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Nucleon-Deuteron Low Energy Parameters

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Abstract

Momentum space Faddeev equations are solved for nucleon-deuteron scattering and effective range parameters are calculated. A reverse trend is found in the two spin states by ${}^4a_{nd} < {}^4a_{pd}$ and ${}^2a_{pd} < {}^2a_{nd}$ which is in agreement with a configuration space calculation, but in conflict with all existing experiments. The Coulomb contributions to the effective range are small in quartet but sizeable in doublet scattering.

Proton-deuteron (p-d) low-energy scattering parameters have been investigated recently both in theory and experiments. One main impetus for p-d calculations that include the Coulomb force properly has come from configuration space Faddeev calculations. Important contributions to the theoretical development of this method are due to Merkuriev[1]; in a further step he has shown numerical results, namely p-d S-wave phase shifts between E_p = 1.5 and 14 MeV[2]. Within this framework Kvitsinsky[3] and lately Friar et al.[4] have calculated p-d scattering lengths both using the same S-wave N-N interactions of Malfliet and Tjon[5]. Their results, however, are not consistent both in the quartet and doublet case and yield even different signs of the Coulomb correction for doublet scattering. On the experimental side recent measurements of differential cross sections and proton analyzing powers at proton energies as low as 0.4 MeV (for $d\sigma/d\Omega$) have been included by Huttel et al.[6] in their p-d phase shift analysis below break-up threshold. The low energy expansion coefficients found in this analysis confirm the quartet scattering length of earlier analyses[7,8] and support the bigger doublet value of Ref.8. In general the experiments are in conflict with the calculation of Friar et al., in particular, the relation ${}^2a_{sc} > {}^2a_s$ as suggested by the phase shift analyses qualitatively disagrees with this zero energy Faddeev calculation. Consequently the p-d values substantially deviate from the

Phillips line[9] in contrast to former results both in experiments[7] and theory[10].

Given this contraversial situation we want to add another calculation of low-energy p-d parameters, but this time within the framework of momentum space Faddeev equations. Such a method was developed by Alt et al.[11,12] and was applied, among others, for calculating p-d quartet low energy parameters[13]. The difficulties this method encounters above break-up threshold are discussed in some detail by Chandler and Kok[14], but here we avoid these problems since we are mainly interested in energies close to the elastic threshold.

To calculate the effect of the Coulomb distortion to nucleon-deuteron (N-d) scattering we follow the procedure outlined in Ref.11, but the numerical method was adapted to better account for the on-shell Coulomb singularity. The screened Coulomb distorted amplitude is obtained by subtraction of the pure Coulomb amplitude from the total p-d amplitude. Pushing the (Yukawa-type) screening towards infinity the true Coulomb distorted p-d Faddeev amplitude (T_{sc}) should be approached. In calculating T_{sc} as a function of the screening radius we have studied the stability of the results against the number of mesh points and a variation of the screening radius. To determine the low energy parameters we have applied the low energy expansion to the following functions:

n-d: $$x_s(k^2) = k\,\mathrm{ctg}\,\delta_s(k) \ , \tag{1}$$

where k is the c.m. momentum and δ_s the n-d phase shift.

p-d: $$x_{sc}(k^2) = C_0^{\,2} k\,\mathrm{ctg}\,\delta_{sc}(k) + 2\eta k\left(\eta^2 \sum_{\ell=1}^{\infty} \frac{1}{\ell(\ell^2+\eta^2)} - 0.57721 - \ln\eta\right) \tag{2}$$

with $\eta = \frac{\mu e^2}{\hbar^2 k}$, μ the reduced of the three-nucleon system,

$C_0^{\,2} = \frac{2\pi\eta}{e^{2\pi\eta}-1}$ and δ_{sc} the Coulomb modified nuclear phase.

As nuclear interaction model we emplcy a Yamaguchi potential, the parameters of which were fitted to the $^1S_0$(np,pp) and $^3S_1$(np) scattering lengths and effective ranges[15]. This is certainly not a very realistic potential, but since we are interested mainly in the relation nd versus pd it should, at least, contribute to a qualitative discussion.

Quartet scattering.

For the quartet case a shape independent effective range expansion

$$-\frac{1}{{}^4a_i} + \frac{1}{2}\,{}^4r_i k^2 = x_i(k^2) \qquad i\in\{s,sc\} \tag{3}$$

is sufficient to describe the functions $x_s(k^2)$ and $x_{sc}(k^2)$ respectively up to energies of 2-3 MeV. In fact we have used phases from 0.15 - 3 MeV with the energies below 1 MeV weighted more heavily. Thereby we reproduced the n-d scattering length that has been obtained from the zero energy Faddeev amplitude within

0.01 fm. It should be noted that $x(k^2)$ at higher energies deviates from a linear form demanding the use of a shape parameter to avoid a wrong extrapolation to zero energy.

The scattering length was finally determined for screening radii between 200 and $\sim$700 fm (fig.1), since for larger radii numerical instabilities appear at very low energies. Replacing δ_{sc} in Equs. (2) and (3) by δ_s yields $^4a = -40$ fm making the dependence of $^4a_{sc}$ in fig.1 at small radii plausible (applies also to r_{sc} not shown).

In quartet scattering only asymptotic properties of the N-N interaction are important; since the deuteron binding energy is reproduced by our potential a comparison with experiments is feasible in this channel. Table I contains our results vis a vis other N-d calculations (Eyre et al.[10] calculate approximate Coulomb corrections) and experiments. The difference in $^4a_{sc}$ resulting from the calculations with the local potential is roughly 2 fm. Our a_{sc} together with Alt's calculation support the result of Friar et al. which leaves Kvitsinsky's results questioned$^{+)}$.

The experimental values for a_{sc} are consistently lower than our results which is quite understandable in the case of Ref.7 and 8 where data only down to 1 MeV have been analysed. Neglecting the n-d phases below 1 MeV results in a substantial deviation of the extrapolated a_s from the value found at zero energy. A similar behavior can be expected for p-d scattering. This, however, does not explain the smaller a_{sc} of Ref.6 where data down to 0.4 MeV have been included.

$^{+)}$ Using Merkurievs p-d phase shifts[2] from 1.5 to 10 MeV yield $^4a_{sc}$ = 11.97 fm and $^4r_{sc}$ = 2.31 fm in a shape independent fit; the knowledge that for higher energies a shape parameter has to be introduced suggests to ask for the energies that have actually been used in Kvitsinsky's[3] extrapolation.

Doublet sca

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doublet sca
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N-d low ene

Doublet scattering.

For doublet n-d scattering it is well established that the small scattering length can be related to an anomalous behavior of the effective range function x caused by a pole in it near elastic threshold. Since there is no a priori reason to assume that such a pole would vanish in p-d scattering we have to modify Equ. (3) for doublet scattering[8,17]

$$\left(-\frac{1}{a_i} + \frac{1}{2} r_i k^2\right)/\left(1 + k^2/k_o^2\right) = x_i(k^2) \qquad i \in \{s, sc\} \qquad (4)$$

For realistic nuclear interaction models this pole for n-d scattering is located at small negative energies ($0.006\ fm^{-2} \lesssim k_o^2 \lesssim 0.007\ fm^{-2}$)[10,18], but for too attractive potentials like the one used here this pole moves to positive energies ($k_o^2 = -0.004\ fm^{-2}$). This reflects the fact that in the doublet case due to the long range attraction of the nucleon exchange mechanism the effect of the nucleon-nucleon interaction becomes more influential. Consequently we cannot directly compare our results with the experiments, but rather discuss qualitative features.

The doublet scattering length a_{sc} plotted as a function of the screening radius (fig.1) does not display such a pronounced plateau of stability as in quartet scattering, but rather tends to flatten smoothely towards larger radii encountering the same instability as already found for quartet scattering. The knowledge of the accurate position of the pole is essential for applying a low energy expansion, i.e. once the pole is determined the function $(1+ \frac{k^2}{k_o^2}) x(k^2)$ is found to be linear up to energies of $\sim$3 MeV. Our results for the doublet N-d low energy parameters are shown in table II together with other values from theory and experiment. The "anomaly" in the low energy expansion has been associated with the effect of a virtual bound state[19-21]. Employing relations given in ref.20 we obtain with our values of a,r and k_o virtual state energies of

$\hat{E}_{nd} = -0.41$ MeV $\qquad\qquad$ $\hat{E}_{pd} = -0.50$ MeV

In terms of the nucleon exchange mechanism this result can be interpreted in the following way: the strong (variing) attractive force that causes, according to Efimov[21], the virtual state, is weakened by the repulsive long range Coulomb force thus allowing the pole to move deeper into the unphysical region. Efimov also connected this possible virtual state with the doublet effective range. For positive ^2r a virtual state may exist due to the strongly variing attractive force at distances smaller than the triplet scattering length, whereas for negative ^2r no such state should be detected. This statement seems to be supported by the values given in table II.

The disagreement of the configuration space calculation of Friar et al. with other calculations as well as with experiments persists in doublet scattering. Our calculational results, however, display the same trend as given by Friar et al., namely, that $^2a_{sc}$ is smaller than ^2a_s. If we plot the ^{3}He binding energy of ref.22 and $^2a_{sc}$ of ref.4 together with our ^{3}He binding energy and $^2a_{sc}$ (fig.2) a deviation of these points from the Phillips line which represents an approximate linear relationship of triton binding energies and doublet scattering lengths is obvious. Universality[21] does not seem to apply simultaneously to the three nucleon system with and without the Coulomb force, even if the Phillip line is substituted by a more realistic narrow band. The results of Kvitsinsky and to some extent the approximation of ref.10

(in comparing with this work we note that a shape parameter has been used therein), on the other hand, contradict this finding and lie, together with the experimental values of ref.7 on the Phillips band. However, there is quite a spread in the experimental results. The very recent analysis of Huttel et al. favours even bigger values for ${}^2a_{sc}$ than already given by Arvieux implying the experimental ${}^2a_{sc}$ to be significantly larger than ${}^2a_s$.

The effective range ${}^2r_{sc}$ of ref.6 is negative and consequently the virtual state should be absent which is the case, indeed; no pole term was applied in their effective range expansion. Considering the rather big positive ${}^2r_s$ (100-500 fm) some realistic potentials yield[24] it is not quite clear whether the Coulomb force could be responsible for a change in the sign of the effective range.

Employing momentum space Faddeev equations that also include the Coulomb force we have found a relation between p-d and n-d scattering lengths which is, at least, qualitatively in agreement with a configuration space calculation. A substantial deviation of ${}^2a_{sc}$ from the Phillips line is apparent and opposite in trend than the latest experimental value.

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Figure Caption

Figure 1: Doublet and quartet p-d scattering lengths as a function of the screening radius.

Figure 2: N-d doublet scattering lengths plotted against trinucleon binding energies. Full circles denote nd-^{3}H and full squares pd-^{3}He quantities. Results without reference number belong to our work. The dashed line should indicate the Phillips line.

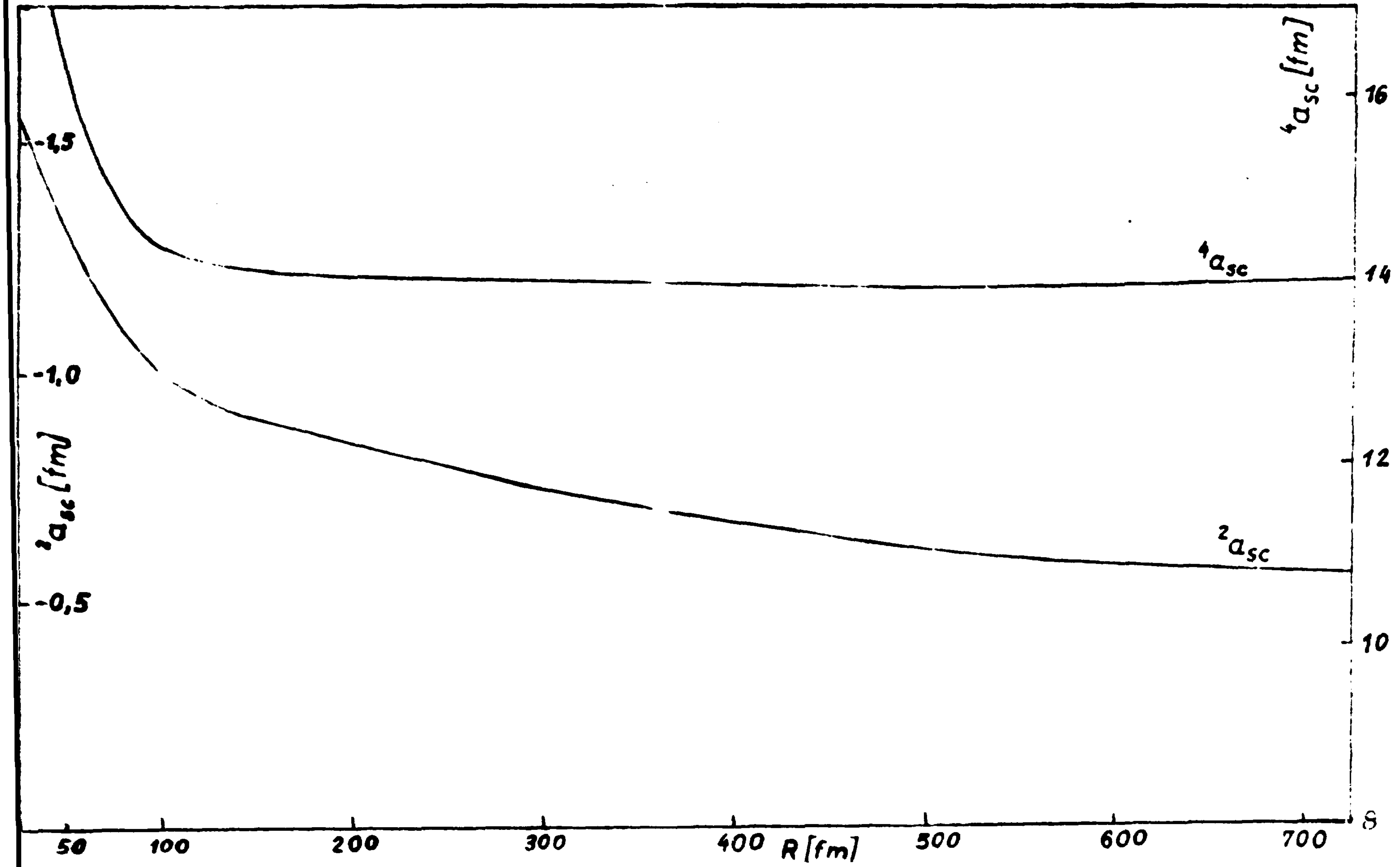
Table I Low energy quartet N-d parameters in fm

		$^4a_{nd}$	$^4a_{pd}$	$^4r_{nd}$	$^4r_{pd}$
Calculated	present work	6.30±0.01	14.0±0.2	1.96±0.04	1.97±0.06
	Friar et al. [Ref.4]	6.44	13.8		
	Kvitsinsky [Ref.3]	6.4	11.96		
	Eyre et al. [Ref.10]	6.3	10.9	1.99	1.34
	Alt [Ref.13]	6.32	13.3		1.9
Experimental	Dilg et al. [Ref.17]	6.35±0.02			
	Van Oers et al. [Ref.7]		$11.4^{+1.8}_{-1.2}$		2.05±0.25
	Arvieux [Ref.8]		11.9		2.44
	Huttel et al. [Ref.6]		11.11±0.24		2.64±0.1

Table II		Low energy doublet N-d parameters in fm					
		$^2a_{nd}$	$^2a_{pd}$	$^2r_{nd}$	$^2r_{pd}$	$k_o^2(nd)[fm^{-2}]$	$k_o^2(pd)[fm^{-2}]$
Calculated	present work	−0.56±0.04	−0.68±0.17	210±5	113±30	−0.004	−0.0043
	Friar et al. [Ref.4]	0.7	0.15				
	Kvitsinsky [Ref.3]	0.62	1.03				
	Eyre et al. [Ref.17]	0.78	1.8	−174	−76	0.0047	0.007
Experimental	Dilg et al. [Ref.17]	0.65±0.04					
	Van Oers et al. [Ref.7]		1.3±0.2				
	Arvieux [Ref.8]		2.73		2.29		
	Huttel et al. [Ref.6]		4.0±0.66		−2.8±1.0		

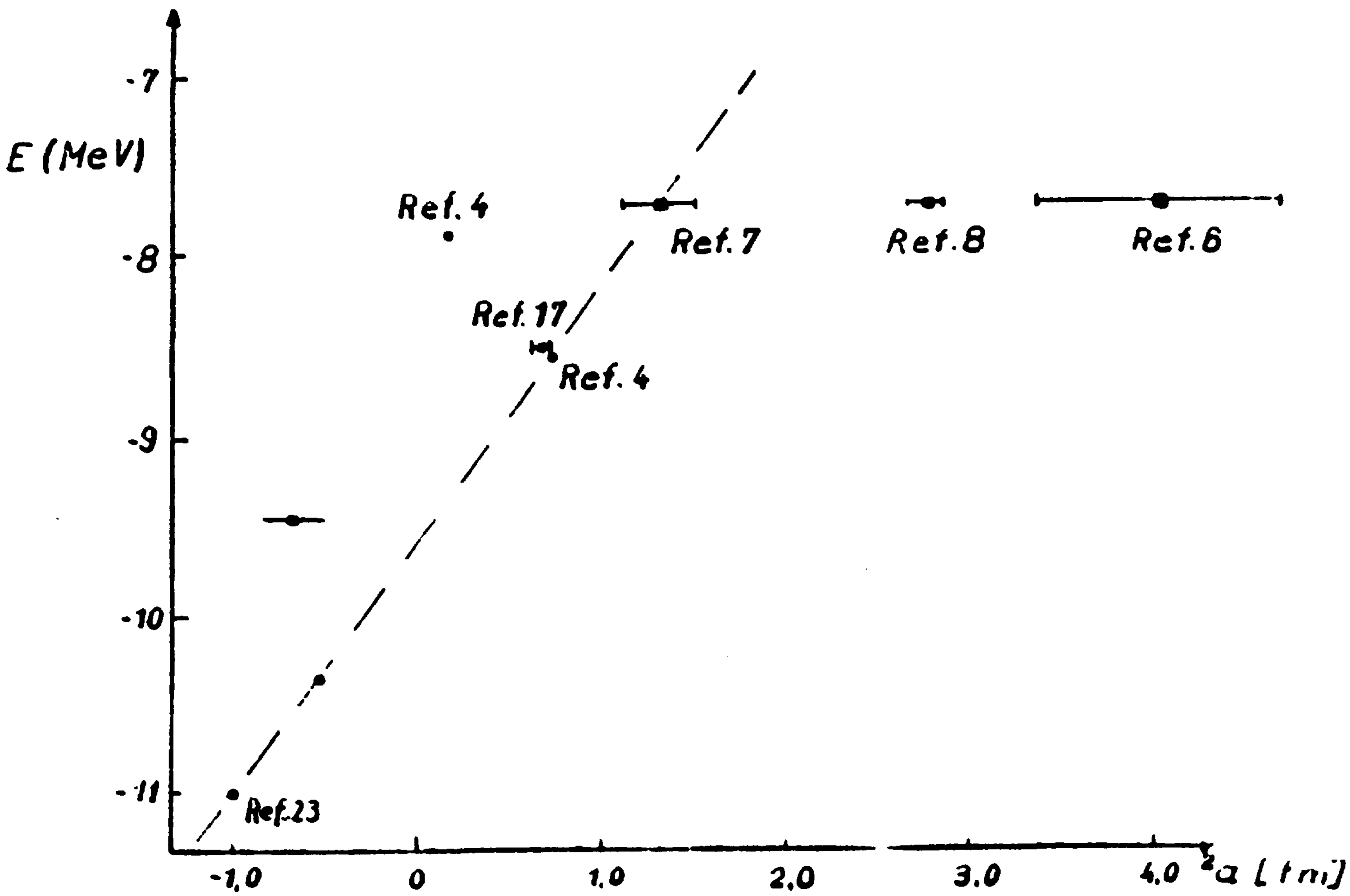
Figure 1

Figure 1



-1,5
-1,0
-0,5
$^2a_{sc}$ [fm]
$^4a_{sc}$ [fm]
16
14
12
10
8
$^4a_{sc}$
$^2a_{sc}$
50
100
200
300
400
R [fm]
500
600
700

Fig. 2



E (MeV)
-7
-8
-9
-10
-11
Ref. 4
Ref. 7
Ref. 8
Ref. 6
Ref. 17
Ref. 4
Ref. 23
-1,0
0
1,0
2,0
3,0
4,0