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A SKETCH TO THE GEOMETRICAL N=2-d=5 YANG-MILLS THEORY OVER A SUPERSYMMETRIC GROUP-MANIFOLD - I *

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ABSTRACT

This work concerns the search and the construction of a geometrical structure for a supersymmetric N=2-d=5 Yang-Mills theory on the group manifold. From criteria established throughout this paper, we build up an ansatz for the curvatures of our theory and then solve the Bianchi identities, whose solution is fundamental for the construction of the geometrical action.

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I. INTRODUCTION

In this work we build up the general geometrical structure for the supersymmetric $N = 2 - d = 5$ Yang-Mills theory (S.Y.M. $N = 2 - d = 5$), where N is the number of generations of supersymmetry transformations and d stands for the dimension of the space-time manifold. The criteria followed for our construction shall be established throughout this work, however the path taken is the geometrical approach of supersymmetric theories over a group manifold [1-3]. Such a scheme allows an elegant formulation of supergravity and gives results in agreement with those obtained in other approaches. Even the most interesting case of the $d = 11$ supergravity(whose dimensional reduction could give results of remarkable phenomenological interest) may be treated geometrically by means of the concept of Cartan's integrable system, which generalizes the idea of group [4,5].

Even if there exists a systematic method to treat supergravity, an analogous formulation has not yet been established for the Yang-Mills theories over a group manifold. They are said to be "impure", contaminated by the presence of 0-form matter fields. In a "pure" theory, instead, the only fields which are present are the 1-forms μ^A, which constitute the pseudo-connection. On the other hand, there exists also for the Yang-Mills theories a constructive scheme which is rather precise, even though, in some steps, there is no other way than to proceed by trials. This does not mean that such theories are less interesting. First of all, geometrical Yang-Mills theories over a group manifold can naturally be coupled to supergravity, if the latter is a pure theory [6]. For instance, the globally supersymmetric $N = 2 - d = 5$ Yang-Mills theory becomes locally supersymmetric in the geometrical formalism by adjoining the action of the $N = 2 - d = 5$ supergravity [3], which is the only other known "pure" theory of supergravity (besides the $N = 1 - d = 4$ supergravity).

Another motivation for the study of such theories is their possible connection with other cases (for example, with pure supergravity theories formulated in higher dimensional space times) through the technique of dimensional reduction, improving therefore the knowledge of the various supersymmetric actions. It should also be stressed that, before performing the dimensional reduction, this theory exhibits similarities with the supersymmetric $N = 2 - d = 4$ Yang-Mills theory [7].

The present work has been divided as follows. Sec.II:Remarks on mathematical achievements. We make here a brief review of the differential forms and show their importance for a geometrical formulation. Here, we also define the supergroup and study the composition of the supermultiplet, the Dirac algebra and the reason for the use of Dirac spinors. Sec.III: Bianchi identities. From the "rheonomy criticism" [2] and other indispensable criteria, like for instance, the considerations of the degree of the forms and the fermionic or bosonic character of the object under consideration, we establish an ansatz for the curvature. The explicit determination of the parameters of the curvatures so constructed is possible through the solution of the Bianchi identities. Sec.IV contains the main conclusions of our work. An appendix is also given.

It is worthwhile to recall that a supersymmetric five-dimensional Yang-Mills theory has already been studied by Zizzi [8] using, however, a different formalism without the utilization of a geometric background.

The authors shall be very glad if the present work may constitute a pedagogical introduction to the understanding of the geometrical formalism for the study of supersymmetric theories through the use of the calculus with forms.

II. REMARKS ON MATHEMATICAL ACHIEVEMENTS

2.1 On differential forms, vierbein and Cartan's connection in a manifold

A first attempt in formulating a gauge structure for the theory of gravitation had been accomplished by E. Cartan, already in the twenties, in a series of works published in "Annales de l'Ecole Normale Supérieure" entitled "Sur les variétés à connexions affines et la théorie de la relativité généralisée" [9]. They undoubtedly constitute the purpose and the starting point for the more recent geometrical attempts for gravity and supergravity [1,2]. These works remained unknown to most physicists, till Kibble [10], in the sixties, proposed a formulation equivalent to that of Cartan, in his attempt of treating Einstein's gravitation as a Yang-Mills theory.

To understand both Cartan's theory and Kibble's formulation, it is necessary to have an understanding of differential forms, the concepts of vierbein and connection on a differentiable manifold. They will be briefly analysed in the next paragraph, mainly because in our theory on a group manifold we have used the differential forms as the geometrical elements. The great advantage of the calculus with the forms is that it allows us to deal with entities, the spinors for example, which cannot be treated in the usual absolute differential calculus.

Let M be a differentiable manifold. At each tangent plane, we can define a basis of n vectors $\vec{e}_i$. They span the so-called local frame. If M is the space-time manifold, the vectors $\vec{e}_i$ constitute a locally inertial frame. An infinitesimal displacement in the manifold M, $d\vec{Q}$, may be described as

$$d\vec{Q} = A^i \vec{e}_i \quad . \qquad (2.1)$$

The coefficients A^i are linear in the differentials of co-ordinates, x^μ, of the manifold and then

$$A^i = A^i_\mu \, dx^\mu \quad . \qquad (2.2)$$

The A^i's are the so-called 1-forms. Let us denote them by vierbein. For the sake of accuracy, some authors use the word fünfbein in the case $n > 4$ and vierbein only if $n = 4$. We shall here use only the latter. The A^i_μ's are the components of the vierbein with respect to a dual basis in cotangent space. The infinitesimal displacement in M may have the tangent vectors as a reference:

$$\frac{\partial}{\partial x^\mu} = \vec{\partial}_\mu \qquad (2.3)$$

then

$$d\vec{Q} = A^\mu \vec{\partial}_\mu = dx^\mu \vec{\partial}_\mu \qquad (2.4)$$

and consequently,

$$A^\mu \vec{\partial}_\mu = A^i \vec{e}_i = A^i_\mu A^\mu \vec{e}_i \quad , \qquad (2.5)$$

$$\therefore \quad \boxed{\vec{\partial}_\mu = A^i_\mu \vec{e}_i} \quad . \qquad (2.6)$$

Therefore

$$\boxed{e_i = A^\mu_i \vec{\partial}_\mu} \quad . \qquad (2.7)$$

The conclusion is that the components of the vierbein are the elements of a matrix which allow to pass from an orthonormal local frame to the co-ordinate system. In what follows the vierbein and its components will be denoted by V^i and V^μ_i .

We can define in M the Cartan connection, denoted by ω^{ij}. In passing from x^μ to $x^\mu + dx^\mu$, the local orthonormal frame, $\vec{e}_i$, changes by

$$d\vec{e}^{\,i} = \omega_i^{\ j}\, \vec{e}_j \quad , \tag{2.8}$$

where ω^{ij} is linear in the differentials of the co-ordinates, that is

$$\omega^{ij} = \omega^{ij}_\mu \; dx^\mu \quad . \tag{2.9}$$

Finally, the torsion and the curvature of the manifold will be defined as 2-forms:

$$R^i = dv^i - \omega^{ij} \wedge v_j \quad , \tag{2.10}$$

$$R^{ij} = d\omega^{ij} - \omega^{ik} \wedge \omega_k^{\ j} \quad . \tag{2.11}$$

We have here introduced two operators, "d" and "$\wedge$", both essential for the calculus with the forms. "d" is the exterior derivative and "$\wedge$" is the exterior product.

The 2-forms $(R^{ij}, R^i) = R^A$ can be decomposed either with respect to the basis of co-ordinates (Greek indices) or with respect to the vierbein (Latin indices):

$$\begin{aligned} R^A &= R^A_{\ \mu\nu}\; dx^\mu \wedge dx^\nu \\ &= R^A_{\ ij}\; v^i \wedge v^j \end{aligned} \tag{2.12}$$

It is interesting to notice that a local description in terms of the vierbein is far more convenient if one wishes to introduce spinor fields on the manifold M, as spinors are defined with respect to the group $SO(1,n)$ rather than $GL(n+1)$ (group of non-singular linear transformations in tangent space).

Let us come back for a moment to the exterior product. If δ and η are two forms of degree $\underline{p}$ and $\underline{q}$ and grading $\underline{a}$ and $\underline{b}$ respectively, the exterior product (also well known as the wedge product) will be

$$\delta \wedge \eta = (-1)^{ab+pq}\, \eta \wedge \delta \tag{2.13}$$

Let us recall that the grading distinguishes bosonic and fermionic forms: $\underline{a}$ and $\underline{b}$ are zero in the case where the forms are bosonic and 1 whenever the forms are fermionic. Then, depending on the bosonic or fermionic

character of the form under consideration, the wedge product may be commutative. A κ-form, where κ stands for the degree of the form, may be written as

$$\eta = A_{\mu_1} \cdot A_{\mu_2} \cdots A_{\mu_\kappa}\, dx^{\mu_1} \wedge dx^{\mu_2} \wedge \cdots \wedge dx^{\mu_\kappa} \tag{2.14}$$

It is easy to see, through the tensor calculus, that this quantity (2.14) is invariant under a change of basis. Indeed, under a transformation of co-ordinates, the form (2.14) becomes

i) $$A_{\mu_1} \cdot A_{\mu_2} \cdots A_{\mu_\kappa} = \Lambda^{s_1}_{\ \mu_1} \cdot \Lambda^{s_2}_{\ \mu_2} \cdots \Lambda^{s_\kappa}_{\ \mu_\kappa} \cdot A_{s_1} \cdot A_{s_2} \cdot A_{s_\kappa} \;;$$

ii) $$dx^{\mu_1} \wedge dx^{\mu_2} \wedge \cdots \wedge dx^{\mu_\kappa} = \Lambda^{\mu_1}_{\ s_1} \cdot \Lambda^{\mu_2}_{\ s_2} \cdots \Lambda^{\mu_\kappa}_{\ s_\kappa} \cdot dx^{s_1} \wedge dx^{s_2} \wedge \cdots \wedge dx^{s_\kappa} .$$

then

$$\begin{aligned} &A_{\mu_1} A_{\mu_2} \cdots A_{\mu_\kappa}\, dx^{\mu_1} \wedge dx^{\mu_2} \wedge \cdots dx^{\mu_\kappa} \\ &= \Lambda^{s_1}_{\ \mu_1} \Lambda^{s_2}_{\ \mu_2} \cdots \Lambda^{s_\kappa}_{\ \mu_\kappa} \cdot \Lambda^{\mu_1}_{\ \tau_1} \cdot \\ &\quad \cdot \Lambda^{\mu_2}_{\ \tau_2} \cdots \Lambda^{\mu_\kappa}_{\ \tau_\kappa}\, A_{s_1} A_{s_2} \cdots A_{s_\kappa} \cdot \\ &\quad \cdot dx^{\tau_1} \wedge dx^{\tau_2} \wedge \cdots \wedge dx^{\tau_\kappa} . \end{aligned} \tag{2.15}$$

$$\Lambda^{s_i}_{\ \mu_\kappa} \Lambda^{\mu_\kappa}_{\ \tau_j} = \delta^{i\kappa}_{\kappa j} \begin{cases} 0, & \text{for } i \neq j \text{ and/or } \kappa \neq K \\ 1, & \text{for } i = j \text{ and } \kappa = K \end{cases} \tag{2.16}$$

One therefore has that (2.14) is invariant under a change of basis, that is,

$$\eta = A_{\mu_1} A_{\mu_2} \ldots A_{\mu_k} dx^{\mu_1} \wedge dx^{\mu_2} \wedge \ldots \wedge dx^{\mu_k}$$
$$= A_{S_1} \cdot A_{S_2} \ldots A_{S_k} dx^{S_1} \wedge dx^{S_2} \wedge \ldots \wedge dx^{S_k}. \quad (2.17)$$

This is another important feature of the forms: they behave as scalars under change of co-ordinates. In this sense, we shall say that the forms are considered as geometrical objects.

We can finally summarize three of the most predominent conveniences for the use of the forms in a geometrical theory:

1) its components, A_i^{μ} , allow the passage from a local orthonormal frame to a general co-ordinate system;
2) a natural introduction of spinor fields, defined only with respect to transformations of SO(1,n);
3) their invariance with respect to a change of basis.

2.2 Choice of the supergroup of the theory; Dirac's algebra and composition of the supermultiplet

The first step of the theory is the choice of the group on which the theory is to be formulated. The group, in its turn, is determined from the d = 5 supergravity, since the geometrical S.Y.M. N = 2 - d = 5 can immediately be coupled to the N = 2 - d = 5 supergravity. In the case of the supergravity, one can deduce the nature of the group upon analysis of the field content of the theory. The group is characterized by the dual generations of the 1-forms ω^{ab} (SO(1,4) Lorentz connection with ten parameters), V^a (vierbein associated to the graviton with five parameters), B (spin-1 field with five parameters) and ψ_A (gravitini with four parameters); it is therefore a 24-parameters group. The group can then be identified as SU(2,2/1) or one of its contractions [3].

The group of the N = 2 - d = 5 S.Y.M. turns out to be the direct product between that of the supergravity (which we shall assume in its contracted form) and a general gauge group $\mathcal{G}$:

$$G = \mathcal{G} \otimes \overline{SU(2,2/1)} \quad (2.18)$$

G is the Lie group, whose r-dimensional Lie algebra is given by

$$[t_\alpha , t_\beta] = C^\theta{}_{\alpha\beta}\, t_\theta$$
$$(\alpha, \beta, \theta = 1, 2 \ldots, r) \quad (2.19)$$

It is important to stress that our choice is not dictated by the supermultiplet of fields of the theory, but from the requirement that it can be coupled to the supergravity. From the group SU(2,2/1) (used in the Poincaré version, without cosmological constant) of the supergravity, we shall use instead of the commutators between the generators, the expressions of the curvatures

$$R^{ab} = d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b \quad (2.20)$$
$$R^a = \mathcal{D} V^a - \frac{i}{2} \overline{\psi}_A \Gamma^a \psi_A \quad (2.21)$$
$$R^{\oplus} = dB - i\, \overline{\psi}_A \wedge \psi_A . \quad (2.22)$$
$$\rho_A = \mathcal{D} \psi_A \quad ; \quad (2.23)$$

where

$$\mathcal{D} V^a = dV^a - \omega^{ab} \wedge V_b \quad (2.24)$$
$$\mathcal{D} \psi_A = d\psi_A + \frac{i}{2} \omega^{ab} \wedge \Sigma_{ab} \psi_A \quad (2.25)$$

$\mathcal{D}$ being the exterior covariant derivative.

The set of curvatures is completed by $\mathcal{F}^\alpha$, the curvature associated to the gauge group $\mathcal{G}$. If the A^α's are 1-forms dual to the generators of $\mathcal{G}$, one has that

$$\mathcal{F}^\alpha = dA^\alpha + \frac{1}{2} C^\alpha{}_{\beta\theta} A^\beta \wedge A^\theta \quad (2.26)$$

Therefore, in the S.Y.M. N = 2 - d = 5, the field A^α is the only component of the supermultiplet giving a contribution to its group structure.

In an arbitrary space-time, the Dirac matrices satisfy the fundamental relation:

$$\{ \Gamma_a , \Gamma_b \} = 2 \eta_{ab} \tag{2.27}$$

where η_{ab} is the Minkowski metric tensor. For $d = 5$, in particular, we choose

$$\eta_{ab} = \mathrm{diag.}(+1,-1,-1,-1,-1) \tag{2.28}$$

In $d = 5$, the matrices Γ_a are 4×4 and the spinors possess four complex parameters being defined then by eight real parameters. In this case, however, it is not possible to have a spinor which becomes purely real in suitable representation of the Γ_a's. This is possible in $d = 4$, when Majorana spinors can be defined:

iii) $\psi^c = \psi$,

where

iv) $\psi^c = C \bar{\psi}^T$,

C being the charge conjugation matrix whose most remarkable properties can be found in the appendix.

Upon use of the definition iv) and some of the properties of the matrices Γ and C (Appendix 2,3), we can deduce

$$\begin{aligned}(\psi^c)^c &= C \overline{\psi^c}^T = C[(C\Gamma_0^T\psi^*)^+\Gamma_0]^T \\ &= C(\Gamma^T C)\Gamma_0 \psi = CC\Gamma_0\Gamma_0\psi \\ &= -\psi .\end{aligned} \tag{2.29}$$

This then shows that Majorana spinors cannot be defined on a 5-dimensional Minkowski-like space time. We shall work with Dirac's spinors which, in $d = 5$, can be decomposed in the following way:

$$\psi = \psi_1 + \psi_2 \quad , \tag{2.30}$$

where ψ_1 and ψ_2 are such that

$$\psi_1^c = C \bar{\psi}_1^T = \psi_2 \quad , \tag{2.31}$$

$$\psi_2^c = C \bar{\psi}_2^T = -\psi_1 \quad . \tag{2.32}$$

We consequently have that

$$\begin{aligned}(\psi^c)^c &= (\psi_1^c)^c + (\psi_2^c)^c = \psi_2^c - \psi_1^c = \\ &= -\psi_1 - \psi_2 = -\psi .\end{aligned} \tag{2.33}$$

We can now introduce a more useful notation

$$\psi_A{}^c = C \bar{\psi}_A{}^T = \epsilon_{AB} \psi_B \qquad A, B = 1, 2 . \tag{2.34}$$

ϵ_{AB} are the entries of the matrix

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \quad \epsilon^2 = -1 , \quad \epsilon_{AB} = -\epsilon_{BA} . \tag{2.35}$$

As the gauge field A^α (remember that α stands for the group index) has zero mass and spin 1, it is associated to $d - 2 = 3$ degrees of freedom on mass shell. In a supersymmetric theory the number of bosonic and fermionic degrees of freedom must be equal. Therefore, the supermultiplet should contain still another spinor, λ^α (α is not the spinorial index), to which there correspond 8/2 degrees of freedom (the factor 1/2 comes from the Dirac equation obeyed by λ_A^α). At this stage, another bosonic degree of freedom is missing which can be introduced by adjoining a scalar field ϕ^α to the other fields of the theory. All fundamental 1-forms have already been defined by choosing the group; then, λ_A^α and ϕ^α are introduced as 0-forms; for this reason the theory is defined to be "impure", to stress the difference with respect to those in which all fields are contained in the pseudo connection. Even though the physical content of the theory is expressed, in absence of a coupling to supergravity, only by fields λ_A^α, ϕ^α, A^α having in mind a first order formalism, two new fields, ϕ_a^α and $F_{ab}{}^\alpha$, both 0-forms, shall be introduced. They are nothing but covariant derivatives of the bosonic field. Moreover, one assumes that all fields (not only the gauge field A^α) have indices in the Lie algebra of $\mathcal{G}$.

III. THE CURVATURE RHEONOMY CRITERIUM

3.1 Rheonomy criterium and geometrical construction of the curvature

Our main purpose in this work is to set all the structure required for the attainment of the action for the supermultiplet λ^α_A, A^α, φ^α in the first-order formalism (upon use of the other two fields ϕ^α_a and F^α_{ab} which shall be varied independently) and in a purely geometrical fashion. By this we mean that the only operations involved in the construction of the action are the exterior differentiation and the $\wedge$-product, both mapping forms into forms. The second requirement automatically guarantees the invariance under general co-ordinate transformations (a form is indeed a scalar under these transformations, Subsec.2.1).

Another consequence of the purely geometrical nature of the action is the absence of the Hodge operator, $\mathcal{F}^\alpha \mapsto \mathcal{F}^{\alpha *}$ ($F_{\mu\nu} \to \epsilon_{\mu\nu\rho\sigma\tau} F^{\rho\sigma\tau}$) used in general in the construction of the kinetic term for A^α_μ; this leads, at the level of the equations of motion, to no reference to the dimensions of the space on which the forms are defined. We can therefore think of them as being defined on a manifold higher than the space-time manifold M^5. As our manifold, we choose that of the group G. Let us in the sequel see what this means and which are the advantages of such an interpretation.

The equations of motion, besides the fields, shall contain also the curvatures of $\overline{SU(2,2/1)}$, $\mathcal{F}$ and the covariant derivative of λ^α_A, φ^α, F^α_{ab}, ϕ^α_a. For simplicity, let us assume vanishing supergravity curvatures; moreover, we shall use in what follows the expression curvature to indicate both $\mathcal{F}^\alpha$ and the covariant defivatives of the 0-forms. The group G has (r + 24) parameters corresponding to the local basis formed by A^α (r parameters, V^a (5), ω^{ab} (10), fermions ψ_A (4) and B (5). Then, the 2-form $\mathcal{F}^\alpha$ may generally be developed in the basis of all possible 2-forms obtained by combining two by two the precedent 1-forms and the covariant derivative 1-forms. However, the theory will be developed with the use of a more limited basis. We should recall at this point the mechanism taking place in the case of the supergravity [3] and to fix ideas we consider the N = 2 - d = 5 theory.

The equations of motion imply that

$$R^a(ab)\; B = 0 \qquad , \qquad (3.1)$$

$$R^A \oplus B = 0 \qquad . \qquad (3.2)$$

A,B being indices of $\overline{SU(2,2/1)}$, that is, the curvature does not have components along ω^{ab} and B. It is then said that the theory is factorized with respect to the group SO(1,4) $\otimes$ U(1); for this reason, the group $\overline{SU(2,2/1)}$ acquires a bundle structure with fibre H represented by SO(1,4) $\otimes$ U(1) and the basis space being quotient G/H, identifiable with the superspace. In other words, the first condition implies that ω^{ab} is a connection on a principal bundle with basis space given by $\overline{SU(2,2/1)}$/SO(1,4) and gauge group SO(1,4); the second condition implies that also B is a connection on a principal bundle (the basis space in this case is $\overline{SU(2,2/1)}$/U(1) and the gauge group is U(1)).

As we wish to have our theory with the same bundle structure as the corresponding supergravity, enriched with the group $\mathcal{G}$, that is, we want the fibre to be

$$H = \mathcal{G} \otimes SO(1.4) \otimes U(1) \qquad (3.3)$$

and the basis space G/H. We shall impose that

$$R^A(ab)\; B = 0 \qquad (3.4)$$

$$R^A \oplus B = 0 \qquad (3.5)$$

$$R^A{}_{aB} = 0 \qquad (3.6)$$

A,B are indices of $\mathcal{G} \otimes \overline{SU(2,2/1)}$. Remember that $\overline{SU(2,2/1)}$ is a contract group. Therefore, we shall develop the curvatures making use only of the thirteen 1-forms V_a and ψ_A, which constitute a basis for the cotangent space (dual) of the superspace on which the fields are defined.

We notice that the action will turn out to be gauge invariant with respect to the fibre H, that is to say,

$$\mathcal{G} \otimes SO(1,4) \otimes U(1) \quad . \qquad (3.7)$$

As we shall see the construction of the theory follows a way which is different from that of the N = 2 - d = 5 supergavity, whose formalization developed in [2] and based on the concept of Chevalley's cohomology, allows for the obtention of the action without a preliminary study of the curvatures. In the Yang-Mills case, we take as our starting point just the curvatures. In a future work, we may try to construct the action by trials, by requiring that the equations of motions are non-trivially satisfied, in zero supgeravity, by these curvatures. By non-trivial we mean that one requires one solution in which not all the curvatures are zero.

Besides the factorization, another property, also very important for the theories on a group manifold, must be considered in setting the curvatures: the rheonomy [2]. It is worthwhile to come back once again to the case of N = 2 - d = 5 supergravity. For this theory, the requirement of non-triviality implies that the components "in" of the curvature (that is, those components with respect to the basis $\underline{VV}$) are related to the components "out" (with respect to the basis $\underline{\psi V}$ or $\underline{\psi\psi}$):

$$R^{A}{}_{\alpha B} = C^{a/ij}_{\alpha B/c}\, R^{c}{}_{ij} \quad , \tag{3.8}$$

whose $C^{A/ij}_{\alpha B/c}$ are constants.

From the way it has been formulated, our theory lies in superspace, that is, the fields depend on X^{μ} and θ_A (pseudo-Majorana fermionic co-ordinates). We can consider the space-time M^5 as a hypersurface embedded in superspace.

The components "in" of the curvature are substantially the derivatives of the pseudo-connection along M^5, the components "out" being those along orthonormal directions. The rheonomy therefore is equivalent to the possibility of developing all the dynamics in space-time, giving the theory a physical meaning. In order that the S.Y.M. N = 2 - d = 5, formulated in superspace, also acquires a physical meaning, it will be necessary that the derivatives "out" may be expressed in terms of the derivatives "in".

We finally have at our disposal all the elements sufficient for the formulation of explicit expressions for $\mathcal{F}^{\alpha}$ and for the covariant derivatives. The following criteria are obviously indispensable

- respect of the fermionic or bosonic character of the object under consideration [appendix , Table II];
- consideration of the degree of the forms [appendix , Table II];
- Lorentz covariance
- respect of the character of reality [appendix, Table I];
- dimensional analysis.

Moreover, the two fundamental properties of a theory on a group manifold are

- factorization, which is associated to the use of the restricted basis, V^a and ψ_A and
- rheonomy.

From these criteria, we impose the following ansatz for the curvatures:

$$\mathcal{F} = dA = F_{ab}\, V^a \wedge V^b + ic\, \bar\lambda_A \wedge \Gamma_m \psi_A \wedge V^m + id\, \epsilon_{AB} \bar\lambda_A \wedge \Gamma_m \psi_B \wedge V^m + if\, \varphi \wedge \bar\psi_A \wedge \psi_A . \tag{3.9}$$

$$\mathcal{D}\lambda_A = \Lambda_{mA} \wedge V^m + ig F_{ab} \wedge \Sigma^{ab} \psi_A + h\, \Phi_a \wedge \Gamma^a \psi_A + it F_{ab} \wedge \epsilon_{AB} \Sigma^{ab} \psi_A + z \Phi_a \wedge \Gamma^a \epsilon_{AB} \psi_B . \tag{3.10}$$

$$\mathcal{D}\varphi = \Phi_a \wedge V^a + ik\, \bar\lambda_A \wedge \psi_A + il\, \epsilon_{AB} \bar\lambda_A \wedge \psi_B \tag{3.11}$$

$$\mathcal{D}F^{ab} = G_m{}^{ab} \wedge V^m + in\, \bar\Lambda_A^{[a} \wedge \Gamma^{b]} \psi_A + ip\, \epsilon_{AB} \bar\Lambda_A^{[a} \wedge \Gamma^{b]} \psi_B . \tag{3.12}$$

where c, d, f, g, h, t, z, k, l, n, p are parameters to be determined, and

$$\Lambda_{mA} = \mathcal{D}_m \lambda_A \quad , \tag{3.12.1}$$

$$G_m{}^{ab} = \partial_m F^{ab} \quad , \tag{3.12.2}$$

$$\bar\Lambda_A = \partial \bar\lambda_A \quad . \tag{3.12.3}$$

3.2 Determination of the parameters of the curvatures through the Bianchi identities

To find the parameters we shall use the integrability conditions

$$\mathcal{D}\mathcal{F} = 0 \tag{3.13}$$

$$\mathcal{D}\mathcal{D}\lambda_A = 0 \tag{3.14}$$

$$\mathcal{D}\mathcal{D}\varphi = 0 \tag{3.15}$$

$$\mathcal{D}\,\mathcal{D}\,F^{ab} = 0 \tag{3.16}$$

In the case of the curvatures, such a kind of identities are usually called Bianchi identities. We shall adopt this nomenclature for the preceding equations, even though the last three have more than one covariant derivative. We also notice that in general the right-hand side of identities (3.14), (3.15), (3.16) exhibits terms containing the curvatures of G and the structure constants of the gauge group. The Yang-Mills fields have indices in the adjoint representation of the gauge group G; in our treatment such terms are absent as we wish to determine the parameters in zero supergravity and we consider, for the sake of simplicity, the abelian case. Let us then calculate the Bianchi identities from(3.9), (3.10), (3.11) and (3.12).

From (3.12), we have that

$$\begin{aligned}\mathcal{D}\,\mathcal{D}\,F^{ab} = {} & \mathcal{D}(G_m{}^{ab})\wedge V^m + \\ & + \frac{i}{2} G_m{}^{ab}\wedge \bar\psi_A \wedge \Gamma^m \psi_A - i n\, \mathcal{D}\bar\Lambda_A^{[a}\wedge \Gamma^{b]}\psi_A + \\ & + i p\, \mathcal{D}\bar\Lambda_A^{[a}\wedge \Gamma^{b]}\psi_B = 0\end{aligned} \tag{3.17}$$

where we have already used the fact that $DV^m = \frac{i}{2}\psi_A \wedge \Gamma^m \psi_A$ and $D\psi_A = 0$ if the curvatures of $\overline{SU(2,2/1)}$ vanish.

Upon substitution of the expressions of $\mathcal{D} G_m^{ab}$ and $D\bar\Lambda_A^a$, we obtain

$$\begin{aligned}& \mathcal{D}_n G_m{}^{ab}\wedge V^n \wedge V^m + i n\, \mathcal{D}_m \bar\Lambda_A^{[a}\wedge\Gamma^{b]}\psi_A \wedge V^m + \\ & + i p\, \mathcal{D}_m \bar\Lambda_B^{[a}\wedge \Gamma^{b]}\psi_B \wedge V^m + \\ & + \frac{i}{2} G_m{}^{ab}\wedge \bar\psi_A \wedge \Gamma^m \psi_A + \\ & \qquad + i n\, \mathcal{D}^{[a}\bar\Lambda_{mA}\wedge V^m \wedge \Gamma^{b]}\psi_A + \\ & + n g\, \mathcal{D}^{[a} F_{lm}\wedge \bar\psi_A \wedge \Sigma^{lm}\Gamma^{b]}\psi_A + \\ & + i n h\, \mathcal{D}^{[a}\Phi_l \wedge \psi_A \Gamma^l \Gamma^{b]}\psi_A + \\ & + n t\, \epsilon_{AB}\mathcal{D}^{[a} F_{lm}\wedge \psi_B \Sigma^{lm}\Gamma^{b]}\psi_A + \\ & + i n z\, \epsilon_{AB}\mathcal{D}^{[a}\Phi_l \wedge \psi_B \Gamma^l \Gamma^{b]}\psi_A + \\ & + i p\, \epsilon_{AB}\mathcal{D}^{[a}\bar\Lambda_{mA}\wedge V^m \wedge \Gamma^{b]}\psi_B +\end{aligned}$$

$$\begin{aligned}& + p g\, \epsilon_{AB}\mathcal{D}^{[a} F_{lm}\wedge \bar\psi_A \wedge \Sigma^{lm}\Gamma^{b]}\psi_B + \\ & + i p h\, \epsilon_{BC}\mathcal{D}^{[a}\Phi_l \wedge \bar\psi_A \wedge \Gamma^l\Gamma^{b]}\psi_C + \\ & + p t\, \epsilon_{AB}\epsilon_{AC}\mathcal{D}^{[a} F_{lm}\wedge \bar\psi_B \wedge \Sigma^{lm}\Gamma^{b]}\psi_C + \\ & + i p z\, \epsilon_{AB}\epsilon_{AC}\mathcal{D}^{[a}\Phi_l \wedge \bar\psi_B \Gamma^l\Gamma^{b]}\psi_C \\ & = 0 .\end{aligned} \tag{3.18}$$

The projection on two fünfbeins leads to

$$\mathcal{D}_{[n}\,\mathcal{D}_{m]}\,F^{ab} = 0 \quad ; \tag{3.19}$$

this is an identity, since

$$\mathcal{D}_n , \mathcal{D}_m] = 0 \ . \tag{3.20}$$

By picking those containing $\Sigma\Gamma$ and $\Gamma\Gamma$ and by developing them in the basis of Dirac matrices [appendix 1, 2] , one finds

i) $$\mathcal{D}^m F^{ab} - 2(ng + pt)\,\mathcal{D}^{[a} F^{b]m} = 0$$

which coincides with the homogeneous Maxwell equations

$$G[ab/m] = 0 \text{ and one then has} \tag{3.21}$$

$$ng + pt = 1 \tag{3.22}$$

and

ii) $$\begin{aligned}& \frac{1}{2}(pg - nt)\,\epsilon^{cdlm[b}\mathcal{D}^{a]} F_{cd} + \\ & + 2(ph - nz)\,\eta^{n[b}\mathcal{D}^{a]}\Phi^l = 0\end{aligned}$$

In order that the relationship ii) be satisfied without requiring the validity of non-physically acceptable relations between the derivatives of different fields, it is necessary that

$$pg - nt = 0 \tag{3.23}$$

$$ph - nz = 0 \tag{3.24}$$

We have therefore obtained the first three relations among the parameters of the curvatures. By using (3.11) and working in zero supergravity, one has

$$
\begin{aligned}
\mathcal{D}\mathcal{D}\varphi = {} & \mathcal{D}_a \Phi_b \wedge V^a \wedge V^b + ik \mathcal{D}_a \bar{\lambda}_A \wedge \psi_A \wedge V^a + \\
& + i\ell \epsilon_{AB} \mathcal{D}_a \bar{\lambda}_A \wedge \psi_B \wedge V^a + \frac{i}{2} \Phi_a \wedge \bar{\psi}_A \wedge \Gamma^a \psi_A + \\
& + ik \bar{\Lambda}_{mA} \wedge V^m \wedge \psi_A + \\
& + kg F_{ab} \wedge \bar{\psi}_A \wedge \Sigma^{ab} \psi_A + ikh \Phi_a \wedge \bar{\psi}_A \wedge \Gamma^a \psi_A + \\
& + kt \epsilon_{AB} F_{ab} \wedge \bar{\psi}_B \wedge \Sigma^{ab} \psi_A + \\
& + ikz \epsilon_{AB} \Phi_a \wedge \bar{\psi}_B \wedge \Gamma^a \psi_A + i\ell \epsilon_{AB} \bar{\Lambda}_{mA} \wedge V^m \wedge \psi_B \\
& + \ell g \epsilon_{AB} F_{ab} \wedge \bar{\psi}_A \wedge \Sigma^{ab} \psi_B + i\ell h \epsilon_{AB} \Phi_a \wedge \bar{\psi}_A \wedge \Gamma^a \psi_B + \\
& + \ell t \epsilon_{AB} \epsilon_{AC} F_{ab} \wedge \bar{\psi}_B \wedge \Sigma^{ab} \psi_C + \\
& + i\ell z \epsilon_{AB} \epsilon_{AC} \Phi_a \wedge \bar{\psi}_B \wedge \Gamma^a \psi_C \\
& = 0 \qquad (3.25)
\end{aligned}
$$

where use has been made of the developments for $\mathcal{D}\phi^a$ and $\mathcal{D}\bar{\Lambda}_A$. As in the preceding case, one does not get any information from the projections over $\underline{VV}$ and $\underline{\psi V}$. We have only to examine the terms containing two ψ's. By separating those with $\bar{\psi}_A \Gamma_a \psi_A$ and $\bar{\psi}_A \Sigma^{ab} \psi_A$, one gets

$$
\begin{aligned}
& \frac{i}{2} \Phi_a \wedge \bar{\psi}_A \wedge \Gamma^a \psi_A + ikh \Phi_a \wedge \bar{\psi}_A \wedge \Gamma^a \psi_A \\
& + i\ell z \epsilon_{AB} \Phi_a \wedge \bar{\psi}_B \wedge \Gamma^a \psi_A \\
& = 0 \qquad (3.26)
\end{aligned}
$$

Whereby it follows that

$$\frac{1}{2} + kh + \ell z = 0 \qquad (3.27)$$

and

$$
\begin{aligned}
& kt \epsilon_{AB} F_{ab} \wedge \bar{\psi}_B \wedge \Sigma^{ab} \psi_A + \\
& + \ell g \epsilon_{AB} F_{ab} \wedge \bar{\psi}_A \wedge \Sigma^{ab} \psi_B = 0 \qquad (3.28)
\end{aligned}
$$

and then a new equation for the parameters

$$Kt - g = 0 \qquad (3.29)$$

By following a reasoning analogous to the previous two which we have presented, we develop the Bianchi identities for $\mathcal{D}A$ and $\mathcal{D}\lambda_A$. We can thereby find the following equations for the parameters:

$$2fk - c = 0 \qquad (3.30)$$

$$2f\ell - d = 0 \qquad (3.31)$$

$$m = c \text{ and } p = d \qquad (3.32)$$

$$ch + f + dz = 0 \qquad (3.33)$$

$$1 + cg + dt = 0 \qquad (3.34)$$

$$dh - cz = 0 \qquad (3.35)$$

$$dg - ct = 0 \qquad (3.36)$$

obtained from the identity $\mathcal{D}\mathcal{D}A = 0$ and

$$4 + 3dt + 2z + 3cg + 2kh = 0 \qquad (3.37)$$

$$3ct + 2kz - 3dg - 2\ell k = 0 \qquad (3.38)$$

$$cg - 2kh = 0 \qquad (3.39)$$

$$dg - 2\ell h = 0 \qquad (3.40)$$

$$ct - 2kz = 0 \qquad (3.41)$$

$$dt - 2\ell z = 0 \qquad (3.42)$$

obtained from the identity $\mathcal{D}\mathcal{D}\lambda_A = 0$.

The set of equations for the parameters has then been fulfilled. A complete development of $\mathcal{D}\mathcal{D}A$ and $\mathcal{D}\lambda_A$ is given in the appendix

The analysis of the algebraic system for the parameters of the curvatures leads to the following conclusions.

Firstly, there is no incompatibility among the equations (3.22) and (3.42). This is a signal that the hypothesis made for the curvatures are acceptable: a more restrictive hypothesis could imply the non-solvability of the system.

Secondly, there are two free parameters. Two degrees of freedom can be associated to the arbitrariness in the definition of the fields λ_A and φ. So, we can fix c = 1 and f = 1. A third free parameter can be explained by the O(2) symmetry presented by the theory. This means that we can assume d = 0. Actually, for d = 0, all terms with $\psi_A^c = \epsilon_{AB}\psi_B$ disappear from the ansatz. With the above choices, the solution of the system is as follows:

$$c = 1 \tag{3.43}$$

$$d = 0 \tag{3.44}$$

$$f = 1 \tag{3.45}$$

$$m = 1 \tag{3.46}$$

$$k = \frac{1}{2} \tag{3.47}$$

$$p = d = 0 \tag{3.48}$$

$$g = h = -\frac{1}{1 + d^2} = -1 \tag{3.49}$$

$$t = z = -\frac{d}{1 + d^2} = 0 \tag{3.50}$$

$$\ell = \frac{d}{2} = 0 \tag{3.51}$$

With these solutions, the expressions for the curvatures can be written in the form:

$$dA = F_{ab} V^a \wedge V^b + i \bar{\lambda}_A \wedge \Gamma_m \psi_A \wedge V^m + i \varphi \wedge \bar{\psi}_A \wedge \psi_A . \tag{3.52}$$

$$\mathcal{D}\lambda_A = \Lambda_{mA} \wedge V^m - i F_{ab} \wedge \Sigma^{ab} \psi_A + - \not{\Phi}_a \wedge \Gamma^a \psi_A . \tag{3.53}$$

$$\mathcal{D}\varphi = \not{\Phi}_a V^a + \frac{i}{2} \bar{\lambda}_A \wedge \psi_A . \tag{3.54}$$

$$\mathcal{D}F^{ab} = G_m{}^{ab} \wedge V^m + i \bar{\Lambda}_A{}^{[a} \wedge \Gamma^{b]} \psi_A . \tag{3.55}$$

IV. CONCLUSIONS

The results achieved in this work can be summarized as follows.

A. The determination, through the analysis of the Bianchi identities, of a compatible system of equations for the parameters of the curvatures of the theory indicates that the hypothesis made for them are acceptable (Subsec.3.1). Three of these parameters result indetermined (two can be fixed, the other can be taken zero due to the O(2) symmetry).

B. The Bianchi identities are satisfied by the curvatures if the Dirac equation and the homogeneous Maxwell equations hold:

$$\Gamma^m \Lambda_{mA} = 0 \quad ; \quad G_{[ab/m]} = 0$$

This result is related to the closure of the algebra of supersymmetry transformations "on-shell". A possible development of the theory is the so-called "off-shell" formulation, interesting under the point of view of quantization. Finally, through the explicit determination of the curvatures by means of the Bianchi identities we shall be able to build up, in a future report, the geometrical action of N = 2 - d = 5 supersymmetric Yang-Mills theory, which will result geometrical and then be readily coupled to the N = 2 - d = 5 supergravity [6].

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APPENDIX A

1. $\Gamma_a \rightarrow$ Dirac's matrix in $d = 5$

$$\{\Gamma_a, \Gamma_b\} = 2\,\eta_{ab} \tag{A.1}$$

$$\eta_{ab} = \text{diag.}(+,-,-,-,-,) \qquad a,b = 0,1,2,3,4. \tag{A.2}$$

2. The matrices Γ^a ca be defined from the $d = 4$-Dirac's matrices. Indeed,

$$\Gamma^a = i\,\gamma^5\,\gamma^a \qquad (a = 0,1,2,3) \tag{A.3}$$

$$\Gamma^4 = i\,\gamma^5 \;. \tag{A.4}$$

It is convenient to write down some useful properties of the Γ matrices in five dimensions which have been used throughout this work. Other properties can be found in Ref.3

$$\Gamma_0^{\,+} = \Gamma_0 \;, \tag{A.5}$$

$$\Gamma_i^{\,+} = -\Gamma_i \;;\; (i=1,2,3,4) \;, \tag{A.6}$$

$$\Gamma_0\,\Gamma_a\,\Gamma_0 = \Gamma_a^{\,+} \;, \tag{A.7}$$

$$\Gamma_0\,\Sigma_{ab}\,\Gamma_0 = \Sigma_{ab}^{\,+} \;, \tag{A.8}$$

$$[\Gamma_a, \Sigma_{bc}] = i(\eta_{ab}\Gamma_c - \eta_{ac}\Gamma_b) \tag{A.9}$$

$$\{\Gamma_a, \Sigma_{bc}\} = \epsilon_{abc\ell m}\,\Sigma^{\ell m} \tag{A.10}$$

$$[\Sigma_{ab}, \Sigma_{cd}] = -i(\eta_{ac}\Sigma_{bd} + \eta_{bd}\Sigma_{ac} - \eta_{ad}\Sigma_{bc} - \eta_{bc}\Sigma_{ad}) \tag{A.11}$$

$$\{\Sigma_{ab}, \Sigma_{cd}\} = \frac{1}{2}(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc})\mathbf{1} + \frac{1}{2}\epsilon_{abcdk}\Gamma^k. \tag{A.12}$$

$$\Sigma_{ab} = \frac{i}{4}[\Gamma_a, \Gamma_b]. \tag{A.13}$$

3. C: charge conjugation matrix:

$$C^2 = -\mathbf{1} \tag{A.14}$$

$$C^T = -C \tag{A.15}$$

$$C\,\Gamma^a\,C^{-1} = \Gamma^{a^+} \tag{A.16}$$

$$C\,\Sigma^{ab}\,C^{-1} = (-\Sigma^{ab})^+ \tag{A.17}$$

$$\Gamma_0^+\,C = C\,\Gamma_0 \;. \tag{A.18}$$

4. On the combinations between the 1-forms ψ and the 0-forms λ of the five-dimensional space.

Let us consider, for instance,

$$\bar{\psi}_A \wedge \Sigma^{ab}\,\psi_A \;. \tag{A.19}$$

From the definition (2..34) in Subsec.2.2 we have that

$$(\psi_A{}^C)^T\,C = \bar{\psi}_A \;; \tag{A.20}$$

$$-C\,(\overline{\psi}_A^{\,c})^T = \psi_A \tag{A.21}$$

Therefore

$$\begin{aligned}\overline{\psi}_A \wedge \Sigma^{ab}\psi_A &= -(\psi_A^{\,c})^T \wedge C\,\Sigma^{ab} C\,(\overline{\psi}_A^{\,c})^T \\ &= -(\psi_A^{\,c})_\alpha (\Sigma^{ab\,T})_{\alpha\beta} (\overline{\psi}_A^{\,c})_\beta = \\ &= -\overline{\psi}_A^{\,c} \wedge \Sigma^{ab}\psi_A^{\,c} = -\overline{\psi}_A \wedge \Sigma^{ab}\psi_A = 0\end{aligned} \tag{A.22}$$

where we have used the definition of the wedge product and the definitions of Appendix 3

Using an analogous reasoning, it is possible to find combinations between the 1-forms ψ and the 0-forms λ which are vanishing or not. A complete list is shown in Table I. Another important consideration about bilinears on λ and ψ concerns their real or imaginary character. As an example, let us take

$$\overline{\lambda}_A \wedge \Gamma^a \psi_A \qquad , \tag{A.23}$$

which is surely different from zero in general, since it is the combination of two different fermions. Let us study its reality. One has

$$\begin{aligned}(\overline{\lambda}_A \wedge \Gamma_a \psi_A)^* &= \\ &[(\lambda_A^*)_\alpha . (\Gamma_0)_{\alpha\beta} . (\Gamma_a)_{\beta\gamma} . (\psi_A)_\gamma]^* = \\ &= (\psi_A^*)_\gamma (\Gamma_a^\dagger)_{\beta\gamma} (\Gamma_0^\dagger)_{\alpha\beta} (\lambda_A)_\alpha = \\ &= \psi_A^\dagger \wedge \Gamma_a^\dagger \Gamma_0^\dagger \lambda_A = \\ &= \overline{\psi}_A \Gamma_0 \Gamma_a^\dagger \Gamma_0 \lambda_A = \\ &= \overline{\psi}_A \wedge \Gamma_a \lambda_A .\end{aligned} \tag{A.24}$$

After having used the complex conjugation let us exploit the properties of symmetry of $C\,\Gamma^a$ and (anti)commutation of the usual wedge product:

$$\begin{aligned}\overline{\psi}_A \wedge \Gamma_a \lambda_A &= -(\psi_A^{\,c})^T \wedge C\,\Gamma_a C\,(\overline{\lambda}_A^{\,c})^T = \\ \psi_A^{\,T} \wedge \Gamma_a^{\,T} \overline{\lambda}_A^{\,T} &= (\psi_A)_\alpha \wedge (\Gamma_a)_{\beta\alpha} (\overline{\lambda}_A)_\beta \\ &= -(\overline{\lambda}_A)_\beta \wedge (\Gamma_a)_{\beta\alpha} (\psi_A)_\alpha = \\ &\quad -\overline{\lambda}_A \wedge \Gamma_a \psi_A .\end{aligned} \tag{A.25}$$

$$(\overline{\lambda}_A \Gamma_a \psi_A)^* = -\overline{\lambda}_A \wedge \Gamma_a \psi_A . \tag{A.26}$$

the bilinear we are considering is purely immaginary. In an analogous way, we can find others results which are also shown in Table I.

Let us conclude by considering the objects

$$\psi_A^{\alpha} \wedge \psi_B^{\beta} \qquad \text{and} \qquad \psi_A^{\alpha} \wedge \psi_B^{\beta} \wedge \psi_C^{\gamma} \quad , \tag{A.27}$$

where α,β,γ are now spinorial indices. The first expression defines a 36-component object, the last a 120-component object. Actually, due to the commutativity of the product $\wedge$, the above expressions define rank-2 and -3 symmetric tensors, where each index (α, A) assumes 8 values (α= 1,2,3,4; A=1,2).

Now by using the general formula for a rank-m symmetric tensor on n dimensions, one obtains (n = 8):

$$\begin{pmatrix} n+m-1 \\ m \end{pmatrix} = 36 \quad , \text{if } m = 2 \tag{A.28}$$

and

$$= 120 \quad , \text{if } m = 3$$

For this combinations, it is possible to give developments in terms of currents

$$\psi_A \wedge \overline{\psi}_B = \frac{1}{2}\Sigma_{ab} X^{ab}_{BA} + \frac{1}{8}\delta_{AB}\left(\Gamma_a X^a + + 1 X^{\oplus}\right) \qquad (A.29)$$

$$\psi_A \wedge \overline{\psi}_B \wedge \psi_C = \frac{1}{2}\delta_{BC}\psi_A \wedge X^{\oplus} = -\frac{i}{2}\delta_{BC}\Theta_A \qquad (A.30)$$

$$\psi_A \wedge \overline{\psi}_B \wedge \Gamma^a \psi_C = \frac{1}{2}\delta_{BC}\psi_A \wedge X^a = = -\frac{i}{2}\delta_{BC}\left(\Theta_A^a + \frac{1}{5}\Gamma^a \Theta_A\right) \qquad (A.31)$$

where

$$X^{\oplus} = \overline{\psi}_A \wedge \psi_A \qquad (A.32)$$

$$X^a = \overline{\psi}_A \wedge \Gamma^a \psi_A \qquad (A.33)$$

$$X^{ab}_{BA} = \overline{\psi}_A \wedge \Sigma^{ab} \psi_B \qquad (A.34)$$

$$\Theta_A = -i\,\psi_A \wedge X^{\oplus} \qquad (A.35)$$

(A.32), (A.33), and (A.34) are the irreducible basis for the combination $\psi \wedge \psi$.

5. Upon substitution of the expression for $\mathfrak{D}F_{ab}$, $\mathfrak{D}\overline{\lambda}_A$ and $\mathfrak{D}\varphi$ we have that

$$\begin{aligned}
\mathfrak{D}\mathfrak{D}A = {} & \mathfrak{D}_m F^{ab} \wedge V^m \wedge V_a \wedge V_b + \\
& + im\,\mathfrak{D}^a \overline{\lambda}_A \wedge \Gamma^b \psi_A \wedge V_a \wedge V_b + ip\,\epsilon_{AB}\mathfrak{D}^a \overline{\lambda}_A \wedge \Gamma^b \psi_B \wedge V_a \wedge V_b + \\
& + i\,F_{ab} \wedge \overline{\psi}_A \wedge \Gamma^a \psi_A \wedge V^b + \\
& + ic\,\mathfrak{D}^a \overline{\lambda}_A \wedge V_a \wedge \Gamma^b \psi_A \wedge V_b + \\
& + cg\,F_{ab} \wedge \overline{\psi}_A \wedge \Sigma^{ab}\Gamma_m \psi_A \wedge V^m + \\
& + ich\,\Phi_a \wedge \overline{\psi}_A \wedge \Gamma^a \Gamma^b \psi_B \wedge V_b + \\
& + ct\,\epsilon_{AB} F_{ab} \wedge \overline{\psi}_B \wedge \Sigma^{ab}\Gamma_m \psi_A \wedge V^m + \\
& + icz\,\epsilon_{AB}\Phi_a \wedge \overline{\psi}_B \wedge \Gamma^a \Gamma^m \psi_A \wedge V_m + \\
& + \frac{c}{2}\overline{\lambda}_A \wedge \Gamma_m \psi_A \wedge \psi_B \wedge \Gamma^m \psi_B + \\
& + id\,\epsilon_{AB}\mathfrak{D}^a \overline{\lambda}_A \wedge V_a \wedge \Gamma_b \psi_B \wedge V^b + \\
& + dg\,\epsilon_{AB} F_{ab} \wedge \overline{\psi}_A \wedge \Sigma^{ab}\Gamma^m \psi_B \wedge V_m + \\
& + idh\,\epsilon_{AB}\Phi_a \wedge \overline{\psi}_A \wedge \Gamma^a \Gamma^m \psi_B \wedge V_m + \\
& + dt\,\epsilon_{AB}\epsilon_{AC} F_{ab} \wedge \overline{\psi}_B \wedge \Sigma^{ab}\Gamma^m \psi_C \wedge V_m + \\
& + idz\,\epsilon_{AB}\epsilon_{AC}\Phi_a \wedge \overline{\psi}_B \wedge \Gamma^a \Gamma^m \psi_C \wedge V_m + \\
& + \frac{d}{2}\epsilon_{AC}\overline{\lambda}_A \wedge \Gamma_m \psi_C \wedge \overline{\psi}_B \wedge \Gamma^m \psi_B + \\
& + if\,\Phi_a \wedge V^a \wedge \overline{\psi}_A \wedge \psi_A - fk\,\overline{\lambda}_A \wedge \psi_A \wedge \overline{\psi}_B \wedge \psi_B + \\
& - fl\,\epsilon_{AC}\overline{\lambda}_A \wedge \psi_C \wedge \overline{\psi}_B \wedge \psi_B = 0
\end{aligned} \qquad (A.36)$$

$$\begin{aligned}
\mathfrak{D}\mathfrak{D}\lambda_A = {} & \mathfrak{D}\Lambda_{mA} \wedge V^m + \frac{i}{2}\Lambda_{mA} \wedge \overline{\psi}_B \wedge \Gamma^m \psi_B \\
& + ig\,\mathfrak{D}F^{ab} \wedge \Sigma_{ab}\psi_A + h\,\mathfrak{D}\Phi_a \wedge \Gamma^a \psi_A + \\
& + it\,\epsilon_{AB}\mathfrak{D}F_{ab} \wedge \Sigma^{ab}\psi_B + \\
& + z\,\epsilon_{AC}\mathfrak{D}\Phi_a \wedge \Gamma^a \psi_C = 0
\end{aligned} \qquad (A.37)$$

obtained with vanishing curvatures of $\overline{SU(2,2/1)}$. Now by replacing the expressions of $\mathfrak{D}\Lambda_{mA}$, $\mathfrak{D}\phi_a$ and $\mathfrak{D}F_{ab}$, one finds

$$
\begin{aligned}
\mathcal{D}\mathcal{D}\lambda_A = {} & \mathcal{D}_n \Lambda_{mA} \wedge V^n \wedge V^m + \\
& + i g \mathcal{D}_m F_{ab} \wedge \Sigma^{ab}\psi_A \wedge V^m + \\
& + h \mathcal{D}_m \phi_a \wedge \Gamma^a \psi_A \wedge V^m + \\
& + i t \epsilon_{AB} \mathcal{D}_m F_{ab} \wedge \Sigma^{ab}\psi_B \wedge V^m + \\
& + z \epsilon_{AB} \mathcal{D}_m \phi_a \wedge \Gamma^a \psi_B \wedge V^m + \\
& + \frac{i}{2} \Lambda_{mA} \wedge \bar\psi_B \Gamma^m \wedge \psi_B + \\
& + i g \mathcal{D}_m F_{ab} \wedge V^m \wedge \Sigma^{ab}\psi_A + \\
& - c g \mathcal{D}_a \bar\lambda_B \wedge \Gamma_b \psi_B \wedge \Sigma^{ab}\psi_A + \\
& - d g \epsilon_{BC} \mathcal{D}_a \bar\lambda_B \wedge \Gamma_b \psi_C \wedge \Sigma^{ab}\psi_A + \\
& + h \mathcal{D}_a \phi_b \wedge V^b \wedge \Gamma^a \psi_A + \\
& + i k h \mathcal{D}_a \bar\lambda_B \wedge \psi_B \wedge \Gamma^a \psi_A + \\
& + i l h \epsilon_{BC} \mathcal{D}_a \bar\lambda_B \wedge \psi_C \wedge \Gamma^a \psi_A + \\
& + i t \epsilon_{AB} \mathcal{D}_m F^{ab} \wedge V^m \wedge \Sigma_{ab}\psi_B + \\
& - c t \epsilon_{BC} \mathcal{D}^a \bar\lambda_B \wedge \Gamma^b \psi_B \wedge \Sigma_{ab}\psi_C + \\
& + z \epsilon_{AB} \mathcal{D}_a \phi_b \wedge V^b \wedge \Gamma^a \psi_B + \\
& - d t \epsilon_{BC} \epsilon_{AD} \mathcal{D}^a \bar\lambda_B \wedge \Gamma^b \psi_C \wedge \Sigma_{ab}\psi_D + \\
& + i k z \epsilon_{AC} \mathcal{D}_a \bar\lambda_B \wedge \psi_B \wedge \Gamma^a \psi_C + \\
& + i l z \epsilon_{AC} \mathcal{D}_a \bar\lambda_B \wedge \psi_B \wedge \Gamma^a \psi_C = 0
\end{aligned}
\tag{A.38}
$$

We would like to stress once more that the equations for the parameters (3.22) to (3.42), are obtained in the hypothesis of validity of Dirac's equation

$$\Gamma^m \Lambda_{mA} = 0 \quad . \tag{A.39}$$

This allows the elimination of several terms. Ours is then an "on-shell" formulation. An "off-shell" formulation, which is more interesting in a quantum framework, requires the introduction of the so called auxiliary fields.

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TABLE I

Identities for some currents in 5 dimensions		Reality character of some currents in 5 dimensions	
Vanishing	Non-Vanishing	Real	Imaginary
$\bar{\psi}_A \Sigma^{ab} \psi_A$	$\bar{\psi}_A \wedge \psi_A$	$\bar{\lambda}_A \wedge \Sigma^{ab} \psi_A$	$\bar{\psi}_A \wedge \psi_A$
$\bar{\psi}_A{}^C \wedge \psi_A$	$\bar{\psi}_A \wedge \Gamma^a \psi_A$		$\bar{\psi}_A \wedge \Gamma^a \psi_A$
$\bar{\psi}_A{}^C \wedge \Gamma^a \psi_A$	$\bar{\psi}_A{}^C \wedge \Sigma^{ab} \psi_A$		$\bar{\lambda}_A \wedge \psi_A$
$\bar{\lambda}_A \wedge \lambda_A$	$\bar{\lambda}_A \wedge \Sigma_{ab} \lambda_A$		$\lambda_A \wedge \Gamma^a \psi_A$

TABLE II

Grading and degree of the following fields

	Grading		Degree	
	π		$=$	
	0	1	0	1
	λ_A (fermion)	φ (boson)	Λ_A (fermion)	A (boson)
		A (boson)	φ (boson)	
		V (boson)	λ_A (fermion)	V (boson)
Fields		ψ_A (fermion)	Φ_a (boson)	
	Λ_A (fermion)	Φ_a (boson)	F^{ab} (boson)	ψ_A (fermion)
		F^{ab} (boson)		

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