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ABSTRACT

It is shown that by applying Nelson's stochastic quantization scheme to free linearized gravitational field tensor one can associate with the resulting stochastic system a stochastic tensor field which coincides with the "space" part of the Riemannian tensor in Euclidean space-time. However, such a stochastic field fails to satisfy the Markov property. Instead, it satisfies the reflection positivity. The Markovian structure of the stochastic fields associated with the electromagnetic field is also discussed.

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INTRODUCTION

The main purpose of this paper is to extend Nelson's stochastic quantization scheme [1] to free linearized gravitational field and to study the Markovian structure of this field as well as that of the stochastic electromagnetic field obtained by Guerra and Loffredo [2]. Although Markov property is not an essential requirement in the constructive field theory, the recent work of Jona-Lasinio et al. [3] has indicated that the strong Markov property may play an important role in the semiclassical approximation calculation in stochastic (and Euclidean) field theory. Our result indicates that stochastic fields associated with the higher spin boson fields fail to satisfy the Markov property. Instead, these fields satisfy reflection (O-S) positivity which is the usual requirement in the constructive field theory [4].

STOCHASTIC QUANTIZATION OF LINEARIZED RIEMANNIAN TENSOR

Due to the presence of gauge ambiguities it is convenient to consider stochastic quantization of higher spin massless field in terms of field tensor as far as free field is concerned (stochastic gauge fields in Nelson's framework will be discussed elsewhere). Here we shall follow the method similar to the one introduced by Guerra and Loffredo [2]. Consider the classical linearized gravitational field in vacuum represented by the Riemannian tensor $R_{\mu\nu\lambda\sigma}(\underset{\sim}{x},t)$, $\mu, \nu, \lambda, \sigma = 0,1,2,3$ and $(\underset{\sim}{x},t) \in \mathbb{R}^4$, four-dimensional Minkowski space-time. Einstein's equations in the weak field approximation in vacuum are

$$R_{\mu\nu}(\underset{\sim}{x},t) = 0 \ , \quad R(\underset{\sim}{x},t) = 0 \quad . \tag{1a}$$

where

$$R_{\mu\nu} = g_{\sigma\lambda} R_{\sigma\mu\nu\lambda} \quad , \quad R = g_{\mu\nu} R_{\mu\nu} \tag{1b}$$

and $g_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$. $R_{\mu\nu\sigma\lambda}$ also satisfies the following equations:

$$R_{\sigma\mu\nu\lambda} = - R_{\mu\sigma\nu\lambda} = R_{\mu\sigma\lambda\nu} = R_{\nu\lambda\sigma\mu} \quad , \tag{1c}$$

$$R_{\mu\nu\lambda\sigma} + R_{\mu\sigma\nu\lambda} + R_{\mu\lambda\sigma\nu} = 0 \tag{1d}$$

and the Bianchi identity

$$\varepsilon_{\mu\nu\lambda\sigma} \, \partial_\nu \, R_{\eta\xi\lambda\sigma} = 0 \ , \tag{1e}$$

where $\epsilon_{\mu\nu\lambda\sigma}$ is a completely antisymmetric tensor with $\epsilon_{0123} = 1$.

To stochastic quantize the field we introduce the tensor analogues of electric and magnetic fields. Define the following 3-tensors

$$E_{ij}(\underset{\sim}{x},t) = R_{i0j0}(\underset{\sim}{x},t) \tag{2a}$$

$$B_{ij}(\underset{\sim}{x},t) = \frac{1}{2}\,\epsilon_{ik\ell}\, R_{k\ell j0}(\underset{\sim}{x},t) \tag{2c}$$

where $i,j,k,\ell = 1,2,3$. Both the tensors E_{ij} and B_{ij} are symmetric in vacuum. E_{ij} is traceless as a result of the vacuum condition, and the tracelessness of B_{ij} follows from the cyclic identity (1d). The Hamiltonian of the system is given by

$$H = (2)^{-1}\int (E_{ij}E_{ij} + B_{ij}B_{ij})\, d\underset{\sim}{x} \tag{3}$$

To derive the equations of motion for E_{ij} and B_{ij} one first notes that Einstein's equation (1a), together with the Bianchi identity, gives

$$\partial_\mu R_{\mu\nu\rho\sigma} = 0 \tag{4}$$

This equation implies the following set of equations:

$$\partial_i R_{i0j0} = 0 = \partial_i R_{0ijk} \tag{5a}$$

$$\partial_0 R_{0j0k} = \partial_i R_{ij0k} \tag{5b}$$

$$\partial_0 R_{0jk\ell} = \partial_i R_{ijk\ell} \tag{5c}$$

From Eqs.(2) and (5) one obtains a set of equations analogous to Maxwell equations:

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{E}_i = 0\,, \qquad \underset{\sim}{\nabla}\cdot\underset{\sim}{B}_i = 0 \tag{6}$$

$$(\underset{\sim}{\nabla}\times\underset{\sim}{E}_i)_j = -\frac{\partial B_{ij}}{\partial t} \tag{7}$$

$$(\underset{\sim}{\nabla}\times\underset{\sim}{B}_i)_j = \frac{\partial E_{ij}}{\partial t} \tag{8}$$

where we have adopted the following notations: $\underset{\sim}{E}_i = \{E_{ij}\}$, $\underset{\sim}{\nabla}\cdot\underset{\sim}{E}_i = \partial_i E_{ij}$ and $(\underset{\sim}{\nabla}\times\underset{\sim}{E}_i)_j = \epsilon_{jk\ell}\,\partial_k E_{\ell i}$, ϵ_{ijk} is a totally antisymmetric tensor with $\epsilon_{123} = 1$. Eqs.(7) and (8) are the dynamic equations of the system with Eq.(6) as the constraints.

Using the same argument as given in Ref.2 stochastic quantization scheme of Nelson requires the "magnetic" tensor field component B_{ij} be promoted to a random Markov process, and the "electric" component E_{ij} splits into a pair of functionals of B_{ij}, denoted by $E^{(\pm)}_{ij}(B;\underset{\sim}{x},t)$. The stochastic system $\{B_{ij}(\underset{\sim}{x},t),\ E^{(\pm)}_{ij}(B,\underset{\sim}{x},t)\}$ satisfies the following equations:

$$dB_{ij}(\underset{\sim}{x},t) = -(\nabla\times\underset{\sim}{E}^{(\pm)}_i)_j\,(B;\underset{\sim}{x},t) + d\,W_{ij}(\underset{\sim}{x},t)\,, \tag{9}$$

$$(D_{(\pm)}\,B_{ij})\,(\underset{\sim}{x},t) = -(\nabla\times\underset{\sim}{E}^{(\pm)}_i)_j\,(B;\underset{\sim}{x},t) \tag{10}$$

$$(2)^{-1}\,[D_{(+)}E^{(-)}_{ij} + D_{(-)}E^{(+)}_{ij}] = (\nabla\times\underset{\sim}{B}_i)_j \tag{11}$$

$D_{(+)}$ and $D_{(-)}$ are, respectively, the mean forward and backward derivatives defined as follows:

$$D_{(+)}B_{ij}(\underset{\sim}{x},t) = \lim_{\Delta t\downarrow 0}\,(\Delta t)^{-1}E_t\left[B_{ij}(\underset{\sim}{x},t+\Delta t) - B_{ij}(\underset{\sim}{x},t)\right], \tag{12a}$$

$$D_{(-)}B_{ij}(\underset{\sim}{x},t) = \lim_{\Delta t\downarrow 0}\,(\Delta t)^{-1}E_t\left[B_{ij}(\underset{\sim}{x},t) - B_{ij}(\underset{\sim}{x},t-\Delta t)\right], \tag{12b}$$

where E_t is the conditional expectation with respect to the σ-algebra generated by $\{B_{ij}(\underset{\sim}{x},t)\}$. The Wiener process W_{ij} is defined by the following mean and covariance:

$$\langle dW_{ij}(\underset{\sim}{x},t)\rangle = 0$$

$$\langle dW_{ij}(\underset{\sim}{x},t)\,dW_{i'j'}(\underset{\sim}{x}',t)\rangle = (2\pi)^{-3}\int \exp[i\underset{\sim}{p}\cdot(\underset{\sim}{x}-\underset{\sim}{x}')]\,Q_{ij,i'j'}(\underset{\sim}{p})\,d\underset{\sim}{p} \tag{13a}$$

with

$$Q_{ij,i'j'}(\underset{\sim}{p}) = (4)^{-1}\left[d_{ii'}d_{jj'} + d_{ij'}d_{ji'} - d_{ij}d_{i'j'}\right] \tag{13b}$$

and

$$d_{ij} = \delta_{ij} - p_ip_j\underset{\sim}{p}^{-2}\,. \tag{13c}$$

Here Eq.(13) replaces the usual simple delta function normalization because of the transverse and traceless constraints on B_{ij}.

The stochastic differential Eq.(9) can be solved as in Ref.2, giving the ground state stochastic process associated with B_{ij} as a Gaussian random field with zero mean and covariance

$$\langle B_{ij}(\underset{\sim}{x},t)B_{i'j'}(\underset{\sim}{x}',t')\rangle = (2\pi)^{-4}\int \exp[ip\cdot(x-x')]\,Q_{iji'j'}(\underset{\sim}{p})\,\underset{\sim}{p}^4\,p^{-2}d^4p \;, \tag{14}$$

where $p = (\underset{\sim}{p},p_4)\in\mathbb{R}^4$, $p^2 = \sum_{\alpha=1}^{4} p_\alpha^2$, $p\cdot(x-x') = \underset{\sim}{p}\cdot(\underset{\sim}{x}-\underset{\sim}{x}') + p_4(t-t')$, and d^4p is the Lebesque measure on $\mathbb{R}^4$. Here we shall clarify the identification of the stochastic Markov process and the Gaussian random field. To be more precise one should consider that Nelson's stochastic quantization requires B_{ij} to be a Markov process $t\mapsto B^t_{ij}$, satisfying the stochastic differential equation of Ito type (9). Now consider the orthogonal splitting $\mathbb{R}^4 = \mathbb{R}^3\times\mathbb{R}^1$, $x = (\underset{\sim}{x},t)$. Then we can identify the Gaussian random field $B_{ij}(\cdot,t)$ with B^t_{ij} considered as a process $t\mapsto B_{ij}(\cdot,t)$ with values in $\mathcal{S}'_r(\mathbb{R}^3)$, Schwartz space of real tempered distributions. In this way we can write $B^t_{ij}(\underset{\sim}{x}) = B_{ij}(x)$. (For a more precise treatment see Refs.5 and 6.)

PROPERTIES OF STOCHASTIC TENSOR FIELDS

In order to see the relation between the stochastic tensor field $B_{ij}(\underset{\sim}{x},t)$ and the tensor field $R_{\alpha\beta\gamma\delta}(x)$, $\alpha,\ \beta,\ \gamma,\ \delta\ = 1,2,3,4$, in Euclidean space-time, it is convenient to consider another Gaussian tensor field defined by the following 3-tensor of rank four:

$$R_{ijmn}(\underset{\sim}{x},t) = -\,\varepsilon_{ijk}\,\varepsilon_{mn\ell}\,B_{k\ell}(\underset{\sim}{x},t) \;. \tag{15}$$

Then one obtains a result similar to that for electromagnetic field [2].

<u>Proposition 1</u>

If x_4 is taken as equal to t then the stochastic tensor field $R_{ijmn}(\underset{\sim}{x},t)$ defined by Eq.(15) coincides with the Euclidean tensor field $R_{\alpha\beta\gamma\delta}(\underset{\sim}{x},x_4)$ restricted to $\alpha,\ \beta,\ \gamma,\ \delta = 1,2,3$.

<u>Proof</u>

Direct computation gives the covariance of R_{ijmn} as

$$\begin{aligned}\langle R_{ijmn}(\underset{\sim}{x},t)\,R_{i'j'm'n'}(\underset{\sim}{x}',t')\rangle &= \varepsilon_{ijk}\,\varepsilon_{mn\ell}\,\varepsilon_{i'j'k'}\,\varepsilon_{m'n'\ell'}\,\langle B_{k\ell}(\underset{\sim}{x},t)B_{k'\ell'}(\underset{\sim}{x}',t')\rangle \\ &= (2\pi)^{-4}\int \exp[ip\cdot(x-x')]\,Q_{ijmn,i'j'm'n'}(\underset{\sim}{p})\,p^{-2}d^4p\end{aligned} \tag{16a}$$

with

$$\begin{aligned}Q_{ijmn,i'j'm'n'}(\underset{\sim}{p}) = (4)^{-1}\Big[&\Big(\hat{\sum_{iji'j'}}\delta_{ii'}p_jp_{j'}\Big)\Big(\hat{\sum_{mnm'n'}}\delta_{mm'}p_np_{n'}\Big) + \\ &+\Big(\hat{\sum_{ijm'n'}}\delta_{im'}p_jp_{n'}\Big)\Big(\hat{\sum_{i'j'mn}}\delta_{i'm}p_{j'}p_n\Big) - \Big(\hat{\sum_{ijmn}}\delta_{im}p_jp_n\Big)\Big(\hat{\sum_{i'j'm'n'}}\delta_{i'm'}p_{j'}p_{n'}\Big)\Big] \;,\end{aligned} \tag{16b}$$

where the sum $\hat{\sum}$ on an arbitrary 3-tensor of rank four $V_{ij,mn}$ means

$$\hat{\sum_{ijmn}} V_{ij,mn} = V_{im,jn} + V_{jn,im} - V_{jm,in} - V_{in,jm} \;. \tag{17}$$

The covariance given by Eq.(16) is the same as the "space" part (i.e. with $\alpha,\beta,\gamma,\delta$ restricted to 1,2,3) of the two-point Schwinger function of the Euclidean tensor field $R_{\alpha\beta\gamma\delta}(\underset{\sim}{x},x_4)$ provided x_4 is taken as equal to t. Since any Gaussian field is uniquely determined by its mean and covariance, the result follows.

Thus for free electromagnetic field and linearized gravitational field there exists some kind of generalization of the result on stochastic scalar field first obtained by Guerra and Ruggiero [7]. They found that the ground state stochastic scalar field coincides with the corresponding Euclidean-Markov field if one identifies the fourth component of the Euclidean co-ordinate x_4 with the physical time t. In this way, Nelson's stochastic quantization scheme can be regarded as a possible framework for understanding Euclidean fields which otherwise are considered only as useful mathematical objects having no direct link with the physical world. In view of the recent interest on the stochastic quantization of gauge fields in Euclidean space-time [8,9], it may be worthwhile to make a more detailed study of the interpretation of Guerra and Ruggiero for higher spin fields.

We shall first consider the Markovian structure of the stochastic fields associated with the free electromagnetic and linearized gravitational fields. There exist various ways of characterizing the Markovian structure (see [10] for details). For our purpose the following result is suitable.

Proposition 2 (Pitt [11])

A homogeneous Gaussian random field is Markovian with respect to all half spaces if and only if the inverse of its spectral density agrees a.e. with the restriction onto $\mathbb{R}^4$ of an entire function of minimal exponential type.

This result was also obtained independently by Kotani and Okabe [12] using the theory of hyperfunctions. It also holds for random tensor fields [13].

Before we apply this result to study the Markovicity of stochastic fields we make the following observation. First, note that the definition of the Markov property given by Nelsen [14] for Euclidean quantum fields has been shown [15] to be equivalent to that of Pitt. For notational simplicity we shall replace $(\underset{\sim}{x},t)$ by (x) so all stochastic fields are to be represented as $\phi_{ij\ldots}(x)$. The stochastic fields under consideration are homogeneous. That is, if $\tau_a : x \to x + a,\ a \in \mathbb{R}^4$ represents space-time translation in $\mathbb{R}^4$, then a Gaussian random field is homogeneous if

$$\langle \Phi_{ij\ldots}(\tau_a x)\,\Phi_{i'j'\ldots}(\tau_a x')\rangle = \langle \Phi_{ij\ldots}(x)\,\Phi_{i'j'\ldots}(x')\rangle . \qquad (18)$$

Finally we note that the spectral density matrices of the stochastic fields are all in the form of a rational function of matrix polynomials $Q_{ij\ldots i'j'}(p)/p^2$. This property reduces the necessary and sufficient condition for Markovicity of Pitt et al. to the requirement that the inverse of the spectral density be a matrix polynomial.

We shall first discuss the Markov property for the stochastic Gaussian fields associated with the free electromagnetic field. Stochastic quantization of electromagnetic field [2] gives rise to a stochastic Markov process associated to the magnetic field component $B_j(x)$ and a pair of functionals $E_j^{(\pm)}(B,x)$. The covariance matrix of $B_j(x)$ is given by

$$\langle B_j(x)\,B_{j'}(x')\rangle = (2\pi)^{-4}\int \exp[ip\cdot(x-x')](\underset{\sim}{p}^2\delta_{jj'} - p_j p_{j'})\,p^{-2}\,d^4p \qquad (19)$$

First we prove the following lemma:

Lemma

Let $S_{jj'}(\underset{\sim}{p})$ be any homogeneous matrix polynomial of degree 2 in p_j, $j = 1,2,3$. Then the rational function $S_{jj'}(\underset{\sim}{p})/p^2$, $p^2 = \underset{\sim}{p}^2 + p_4^2$ cannot have a matrix polynomial inverse.

Proof

Assuming the contrary and let the inverse be denoted by $T_{ij}(p)$, a matrix polynomial in p_1, p_2, p_3 and p_4. Then we have

$$(\underset{\sim}{p}^2 + p_4^2)\,\delta_{jj'} = S_{ji}(\underset{\sim}{p})\,T_{ij'}(p) \ . \qquad (20)$$

The (1-1) element of the above equation is

$$\underset{\sim}{p}^2 + p_4^2 = S_{11}(\underset{\sim}{p})\,T_{11}(p) + S_{12}(\underset{\sim}{p})\,T_{21}(p) + S_{13}(\underset{\sim}{p})\,T_{31}(p) \ . \qquad (21)$$

Comparing the coefficients of the p_4^2 terms in Eq.(21) gives

$$1 = S_{11}(\underset{\sim}{p})\,T'_{11}(p) + S_{12}(\underset{\sim}{p})\,T'_{21}(p) + S_{13}(\underset{\sim}{p})\,T'_{31}(p) \ , \qquad (22)$$

where the T'_{i1}, $i = 1,2,3$ are the coefficients (in polynomials of p_j, $j = 1,2,3$) of p_4^2 terms in T_{i1}, $i = 1,2,3$. By assumption at least one $T'_{i1} \neq 0$. Since S_{1j}'s are homogeneous in p_j of degree 2, therefore the right-hand side of Eq.(22) cannot be a non-zero constant.

Now we are ready to consider the Markovian structure of the stochastic magnetic field and the stochastic tensor field defined by $F_{ij} = \varepsilon_{ijk}B_k$.

Proposition 3

The stochastic fields $B_j(x)$ and $F_{ij}(x)$ do not satisfy the Markov property.

Proof

We cannot apply the lemma directly because the spectral density for B_j given by

$$\frac{Q_{jj'}(\underset{\sim}{p})}{p^2} = \frac{(\underset{\sim}{p}^2\delta_{jj'} - p_j p_{j'})}{p^2} \qquad (23)$$

is a singular matrix, hence its inverse does not exist. The singularity of the covariance matrix means that not all the components of B_j are independent, as this is also shown by the constraint $\underset{\sim}{\nabla}\cdot\underset{\sim}{B} = 0$. There exists an equivalent stochastic field with reduced degrees of freedom such that the covariance matrix is non-singular.

If we consider B_j as a generalized random vector field over $\mathcal{S}_r^3(\mathbb{R}^4)$, 3-fold Cartesian product of the real Schwartz space of test functions; then the transverse condition $\nabla\cdot\underset{\sim}{B} = 0$ allows one to regard B_j as defined over the subspace of $\mathcal{S}_r^3(\mathbb{R}^4)$ with elements $\underset{\sim}{f}$ satisfying $\sum_{i=1}^{3} \partial_i f_i = 0$ $\left\{\text{or } \sum_{i=1}^{3} p_i \tilde{f}_i = 0 \text{ where } \tilde{f}_i \text{ is the Fourier transform of } f_i\right\}$. The spectral density now reduces to $\underset{\sim}{p}^2\, \delta_{jj'} p^{-2}$, which is non-singular and has a non-polynomial inverse $p^2\delta_{jj'}/\underset{\sim}{p}^2$. By Proposition 2 we conclude that B_j is non-Markovian.

For the stochastic tensor field $F_{ij}(x)$, the spectral density is given by

$$\frac{Q_{ij,i'j'}(\underset{\sim}{p})}{p^2} = \frac{\varepsilon_{ijk}\,\varepsilon_{i'j'k'}\,Q_{kk'}(\underset{\sim}{p})}{p^2}$$

$$= \left(p_i p_{i'}\delta_{jj'} + p_j p_{j'}\delta_{ii'} - p_i p_{j'}\delta_{i'j} - p_j p_{i'}\delta_{ij'}\right)/p^2 \, . \tag{24}$$

Here one can define the "one-particle" Hilbert space N associated to F_{ij} as the Hilbert space of antisymmetric 3 x 3 matrices $\underset{\sim}{f}$ whose elements f_{ij}, $i,j = 1,2,3$ belong to $\mathcal{S}_r(\mathbb{R}^4)$, with inner product

$$\langle \underset{\sim}{f}, \underset{\sim}{g}\rangle_N = \sum_{i,j,i',j'=1}^{3} \int \tilde{f}_{ij}(p)\, \frac{Q_{ij,i'j'}(\underset{\sim}{p})}{p^2}\, \tilde{g}_{i'j'}(p)\, d^4p \tag{25}$$

By the antisymmetric property this inner product can be expressed in the following form:

$$\langle \underset{\sim}{f}, \underset{\sim}{g}\rangle_N = \sum_{\substack{(i,j)\in I\\ (i',j')\in I}} \int \tilde{f}_{ij}(p)\, \frac{Q_{ij,i'j'}(\underset{\sim}{p})}{p^2}\, \tilde{g}_{i'j'}(p)\, d^4p \tag{26}$$

with $I = \{(1,2), (1,3), (2,3)\}$ and the matrix $Q_{ij,i'j'}$ for $(i,j)\in I$, $(i',j')\in I$ is given by

$$\begin{array}{cccc} (i',j') = (1,2) & (1,3) & (2,3) & (i,j) \\ \left\{\begin{array}{ccc} p_1^2 + p_2^2 & p_2p_3 & -p_1p_3 \\ p_2p_3 & p_1^2 + p_3^2 & p_1p_2 \\ -p_1p_3 & p_1p_2 & p_2^2 + p_3^2 \end{array}\right\} & & & \begin{array}{c} (1,2) \\ (1,3) \\ (2,3) \end{array} \end{array}$$

Each element in this matrix is a homogeneous polynomial in p_j's of degree 2. Now we can apply our lemma to deduce that $Q_{ij,i'j'}(\underset{\sim}{p})\, p^{-2}$ does not have a matrix polynomial inverse and again by Proposition 2 F_{ij} is non-Markovian.

Although one can identify the stochastic tensor field F_{ij} with the "space" part of the Euclidean electromagnetic field $F_{\alpha\beta}$ if x_4 is taken as equal to t [2], the two fields differ in the Markovian structure. It has been shown by several authors [16-19] that $F_{\alpha\beta}$ satisfies the Markov property. This is not really surprising. Intuitively one can view the situation as follows. Consider $F_{ij}(f)$, as a generalized random tensor field, $f\in\mathcal{S}_r(\mathbb{R}^4)$ with support in an open set, say $\Lambda \subset \mathbb{R}^4$, then the associated boundary Hilbert space $N(\partial\Lambda)$ (corresponds to the "present") is not large enough to contain sufficient information to enable the exterior (the "future") to be independent of the interior (the "past"). The "space" part F_{ij} as well as the "time" part $F_{j4} = -F_{4j}$ are necessary to ensure Markovicity.

The non-Markovicity of the stochastic tensor fields associated with the free linearized gravitational field can also be verified in a similar way.

Proposition 4

The stochastic tensor fields B_{ij} and R_{ijmn} do not satisfy the Markov property.

Proof

For B_{ij}, the spectral density is $Q_{ij,i'j'}(\underset{\sim}{p})\, p^{-2}$, with $Q_{ij,i'j'}(\underset{\sim}{f})$ given by Eq.(14), which can be regarded as a direct (or Kronecker) product of matrices. $Q_{ij,i'j'}$ is singular in this matrix representation. B_{ij} can be defined over subspaces of $\mathcal{S}_r^9(\mathbb{R}^4)$ with elements $\underset{\sim}{f} = \{f_{ij},\ i,j = 1,2,3\}$ satisfying the conditions $\sum_{i=1}^{3} f_{ii} = 0$ and $\sum_{i=1}^{3} \partial_i f_{ij} = 0$. One can easily show that the spectral density now becomes $\underset{\sim}{p}^4(\delta_{ii'}\,\delta_{jj'} + \delta_{ij'}\,\delta_{i'j})/(4p^2)$ which clearly does not possess a matrix polynomial inverse. R_{ijmn}

can be shown to be non-Markovian using the same method as for F_{ij}. We omit the details, except we remark that one needs a result similar to the lemma given for matrix elements of Q homogeneous of degree 4 in p_j's.

The Markov property is a rather strong requirement in the constructive field theory. The appropriate property is the reflection or Osterwalder-Schrader positivity [4], which can be considered as a kind of weak Markovian structure. One can define reflection positivity as follows. Let θ be the unitary time-reflection operator

$$\theta : (\underset{\sim}{x},t) \rightarrow (\underset{\sim}{x},-t) \quad , \tag{27a}$$

$$(\theta f)(\underset{\sim}{x},t) \rightarrow f_\theta(\underset{\sim}{x},t) = f(\underset{\sim}{x},-t) \quad . \tag{27b}$$

Formally we have for any tensor field $\Phi_{\alpha\beta\ldots}$

$$\theta\, \Phi_{\alpha\beta\ldots}(f) = (-1)^{\delta_{\alpha 4}+\delta_{\beta 4}+\ldots}\, \Phi_{\alpha\beta\ldots}(f_\theta) \quad . \tag{28}$$

Clearly for stochastic tensor field $\Phi_{ij\ldots}$, $i,j,\ldots = 1,2,3$,

$$\theta\, \Phi_{ij\ldots}(f) = \Phi_{ij\ldots}(f_\theta) \quad , \tag{29}$$

Suppose Q is a continuous bilinear form on $\mathcal{S}_r(\mathbb{R}^4) \times \mathcal{S}_r(\mathbb{R}^4)$ then Q is said to satisfy reflection positivity if for every $f \in \mathcal{S}_r(\mathbb{R}^4)$ with support in $\mathbb{R}^4_+ = \{x \in \mathbb{R}^4 : t > 0\}$, $\langle \theta f, Qf \rangle \geq 0$. One can show that all the stochastic fields under consideration are reflection positive.

<u>Proposition 5</u>

The stochastic fields B_j, F_{ij}, B_{ij} and R_{ijmn} satisfy reflection positivity.

<u>Proof</u>

Suffice to show that the following continuous bilinear form on $\mathcal{S}^n_r(\mathbb{R}^4) \times \mathcal{S}^n_r(\mathbb{R}^4)$ is reflection positive

$$C(\underset{\sim}{f},\underset{\sim}{g}) = \left\langle \underset{\sim}{f}, \frac{Q}{-\Delta}\underset{\sim}{g} \right\rangle$$

$$= \sum_{\substack{i_1\cdots i_n=1 \\ j_1\cdots j_n}}^{3} \int f_{i_1\cdots i_n}(x)\, \frac{Q_{i_1\cdots i_n, j_1\cdots j_n}(\partial)}{-\Delta}\, g_{j_1\cdots j_n}(x)\, d^4x \; , \tag{30}$$

where n is an integer, its value depends on the rank of the stochastic tensor field, $\Delta = \sum_{i=1}^{4} \partial_i^2$ is the Laplacian operator on $\mathbb{R}^4$, and $Q(\partial)$ is a matrix polynomial in differential operator ∂_i, $i = 1,2,3$. $Q(\partial)$ is θ-invariant since it commutes with θ. Now the rest of the proof follows exactly like the scalar Euclidean massless field with covariance kernel $\Delta^{-1}(x-x')$ (see Glimm and Jaffe [4], p.105, Proposition 6.25).

Reflection positivity allows the analytic continuation of a theory in imaginary time to quantum theory in real time. Actually, it ensures the positivity of the inner product in the Hilbert space on which the Minkowski quantum field acts.

CONCLUSION

Nelson's stochastic quantization scheme when applied to the linearized gravitational field in vacuum gives rise to a stochastic system $\{B_{ij}(\underset{\sim}{x},t), E^{(\pm)}_{ij}(B,\underset{\sim}{x},t)\}$ which is not exactly equivalent to the corresponding Euclidean field. Although the 3-tensor $R_{ijmn}(\underset{\sim}{x},t)$ can be identified as the "space" part of the Euclidean field $R_{\alpha\beta\gamma\delta}(x)$ by taking $x_4 = t$, it is non-Markovian. Furthermore, the stochastic fields are not Euclidean-covariant. The non-equivalence of stochastic fields and the corresponding Euclidean fields is further confirmed by the fact that for the positive temperature case, stochastic field differs from Euclidean field even for the scalar case [20,21].

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