing powers: In general analyzing powers are determined by comparing cross sections obtained for various polarization states (including the unpolarized one!) with each other (see eq. 1.20). Invariance of the interaction against a parity operation reduces the number of non zero analyzing powers considerably. First rank or vector analyzing powers are determined in experiments with vector polarized beams polarized along the normal to the scattering plane. If we label a purely vector polarized beam (eq. 1.15) by the sign "⅋" and the corresponding cross section by \( \sigma ( \tau ) \) (fig. 1.4) than the vector analyzing power \( i T_{11} \) in the coordinate system A (Madison convention) is obtained by

\[
i T_{11} = \sqrt{\frac{3I}{2I^2}} \left( \frac{\sigma (\tau)}{\sigma} - 1 \right)
\]  

(1.24)

In the transverse coordinate system the vector analyzing power for such an experiment is labeled by \( T_{10} \). It can be obtained from eq. 1.24 applying eq. 1.9 with the proper Euler angle of rotation defined through the definition of the coordinate systems (figs. 1.3).
Second rank or tensor analyzing powers are obtained with purely second rank tensor polarized (aligned) beams (eq. 1.15). If the alignment axes of such a beam is indicated by the self-explanatory symbol "++" then from fig. 1.5 we obtain expressions for the tensor analyzing power in terms of the cross section $\sigma(P)$, $P$ denoting the direction of the alignment axis (Hae 74). The factor in front of the ratios are the one for $I = 3/2$. The corresponding factors for $I = 1$ can be found elsewhere (Hae 74).

In practice, partially polarized beams are used in experiments, in which case the ratios of cross sections defined in eq. 1.24 and fig. 1.5 have to be normalized to the actual beam polarization (see eq. 1.20). The absolute determination of the beam polarization requires polarization standards for beams of rank $K$. Our present knowledge of standards for polarized HI beams are summarized in (Zup 79).
Fig. 1.5
(from W. Haeberl, Hae 74)
PARITY (Sim 74): We assume that parity is conserved. The $T_{kq}$ defined in coordinate system A (eq. 1.21) obey the following symmetry relations:

$$T_{kq} = (-1)^q T_{k-q}$$ \hspace{1cm} (1.25)

i.e.,

$$T_{k0} = 0 \quad \text{for } k \text{ odd.}$$ \hspace{1cm} (1.26)

With the hermiticity condition (eq. 1.17),

$$T_{kq} \text{ is real for } k \text{ even}$$

$$T_{kq} \text{ is pure imaginary for } k \text{ odd}$$ \hspace{1cm} (1.27)

The same equations hold for $T_{kq}$ defined in coordinate system B (eq. 1.22).

For the $T_{kq}$ defined in the 'transverse' coordinate system C (eq. 1.23)

$$T_{kq} = (-1)^q T_{k-q}$$ \hspace{1cm} (1.28)

holds. Therefore

$$T_{kq} = 0 \quad \text{for } q \text{ odd}$$

Using the hermiticity condition (eq. 1.17)

$$T_{k0} \text{ is real}.$$ \hspace{1cm} (1.27)

$T_{kq}$ are complex, for $q$ even and vanish for $q$ odd.

1.3. Symmetry Properties and Spin Dependence of Interaction

We restrict our discussion to the elastic scattering of spin 1 particle from a spin 0 target. We shall work with the $M$ matrix which
connects the initial spin state with the final one:

\[ |\text{Im}',\text{fin}\rangle = M |\text{Im}, \text{fin}\rangle \tag{1.30} \]

Besides a normalization constant it is identical to the well-known T-matrix. The matrix elements of the M-matrix, \( M_{m',m} \), depend on the initial and final momenta \( \hbar k_{\text{fin}} \) and \( \hbar k_{\text{fin}} \). In the discussion of observables, cross sections and analyzing powers, and in the discussion of their spin dependence we take full advantage of irreducible spherical tensors. Knowing the M-matrix the product of differential cross section \( \sigma \) and analyzing power \( T_{kq} \) is obtained from

\[ \sigma T_{kq} = \frac{1}{I^2} \text{Tr}(M^* T_{kq} M) \tag{1.31} \]

Putting \( k = q = 0 \) we obtain

\[ \sigma = \frac{1}{I^2} \text{Tr}(M^* M) \tag{1.32} \]

For the discussion of the spin dependence of the interaction it is advantageous to project from M its spherical components \( M_{kq} \) applying the spherical spin operators (eq. 1.4)(Ho Jo 71).

\[ M_{kq} = \frac{1}{I} \text{Tr}(\tau_{kq} M) \tag{1.33} \]

In terms of the spherical components of the M-matrix the product of differential cross section and tensor analyzing power can be expressed as (Ho Jo 71)

\[ \sigma T_{kq} = \sum_{k'q'} (-1)^{k-k'} \hat{\tau}^{k-k'} W(Ik' Ik'';Ik)(k'q'k''q'q)(k'q'k''q'q)(k'q'k''q'q)(k'q'k''q'q) \]

\[ \tag{1.34} \]
The $M_{kq}$ may be expressed in any of the coordinate systems defined earlier. The spherical components of the $M$-matrix obey now the following symmetry relations.

Parity conservation:

\[
M_{k-q} = (-1)^{k+q} M_{kq}, \text{ coordinate system } A
\]

\[
\bar{M}_{k-q} = (-1)^{k+q} \bar{M}_{kq}, \text{ coordinate system } B
\]

For both systems it follows that

\[
M_{ko} = 0, \bar{M}_{ko} = 0 \text{ for } k = \text{ odd}
\]

In the transverse coordinate system (System C) parity conservation requires

\[
T_{M_{kq}} = (-1)^{q} T_{M_{kq}}
\]

from which it follows that

\[
T_{M_{kq}} = 0 \text{ for } q = \text{ odd}
\]

In the case of elastic scattering we can apply the additional constraint of time reversal invariance. It implies

\[
M_{kq}(k_{f1n}, k_{f1n}) = (-1)^{k} M_{kq}(-k_{f1n}, -k_{f1n})
\]

It is obvious that the implications of time reversal invariance can be discussed more easily in a coordinate system for the definition of which only symmetric or anti-symmetric expressions of $k_{f1n}$ and $k_{f1n}$ are used. Therefore, coordinate system B is used now. Equation 1.39 is then
equivalent to

\[ \mathcal{M}_{kq} = \mathcal{M}_{k-q} \]  

(1.40)

which implies together with requirements from parity conservation (eq. 1.35)

\[ \mathcal{M}_{kq} = 0 \quad \text{for } k+q \text{ odd} \]  

(1.41)

Remember the bar indicates use of coordinate System B.

Dynamical considerations (Ho Jo 71):

We like to ask which of the \( M_{kq} \) are important. We know that

\[ M(E) = V + (E - H_0 - V + iE)^{-1} V \]  

(1.42)

In the first order Born approximation \( M(E) = V \); we expand \( V \) in the spin space, using again the spherical tensors

\[ V = \sum_{q} V_{kq} \tau_{kq}^{+} = V_{00} \tau_{00} + \sum_{k>0} V_{kq} \tau_{kq}^{+} \]  

(1.43)

\[ = V_c + V_s \]

where \( V_c \) denotes central interaction and \( V_s \) spin interaction.

\[ V_c = V_{00} \tau_{00} \]  

(1.44)

\[ V_s = \sum_{q} (V_{1q} \tau_{1q}^{+} + V_{2q} \tau_{2q}^{+} + \ldots) \]  

(1.45)
In the first order Born approximation

\[ M_{00} = V_{00} + \ldots \]

\[ M_{kq} = V_{kq} + \ldots \]

If we restrict operators in \( V_{kq} \) to terms at the most quadratic in momentum we can identify the various tensor interactions as follows.

\[ M_{00} = V_{00} : \text{central interaction} \]

\[ M_{1q} = V_{1q} : \mathbf{x} \cdot \mathbf{l} \text{ interaction} \]

\[ M_{2q} = V_{2q} : T_R, T_P \text{ and } T_L \text{ interaction} \quad (1.46) \]

\[ T = (\mathbf{l} \cdot \mathbf{R})^2 - I(I+1)/3 \]

\( T_R \) and \( T_L \) will not be discussed in what follows.

\[ M_{3q} = V_{3q} : T_3 = \frac{1}{2} \{ T_R ( \mathbf{z} \cdot \mathbf{I} + \mathbf{z} \cdot \mathbf{I} T_R ) + \frac{3}{2} T_R + \frac{2}{5} \mathbf{z} \cdot \mathbf{I} \} \]

This is a deformed \( \mathbf{z} \cdot \mathbf{I} \) interaction (Mu Gr 82)

For \( I = 1 \ k \leq 2 \) and from eq. 1.41 only \( \overline{M}_{00}, \overline{M}_{11}, \overline{M}_{20}, \overline{M}_{22} \) are not equal to zero. From means of eq. 1.34 we obtain
\[ \sigma = |\bar{M}_{00}|^2 + 2|\bar{M}_{11}|^2 + |\bar{M}_{20}|^2 + 2|\bar{M}_{22}|^2 \]

\[ \sigma_{\bar{T}_{11}} = 2 \text{Im} \left\{ (\bar{M}_{00} \bar{M}_{11}^*) + \frac{1}{\sqrt{6}} \bar{M}_{11} (\bar{M}_{20}^* + \sqrt{6} \bar{M}_{22}^*) \right\} \] (1.47)

\[ \sigma_{\bar{T}_{20}} = 2 \text{Re} (\bar{M}_{00} \bar{M}_{20}^*) - \frac{1}{\sqrt{2}} |\bar{M}_{11}|^2 + \frac{1}{\sqrt{2}} |\bar{M}_{20}|^2 - \frac{1}{\sqrt{2}} |\bar{M}_{22}|^2 \]

\[ \sigma_{\bar{T}_{21}} = -\sqrt{3} \text{Re} \bar{M}_{11} (\bar{M}_{22}^* - \sqrt{3/2} \bar{M}_{20}^*) \]

\[ \sigma_{\bar{T}_{22}} = 2 \text{Re} (\bar{M}_{00} \bar{M}_{20}^*) - \sqrt{3/4} |\bar{M}_{11}|^2 + \text{Re} (\bar{M}_{20} \bar{M}_{22}^*) \]

The central interaction is by far the strongest one. Therefore the leading terms on eqs. (1.4) are

\[ \sigma = |\bar{M}_{00}|^2 \]

\[ \sigma_{\bar{T}_{11}} = 2 \text{Im} (\bar{M}_{00} \bar{M}_{11}^*) \]

\[ \sigma_{\bar{T}_{20}} = 2 \text{Re} (\bar{M}_{00} \bar{M}_{20}^*) \] (1.48)

\[ \sigma_{\bar{T}_{21}} = 0 \]

\[ \sigma_{\bar{T}_{22}} = 2\text{Re} (\bar{M}_{00} \bar{M}_{20}^*) \]

Both vector or tensor analyzing powers are interference terms between central and $2\cdot S$ or $T_R$ interaction. In this approximation $\bar{T}_{21}$ vanishes (see Sect. 2) and $\bar{T}_{20}$ and $\bar{T}_{22}$ carry the same information. A precise measurement of $\bar{T}_{21}$ may give information on the magnitude of higher order terms.
A similar discussion was used for the elastic scattering of spin 3/2 particles (Co Ph 82). The rank 3 interaction was neglected and therefore the structure of equations obtained for the analyzing power was the same as for spin 1. For spin 3/2 beams the measurement of rank 3 analyzing power, e.g. \( T_{31} \), will yield information on the magnitude of deformed potentials.
References


LECTURE II

2. Interactions of Lithium - Simple Ideas

Introductory Remarks

Essential ideas presented in this talk and useful references pertaining to its material are contained in (Fic 81). To start with simple ideas, I would like to give you another picture of how you can think of vector and tensor polarized beams in terms of the moments of the nuclei.

purely vector polarized beam:

Fig. 2.1a

purely aligned beam:

Fig. 2.1b
Figure 2.1a shows a purely vector polarized beam. In this case the magnetic moments of the nuclei are pointing along a common axis (z-axis). Figure 2.1b shows a purely tensor polarized beam: all the quadrupole moments are aligned along a common axis (γ axis). When I talk of a quadrupole moment, it is the spectroscopic moment that I am referring to, not the intrinsic one. It is the spectroscopic moment that we really align in experiments.

I would like to remind you that $^6$Li has a ground-state spin $I = 1$ and is a nearly spherical nucleus. However, we can still define an axis of alignment because $^6$Li has a non-zero quadrupole moment. Lithium-7 has a ground-state spin $I = 3/2$ and a large static deformation. Figure 2.2 displays a sketch of both nuclei. They are represented by homogeneously charged rotational ellipsoids ($\mu$ denotes the magnetic moment).

![Fig. 2.2](image-url)
The organization of Sect. 2 will follow closely the situation in fig. 2.1. For most of the subsections (2.1-2.4), we have the situation of fig. 2.1b in mind: aligned quadrupole moments representing a tensor polarized beam. Except for Coulomb scattering (2.4), it is the mass quadrupole moment which is the relevant quantity (2.1-2.3). Subsections 2.5 and 2.6 deal with inelastic channels, but still with tensor polarization. In this context we will discuss the question of m-substate populations of ejectiles (2.6). The two remaining subsections (2.7 and 2.8) will be devoted to simple ideas pertaining to vector polarized beams (fig. 2.1a). Here we focus on the effect and the source of spin orbit forces.
2.1. Total Reaction Cross Section for Li-51V Interaction (Moe 81)

2.1.a. Data

Measuring the total reaction cross section for a nucleus such as lithium is not straightforward. We did this using two methods (Moe 81): (1) looking at the gamma rays emitted by the reaction residues and (2) looking at the elastic or quasi-elastic scattering. The choice of 51V as a target was made for specific reasons. If too many long-lived β-active nuclei are produced in the reaction, one starts measuring gamma rays from residues produced in the target two or three hours earlier. Figure 2.3 shows all the nuclei formed in the 7Li + 51V reaction through various channels. It is clear that the contaminants are few in number. This problem has been analyzed in detail, and we are sure that the error bars in our data take into account this effect. Gamma-ray spectra and their analysis to obtain the total reaction cross section σr and the corresponding analyzing power T^p_{20} (figs. 2.4 and 2.5) can be found in (Moe 81).

![Fig. 2.3](image-url)
This method of looking at gamma rays has some drawbacks. When I call something a total reaction cross section $\sigma^r$, it is the real cross section with some parts excluded, for example, the transitions to the ground state, the breakup of $^7$Li, and the excitation of $^7$Li and $^{51}$V. We didn't see any gamma rays from the low level excitation of vanadium.

The second way we determined this cross section is known in the literature as the sum-of-differences method (Ho Th 65, Moe 81). Here we define the total reaction cross section as

$$\sigma^r = 2\pi \lim_{\delta \to 0} \left[ \sigma_R(\theta) - \sigma(\theta) \right] \sin \theta d\theta$$  \hspace{1cm} (2.1)$$

and the tensor analyzing power $T_{20}^r$ as

$$\sigma^r T_{20}^r = -2\pi \lim_{\delta \to 0} \int_\delta^\pi \gamma(\theta) T_{20}(\theta) \sin \theta d\theta .$$  \hspace{1cm} (2.2)$$

$\sigma_R(\theta)$ is the Rutherford cross section; $\sigma(\theta)$ and $T_{20}$ the cross section and tensor analyzing power for quasi-elastic scattering. The reaction cross section obtained this way certainly includes the transitions to the ground state, but does not include the $^7$Li and $^{51}$V excitation.

Figure 2.4 shows the data for the quasi-elastic scattering of $^7$Li + $^{51}$V at various energies. These data include the excitation of $^7$Li. Figure 2.5 shows the analyzing powers $T_{20}$ of the corresponding reactions.

For most of the polarization data, we could resolve the inelastic scattering. You might wonder why this is only possible for the polarization data and not for the cross section data. This is because in the case of the cross section data, we have to work with absolute magnitudes. In spectra, the whole lines have to be integrated. For analyzing
power data we have to form ratios only. Therefore, only parts of lines may be used to extract the information.

Figure 2.6 shows the data measured for the total reaction \( \sigma^r \) and its tensor analyzing power \( T_{20}^r \) as a function of the center-of-mass energy \( E \) and its inverse, respectively. The open circles plot the gamma-ray data, and the black ones the quasi-elastic scattering data. Additionally, \( \Delta \sigma^r = \sigma^r \cdot T_{20}^r = c_{al}^r - \sigma^r \) is plotted. \( \sigma^r \) displays, except for the data points at the lowest energy, the well-known \( 1/E \) dependence.
(see later). $T_{20}$ displays large negative values at low energies, and it changes sign when the energy is increased. The energy dependence of $T_{20}$ and/or $\sigma^T$ can be understood in the simple sharp cutoff model. There the spectroscopic deformation of $^7\text{Li}$ will be identified as the main source of $T_{20}$. In order to demonstrate this qualitatively, we investigated, at a few selected energies, $\sigma^T$ for the $^6\text{Li} - ^{12}\text{C}$ reaction too, using quasi-elastic scattering data only. Figure 2.7 shows the angular distributions. As compared to $^7\text{Li}$ (fig. 2.4), the tensor analyzing powers are small, whereas the angular distributions of $\sigma/\sigma^T$ are very similar.

![Figure 2.6](image-url)
The data can be reduced by using the sum-of-differences method, and the resulting cross sections are shown in fig. 2.6 as triangles. The $^6\text{Li}$ cross sections follow the $^7\text{Li}$ values, more or less. The analyzing powers, however, are very different below the barrier. I claim, at this point, that the effects we observed have to do only with the fact that $^6\text{Li}$ and $^7\text{Li}$ have different shapes (fig. 2.2).

I have a few remarks on the polarization of the beam and its monitoring.

The beam polarization is monitored continuously. Just as cross sections are normalized to the number of particles going through the target, we normalize the results of our experiment with the measured average beam polarization (eq. 1.20). In my discussions I have always referred to completely aligned beams, but, in fact, the experiments are
done with beams that are not completely aligned. The fraction of alignment is around 0.4. However, in order to avoid complications in these discussions, I shall not talk about incompletely aligned beams.

The polarization of the $^7\text{Li}$ beam is measured by bombarding hydrogen with lithium and looking for the outgoing alpha particles. At $0^\circ$ and $180^\circ$ the analyzing power for this reaction is independent of energy and is equal to $-1$ (Zup 79). It also does not depend on any reaction mechanism. This has to do with the fact that at $0^\circ$ there can be no change in the $z$ component of the orbital angular momentum. For heavier projectiles we can measure the polarization using Coulomb excitation.

2.1.b Analysis Within the Sharp Cutoff Model

Throughout this analysis (Moe 81) we will treat $^7\text{Li}$ (and $^6\text{Li}$) as simple rotational ellipsoids with a sharp surface and a homogeneous charge and mass distribution. Using a homogeneously charged nucleus with a sharp surface is clearly an idealization, but it makes the presentation of ideas fairly simple.

Figure 2.6 shows the reaction cross section as a function of $1/E$. As long as we have this $1/E$ behavior, the cross section can be parameterized under the sharp cutoff model (No We 80, Bas 80). To discuss this
model we define an impact parameter at infinity (fig. 2.8). This fixes the trajectory shown, and we can also define an angular momentum for this trajectory.

\[ b_0 \text{ is the impact parameter and } R_0 \text{ the distance of closest approach for the grazing trajectory. Angular momentum is defined by} \]

\[ l_0 = b_0 \, p_0 = R_0 \, p_0 , \quad (2.3a) \]

\[ p_0 \text{ being the momentum at the distance of closest approach for a grazing trajectory. The energy } E_\infty \text{ is given by} \]

\[ E_\infty - V(R_0) = \frac{p_0^2}{2\mu} , \quad (2.3b) \]
where $V(R_0)$ is the interaction potential at $R_0$. The distance of closest approach $R_0$ for a grazing trajectory divides the interaction into two parts. When the trajectories of the projectile are such that the distance of closest approach $R > R_0$, then there is no nuclear reaction. Reactions can occur only for $R < R_0$. The total cross section is defined by

$$\sigma^r = \pi b_0^2. \quad (2.4)$$

From eq. 2.3 we obtain

$$\sigma^r = \pi R_0^2 \left(1 - \frac{V(R_0)}{E}\right). \quad (2.5)$$

A fit to the data (fig. 2.9) yields $R_0 = (9.5 \pm 0.3)$ fm and $V(R_0) = (9.6 \pm 1.0)$ MeV.

Now let us see what happens when we use a polarized beam. If we align the $^7\text{Li}$ beam along the incoming momentum and we assume that the alignment axis does not change during the collision, we see for the trajectory of fig. 2.10 that the $R_0$ which belongs to the grazing trajectory is larger than the $R_0$ for an unpolarized projectile. In other words, for an aligned beam the impact parameter and the orbital angular momentum connected with this grazing trajectory become larger. In the case of more central collisions ($b$ and $\phi$ small), the situation is reversed. For an aligned beam the impact parameter and orbital angular momentum become smaller.

We now use the idea of a sharp cutoff model to write down expressions for the analyzing powers. We start by neglecting all effects that come from the fact that the line joining the centers of the nuclei does not go through the point of contact between projectile and target. We
parameterize the reaction cross section for the aligned beam by

$$\sigma_{al} = \pi R_0' \left(1 - \frac{V(R_0')}{E}\right),$$  \hspace{1cm} (2.6)

with

$$R_0' = R_V + R_{Li} \left[1 + \beta Y_{20}(\phi)\right],$$ \hspace{1cm} (2.7)

where $R_V$ is the radius of the vanadium nucleus (Fig. 2.10).

Therefore, in order to calculate this cross section, we need to know the angle $\phi$ for each of the trajectories. The difference in cross section for an unpolarized and an aligned beam is in first order

$$\Delta \sigma = \frac{\partial \sigma}{\partial R} \Delta R = \left(\frac{\partial \sigma}{\partial R} + \frac{\partial \sigma}{\partial Y} \frac{\partial Y}{\partial R}\right) \Delta R,$$ \hspace{1cm} (2.8)

where (eq. 2.7)

$$\Delta R = \beta R_{Li} Y_{20}(\phi).$$ \hspace{1cm} (2.9)

The change in cross section comes from (a) a change in the impact parameter and (b) a change in the interaction potential. Thus we have to
specify the radial dependence of the potential $V(R)$ and the angle $\phi(E)$ before we can calculate $\Delta \sigma$.

We can use a simple Coulomb model to calculate the trajectory. Using

$$ V(R) = \frac{1}{R} , \tag{2.10} $$

the angle $\phi$ is then given by

$$ \tan \phi(E) = \frac{2 \sqrt{E}}{V(R_0)} \cdot \frac{E}{V(R_0)} - 1 . \tag{2.11} $$

It seems at this point that the angle $\phi$ is an uncertain quantity, but this is not the case. The higher we get in energy, the angle $\phi$ approaches $90^\circ$. Hence, most of the errors due to $\phi$ are at low energies. However, at the low energies we know that the interaction is mainly Coulombian. What we do not know, however, is the exact gradient of the potential at higher energies, since nuclear interaction may play a role there. This is the major source of error.

Using eqs. 2.8 and 2.10, we can get a closed form expression for the analyzing power,

$$ T_{20}^r = \frac{\Delta \sigma R}{\sigma R} = \frac{\Delta R}{R_0} \cdot \frac{2E - V(R_0)}{E - V(R_0)} , \tag{2.12} $$

where $\Delta R = \beta R_{Li} Y_{20}(\phi)$. The parameters $R_0$ and $V(R_0)$ were determined by fitting $\sigma_R$. Therefore, the deformation length $\beta R_{Li}$ is the only parameter left to fit $T_{20}^r$. A fit to the data (fig. 2.9) yields

$$ \beta R_{Li} = (-0.28 \pm 0.08) \text{ fm}. $$
The sign of the analyzing power directly reflects the sign of the deformation. This can easily be realized for the limit \(E \gg V(R_0)\). There we know \(\phi(E) \leq 90^\circ\). We then obtain

\[
T_{20} \approx \frac{2\Delta R}{R_0} = \frac{5}{16\pi} \frac{\beta_{R\text{Li}}}{R_0}.
\] (2.13)

Figure 2.9 shows as solid lines the description of the data with the sharp cutoff model. For energies \(E > V(R_0)\) (the barrier), the data are described perfectly. The sharp cutoff model also clearly demonstrates why the tensor analyzing powers vanish for the lithium isotope \(^6\text{Li}\). As stated before, it is the spectroscopic deformation which generates the tensor analyzing powers.

Within the sharp cutoff model, it is not surprising to find large deviations between data, cross sections and analyzing powers, and description for energies below the barrier \([V(R_0) = 9.6\text{ MeV}]\). Within this model the cross section has to vanish. Because of tunneling, it does not. We shall come back to this point in Sect. 3.

A remaining point now is: what does the parameter \(\beta_{R\text{Li}}\) describe? And is it in agreement with quantities we can calculate from the spectroscopic quadrupole moment of \(^7\text{Li}\)? In Lecture III, I will make comparisons of the deformation length we obtain from various measurements.
2.2. Elastic Scattering on $^{58}$Ni - Shape Effect

Most of what I will discuss has been published in (Mor 82). Figure 2.11 shows the $^7$Li and $^6$Li elastic scattering cross sections. We note from the data that the results are very similar. We repeated the experiment with an aligned beam, with the alignment axis along the normal to the scattering plane. Figure 2.11 shows also the tensor analyzing powers $T_{20}$ obtained in this experiment. $T_{20}$ is defined according to fig. 1.5. Note that the numerical factors in the expression are different for spin 1 ($^6$Li) and spin 3/2 ($^7$Li) [fig. 1.5 and (Hae 74) from Sect. 1].

![Graph showing elastic scattering cross sections and tensor analyzing powers.](image)

Fig. 2.11

The tensor analyzing power for $^6$Li is close to zero, while that for $^7$Li is large and negative. We can understand these results qualitatively...
In terms of the deformation of the lithium isotopes. In fig. 2.12 we represent the unaligned $^7$Li by a uniformly charged spheroid (dashed curve) and the aligned $^7$Li by a rotational ellipsoid. We see immediately that around the grazing trajectory the overlap between $^7$Li and $^{58}$Ni is different, depending on whether the $^7$Li is aligned or unpolarized. When there is more overlap, as is the case when $^7$Li is aligned, the reaction cross section is larger, and thus the elastic cross section is reduced, compared to the experiment with an unpolarized beam. This results in a negative analyzing power. We do not see this effect for $^6$Li because of its almost vanishing deformation. In this whole treatment we neglect effects of dynamic deformation and dynamic reorientation. These effects could become important for heavy nuclei where low-lying collective modes of excitation exist.

Figure 2.13 shows a complete set of tensor analyzing powers for the $^7$Li + $^{58}$Ni elastic scattering. It is sufficient to measure three of the four analyzing powers plotted, because of the following relation:

$$2 \, T_{20} = -T_{20} - \sqrt{6} \, T_{22}.$$  \hspace{1cm} (2.14)
In fig. 2.12 we interpreted the observation for $T_{20}$ in terms of the aligned deformed mass distribution of the $^7$Li projectile. With similar arguments, the data observed for the other tensor analyzing power $T_{2q}$ can be understood. As a further example, fig. 2.14 shows the alignment axes of $^7$Li for the measurement of $T_{21}$. You can see that at the distance of closest approach for the two different alignments, there is a huge difference in the overlap of the interacting nuclei. One can also see that according to its definition (fig. 1.5), the analyzing power $T_{21}$ will be positive for a negatively deformed nucleus. We can also understand from this simple picture why $T_{20}$ is negative at small angles and then becomes positive as the angle increases.
I hope I have convinced you that it is the spectroscopic deformation of \( _7\text{Li} \) which is responsible for the various effects we have discussed.

To finish this point, I add another piece of evidence and remind you that the effect of the aligned Q moments of \( _7\text{Li} \) can be treated formally as a \( T_R \)-tensor interaction. In terms of the \( M_{k\ell} \) defined in Chap. 1, this interaction influences in first order the elements \( M_{2\ell} \) only (eq. 1.46). Then we recall from Sect. 1 the relations between tensor analyzing powers \( \tau_{k\ell} \) and \( \overline{\tau}_{k\ell} \) both defined in the "barred" coordinate system (fig. 1.3b, eqs. 1.22 and 1.48).

\[
\frac{d\sigma}{d\Omega} = (\overline{M}_{00})^2 ;
\]

\[
\frac{d\sigma}{d\Omega} \tau_{20} = 2\text{Re}(\overline{M}_{00} \overline{M}_{20}^*) ;
\]

\[
\frac{d\sigma}{d\Omega} \tau_{21} = 0 ;
\]

\[
\frac{d\sigma}{d\Omega} \tau_{22} = 2\text{Re}(\overline{M}_{00} \overline{M}_{22}^*) .
\]

We see that with a first-order treatment of a \( T_R \) interaction, \( \tau_{21} = 0 \). Again, this can be understood qualitatively by looking at the meaning of a \( \tau_{21} \) measurement in terms of aligned Q moments of the \( _7\text{Li} \).
beam (Fig. 2.15). It is clear that the overlaps between lithium and nickel are on first order the same for the two beam alignments.

As for the barred coordinate system, the $\bar{z}$ axis changes with scattering angle, and $T_{21}$ cannot be determined directly. However, it can be calculated from the measured tensor analyzing powers $T_{2q}$ (Co Ph 82).

$$T_{21} = \sqrt{3/8} T_{20} \sin \theta + T_{21} \cos \theta - 1/2 T_{22} \sin \theta.$$  \hspace{1cm} (2.16)

Figure 2.16 displays the data confirming $T_{21} = 0$ to a large extent.

Parallel to more formal arguments, we have used, up to now, "intuitive pictures" in terms of "more or less overlap between $^7$Li and the target." We will finish this section by considerations which will connect our formal reasoning with these hand-waving pictures. To start with, I remind you that the "shape effect model" in its original form (Fic 81, Mor 82) claims for elastic scattering, in a situation as sketched in fig. 2.17, that the change in cross section due to the
alignment of the projectile along an axis \( \mathbf{P} \) is proportional to the change in distance between the surfaces of projectile and target at the distance of closest approach \( (r = r_0) \). This assumption establishes relations between the analyzing power \( T_{20} \) and the analyzing powers \( T_{2q}(\theta) \) \((q = 0, 1, 2)\), which can be summarized in the form,

\[
T_{2q}(\theta) = -\sqrt{16\pi/5} \times Y_{2q} \left( \frac{\pi + \theta}{2} \right) \cdot T_{20}(\theta).
\]  

(2.17)

In detail these relations read:

\[
T_{20}(\theta) = (1 - 3 \sin^2 \frac{\theta}{2}) \cdot T_{20}(\theta);
\]

\[
T_{21}(\theta) = -\sqrt{3/2} \sin \theta \cdot T_{20}(\theta);
\]

\[
T_{22}(\theta) = -\sqrt{3/2} \cos^2 \frac{\theta}{2} \cdot T_{20}(\theta).
\]  

(2.18)
Before applying these relations to the present data, a general derivation will be presented which indicates the necessary physical assumptions entering the relations of eq. 2.17. Moreover, this derivation clarifies that the relations above might be applicable to reactions other than just elastic scattering.

First, I remind you about the decomposition of the $T$ matrix in terms of its spherical components $M_{kq}$ (Sect. 1.3) and, in particular, about Eq. 1.34 which gives a connection between $\sigma T_{kq}$ and the $M_{kq}$'s. To start our considerations, the $z$ axis is chosen as to point always along the alignment axis $\hat{p}$. Then from the second rank tensor analyzing powers, only $T_{20}$ doesn't vanish (fig. 1.5). If now, according to earlier arguments, the tensor interaction is considered weak with respect to the central one, the leading term in eq. 1.34 reads

$$\sigma(\theta) T_{20}(\theta, \hat{p}) = 2 \text{Re}[M_{00}(\theta) V_{20}^*(\theta, \hat{p})] , \quad (2.19)$$

whereby the spherical component $M_{20}$ was replaced in a Born-approximation treatment of the interaction by the spherical component of the potential $V_{20}$ itself. The tensor analyzing power $T_{20}$ and the spherical component of the potential $V_{20}$ are written as functions of $\hat{p}$ to indicate the choice of the $z$ axis. On the basis of our experimental experience, the shape effect model assumes that the potential is of $T_R$ type and acts at the distance of closest approach $r = r_0$ only. Therefore, $V_{20}(\theta, \hat{p})$ is proportional to $P_2(\hat{p} \cdot \hat{r}_0)$, $P_2$ being a Legendre polynomial. Denoting the proportionality function by $v_2(\theta)$, eq. 2.19 reduces to

$$\sigma(\theta) T_{20}(\theta, \hat{p}) = 2 \text{Re} M_{00}(\theta) v_2^*(\theta) P_2(\hat{p} \cdot \hat{r}_0) . \quad (2.20)$$
The final formulae (eq. 2.17) are now derived in three steps. First, \( \mathbf{P} \) is chosen along \( \mathbf{N} \), perpendicular to \( \mathbf{r}_0 \). This leads to

\[
\sigma(\theta) T_{20}(\theta, \mathbf{N}) = \sigma(\theta) T_{20}(\theta) = -\text{Re} M_{00}(\theta) \nu_2^*(\theta) \tag{2.21}
\]

and to the general relation (eqs. 2.20 and 2.21),

\[
T_{20}(\theta, \mathbf{P}) = -2T_{20}(\theta) P_2(\mathbf{P} \cdot \mathbf{P}_0) . \tag{2.22}
\]

Equation 2.22 is valid for any choice of \( \mathbf{P} \). Therefore, a most simple choice, \( \mathbf{P} = \mathbf{r}_0 \), will allow deriving the equation,

\[
T_{20}(\theta, \mathbf{r}_0) = -2T_{20}(\theta) . \tag{2.23}
\]

Finally, the analyzing powers \( T_{2q}(\theta) \) are obtained by a rotation from the quantization axis (\( \mathbf{r}_0 = \mathbf{P} \)) to the \( \mathbf{z} \) axis (\( \mathbf{z} = \mathbf{k}_1 \mathbf{n} \)) chosen in the Madison convention. The rotation angle is negative since the \( \mathbf{y} \)-axis is defined along the normal to the scattering plane, \( \mathbf{N} \).

\[
T_{2q}(\theta) = D_{02}^2(0, -\frac{\pi + \theta}{2}, 0) T_{20}(\theta, \mathbf{r}_0) \tag{2.24}
\]

\[
= -\sqrt{16\pi/5} \gamma_{2q} \left( -\frac{\pi + \theta}{2}, 0 \right) T_{20}(\theta) .
\]

These are the equations to be proved. In summary, a first-order treatment of a tensor interaction of \( T_R \) type leads to the "shape effect model" if the interaction is considered at the distance of closest approach only. The derivation presented did not need any kind of special assumption on the type of reaction. Therefore, the relations of eq. 2.17 are supposed to hold for inelastic scattering and transfer reactions as well, if the assumptions are fulfilled and the momentum transfer is not too large.
Figures 2.13 and 2.18 show our results. The curves through the measured points for $T_{T20}(\theta)$ were drawn by hand, and eq. 2.18 was used to obtain the curves for $T_{20}$, $T_{21}$, and $T_{22}$. Because of the relation (eq. 2.14) between $T_{T20}$, $T_{20}$, and $T_{22}$, one relation in eq. 2.18 is redundant. Besides that trivial agreement, we can see that the trends of the data are well described. Hence, if the interactions are dominated by tensor forces, the tensor analyzing powers are related to each other geometrically, and the interesting information lies in the deviations of the experiments from the predictions of our simple model.

![Graphs showing data](image)

**Fig. 2.18**

Note added in proof: For $^7\text{Li}$-$^12\text{C}$ around 20 MeV, strong deviations from the shape effect relations (eqs. 2.17 and 2.18) have been observed (Mor 83).
2.3. Diffraction and Optical Model Analyses (Mor 81)

2.3.a. Analysis Within the Diffraction Model

Since in fig. 2.18 \( \sigma/\sigma_R \) behaves like a Fresnel distribution, we can also approach our problem using a parameterized \( S \) matrix (Fra 78). We start by defining

\[
\lambda = \lambda + 1/2
\]  

(2.25)

and factorize the \( S \) matrix in terms of

\[
S(\lambda) = S(N)(\lambda)e^{2i\sigma(\lambda)},
\]  

(2.26)

where the \( S \) matrix is written as a product of a nuclear part \([S(N)(\lambda)]\) and in terms of a Coulomb phase. We can parameterize the nuclear part as (Fra 78)

\[
S(N)(\lambda) = \frac{1}{1 + \exp \left(-\frac{\lambda - \Lambda}{\Delta}\right)}.
\]  

(2.27)

Fig. 2.19
The parameterization is shown in fig. 2.19. We ran a computer code to get values for \( \Delta \) and \( \Delta \) that fit the cross section data, and we obtained (fig. 2.20)

\[
\begin{align*}
\Delta &= 12 , \\
\Delta &= 0.8
\end{align*}
\]

for the 20.3-MeV data of \( ^7\text{Li}-^{58}\text{Ni} \) elastic scattering.

Fig. 2.20

For the calculation of \( T_{20} \), we follow the ideas of fig. 2.12. We know that the only effect of an alignment of \( ^7\text{Li} \) along \( N \) is a change \( \Delta R \) of the interaction radius. Hence (fig. 1.5),

\[
T_{20} = \frac{\sigma_{a1} - \sigma}{\sigma} = \frac{\sigma(R + \Delta R) - \sigma(R)}{\sigma(R)} ,
\]

(2.28)

where \( \Delta R = \beta_{\text{Li}} Y_{20}(\pi/2) = -\sqrt{5}/16 \pi \beta_{\text{Li}} \). R is the nuclear separation at
the distance of closest approach. We need to relate $R$ with $\Lambda$. If Coulomb effects are included,

$$\Lambda = kR \sqrt{1 - 2 \eta/kR}, \quad (2.29)$$

where $\eta$ is the Sommerfeld parameter and $\Lambda$ is given by

$$\Lambda = kd \sqrt{1 + (\eta/\Lambda)^2}, \quad (2.30)$$

where $d$ is the diffuseness in the angular momentum (Fra 78).

The only adjusted parameter in this analysis is $\beta R_{L1}$. The results are shown in fig. 2.20: $\beta R_{L1} = -0.47$ fm. It is clear that the analysis describes the trends of the data. This is the first example of a dynamical calculation of the analyzing powers.

2.3.b. Analysis Within the Optical Model

In the last subsection we looked at the data using a parameterized $S$ matrix. The only adjusted parameter needed to describe the tensor analyzing powers was $\beta R_{L1}$. We can now go ahead and carry out an analysis in terms of an optical model.

We start with an ansatz by writing down the interaction in a form,

$$U(r, \beta) = U(x) = \frac{U_0}{(1 + \exp x)}, \quad (2.31)$$

where $x = \{r - R_{N1} - R_{L1}(1 + \beta \sqrt{5/4 \pi} T_R)/a$ and $T_R$ is defined in eq. 1.46. Since $\beta R_{L1}$ is small with respect to $R_{N1} + R_{L1}$, we can expand the potential around $x_0 = x(\beta = 0)$. We obtain

$$U(x) = U(x_0) - \frac{dU}{dx} \bigg|_{x=x_0} \sqrt{5/4 \pi} \frac{\beta R_{L1}}{a} T_R. \quad (2.33)$$
$\beta R_{Li}/a$ is the strength parameter and goes to infinity in the limit of a sharp surface. The extent to which the tensor interaction influences the cross section is strongly connected with the diffuseness of the surface.

Fig. 2.21

We start by fitting the unpolarized $^7$Li-$^{58}$Ni scattering data shown in fig. 2.21 to get a set of parameters for the central potential $U(x_0)$. We then fit the tensor and analyzing powers plus the unpolarized data. The only additional adjustable parameter is $\beta R_{Li}$. The curves A, B, and C refer to different forms of the central potential. For details see (Mor 81). In this model we have used the same deformation parameters for the real and imaginary parts of the optical potential. In principle,
however, it might be possible that the real and imaginary parts show different deformations (see Chap. 4).

Figure 2.22 shows quasi-elastic scattering data taken at 14 MeV. The curves are qualitatively similar to those of Fig. 2.21. There are deviations from theory in the analyzing power measurements, and these deviations are seen in all the curves. Their meaning is not clear presently.

![Figure 2.22]

Table 2.1 lists the values obtained for $\beta_{Q_1}$ from various measurements. The number quoted in the last line was obtained by probing the $^7$Li electric quadrupole moment by the field gradient of the Coulomb field under the assumption that $^7$Li is a homogeneously charged nucleus with a sharp surface (see Sect. 2.4). The other numbers represent values
Table 2.1

<table>
<thead>
<tr>
<th>$E_{L1}$/MeV</th>
<th>Target</th>
<th>Measured Quantity</th>
<th>Model</th>
<th>$\beta_{RL1}$/fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.2</td>
<td>$^{58}$Ni</td>
<td>quasi elastic</td>
<td>optical</td>
<td>-0.53</td>
</tr>
<tr>
<td>20.1</td>
<td>$^{58}$Ni</td>
<td>elastic</td>
<td>optical</td>
<td>-0.27</td>
</tr>
<tr>
<td>18 to 22</td>
<td>$^{58}$Ni</td>
<td>elastic</td>
<td>diffraction</td>
<td>-0.47</td>
</tr>
<tr>
<td>9 to 17</td>
<td>$^{51}$V</td>
<td>reaction</td>
<td>sharp cutoff</td>
<td>$-0.28 \pm 0.08$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q moment</td>
<td>homogeneously charged nucleus</td>
<td>$-0.50 \pm 0.10$</td>
</tr>
</tbody>
</table>

we obtained from the analysis of quasi-elastic, elastic, and reaction data. It is obvious that the agreement between the numbers is not very good. It is, however, not clear what the magnitudes of the errors quoted on these numbers should be. Nevertheless, one can make a few remarks about these results. First, all these analyses were performed in a single-channel approximation. That, in my opinion, is a problem. I will show you a specific example of the Coulomb scattering of aligned $^7$Li that will illustrate this point. Second, into each of the analyses we have to put in a value for the deformation parameter $\beta$ and this parameter certainly has different meanings for different models. In the sharp cutoff model, $\beta$ refers to the deformation at the sharp cutoff radius. In the diffraction model, $\beta$ refers to the deformation connected with the grazing angular momentum. In the optical model we parameterize the potential with the deformation parameter defining the point where the potential drops to half its value at the center of the nucleus.
Therefore, the problems we face here are similar to those encountered in the determination of the RMS radii from hadronic scattering. We must have a detailed knowledge of the potential because of the strong absorption. However, there is a significant point to note in the numbers in Table 2.1: they all have the same negative sign due to the negative quadrupole moment of $^7$Li.

Note Added: Very recent calculations show that coupling effects can be neglected in the angular range of the data (Ohn 82).
2.4. Coulomb Scattering of Aligned $^7\text{Li}$

This experiment was performed with the intention of obtaining, independent of the classical methods, a precise value for the spectroscopic quadrupole moment of $^7\text{Li}$. We have also learned from this experiment about the role of coupling to inelastic channels. The data have been taken from Ege 80 and Wei 82. The theory for the experiment was taken from (Ald 73).

Table 2.2. Spectroscopic Quadrupole Moment of $^7\text{Li}$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Theory</th>
<th>Qel.mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wharton et al. 1962</td>
<td>Kahalas and Nesbet 1963</td>
<td>-45</td>
</tr>
<tr>
<td></td>
<td>Brown and Matsen 1964</td>
<td>-43.7</td>
</tr>
<tr>
<td>(Molecular spectroscopy on $^7\text{LiH}$)</td>
<td>Bender and Davidsen 1969</td>
<td>-43.7</td>
</tr>
<tr>
<td></td>
<td>Cade et al. 1970</td>
<td>-34.3</td>
</tr>
<tr>
<td></td>
<td>Bender 1970</td>
<td>-43.7</td>
</tr>
<tr>
<td></td>
<td>Green 1971</td>
<td>-36.6</td>
</tr>
<tr>
<td>Orth et al. 1975</td>
<td>Nesbet 1970</td>
<td>-41.3</td>
</tr>
<tr>
<td>(Atomic beam spectroscopy)</td>
<td>Hameed and Foley 1972</td>
<td>-40.5</td>
</tr>
<tr>
<td></td>
<td>Garpman et al. 1975</td>
<td>-41.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-41 (6)</td>
</tr>
<tr>
<td>Egelhof et al. 1980</td>
<td>Alder et al. 1973</td>
<td>-34 (6)</td>
</tr>
<tr>
<td>(Coulomb scattering of aligned $^7\text{Li}$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There are two sources in the literature (Loe 70) for the value of the quadrupole moment of $^7\text{Li}$. One is an experiment in molecular spectroscopy which measures the hyperfine splitting of $^7\text{LiH}$. The problem in
such experiments is in obtaining a value for the quadrupole moment of $^7\text{Li}$ from the measured frequency shifts. For this, the value of the electric field gradient at the nucleus has to be known precisely. Several theories exist that calculate the quadrupole moment from the data, and the numbers obtained are shown in Table 2.2. The calculations of Green, using Hartree-Fock wavefunctions, are the most accurate, and it is this value that is quoted in the recent Nuclear Data Tables.

A second experiment (Ort 75) looks at the quadrupole splittings in the $3p_{3/2}$ excited level of lithium. The problem here is that the energy splittings are of the same order as the widths of the excited states. The calculations of the field gradient are in this case, however, more accurate.

Our experiment (Ege 80) used a very different approach, namely, the Coulomb scattering of aligned $^7\text{Li}$ nuclei. The advantage of Coulomb scattering is that the interaction potential, and therefore the field gradient, is known exactly. We can write down the potential for the interaction as

$$V_{cb} = \frac{Z_1 Z_2 e^2}{r} + \frac{\pi}{5} \frac{Q \cdot Z_2}{r^3} \gamma_{20}(\hat{S} \cdot \hat{n}) , \quad (2.34)$$

in terms of a central and a quadrupole potential. Figure 2.23 shows the various vectors and angles of eq. 2.34. We can use, for example, a suitable optical model code to calculate the tensor analyzing powers which we are interested in. However, we followed another approach (Ald 73) more suitable to discuss the optimum conditions for the experiment. For this purpose we introduce a strength parameter which is proportional to the quadrupole moment,
where 2a is, as usual, the distance of closest approach for central collisions. We can see from eq. 2.35 that in order to have a large strength parameter, we need to reach a small "a." On the other hand, decreasing "a" too much, we are faced with the problems of nuclear interaction. To avoid nuclear interaction, we need to work very much below the Coulomb barrier.

For spin 3/2 particles up to second order (Ald 73), the cross sections can be written as

\[ \sigma(\theta) = \sigma_R(\theta)[1 + \tilde{f}_{20}(\theta)] ; \]  

whence the analyzing power for Coulomb scattering, \( r_{20}^{\text{Cb}} \), can be obtained as

\[ \frac{\sigma(\theta)}{\sigma_R(\theta)} - 1 = r_{20}^{\text{Cb}} = \tilde{f}_{20}(\theta) . \]
The function $\tilde{r}_{20}(\theta)$ can be calculated from Coulomb integrals and has been plotted in fig. 2.24. Typical data are shown in fig. 2.25 (Wei 82). Other data can be found in (Ege 80).

![Fig. 2.24](image)

If we take the elastic scattering data and use eqs. 2.36 and 2.37 to calculate a strength parameter and the quadrupole moment, we obtain a value for the quadrupole moment which is about twice as large as that quoted in the literature. The reason for this is inelastic scattering to the first excited state of $^{7}$Li.

In a simple semiclassical approach, the Rutherford cross section is related to the elastic and inelastic cross section by

$$\sigma_R(\theta) = \sigma_{el}(\theta) + \sigma_{in}(\theta) .$$

In terms of the analyzing powers, changes in the cross sections due to the alignment of the beam are given by $(1 + T_{20})\sigma$ (fig. 1.5), and hence

$$(1 + T_{20}^{Cb})\sigma_R = (1 + T_{20}^{el}) \sigma_{el} + (1 + T_{20}^{in}) \sigma_{in}$$

in a semiclassical approach. Subtracting eq. 2.38 from eq. 2.39, we obtain

$$\sigma_R T_{20}^{Cb} = \sigma_{el} T_{20}^{el} + \sigma_{in} T_{20}^{in} .$$
Therefore,

\[ T_{20}^{\text{Cb}} = \frac{e_1}{\sigma_R} T_{20}^{\text{el}} + \frac{\text{in}}{\sigma_R} T_{20}^{\text{in}} = T_{20}^{(\text{el}+\text{in})}. \]  

(2.41)

\( T_{20}^{\text{el}+\text{in}} \) is the tensor analyzing power for the sum of elastic and inelastic scattering. Data for this quantity are displayed in fig. 2.25 also. According to eq. 2.41 the spectroscopic Q moment can be extracted from here, rather than from \( T_{20}^{\text{el}} \). However, before doing so, we may try to understand why \( T_{20}^{\text{el}} \) and \( T_{20}^{\text{el}+\text{in}} = T_{20}^{\text{Cb}} \) are quite different, even if \( \sigma_1 \) and \( \sigma_1^{\text{el}+\text{in}} = \sigma_R \) are almost equal. In the semiclassical approximation, the Rutherford cross section is the sum of the elastic and inelastic cross sections. If we look at the cross section data for the \( ^7\text{Li} + ^{120}\text{Sn} \) reaction at \( E_{\text{Li}} = 15.3 \text{ MeV} \), the inelastic scattering would be only a small fraction of the total yield. However, if we look at the change in cross sections (fig. 2.25), the inelastic part is no longer negligible.
In eq. 2.40, $\sigma_{el}$ and $T_{20}^{in}$ turn out to be large quantities, and $T_{20}^{in}$ and $\sigma_{in}$, to be small quantities. Therefore, even for $\sigma_{el} \gg \sigma_{in}$ the two terms on the right-hand side of eq. 2.40 are about equal in magnitude. From eq. 2.41 it is clear that the data we should use in order to get a value for the quadrupole moment are those that include both the elastic and inelastic scattering.

If the data of fig. 2.25 are used without any further corrections, a quadrupole moment for $^7\text{Li}$ of $Q = (-37 \pm 7) \text{em}^2$ is obtained from the elastic + inelastic data. Necessary corrections will change this value and its error. From the earlier experiment, $Q = (-34 \pm 6) \text{em}^2$ was obtained (Ege 82).

To prove our method, we performed another consistency check. We took the elastic + inelastic curve plotted in fig. 2.25 to obtain a value for the quadrupole moment. We calculated $\sigma_{in}$ and $T_{20}^{in}$ from Coulomb excitation codes, and from eq. 2.41 we could then obtain a value for $T_{20}^{el}$ by inserting $T_{20}^{el+in}$ from experiment and taking $\sigma_{el} = \sigma_R$. (A value for the BE2 for the Coulomb excitation of lithium is also needed for this to be possible.) The values obtained for $T_{20}^{el}$ in this way agree quite well with the data (solid curve in fig. 2.25).

In other words, from the elastic and the quasi-elastic data, we can get values for both the quadrupole moment and the BE2 value. This measurement also demonstrates the need for a coupled-channel analysis. Up to now, I have not mentioned that all data have to be corrected for effects caused by the polarizability of $^7\text{Li}$. With effects of sufficient accuracy, this third effect could probably be isolated too.
I have shown you Coulomb scattering data (fig. 2.25) for only the \( {\text{Li}} + {^{120}\text{Sn}} \) reaction. We have performed similar measurements using lead targets as well.

This data bears significance, not only in nuclear physics but in atomic physics too. We can use our numbers for the quadrupole moment to calculate field gradients in atoms and to check the predictions of various theories.

References

The references for Lecture II are listed at the end of Sect. 2 which is the end of Lecture III.
2.5. Inelastic $^7$Li Scattering (Mor 82)

Figure 2.26 shows data for the $^{58}$Ni($^7$Li,$^7$Li$^*$)$^{58}$Ni inelastic scattering reaction at 20.3 MeV (from Mor 82). The cross sections have been analyzed by the computer program DWIS, completely neglecting spins. The analyzing powers shown in fig. 2.26 have been calculated by the de Boer-Winther code for pure Coulomb.
excitation. Since at this energy nuclear excitation is certainly present, the good agreement between data and calculations is surprising. It is the aim of the following considerations to show that simple constraints common to both calculations influence the results of these calculations considerably. In order to show this, we use a semiclassical treatment.

In a semiclassical picture of such reactions, the transfer of angular momentum takes place preferentially with $\Delta M_L' = 0$ (where the prime indicates that the quantization axis is along the recoil momentum). For the rest of this analysis, we shall stick to this classical picture except in the treatment of spin coupling, which shall be treated along the quantum mechanical rules. The $\Delta M_L' = 0$ selection rule has significance in the case of $^7\text{Li}$ because the excited state in $^7\text{Li}$ has spin $1/2^-$. Figure 2.27 shows the two lowest energy levels in $^7\text{Li}$.

![Fig. 2.27a](image)

![Fig. 2.27b](image)

Since the first excited state has spin $1/2$, only the $\pm 1/2$ substates in the ground state can contribute to this excitation if $\Delta M_L' = 0$. If we start with a beam that is purely aligned along the $z'$ axis (defined in fig. 2.27b), the beam has half its population in the $N = 3/2$ substate,
and the other half in the $N' = 3/2$ and no population in the $N' = \pm 1/2$ substates (Sect. 1.1). This implies:

$$t_{20}' = 1.$$ \hspace{1cm} (2.42)

We know

$$\sigma'_\text{pol} = \sigma'(1 + t_{20}' l_{20}').$$ \hspace{1cm} (2.43)

Since there is no population in the $N' = \pm 1/2$ states, under the assumption of the selection rule $\Delta M_L' = 0$,

$$\sigma'_\text{pol} = 0.$$ \hspace{1cm} (2.44)

This implies

$$T_{20}'(\theta) = -1$$ \hspace{1cm} (2.45)

and

$$T_{21}' = T_{22}' = 0.$$ \hspace{1cm} (2.46)

We can now transform to the other frames of reference (eqs. 1.9, 1.21, 1.23) to obtain the dashed lines in fig. 2.26. The dashed lines certainly describe the data within the error bars shown. It is the semiclassical character of the reaction together with the spin value of the excited state of $1/2$ which determine the angular distribution of the tensor analyzing power. This may explain why the polarization data agrees reasonably with the Coulomb-excitation calculations, even in the presence of nuclear excitation. The excitation is dominated by the same selection rule independent of the interaction.

2.6. A Side Aspect: $m$ Substate Population of Ejectiles, Not Necessarily Li

The method presented in the previous section can be used to calculate $m$-substate populations of ejectiles, in particular in angular regions where a large enough momentum and/or angular momentum mismatch
allows this classical method. Again angular momentum coupling is performed along quantum mechanical rules.

2.6.1. Transfer reaction. Figure 2.28 displays data (Wus 79)

![Graph showing data for transfer reaction](image)

Fig. 2.28

obtained for $\sigma$ and the $m$-state population $\rho_{1/2, 1/2}$ of $^{15}N^*$ in the $^{88}\text{Sr}(^{16}O, ^{15}N^*(3/2^-, 6.33 \text{ MeV})^{89}\text{Y}}$ reaction initiated with 96 MeV $^{16}O$ ions. $\rho_{1/2, 1/2}$ denotes the probability to find the $^{15}N^*$ ions in the $m = + 1/2$ and $- 1/2$ sublevel with respect to the final momentum. These probabilities were measured using the $\gamma$-recoil method (Boh 78). The data were taken in an angular range where the transfer cross section decreases exponentially with the scattering angle, and therefore a classical description of the reaction is not unreasonable. If $N_m$
denotes occupation probabilities with respect to the final momentum \( z = \rho_{\text{fin}} \)

\[
\rho_{1/2 \ 1/2} = N_{1/2} + N_{-1/2}
\]  

\( \rho_{1/2 \ 1/2} \) is put into parentheses to indicate that the definition of \( \rho \) is not the one for the density matrix \( \rho_{\text{mm}} \), of Sec. 1. Since the alignment \( \tilde{t}_{20} \) of the \( ^{15}N^* \) is defined as \( (I = 3/2!) \)

\[
\tilde{t}_{20} = (N_{3/2} + N_{-3/2}) - (N_{1/2} + N_{-1/2}),
\]  

we obtain with \( \sum N_m = 1 \)

\[
\tilde{t}_{20} = 1 - 2\rho_{1/2 \ 1/2}
\]  

\( \rho_{1/2 \ 1/2} = \frac{1}{2}(1 - \tilde{t}_{20}) \).  

The determination of \( \rho_{1/2 \ 1/2} \) is therefore equivalent to the determination of the alignment of the ejectile.

In order to calculate \( \tilde{t}_{20} \) along the method developed in the previous section, we remember that this state is formed by a proton pickup from \( ^{16}O \) with

\[
\ell = 1, \ s_p = 1/2, \ j = 3/2, \ \ j = \ell + s
\]  

denoting the spins involved. Instead of using a wave function of the hole, we use a statistical tensor \( t_{kq}(j) \) to describe its polarization status according to the vector coupling (eq. 2.50) (Br Sa 71)

\[
t_{kq}(j) = \Sigma(k_q k_s q_s | k q) t^{(\lambda)} (s) \text{ } t_{k q}(j) t_{k q}(s).
\]  

This relation can be used in any coordinate system. \( t_{k q}^{(\lambda)} \) and \( t_{k q}^{(s)} \) describes the polarization of the angular momentum \( \lambda \) and of the proton.
spins of the transferred proton or our knowledge of it!

Concerning $l$, we assume now that, according to a fully classical picture, the angular momentum is transferred along the normal to the scattering plane. (The sign is not interesting since only second rank tensors shall be calculated.) For our purpose, the transverse system (eq. 1.22) is the most appropriate one. According to the definition of spherical tensors (eq. 1.11), we obtain with the assumption above

$$T_{l}^{j 0}(\ell) = \hat{\ell}(\ell, \ell, -\ell, |k, 0) = \hat{\ell}k\left(\begin{array}{ccc}
\ell & l & k \\
0 & 0 & 0
\end{array}\right).$$

the latter being a $3j$-symbol (Br Sa 71).

Since the polarization of the proton is not observed and no kinematical constraint exists for it (except coupling with $l = 1$ to $j = 3/2$),

$$T_{k s q s}^{j q s}(\ell) = \delta_{k s}^{0} \delta_{q s}^{0}.$$ 

Inserting eqs. 2.52 and 2.53 into eq. 2.51, we obtain

$$T_{t 0}(j) = \hat{\ell}^{2} (-1)^{k+s+j+\ell} \left(\begin{array}{ccc}
\ell & l & k \\
\ell-\ell & 0 & l \times s
\end{array}\right).$$

the symbols in $\{ \}$ being $9j$ and $6j$ symbols, respectively (Br Sa 71).

The experiment measured $\tilde{t}_{20}(j)$. It is related to $T_{t 20}(j)$ by a rotation (eq. 1.9) according to the definitions of the coordinate systems involved

$$\tilde{t}_{k 0}(j) = D_{0 0}^{2}(0, \frac{\pi}{2}, \frac{\pi}{2}) T_{t 0}(j)$$

$$= -\frac{1}{2} T_{t 0}(j).$$

Inserting the quantum numbers involved (eq. 2.50), we obtain
\[ t_{20}(3/2) = -\frac{1}{2} \times 2^3 S^3 (-1) \left( \begin{array}{c c c} 1 & 1 & 2 \\ 1 & 1 & 1/2 \end{array} \right) \left( \begin{array}{c c c} 3/2 & 3/2 & 2 \\ 1 & 1 & 1 \end{array} \right) \]

\[ = -\frac{1}{16} \]

and therefore

\[ \rho_{1/2, 1/2} = \frac{1}{2} (1 - t_{20}) = \frac{5}{8} = 0.625. \] (2.57)

The dotted curve in Fig. 2.28 displays an excellent agreement with the observed data. It seems to be even better than the results of the DWBA calculations. At this point, it is necessary to remark that assumption eq. 2.52 can easily be improved using, instead of the classical assumption, a semiclassical description of transfer reactions and inelastic processes as done by P. Bond (Bon 80).

2.6. Inelastic scattering. Figure 2.29 displays data for the m substate population of \(^{12}\text{C}^* (2^+, 4.44\text{ MeV})\) excited by the \(^{88}\text{Sr}(^{12}\text{C}, ^{12}\text{C}^*)^{88}\text{Sr}\) g.s. at \(E_C = 80\text{ MeV}\) (Ing 82). Beyond 30°, the angular distributions again display a classical behavior and the method of Subsect. 2.5 should work to predict the \(\rho_{mn}\). Certainly for angles below 30° where interference effects, in particular a strong nucleus-Coulomb interference is visible, our crude method to calculate the \(\rho_{mn}\) cannot work, and indeed, it doesn't!

We start our considerations with the assumption which proved useful in the previous section

\[ \Delta m_L' = 0 \text{ along } z' = q, \] (2.58)

where \(q\) is the recoil momentum. The \(z'\)-component of the excited \(^{12}\text{C}\) spin \(I = 2\) has to be \(M = 0\) since the ground state spin of \(^{12}\text{C}\) is zero! Hence
The m-substate populations were measured in a coordinate system with 
\[ \tilde{z} = \hat{P}_{\text{fin}} \] which is related by a rotation of \( \delta \) around the normal to the 
scattering plane \( \hat{N} = \hat{y}' = \tilde{y}' \) (fig. 2.30) to the coordinate system used 
in eq. 2.59. Therefore

\[ \tilde{t}_{ko}(I) = D_{00}^k(0,\delta,0)t'_{ko}(I). \]  

Now remember (eq. 1.14)
III-9

\[ \tilde{N}_M = \frac{1}{I} \sum_k (-1)^{I-M} \langle I-I| k_0 \rangle \tilde{t}_{k_0} (I) \]

\[ = \frac{\hat{k}}{I} \sum_k (-1)^{I-M} \langle I-I| k \rangle \tilde{t}_{k_0} (I) \]

(2.61)

Fig. 2.30

and we get, inserting eqs. 2.59 and 2.60 into eq. 2.61

\[ \tilde{N}_M = (-1)^M \sum_k \hat{k}^2 \begin{pmatrix} I & I & k \\ 0 & 0 & 0 \end{pmatrix} D_{k_0}^{*}(0,\delta,0) \]

(2.62)

\[ = (-1)^M D_{I_0}^{*}(0,\delta,0)D_{I_0}^{*}(0,\delta,0) = |C_{IM}(\delta,0)|^2. \]

The last equation is obtained using relations in Appendices IV and V of (Br Sa 71). The \( C_{IM} \) are, besides a factor, spherical harmonics!

Summing eq. 2.62 over M, we obtain

\[ \sum_M \tilde{N}_M = \sum |C_{IM}|^2 = 1 \]

(2.63)

as to be expected since the \( \tilde{N}_M \) were defined as probabilities!
If the Q-value of the reaction \((Q = 4.4 \text{ MeV})\) is small with respect to \(E_{\text{CM}} \approx 70 \text{ MeV}\)

\[
\delta = \frac{\pi - \theta}{2}
\]  
(2.64)

(see fig. 2.30), and we obtain from eq. 2.62

\[
\begin{align*}
\rho_{00}' &= N_0 = \frac{1}{4} (3\cos^2 \delta - 1) = \frac{1}{4} (1 - 3\sin^2 \frac{\theta}{2}) \\
\rho_{11}' &= \tilde{N}_1 + \tilde{N}_{-1} = 2 \times \frac{3}{2} (\cos \delta \sin \delta)^2 = \frac{3}{2} \sin^2 \theta \\
\rho_{22}' &= \tilde{N}_2 + \tilde{N}_{-2} = 2 \times \frac{3}{8} \sin^2 \delta = \frac{3}{4} \cos^2 \frac{\theta}{2}
\end{align*}
\]  
(2.65)

The dotted lines in fig. 2.29 show that eqs. 2.65 describe the \(m\)-substate populations beyond 30° very well but fail, as expected, to describe the data at smaller angles. It seems that new information can be obtained about this region from such measurements.

2.7. \(^6\text{Li}\) Vector Analyzing Powers, Model Analyses

Our early work with polarized heavy ions began at Heidelberg with studies of the elastic scattering of \(^6\text{Li}\) on \(^{12}\text{C},^{16}\text{O},^{28}\text{Si},\text{and}^{58}\text{Ni}\) at energies around 20 MeV (fig. 2.31). More information on this section could be got from (Wei 76) and (Rus 83).

The analysis of the data can be tried in a simple \(\alpha-d\) folding model without antisymmetrization. The reason that the antisymmetrization can be neglected can be inferred from fig. 2.32. We may calculate, for a reasonable potential, for example, a Woods-Saxon potential, the wavefunctions of \(\alpha-d\) and \(^3\text{He}\) taking into account the actual thresholds (binding energies) of these channels. Because of very different threshold energies, the wavefunctions of these configurations are very
different. To demonstrate this, we can then go ahead and calculate a probability distribution

\[ P(r) = \int_r^\infty |\psi_{A+B}|^2 \, dr \tag{2.66} \]

and this is plotted in fig. 2.32. It is obvious that for large distances, to which we are sensitive, only $^6$Li looks solely as composed of an $\alpha$-$d$ cluster. Therefore the $^6$Li spin-orbit potential was calculated from an $\alpha$-$d$ s-wavefunction folding, according to fig. 2.33, the $\alpha$- and $d$-target optical potentials including spin orbits, over the $^6$Li-wavefunction. The optical potentials were taken from the literature.
Fig. 2.32

Fig. 2.33
The potentials, actually calculated by K.-I. Kubo (for references, see [Wei 76]) are shown in fig. 2.34. The results obtained with standard optical potentials for the central part are displayed as solid lines in

**Fig. 2.34**
fig. 2.31. These calculations describe the gross features of the $1T_{11}$ data, but not their details. In particular, the folding model does not describe the Ni data, if inspected in detail. Moreover, Frahn and Hill (Fr Hi 80) claim that our simple model is inadequate to explain our data because we are using a real spin-orbit potential where we should be using a complex one. We therefore remeasured the vector analyzing powers, now at $E = 20$ MeV. By improving the statistics of our old pioneering data, we hoped to be able to see whether an imaginary spin-orbit potential is necessary and whether the 'simple' folding model is able to describe the data in detail. Both answers are no! The new data are displayed in fig. 2.35.

Firstly, we reanalyzed our new data in terms of Frahn and Hill's model (Fr Hi 80). As before (Sect. 2.3), we write down an S-matrix:

$$S(j)(\lambda) = S_c^{(N)}(\lambda) \exp(2i\sigma(\lambda) + 2i\tau\delta_s(\lambda))$$

$$\lambda = \lambda + \frac{1}{2}, j = \lambda + s \text{ and } \tau = 1, 0, -1 \text{ for } j = \lambda + \tau$$

which consists of a nuclear part and a phase. The phase has a term coming from the Coulomb force ($\sigma$), and a term coming from the spin-orbit force. The nuclear part has the same parameterization as before except for a term which allows for refractive effects,

$$S_c^{(N)}(\lambda) = \frac{1}{1 + \exp(-(\lambda - \Lambda)/\Delta - 1/\tau)}$$

(2.61)

The phase $\delta_s$ is related to the derivative of the nuclear S-matrix. The coupling constant $\kappa$ can be chosen freely and consists of a real and an imaginary part,

$$\delta_s(\lambda) = \kappa_\Delta \frac{\partial}{\partial \lambda} S_c^{(N)}(\lambda, \Lambda, \Delta_s)$$

(2.62)
The results of our fit are shown in Fig. 2.35. We fitted the cross section data with the parameters

\[ \Lambda = 11.3 \quad \Delta = 0.65 \quad \text{and} \quad \alpha = 0.00. \]  

We found that we did not require any refractive effects. We then allowed \( \Lambda_s \) and \( \Delta_s \) to vary independently of \( \Lambda \) and \( \Delta \), but the values we obtained were not very different from those we had obtained for \( \Lambda \) and \( \Delta \):

\[ \kappa = \kappa_\tau + i\kappa_{\tau}. \]
\[ A_s = 11.0 \quad A_s = 0.65 \quad \kappa_r = 0.65 \quad \kappa_f = 0.00. \quad (2.65) \]

The coupling strength \( \kappa \) came out to be real!

Finally, we introduced, artificially, an imaginary coupling strength and the results of the calculation are shown by the dashed curve in fig. 2.35. We can see that the deviations are significant only at very backward angles, and our data are not accurate enough to allow us to distinguish between the two cases.

Figure 2.36 shows the results of an optical model fit. The parametrization was conventional (Rus 83). The solid line represents a fit with a real spin-orbit potential, and the dashed line with a complex one. Figure 2.37 compares the strengths of the real optical spin-orbit potential obtained in the fit with the two available folding model potentials. Besides the Kubo potential, there is one calculated by Petrovich et al. (Pet 78) in a different manner. At a first glance, it is surprising that both folding potentials do not agree for \(^{58}\text{Ni}\) in the surface region (\( r > 8 \text{ fm} \)); however, they agree for \(^{12}\text{C}\) (shown in (Pet 78)). I would guess this disagreement comes from a numerical error. On the other hand, it is also evident that at large radii the optical potential is much larger than either of the folding potentials. There seems to be another problem. We will see that these discrepancies are probably due to coupling to excited channels. A part of the spin-orbit potential is not of static origin (frozen density) but produced by coupling to higher excited states of the projectile (Sect. 3.3).

Since the deformation of \(^6\text{Li}\) is very small and the \(^6\text{Li}\) tensor potential negligible, we also calculated with the fitted spin-orbit potential as the only spin-dependent potential, the tensor analyzing...
powers for the $^6\text{Li} + ^{58}\text{Ni}$ reaction. They are very small and positive, as seen in fig. 2.36. These non-zero tensor analyzing powers are caused by the $\lambda$-$s$ potential in a 'second-order' effect. We recall for $M_{2q} = 0$

$$\frac{d\sigma}{d\Omega}(T_{11}) \propto \text{Im}(M_{00}M_{11}^*)$$

(2.66)

$$\frac{d\sigma}{d\Omega}(T_{20}) \propto |M_{11}|^2$$

(2.67)
Hence, there are contributions to the tensor analyzing power second order in the spin-orbit force. Note added in proof: A microscopic analysis using d-α cluster wavefunctions for \(^6\text{Li}\) identifies the positive quadrupole moment of the deuteron in \(^6\text{Li}\) as the source of the small positive value of \(T_{20}\) in fig. 2.36 (Nis 83).

2.8. \(^7\text{Li}\) Vector Analyzing Powers (Tun 81)

One of the problems with our \(^7\text{Li}\) source was that we could not produce a purely first-rank polarized beam. We had a 1/3 admixture of third-rank polarization. We looked for the effects of this admixture at a few angles in \(^7\text{Li}-^{58}\text{Ni}\) scattering. They were found to be smaller than 10%. However, I would like to make it known that this could have a bearing on our data. A second drawback was the accuracy of our polarization calibration. It is very indirect (for details (Tun 81)).

Our experimental results (fig. 2.38) showed, contrary to the predictions of the simple folding models, that the analyzing power of the \(^7\text{Li} + ^{58}\text{Ni}\) reaction is large and negative instead of small and positive.

In the folding model the \(^7\text{Li}\) consists of a loosely bound α-\(\text{t}\) cluster. Because the triton spin is parallel to the \(^7\text{Li}\) spin, the sign of the \(^7\text{Li}\) and the \(^3\text{H}\) spin-orbit potentials are the same, and this leads to a small but positive vector analyzing power as observed for \(^6\text{Li}\).
For comparison, fig. 2.38 shows the data obtained for $^6$Li and $^7$Li elastic scattering on $^{58}$Ni. We set up an optical model code to fit the $^7$Li data too (not shown). In order to fit the $^7$Li data, we had to work with a $s$-$s$ potential with the "wrong sign". The origin of this "wrong sign" cannot be addressed in a simple way. As for $^6$Li, the solution to the problem is found in channel coupling (Sect. 3.3).

2.9. One Neutron Transfer Reactions (Tun 80, Sa Go 81)

Figure 2.39 shows data obtained for the $^{58}$Ni($^7$Li,$^6$Li)$^{59}$Ni reaction at 20.3 MeV. The vector analyzing powers are almost independent of the detection angle $\theta$. If we then go ahead and plot the analyzing powers over a certain angular range ($85^\circ$ to $100^\circ$) as a function of the Q-value,
we get fig. 2.40, which displays the pronounced Q-value dependence of the vector analyzing power $T_{10} = \sqrt{2} T_{11}$. We can give a simple explanation for the behavior of the vector analyzing power in terms of momentum and angular momentum matching. For doing so $^7\text{Li}$ is treated as a nucleus consisting of a $^6\text{Li}$ core with a neutron going around. For our purpose, a rudimentary picture is sufficient in which all spins are neglected except the angular momentum $\lambda = 1$ of the neutron around the $^6\text{Li}$ core. The spin of $^7\text{Li}$ consists, in this picture, of the neutron angular momentum only. The transfer probability reaches a maximum if momentum and angular momentum is matched to an optimum. Now suppose we have a very large negative Q-value. The neutron gets too high in energy for a well-matched transition. To do its best, it likes to have its angular momentum up, since then this adds to its linear momentum (resulting from the motion according to $\lambda = 1$ (fig. 2.41)). That means
that the transfer is much more favoured for the angular momentum of the neutron "up" (for scattering to the left). We expect a large positive vector analyzing power. For large positive Q-values the situation is just the opposite. Around $Q = 0$, where the one-neutron transfer is well
matched, the vector analyzing power is around zero.

Semiclassical considerations: The above handwaving arguments can be substantiated by stating that in a semiclassical treatment of the transfer process we obtain for a Coulomb trajectory a maximum cross section at the distance of closest approach if the z-component of the angular momentum of the transferred neutron \( \lambda \) obeys the equation

\[
\lambda = -R \frac{1/2 \, mv^2 + Q}{hv}.
\]  

(2.68)

\( R \) being the radius of \( ^7\text{Li} \), \( m \) and \( v \) mass and velocity of the neutron at the distance of closest approach. From this equation, we expect \( T_{10} = 0 \) for \( Q = -1/2 \, mv^2 = -0.5 \text{ MeV} \), which seems reasonable if compared to the data. The solid line in fig. 2.40 stems from a theory formulated by Brink and Hill, which includes the spins of the nucleons properly. It yields (from eq. (2.68))

\[
T_{10} = \frac{11}{5\sqrt{5}} \tanh(2\lambda/kR),
\]  

(2.69)

\( k \) being the wavenumber of the bound neutron in \( ^7\text{Li} \). As the data in fig. 2.40 indicate, this expression describes perfectly the Q-value dependence of the vector-analyzing power in a \( (^7\text{Li},^6\text{Li}) \) transfer reaction. It is worthwhile to mention that in this approach, the only feature that enters eq. 2.69 is the energy (Q value) of the final state and the radius of \( ^7\text{Li} \).

DWBA calculations: Figure 2.42 shows DWBA calculations for the same data (Sa Go 81). They show the same features as before, in particular that the Q-value dependence is much more pronounced than the influence of properties of the final state. In the course of these
calculations the tensor analyzing powers $T_{20}$ were also investigated. They are generally small; however, the calculations indicate a pronounced dependence on the final state properties (fig. 2.43). Whether these properties can be used as a spectroscopic tool has to be investigated in the future.

**Concluding Remarks**

In summary, at this point, we think we know what the spin dependence of the heavy-ion interactions in $^6$Li and $^7$Li are (at least the static part as described by the folding models). We know why the tensor
analyzing powers are large for $^7\text{Li}$ and small for $^6\text{Li}$. What remains to be done is the treatment of the dynamic part in respect to couplings to excited levels.

Concerning spectroscopy, we learned that aligned beams are tools to measure moments, BE(2) values and polarizabilities of those nuclei that are aligned. We can get information on the quadrupole moments using methods that are independent of the more conventional hyperfine splitting measurements. There are other spectroscopic applications, for example, in transfer reactions that I have not emphasized. The reason for this is that our accelerator at Heidelberg is not well suited to make these measurements.

With this I shall conclude our discussions on simple ideas regarding Lithium interactions.
### References

<table>
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<th>Author(s)</th>
<th>Journal/Book</th>
<th>Year</th>
<th>Pages</th>
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<tr>
<td>Fr Hi 80</td>
<td>Frahn and Hill</td>
<td>Ann. Phys. (N.Y.) 124</td>
<td>1980</td>
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<tr>
<td>Loe 70</td>
<td>K. E. G. Löbner, M. Vetter, and V. Hönig</td>
<td>Nucl. Data Tables A7</td>
<td>1970</td>
<td>45%</td>
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<td></td>
<td></td>
<td>This compilation includes all references appearing in Sect. 2.4 without special reference!</td>
<td></td>
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<tr>
<td>Mor 82</td>
<td>Z. Moroz et al.</td>
<td>Nucl. Phys. A381</td>
<td>1982</td>
<td>294</td>
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<tr>
<td>Mor 83</td>
<td>Z. Moroz, K. Rusek, P. Egelhof, S. Kossianids, K.-H. Möbius, E. Stefens, I. Koenig, G. Gravel, and D. Fick</td>
<td>to be published.</td>
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Ort 75  Orth et al., Z. Physik A273 (1975) 221


3. Interactions of Li (Mostly) – More Advanced Ideas

Two essential problems remain in the understanding of the interactions of lithium. One is a microscopic calculation of the potential, including the deformation; the second is that of coupled channels. In the ideal situation we would address both problems simultaneously.

This section of the lecture shall focus on four papers which attempt to solve these problems. It will be finished by a few speculative ideas and remarks on future experiments with heavy ions.

3.1. Double Folding Model for $^7\text{Li}$

Following the approach of K. H. Möbius (Möb 83) the nucleon-nucleon interaction $v$ is folded over the densities of the projectile and the target as shown in eq. 3.1.

$$V(r, \hat{r}\cdot\hat{P}) = \int \rho(r_1, \hat{r}_1\cdot\hat{P})\cdot\rho_T(r_2)\cdot v(r+r_2-r_1)d^3r_1d^3r_2 \quad (3.1)$$

The meaning of the various symbols is obvious from fig. 3.1. The interaction includes a tensor force which depends on the shape of the $^7\text{Li}$. The density of $^7\text{Li}$ is written as

$$\rho(r, \hat{r}\cdot\hat{P}) = \rho_0(r) + Q^m\cdot\rho_2(r)\cdot Y_{20}(\hat{r}\cdot\hat{P}) \quad (3.2)$$

with $Q^m$ the spectroscopic mass quadrupole moment of $^7\text{Li}$ and $Y_{20}$ a spherical harmonic. The quadrupole density $\rho_2$ is normalized according to

$$\int \rho_2(r) r^4 dr = \sqrt{5}/16\pi \quad (3.3)$$
Because of this normalization the spectroscopic mass quadrupole moment $Q^m$ has to be multiplied by $Ze/A$ (for example for $^7\text{Li}$ by $3e/7$) for a comparison with the spectroscopic charge quadrupole moment $Q^c$. The densities $\rho_0(r)$ and $\rho_2(r)$ are taken from (Sue 67). In order to evaluate eq. 4.1 for the $^7\text{Li} + ^{58}\text{Ni}$ and $^{51}\text{V}$ reactions we need the densities of $^{58}\text{Ni}$ and $^{51}\text{V}$. These densities can be written as

$$\rho_T(r) = \rho_T^0 \frac{1}{1 + \exp[(r-R_T)/a_T]} \quad (3.4)$$

The parameters describing the vanadium and nickel nuclei are shown in table 3.1.
Table 3.1

<table>
<thead>
<tr>
<th></th>
<th>$R_T$/fm</th>
<th>$a_T$/fm</th>
<th>$\sqrt{\langle r^2 \rangle}$/fm</th>
<th>$\rho_T^{0}$/fm$^{-3}$</th>
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<td>$^{51}$V</td>
<td>3.985</td>
<td>0.49</td>
<td>3.58</td>
<td>0.1674</td>
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<tr>
<td>$^{58}$Ni</td>
<td>4.10</td>
<td>0.57</td>
<td>3.82</td>
<td>0.1687</td>
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</table>

For the form of the potential $v$ in eq. 3.1 (MOB 83) uses two interactions. One is the simple $\delta$ force between two nuclei, which is written as:

$$v(r) = f \cdot \delta^3(r)$$

The other force is the M3Y force (Sa Lo 79) which is parametrized as:

$$v(r) = N \left\{ \sum_{i=1,2} \frac{v_i e^{-r/a_i}}{4\pi a_i^2 r} + v_3 \delta^3(r) \right\}$$

The only parameters in this model are $Q^m$, $f$ and $N$. $f$ and $N$ can be complex.

We have two sets of data, one for the total reaction cross section ($\sigma^r$ and $T^r_{20}$) and the others for elastic scattering ($\sigma/\sigma_R$ and $T_{2q}$, $q = 0, 1, 2$).

**Total Reaction Cross Section**

As usual the $z$-dependent real interaction potential is decomposed into a sum of Coulomb, centrifugal and nuclear potentials

$$V_z(r, \hat{r}, \hat{p}) = \frac{Z_p Z_T e^2}{r} + \frac{\hbar^2 z(z+1)}{2mr^2} + g_{\text{centr}}(r)$$

$$+ Q^m g_{TR}(r) P_z(\hat{r}, \hat{p})$$
In the above expression, the functions $g$ come from the double folding potential (eq. 3.1). One comes from the monopole contribution and the other from the quadrupole deformation of $^7Li$. The total reaction cross section for unpolarized and aligned beams is calculated as a sum over transmission coefficients

$$
\sigma_{\alpha\beta} = g \sum_{\ell=0}^{\infty} (2\ell+1) T_{\ell} \quad (3.8)
$$

where

$$
T_{\ell} = \frac{1}{1 + \exp \left\{ -2\pi(E-E_{\ell})/\hbar\omega_{\ell} \right\}} \quad (3.9)
$$

The transmission coefficient has been parametrized the usual way using the inverted parabolic potential.

We now have to make an approximation. If we move along a trajectory, the angle between $r$ and $\hat{P}$ (fig. 3.2) will change. We replace this changing angle by its value at the distance of closest approach

$$
\hat{r}_{\text{min}}(\ell,k) \cdot \hat{P} = \cos[\gamma(\ell,k)] \quad (3.10)
$$

The dominant contribution to eq. 3.8 comes from this region.
The parameters in eqs. 3.8 and 3.9 are now defined in terms of the interaction potential, the interaction radius $R_\lambda$ from

$$aV_\lambda[r,\cos\gamma(x,k)]/ar\mid_{r=R_\lambda} = 0,$$  \hspace{1cm} (3.11)

the potential at the top of the barrier from

$$E_\lambda = V_\lambda[R_\lambda,\cos\gamma(x,R)]$$  \hspace{1cm} (3.12)

and the curvature of the parabolic potential from

$$\omega_\lambda = \sqrt{-\frac{1}{\mu} \left(\frac{\partial^2 V_\lambda}{\partial r^2}\right)\mid_{r=R_\lambda}}$$  \hspace{1cm} (3.13)

If we plug in numbers for the various quantities in eqs. (3.8)-(3.12), we get the cross sections for aligned $^7\text{Li}$. In order to obtain the cross section $\sigma^\mu$ for unpolarized particles we have to average over all possible alignment axes of $\hat{\lambda}$.

Figure 3.3 shows the data for the $\gamma$-ray and quasielastic scattering measurements of the $^7\text{Li} + ^{51}\text{V}$ reaction cross section (see Sect. 2.1). Plotted in fig. 3.3 are the results of the two calculations which were performed. The dashed curve was calculated for the zero range interaction, and the solid curve the M3Y force. With this quantum mechanical treatment for the transmission coefficient, one can account for the subbarrier cross section. One might wonder why the $\Delta\sigma$ in fig. 3.3 goes to zero at low energies. The answer is very simple. If one goes lower and lower in energies below the Coulomb barrier, the total reaction cross
section goes to zero, and so does the difference between the aligned and unaligned cross sections. One interesting feature we get out of this analysis is the dependence of the difference in partial cross sections on the angular momentum. Figure 3.4 plots the transmission coefficient
$T_\ell$ and $\Delta\sigma_\ell$ as a function of $\ell$ for different energies. We can see that $\Delta\sigma$ peaks at the grazing angular momentum.

**Elastic scattering**

Figure 3.5 shows the results of calculations with the double folding potential for the angular distributions of $\sigma/\sigma_R$ and $T_{20}$ for elastic and quasielastic scattering of $^7\text{Li}$ or $^{51}\text{V}$. The solid lines and the dashed lines are results with the M3Y and the zero range nucleon-nucleon interaction respectively. The results for the $^7\text{Li}-^{58}\text{Ni}$ scattering at 14.22 MeV and 20.3 MeV are shown in figs. 3.6 and 3.7. The parameters used on the various calculations are listed in Table 3.2 and a compilation of deformation parameters of $^7\text{Li}$ from various sources in Table 3.3. Since the errors, on final analysis, have still to be considered large $(3e/7)Q^m$ agrees satisfactorily with $Q^e$. 
Fig. 3.4

\[ \Delta \sigma^F_\ell = \left( \frac{\pi}{k^2} \right) (2\ell + 1)(T^al_\ell - T_\ell) \]
Fig. 3.5
$^7\text{Li} + ^{58}\text{Ni}$

$E_{^7\text{Li}} = 14.22$ MeV

quasielastic scattering

Fig. 3.6
\( \frac{\sigma}{\sigma_R} \)

Fig. 3.7

\[ \text{\(7\text{Li} + ^{58}\text{Ni}\)}] 

\[ E_{\text{Li}} = 20.3 \text{ MeV} \]

elastic scattering

\[ \theta_{\text{c.m.}} \]

\[ T_{20} \]

\[ T_{21} \]

\[ T_{22} \]
Table 3.2. Model parameters from various analyses.

<table>
<thead>
<tr>
<th>System</th>
<th>E(_{L1}) (MeV)</th>
<th>Data</th>
<th>(\delta) - folding</th>
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<th>M3(^{+}) - folding</th>
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<td></td>
<td>((3e/7)Q^m)</td>
<td>Re(f)</td>
<td>Im(f)</td>
<td>((3e/7)Q^m)</td>
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For the \(7\_L1\-^{51}\_V\) system \(\sigma/\sigma_R\) always denotes the differential cross section for quasielastic scattering divided by the one for Rutherford scattering. The various parameters are explained in the text.
Table 3.3. Deformation parameters of $^7$Li from various sources.

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<thead>
<tr>
<th>System</th>
<th>$E_{LL}$ (MeV)</th>
<th>Analyzed Quantities</th>
<th>Interaction potential</th>
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<tr>
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<td>$\alpha/aR\gamma_0, T\gamma_0$</td>
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<td>M3Y-folding</td>
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</table>

spectroscopic charge quadrupole moment

homogeneously charged nucl. [Mor 82]
3.2. Cluster Folding Model for $^7\text{Li}$ (Mu Gr 82)

We now go to a paper in which a simple $\alpha$-$t$ cluster folding model is used to describe $^7\text{Li}$. (Mu Gr 82) has a detailed discussion of the spin structure of the $^7\text{Li}$ interaction and a discussion of the negative vector analyzing power observed for $^7\text{Li}$ (Fig. 2.38).

![Fig. 1 from (Mu Gr 82)](image)

![Fig. 3.8](image)

We treat $^7\text{Li}$ as a triton and an $\alpha$ particle. The quantum numbers are: $S_t = 1/2$, $S_\alpha = 0$, angular momentum between $t$ and $\alpha$: $l = 1$; $^7\text{Li}$ spin in the ground state $I = 3/2^-$. The potential for the interaction between $N_1$ and $Li$ can be written down as:

$$V(R) = \langle \frac{3}{2} M' | V_{\alpha}(r_{\alpha}) + V_{t}(r_{t}) + \frac{1}{2} t s t V_{so}(r_{t}) | \frac{3}{2} M \rangle$$

(3.11)

$$= F_0(R) + L S F_{so}(R) + T_R F_T(R) + T_3 F_{T3}(R)$$

The coordinates are indicated in fig. 3.8. The $\alpha$ and t potentials can be chosen complex. The orbital angular momenta $t_L$ and $L$ refer to the the coordinate $r_t$ and $R$ respectively. $F_i$ are (complex) form factors.
derived from the integrals in eq. 3.11. The operators $T_R$ and $T_3$
are defined according to eq. 1.46. The folding potential consists
of contributions from the central interaction of the $t$ and $\alpha$ particle
with the nickel target, and the tensor and spin orbit interactions of
the triton particle with the nickel target.

We need now to write down a wavefunction for the $^7\text{Li}$ bound state.
We do this analytically using oscillator wave functions. The oscillator
parameter can be chosen such that $\langle r^2 \rangle$ and $Q$ of $^7\text{Li}$ are correctly
described. It is important to describe the quadrupole moment correctly.
Otherwise we will not get the right tensor analyzing powers. The drawback
of using oscillator wavefunctions is that it gives the wrong asymptotic
behavior, and we know that these interactions are dominated by the
asymptotic part of the wavefunctions. This point needs to be noted.

Figure 3.9 shows the magnitudes of the various form factors contrib-
uting to the potential (eq. 3.11). Clearly, the spin orbit force is
very small in magnitude.

This model describes the tensor analyzing powers fairly well (fig.
3.10). The cross sections are, however, not so well described. The
vector analyzing powers obtained in this calculation are negative, but
are only about half the magnitude of the data points.

This model takes into account the spin structure of the $^7\text{Li}$
interaction correctly (including third-rank terms). It describes the
tensor analyzing power as good as other models but fails, as others
discussed up to now, to describe the vector analyzing power. Therefore
other phenomenon outside the scope of the model discussed up to now have
to be considered. This is done in the following chapter.
Fig. 3.10
3.3. Cluster Folding Model with Channel Coupling

This section focuses its attention on a recent preprint by H. Nishioka et al. (Nis 82) on the spin orbit interaction induced by heavy-ion excitation. This was the first paper which combined folding models and the coupling to inelastic channels. The paper tells us why we observe different signs for the vector analyzing power for $^6\text{Li}$ and $^7\text{Li}$. Somewhat later, an analysis of $^7\text{Li}$ interaction treated in the same spirit was published (Ohn 82). Both papers are the basis of the following discussion. To start with we recall that static folding model calculations for the $^6\text{Li}$ and $^7\text{Li}$ spin-orbit interaction agree in predicting rather small but positive vector analyzing powers $iT_{11}(\theta)$ for the elastic scattering of both nuclei on $^{58}\text{Ni}$ (dashed lines in fig. 3.11). The negative vector analyzing powers obtained in the previous section result with large probability from the fact that these calculations take into account the strong tensor potential in the $^7\text{Li}$-$^{58}\text{Ni}$ interaction. Experimentally, however, large analyzing powers were observed. It can be shown that strong coupling to selected projectile excited states with definite spin values produces an effective spin-orbit interaction with sign and magnitude that reproduces the experimental data. The mechanism to be discussed is very general and not restricted to the loosely bound $^6\text{Li}$ and $^7\text{Li}$ nuclei with their $d-\alpha$ and $t-\alpha$ cluster structure.

Necessary for the generation to this type of effective spin-orbit interaction is the strong (quadrupole) coupling to excited states (whether bound or unbound). The case of $^6\text{Li}$ is considered first. The
Fig. 3.11
folding model calculations mentioned before indicate that the spin dependence of the deuteron-target interaction can be considered as weak. Thus, the deuteron spin $S$ is a constant of motion. In its ground state which is $I^\pi = 1^+$, $^6L_1$ has angular momentum $\ell = 0$ between deuteron and $\alpha$-cluster and $^6L_1$ and deuteron spin are parallel. The excited states to be discussed are those with $\ell' = 2$ leading to a $I'^\pi = 3^+, 2^+, 1^+$ triplet which is split by the spin-orbit force acting between deuteron and $\alpha$-particle. At the low energies involved in the experiments inelastic coupling to the $I'^\pi = 3^+$ state dominates, since it is the lowest lying state of the multiplet. For this state $\ell$ is aligned with the deuteron spin $S$. Since the latter is considered as a constant of motion the excited state spin is aligned with the ground state spin $I$ of $^6L_1$.

Figure 3.12 denotes the angular momentum between $^6L_1$ and target by $L$ and $L'$ for ground and excited state

\[ \begin{array}{ccc}
\hline
& & \\
\hline
\hline
\hline
& & \\
\hline
\end{array} \]

Fig. 3.12
respectively. Then, in the classical limit, vector coupling implies that the orbital angular momentum is \( L' = L - 2 \) for \(^6\text{Li}\) spin \( J \) and orbital angular momentum \( L \) parallel and \( L' = L + 2 \) if both spins are antiparallel (Fig. 3.13). Since the intermediate channel has a negative Q-value, channels with \( L' = L - 2 \) will be strongly favored over the channels with \( L' = L + 2 \) because of centrifugal barrier effects.

These handwaving arguments show that the elastic scattering of \(^6\text{Li}\) via the intermediate \(^3^+\) excited state produces an effective spin-orbit potential which is dominated by the channel in which the \(^6\text{Li}\) ground state spin \( I \) and the orbital angular momentum \( L \) are parallel. It can be shown rigorously that such effective potentials vanish if all intermediate levels of a multiplet are excited according to their statistical weights. Summarizing, it is the spin-orbit force acting in the projectile which
generates in the limit of a classical treatment of the interaction this effective spin-orbit force between projectile and target.

The dash-dotted and solid curves in (Fig. 3.11), are the results of coupled channel calculations with a d-α cluster wave function for $^6$Li. The interaction and coupling potentials were obtained by standard folding procedures. The dash-dotted curve takes only the coupling to the $3^+$ state of the multiplet into account, whereas the solid curve includes the full coupling. The calculations (Nis 82) confirm the above handwaving picture.

For $^7$Li scattering a similar discussion is possible but now triton spin and orbital angular momentum between triton and α-particle which are parallel in the ground state of $^7$Li ($I^{^*} = 3/2^-$) are antiparallel in the first excited state ($I^{^*'} = 1/2^-$). If the spin-orbit interaction between triton and target is neglected and the triton spin considered as a constant of motion, the preferential channel for excitation is now that in which $^7$Li spin $I$ and orbital angular momentum $L$ between $^7$Li and target nucleus are antiparallel. As a result, the effective spin-orbit interaction for $^7$Li has the opposite sign to that for $^6$Li. Coupled channel calculations (Ohn 82) (fig. 3.11), performed similarly to those for $^6$Li, confirm these arguments. The dotted curve is a single channel calculation which includes the tensor potential generated by the strong deformation of $^7$Li. In contrast to $^6$Li this tensor potential is strong enough to affect the vector analyzing powers considerably via second order mechanisms. The dash-dotted curve takes into account the excitation to the $1/2^-$ state only, while the solid curve includes several other excited states in $^7$Li.
These calculations (Ohn 82) also confirm the picture of the tensor potential given before. In the angular range investigated the effective spin-orbit potential is not strong enough to contribute significantly via a second order effect to the strong tensor potential generated by the large deformation of the $^7$Li projectile.

3.4. Coupling to Inelastic Channels - Semiclassical Treatment

The previous chapter demonstrated the importance of coupling to projectile excited states for the understanding of the $^6$Li and $^7$Li vector analyzing powers. We learned from channel coupling effects in Sect. 2.4, which dealt with sub-Coulomb scattering of $^7$Li. There, channel coupling could be handled easily since the cross sections followed, at this energy, the semiclassical relation

$$\sigma_R(\theta) = \sigma_{el}(\theta) + \sigma_{in}(\theta)$$

$\sigma_R$ being the Rutherford cross section. In this section I like to extend this method to a regime where a similar relation holds, however, with $\sigma_{opt}(\theta)$, the cross section for a standard optical potential instead of $\sigma_R(\theta)$.

I shall start with Ar + $^{208}$Pb data taken at GSI (Ar1 80). A special feature of these data are that both the elastic and the inelastic cross sections were measured. The data are shown in fig. 3.14, and show deviations from the Rutherford cross sections at angles much lower than predicted by the optical model (curve 1). These deviations are quantifiable as due to the excitation of the Argon from $0^+$ to $2^+$ at 1.4 MeV.
We find that

\[ \sigma_{\text{opt}}(\theta) = \sigma_{e1}(\theta) + \sigma_{\text{in}}(\theta). \]  

(3.13)

We assume now the very same system, however, with an 'Argon'-nucleus with a ground state spin of \( \frac{3}{2}^+ \) and an excited level at 1.4 MeV of spin \( \frac{7}{2}^+ \).

For this hypothetical nucleus (similar to \(^{23}\text{Na}\) for spin arguments) we can calculate analyzing power using eq. 3.13 and the ideas of Sect. 2.4.
From
\[ \sigma_{\text{opt}} = \sigma_{\text{el}} + \sigma_{\text{in}} \] (3.13)
shall we obtain
\[ T_{T, \text{opt}} = \left( \frac{\sigma_{\text{el}}}{\sigma_{\text{opt}}} \right) T_{T, \text{el}} + \left( \frac{\sigma_{\text{in}}}{\sigma_{\text{opt}}} \right) T_{T, \text{in}} \] (3.14)

following ideas of Sect. 2.4. The tranverse coordinate system (eq. 1.23) was chosen for convenience. If we restrict ourselves to angles below 60°, for which
\[ \frac{\sigma_{\text{opt}}}{\sigma_R} = 1 \] (3.15)
whence,
\[ T_{T, \text{opt}} = 0 , \] (3.16)

if we neglect the small analyzing powers observed in Coulomb scattering (Sect. 2.4). From eq. 3.14 we obtain with eq. 3.16
\[ T_{T, \text{el}} = - \frac{\sigma_{\text{in}}}{\sigma_{\text{el}}} T_{T, \text{in}} . \] (3.17)

It is easy to calculate the analyzing powers for the inelastic scattering in this approximation because we are in the Coulomb excitation regime. From (Zup 79) we can write down the analyzing powers for Coulomb excitation as
\[ T_{T_{kq}}(\lambda) = \hat{s}_a^k \hat{s}_c^l (-1)^{s_a+s_c}s^{\lambda+k}_{a c} \left\{ \begin{array}{ccc} \lambda & \lambda & k \\ s_a & s_a & s_c \end{array} \right\} \left( \begin{array}{ccc} \lambda & \lambda & k \\ \lambda & -\lambda & 0 \end{array} \right) T_{\Pi_{kq}}(\theta, \zeta) \] (3.18)

\[ s_a, s_c \] denoting the spin of ground and excited state, respectively. \( \lambda \) denotes the multipolarity of the transition. \( T_{\Pi_{kq}}(\lambda) \) is a universal function, derived from Coulomb integrals \( R_{\lambda, \mu} \) and depends only on the scattering
angle is and on the adiabaticity parameter
\[ \xi = \eta \cdot \Delta E / 2E \quad (3.19) \]

\[ T_{n_{kq}}(\lambda) = \frac{\sum_{\mu'}(-1)^{\lambda-\mu'}(\lambda \quad \lambda \quad k) \, R_{\lambda \mu} \, R_{\lambda \mu}^*}{(\lambda \quad -\lambda \quad 0) \, \sum_{\mu} |R_{\lambda \mu}|^2} \quad (3.20) \]

Inserting the quantum numbers for our chosen system we get for \( \lambda = 2 \)

\[ T_{10_{k0}}(in) = 2 \times 5(-1)^{1+k} \{ \begin{array}{ccc} 2 & 2 & k \\ 3/2 & 3/2 & 7/2 \end{array} \} \{ \begin{array}{ccc} 2 & 2 & k \\ 2 & -2 & 0 \end{array} \} T_{10_{k0}}(2) (\theta, \xi) \quad (3.21) \]

from which

\[ T_{10_{k0}}(in) = -2 \, T_{10_{k0}}(3, \xi) \]

\[ T_{20_{k0}}(in) = 5/7 \, T_{20_{k0}}(2, \xi) \quad (3.22) \]

\[ T_{30_{k0}}(in) = -\frac{1}{14\sqrt{5}} \, T_{30_{k0}}(2, \xi) \]

is derived. For \( \lambda = 2 \) Fig. 3.15 now plots the functions \( T_{n_{kq}}(2) \) as function of \( \theta \) and \( \xi \).
For our problem the adiabaticity parameter can be calculated as $\xi = 0.35$. For $\theta \leq 60^\circ$ we note from fig. 3.15 with $\xi = 0.35$.

$$- T_{\theta 10}^{(2)} > 0.7$$

$$T_{\theta 20}^{(2)} > 0.7$$

$$- T_{\theta 30}^{(2)} > 0.7$$

(3.23)
Whence for $\theta \leq 60^\circ$

\[ T_{10}^{in} \geq 1.4 \]

\[ T_{20}^{in} \geq 0.5 \] \hspace{1cm} (3.24)

\[ T_{30}^{in} \geq 0.02 \]

The analyzing powers for the elastic scattering are then obtained from eq. 3.17

\[ T_{k0}^{el} = -\frac{\sigma_{fn}^{in}}{\sigma_{el}} T_{k0}^{in} \] \hspace{1cm} (3.25)

At $\theta = 60^\circ$ we recognize from fig. 3.14 that

\[ \frac{\sigma_{fn}^{in}}{\sigma_{el}} = 0.25 \] \hspace{1cm} (3.26)

and obtain

\[ T_{10}^{(60^\circ)} = 0.35 \]

\[ T_{20}^{(60^\circ)} = 0.1 \] \hspace{1cm} (3.27)

\[ T_{30}^{(60^\circ)} = 0.0 \]

Thus this semiclassical model also gives as a result quite large vector analyzing powers for the elastic channel if this channel is coupled largely to inelastic ones.
3.5. **Outlook**

I hope the fits and calculations will enable predictions as nice as obtained for spinless nuclei! Figure 3.16 shows such fits and predictions for spinless particles obtained ten years ago at Oak Ridge. One has still to speculate how the model used in fig. 3.16 can be extended to pions with spin and how the analyzing powers will then look.

![Diagram](image1)

![Diagram](image2)

![Diagram](image3)

**Fig. 3.16**
References

LECTURE V

4. Polarized Heavy Ions – Gone Astray

4.1. Investigation of Parity Violation

An experiment investigating the parity violation effects with longitudinally polarized $^6$Li uses the reaction

$$ ^4\text{He} + ^6\text{Li} \rightarrow ^{10}\text{B}^* + \gamma\text{-quanta} + ^{10}\text{B} $$

The reaction is studied at $E_{^6\text{Li}} \sim 2$ MeV. In $^{10}\text{B}$, a $2^+$ and a $2^-$ level are very close to each other. One can look for parity mixing by measuring the vector analyzing power for longitudinally polarized $^6$Li ions. The effect predicted originally is a part in $10^3$, and seems not too difficult to be observed. However if a gas target is used, the polarization of $^6$Li is affected by the hyperfine interaction produced when the $^6$Li nuclei pick up electrons while passing through the helium gas target. Hence, the polarization of the $^6$Li at the actual target nucleus is not known. This problem has not been solved satisfactorily. Various alternatives are being investigated. For a recent reference see (Pap 82).

4.2. Polarized Recoil Nuclei and their Relaxation in Host Lattices (Koe 81)

Reactions of the type

$$ T(Li,X)^R $$

were investigated, $T$ denoting the target nucleus, and $R$ the recoil nucleus.
The vector polarization of \( R \) was measured by looking at the asymmetry of the \( \beta \) particle emitted by the stopped recoil nuclei. In such measurements, when the vector polarization of the lithium is switched, the polarization of the recoil changes sign too.

Figure 4.1 shows the experimental setup. There is a magnetic field provided to 'keep' the polarization of the beam. The \( \beta \) detectors are two thick silicon counters. Absorbers are placed in front of the counters to stop the heavy particles. What is measured is the asymmetry ratio

\[
\varepsilon = \frac{1}{2} \frac{N_+ (0) - N_+ (\pi)}{N_+ (0) + N_+ (\pi)} - \frac{N_+ (\pi) - N_+ (\pi)}{N_+ (\pi) + N_+ (\pi)}
\]  

(4.1)

\( + \) and \( \pi \) denoting the direction of the vector polarization of the Li beam. \( N(0) \) and \( N(\pi) \) are the counts of the upper and lower counter respectively.
The experiment uses a pulsed beam, and the particles are measured with the beam off. A typical result of a measurement is shown in Fig. 4.2. The decay curve shows two $\beta$ activities, one for $^{12}\text{B}$ and the other for $^{8}\text{Li}$. The asymmetry parameter $c$ is plotted in Fig. 4.3. The observed effect is around 2%. A question that comes to mind at this point is, what magnitude of polarization does a 2% $\beta$ asymmetry reflect? This question cannot be answered properly (see Koe 81), and we can only place a lower limit on the polarization. A $\beta$ asymmetry of 2% translates to an $^{8}\text{Li}$ polarization of at least 20%. Some of the results are shown in fig. 4.4. These data points have been normalized to a $\text{Li}$ beam polarization of one ($P_z$, eq. 1.12a). The beam polarizations actually used in the experiments were:

\[
\begin{align*}
^{6}\text{Li} & \quad P_z = \sqrt{2/3} \quad t_{10} = 0.60 \\
^{7}\text{Li} & \quad P_z = \quad t_{10} = 0.45
\end{align*}
\]

Apart from the problems of normalization for the polarization, we had to account for the finite activation and detection intervals in the experiment. We could have done better in the experiment by measuring the energy of the emitted electrons. The $\beta$ spectrum is cut off at some energy due to the absorber. We could not correct for depolarization due to $\gamma$-cascades to the ground state.

We observe in fig. 4.4 that the polarization of $^{8}\text{Li}$ produced in the $^{9}\text{Be} + ^{7}\text{Li}$ and $^{7}\text{Li} + ^{7}\text{Li}$ reactions are very different. This can be understood in terms of very simple ideas, as a result of which we will learn about the kinds of reactions which will produce a large recoil polarization.
We start with the simple case of the $^9$Be($^7$Li, $^8$Be)$^8$Li reaction. The one-nucleon transfer can occur in two ways as shown pictorially in figs. 4.5. Since the reaction is well matched for the $^8$Be ground state ($0_{g.s.} = -0.36$ MeV) we expect most of the transfer to go through it. (The first excited state of $^8$Be is $\sim 3$ MeV.)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig4.5.png}
\caption{Fig. 4.5}
\end{figure}

Since the ground state of $^8$Be is $0^+$, the transferred proton has to have $j = 3/2$. By combining statistical tensors (like in Sect. 2.6) we can calculate the expected polarization of $^8$Li. Such a calculation is very crude since it neglects any kind of dynamical process. It will give only a very rough estimate of the effects. The value of such calculations lies in delineating the influence of spin coupling on the results.
The polarization is given in cartesian coordinates by

\[ P_z(S) = \sqrt{\frac{s+1}{3s}} t_{10}(S). \]  

(4.3)

It was calculated for two states in \(^8\text{Li}\), and the results are shown in Table 4.1:

<table>
<thead>
<tr>
<th>(^8\text{Li})</th>
<th>(I_\pi)</th>
<th>(E_x) MeV</th>
<th>(P_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2^+)</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>(1^+)</td>
<td>0.98</td>
<td>0.41</td>
<td></td>
</tr>
</tbody>
</table>

The observed results are of the same order of magnitude; in fact, they are smaller. It should be remembered that only a lower limit of \(P\) is plotted on fig. 4.4.

From the discussion of the \(^7\text{Li}(^7\text{Li}, ^6\text{Li})^8\text{Li}\) reaction one may learn what factors lower the polarization of the recoil \(^8\text{Li}\) nuclei. The ground state Q value = -5.2 MeV. Hence, it is very probable that angular momentum is transferred in this reaction, which can decrease the \(^8\text{Li}\) polarization. The polarized \(^7\text{Li}\) can either pick up a neutron, or get stripped of a neutron. These transitions can occur with both \(J = 1/2^-\) and \(J = 3/2^-\) neutrons. The corresponding polarizations can be calculated, and the numbers obtained have to be multiplied by a spectroscopic factor. The results of the calculation are shown in Table 4.2.
A point to note is that the contributions from the $j = 1/2^-$ and $j = 3/2^-$ transitions are opposite in sign. Hence, if we want to produce recoil nuclei with large polarizations, we have to avoid using transitions which have a mixed-$j$ transfer.

Another reaction studied was the "five-nucleon transfer" reaction $^7\text{Li}(^7\text{Li},d)^{12}\text{B}$ which probably proceeds mainly as a compound nucleus reaction. The data obtained are shown in figs. 4.7 and 4.8 for the reaction at 16 MeV. The observed polarizations for $^{12}\text{B}$ are summarized in Table 4.3:
These numbers also show that multinuclear transfer reactions or those proceeding through a compound nucleus yield reasonable polarizations for the recoil nuclei.

Other information we can get from the $\beta$ asymmetry measurements are the magnetic moments of the nuclei. If the experimental setup is placed in a region containing an oscillating RF field, the nuclear moments precess in this field. At certain precession frequencies, resonance conditions occur, and this can be detected by a sudden change in the asymmetry parameter $\epsilon$ as shown in fig. 4.9.
\( ^7\text{Li}(^7\text{Li, d})^{12}\text{B} \)

\( E_{\text{Lab}} = 16 \text{ MeV} \)

Fig. 4.8
Fig. 4.9
We studied several other nuclear reactions leading to recoil nuclei with unknown magnetic moments. However, we failed to observe a reasonable asymmetry. Some of the reactions studied are shown in Table 4.4:

<table>
<thead>
<tr>
<th>Projectile</th>
<th>$E_{Li}$/MeV</th>
<th>Target</th>
<th>Host</th>
<th>Recoil Nucleus</th>
<th>$T_{1/2}$/s</th>
<th>X</th>
<th>Q/MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7_{Li}$</td>
<td>10-20</td>
<td>$^{11}_{B}$</td>
<td>Au</td>
<td>$^{17}_{N}$</td>
<td>4.2</td>
<td>p</td>
<td>+8.4</td>
</tr>
<tr>
<td>6 Li</td>
<td>18</td>
<td>$^{13}_{C}$</td>
<td>Au</td>
<td>$^{17}_{N}$</td>
<td>4.2</td>
<td>2p</td>
<td>-5.2</td>
</tr>
<tr>
<td>$^7_{Li}$</td>
<td>16-20</td>
<td>$^{16}_{O}$</td>
<td>Au,Pb</td>
<td>$^{21}_{F}$</td>
<td>4.4</td>
<td>2p</td>
<td>-4.4</td>
</tr>
<tr>
<td>$^6_{Li}$</td>
<td>18</td>
<td>$^{26}_{Mg}$</td>
<td>Au,Pb</td>
<td>$^{30}_{Al}$</td>
<td>3.7</td>
<td>2p</td>
<td>-0.8</td>
</tr>
<tr>
<td>$^6_{Li}$</td>
<td>18</td>
<td>$^{54}_{Fe}$</td>
<td>Au,Pb</td>
<td>$^{59}_{Cu}$</td>
<td>8.2</td>
<td>n</td>
<td>+6.1</td>
</tr>
<tr>
<td>$^6_{Li}$</td>
<td>18</td>
<td>$^{54}_{Fe}$</td>
<td>Au,Pb</td>
<td>$^{58}_{Cu}$</td>
<td>3.2</td>
<td>2n</td>
<td>-6.6</td>
</tr>
</tbody>
</table>

For all these reactions there was no asymmetry detected. There may be several reasons for this. One may be a small polarization transfer. A second reason may be because all these nuclei have low-lying $I=0$ and $I=1/2$ levels. Any gamma cascade going through these low-lying levels results in a total loss of polarization information. On the other hand, we may be producing nuclei that are polarized, but the asymmetry coefficients may be very small. Other explanations could invoke short relaxation times, but that is something we know very little about.
Relaxation times: In previous measurement (fig. 4.3), we had an $e$ which did not vary with time (the time scales of these measurements were much smaller). In relaxation time studies we write $e$ as

$$e(t) = e_0 e^{-t/T_1}$$

(4.4)

where $T_1$ is the relaxation time for vector polarization (rank 1). Some other data are shown in fig. 4.10. Relaxation times obtained with eq. 4.4 are given in Table 4.5.

![Fig. 4.10a](image_url)

![Fig. 4.10b](image_url)
The numbers obtained for Pt and Au disagree with those obtained by [Ha Ma 73]. The reason for this is probably due to the fact that [Ha Ma 73] had low-energy recoil $^{8}$Li. These nuclei could not have penetrated very far into the host, and the measured relaxation rates could have been affected by impurities on the surface.

### 4.3 Nuclear Spin Relaxation on Surfaces

[Boe81, Hor82]

This study intends to investigate similar questions as before: nuclear spin relaxation, but now for atoms absorbed on surfaces. It is for the first time that such investigations are being done for nuclear moments interacting with fluctuating fields on a surface. Interest here began because our ion sources use a surface ionizer.

A thermal nuclear spin polarized atomic beam is incident on a heated W, Re, or Ir surface. The number of atoms which are ionized depend on the surface condition. The setup is shown schematically in fig. 4.11.
There is a strong magnetic field parallel to the normal to the surface. The polarization of the ions desorbing from the surface is measured by a "beam foil polarimeter". Details of the whole apparatus are discussed later.

![Electrostatic Mirror Diagram](image1)

**Fig. 4.11**

We shall first try to understand in a simple way the process of surface ionization. A metal is characterized by a quantity which is known as the work function. Suppose an atom, e.g. sodium, approaches this surface. The atom is characterized by an ionization energy I as shown in fig. 4.12. When the atom approaches the metal and \( I < \phi \) the sodium valence

![Surface Ionization Diagram](image2)

**Fig. 4.12**
electron gains energy by tunneling through the barrier. Suppose there are \( N_0 \) atoms on the surface. The number of atoms that get ionized is given by

\[
\frac{N^+}{N_0} = 0.5 \exp \left( \frac{\phi - I}{kT} \right)
\]

(4.5)

where \( T \) is the temperature of the surface. The factor of 0.5 takes the electron spin into account.

<table>
<thead>
<tr>
<th>Table 4.6</th>
<th>Ionization Energy I/eV</th>
<th>Surface</th>
<th>Work Function ( \phi/eV )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li</td>
<td>5.39</td>
<td>Re(+O₂)</td>
<td>4.95(+ ~ 0.8)</td>
</tr>
<tr>
<td>Na</td>
<td>5.14</td>
<td>In(+O₂)</td>
<td>5.20(+ ~ 0.5)</td>
</tr>
<tr>
<td>K</td>
<td>4.34</td>
<td>W(+O₂)</td>
<td>4.60(+ ~ 1.6)</td>
</tr>
<tr>
<td>Cs</td>
<td>3.89</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6 lists the ionization energies of the alkali elements, and the work functions of a few surface ionizers. If oxygen is added to the surfaces, the work functions can be increased considerably.

Once the ionization occurs, the ions are bound to the metal surfaces by their image charges. In order to free these ions from the surface, the surface is heated. The ions then get, occasionally, sufficient
energy to overcome the binding energy. The number of ions on the surface, as a function of time can then be expressed as

\[ N(t) = N(0) \exp(-Dt) \]  

(4.6)

where

\[ D = D_0 \exp\left(-\frac{E_{\text{des}}}{kT}\right) \]  

(4.7)

$E_{\text{des}}$ is the desorption energy of these ions at the surface, and D is the desorption rate.

Figure 4.13a shows the Marburg version of the apparatus used in making the measurements to be discussed. A few of these (Boe81) were performed with the source of the Heidelberg EN-Tandem (Ste81)
An atomic beam is produced by a heated oven and a collimator system. It is nuclear spin polarized by passing through a sextupole magnet and a weak field transition (WFT). A chopper enables the measurement of the sticking time of the ions on the surface. This source was different from the previous one at Heidelberg in that the surface ionizer was heated indirectly by electron bombardment. An electrostatic mirror is now used to extract the ion beam.

In order to measure the polarization of the desorbed ions e.g., Na⁺, we used a beam foil polarimeter (figs. 4.12, 4.13a) shown schematically in fig. 4.13b. The ions emitted form the polarized source pass through a carbon foil. The ions pick up an electron from the foil. This electron gets captured partly in the 2p levels in preference to the ground state.

\[ \text{Source for Polarized Ions} \quad \text{Na}^+ ({}^1S_0) \quad \text{Na}^0 \quad \text{C-Foil} \quad \lambda/4\text{-Plate} \quad \text{lin. Polarizer} \quad \text{Filter (589 nm)} \quad \text{Photomultiplier} \]

**Fig. 4.13b**
A measurement of the intensity of the polarized light (circular and/or linear) emitted can be related to the polarization of the ion beam. The "asymmetry" $\varepsilon_i$ is parametrized as

$$
\varepsilon_i = \frac{N(\text{pol})}{N(\text{unpol})} - 1 = \bar{A}_i \cdot P_i ; i = z, zz, \ldots
$$

where $P_i$ is the polarization of the beam to be determined, and $\bar{A}_i$ the analyzing power. These are labeled with a bar since they differ by numerical factors from those defined in Sect. 1. They can be calculated theoretically and are listed in Tab. 4.6 together with the moments of the nuclei.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$\mu/\mu_k$</th>
<th>$Q/e\cdot\text{mb}$</th>
<th>$B_{Cr}/\text{mT}$</th>
<th>$\bar{A}_z$</th>
<th>$\bar{A}_{zz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{Li}$</td>
<td>0.82</td>
<td>-0.8</td>
<td>8.2</td>
<td>0.20</td>
<td>0.01</td>
</tr>
<tr>
<td>$^7\text{Li}$</td>
<td>3.26</td>
<td>-38</td>
<td>22.8</td>
<td>0.39</td>
<td>0.10</td>
</tr>
<tr>
<td>$^{23}\text{Na}$</td>
<td>2.22</td>
<td>+110</td>
<td>63.2</td>
<td>0.58</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The efficiency for detecting the vector polarization of sodium is nearly 60%. The efficiencies for detecting tensor polarizations are significantly lower.

As an illustrating example fig. 4.14 shows the vector polarization of the $^{23}\text{Na}$ ions coming off a Re surface as a function of the temperature of the surface. A point to be noted is that the higher the temperature, the shorter the sticking time. The uncertainty in the measurements is
around 10%. We can see that at high temperatures and short sticking times there is very little depolarization of the desorbed atoms. However for low temperature values, i.e., long sticking time, the polarization is destroyed completely.

Figure 4.15 is data taken with an oxidized W surface in the same spirit but with the Heidelberg source. Here we were also able to measure tensor polarizations. $x$ is the ratio of the applied magnetic field to the critical magnetic field $B_c$ (Tab. 4.5). In all cases, this ratio was large. The effects we see are qualitatively the same. We also see that $^6$Li relaxes quite differently from $^7$Li. This suggests that the relaxation processes are intimately connected with the nuclear moments. The tensor polarizations are seen to relax faster than the vector polarizations.

The magnitude of the polarization of the desorbed nuclei is a function of the sticking time, and we try for the moment the following simple ansatz for the polarization as a function of time $t$.

$$P_i(t) = P_i(0) \exp (-\alpha_i \cdot t) \quad i = z, zz \quad (4.9)$$

We have a constant flux of atoms going off the surface, and the quantity measured is the polarization averaged over the sticking time of the atoms. Expressed as an equation the polarization of the desorbed ions is:

$$P_i(T) = \frac{\int P_i(t) \exp(-D \cdot t) dt}{\int e^{-D \cdot t} dt} = \frac{P_i(0)}{1 + \alpha_i(T)/D(T)} \quad i = z, zz \quad (4.10)$$
Fig. 4.14

$23\text{Na}$

*vector polarization (arb. units)*

$T/\text{K}$

Figure 4.14 shows the vector polarization of $23\text{Na}$ as a function of temperature ($T/\text{K}$) in arbitrary units.
If the relaxation rate ($\alpha$) is equal to the desorption rate ($D$), the observed polarization is half that of the incident polarization. Hence, from figs. 4.14 and 4.15, we know that at temperatures where $P_i(T) = 1/2$, the relaxation rate is of the same order of magnitude as the desorption rate.

The desorption rates were measured with the chopper (fig. 4.13) as a function of temperature and fig. 4.16 shows the results of such a measurement. The exponential behavior of $D$ as a function of $1/T$ is evident from the curve. A parametrization for the desorption rate according to
\[ D = D_0 \exp\left(-\frac{E_{\text{des}}}{kT}\right) \]  

(4.7)

gives from fig. 4.16 the desorption energy

\[ E_{\text{des}} = 2.6 \text{ ev} \]

and

\[ D_0 = 10^{13} \text{ s}^{-1} \]

\text{Fig. 4.16}
The results for the relaxation rate $\alpha$ (eq. 4.9) are shown in fig. 4.17. $\alpha(T)$ plotted as a function of $1/T$ turns out to be a straight line. Such a relaxation process can be parametrized with a characteristic energy called for the moment $E_{\text{diff}}$

$$\alpha_i(T) = \alpha_i(0) \exp(E_{\text{diff}}/kT). \quad (4.11)$$

Such a parametrization for the Na-Re interaction gives

$$E_{\text{diff}} = 1.1 \text{ eV}$$

$$\alpha_z(0) = 0.2 \text{ s}^{-1}.$$
All these measurements were made in magnetic fields much larger than the critical field $B_c$ (Tab. 4.6). The measurements were repeated with a Re surface placed in a high vacuum region ($10^{-9}$ torr) as shown in Fig. 4.19. The part of the apparatus right of the two valves is designed for UHV conditions. Results obtained under these conditions were no different from those obtained before. Before we start with a more detailed theoretical description some remarks on surface diffusion and the time scale for the various dynamical process will follow.

Figure 4.18 shows very schematically the surface of the adsorber as a series of potential wells of depth $E_{\text{dif}}$. The adatoms hop around from well to well at a jumping rate given by

$$\Gamma = v_0 \exp \left(-\frac{E_{\text{dif}}}{kT}\right). \quad (4.12)$$

Such a model immediately gives the correct temperature dependence for the diffusion process and the activation energy of $\alpha$ was therefore identified with $E_{\text{dif}}$.

Table 4.7a lists the typical time scales involved in the processes under consideration.

![Fig. 4.18](image)
Table 4.7a

Typical Time Scales

<table>
<thead>
<tr>
<th>Process</th>
<th>Time Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration of surface atoms</td>
<td>$\sim 10^{-13}$ s</td>
</tr>
<tr>
<td>Vibration of adatom or adion</td>
<td>$\sim 10^{-13}$ s</td>
</tr>
<tr>
<td>Electron fluctuations</td>
<td>$\sim 10^{-15}$ s</td>
</tr>
<tr>
<td>Surface diffusion of adatom or adion</td>
<td>$\sim 10^{-10}$ s</td>
</tr>
<tr>
<td>Larmor precession ($B &lt; 1$ T)</td>
<td>$&lt; 10^{-8}$ s</td>
</tr>
</tbody>
</table>

The diffusion process has a time scale very different from the other processes considered. We finish the introductory remarks with a Table (Tab. 4.7b) which containing typical values for various scales.

Table 4.7b

SUMMARY

Re(-O₂) surface $T = 1250K$, $kT = 0.1$ eV

<table>
<thead>
<tr>
<th>Process</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>desorption</td>
<td>$E_{des} = 3$ eV</td>
</tr>
<tr>
<td>diffusion</td>
<td>$E_{dif} = 1$ eV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Reaction Rate</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{23}$Na</td>
<td>$D = 500(100)/s$</td>
<td>$T = 10^9$(smaller)/s</td>
</tr>
<tr>
<td>$^7$Li</td>
<td>$D = 50(10)/s$</td>
<td>$T = 10^9$(smaller)/s</td>
</tr>
</tbody>
</table>
The desorption coefficients in brackets are for oxygen covered surfaces.

We shall now try to understand the processes described in the last few pages in more detail. We start with a master equation (Fra St 68) for the populations $N_m$ of the magnetic substance. The time dependence of $N_m$ is given by

$$\dot{N}_m = \sum_m Q_{mm'} N_{m'} - \sum_m Q_{m'm} N_m$$

(4.13)

where $Q_{m'm}$ are transition rates from magnetic substate to $m$ to $m'$. The interactions are described in terms of a Hamiltonian

$$H = H_0 + H^D + H^Q ,$$

(4.14)
where $H^D$ is the magnetic dipole interaction and $H^Q$ the electric quadrupole interaction. The time dependence in the interaction is written as

$$H(t) = H \cdot f(t) \quad (4.15)$$

where $f(t)$ is a random function. The random function is characterized by a correlation function

$$G(\tau) = \int f(t) f(t + \tau) \, dt \quad (4.16)$$

where $\tau$ is the correlation time related to the jumping time in our case. Instead of the correlation function, we work with its Fourier transform

$$J(\omega) = \int_{-\infty}^{\infty} G(\tau) e^{-i\omega\tau} \, d\tau \quad (4.17)$$

where we treat the problem to the first order in perturbation theory. The transition rate is then given by

$$h_{mm'} = | \langle m | H^D | m' \rangle |^2 J^D(\omega_{mm'}) +$$

$$| \langle m | H^Q | m' \rangle |^2 J^Q(\omega_{mm'}) +$$

$$2 \text{Re} \left( \langle m | H^D | m' \rangle \langle m | H^Q | m' \rangle^* \right) J^{QD}(\omega_{mm'}) . \quad (4.18)$$
The contributions to the transition rate come from a dipole term, a quadrupole term, and a term that accounts for the interference between the dipole and the quadrupole interactions. We obtain from eqs. 4.13, 1.12 and 1.13 a coupled set of equations for $P_z$, $P_{zz}$, and $P_{zzz}$. It can be shown that if we have only a magnetic interaction, the equations decouple, and we obtain

$$
\begin{align*}
    P_z &= -\frac{2}{3} Q P_z \\
    P_{zz} &= -2Q P_{zz} \\
    P_{zzz} &= -4Q P_{zzz}
\end{align*}
$$

(4.19)

and hence,

$$
P_z(t) = P_z(0) \exp\left(-\frac{2}{3} Qt\right)
$$

(4.20)

That is exactly the same dependence we assumed (eq. 4.9) for the description of the data up to now. It seems to indicate that it is the magnetic interaction which cause the depolarization of the nuclear magnets on the surfaces. If so from the relaxation coefficient $\alpha$ extracted from the measurements (fig. 4.17) we get information on the average magnetic field to which the nuclear magnets are exposed on a surface.

For quadrupole interaction conditions for the time averaged field gradient $\overline{V_{ij}} = \frac{\partial^2 v}{\partial x_i \partial x_j}$ can be found for which the coupled eqs. 4.13 decouple too. They read:

$$
\frac{\partial^2 v}{\partial x z^2} + \frac{\partial^2 v}{\partial y z^2} = \frac{\partial^2 v}{\partial x y^2}
$$
Keeping physical pictures in mind (e.g. fig. 4.18) it seems more likely that the quadrupole interaction is responsible for the relaxation of the nuclear magnets on the surface. This has to be clarified in further experiments devoted to this question.

Note added in proof: Recent experiments (Bec 83) on an Ir surface showed a complex picture. The relaxation of $^{23}$Na, with its large Q-moment (Tab. 4.6) is caused mainly by quadrupole interaction whereas the interaction of $^6$Li with its almost vanishing Q-moment (Tab. 4.6) is mainly magnetic. The interaction of $^7$Li is the somewhere in between.
<table>
<thead>
<tr>
<th>Ref</th>
<th>Author(s) and Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bec 83</td>
<td>E. Beckmann, Thesis Universität Marburg 1983 and to be published.</td>
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