CA8306627

TRI-PP-82-30
Jun 1982

Neutrinoless double-beta decay in left-right symmetric models

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Abstract

Neutrinoless double-beta decay is calculated via doubly charged Higgs, which occur naturally in left-right symmetric models. We find that the comparison with known half-lives yields values of phenomenological parameters which are compatible with earlier analyses of neutral current data. In particular, we obtain a right-handed gauge-boson mass lower bound of the order of 240 GeV. Using this result and expressions for neutrino masses derived in a parity non-conserving left-right symmetric model, we obtain $m_{\nu_e} < 1.5$ eV, $m_{\nu_\mu} < 0.05$ MeV and $m_{\nu_\tau} < 18$ MeV.

(submitted to Physical Review D)

The properties of a left-right symmetric model based on a $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group have been developed in several recent publications.[1] Although at low energy the predictions of this model coincide with those of the standard[2] $SU(2)_L \times U(1)$, in general this model contains interesting new features such as the existence of massive right-handed gauge bosons and heavy Majorana neutrinos. If in addition to the usual minimal Higgs structure there is at least one additional Higgs multiplet of high dimensionality containing doubly charged scalars in either of the models, then the phenomenology becomes more exotic.[1,3] For example, coupling to ordinary fermions allows tree-level $\Delta L = 2$ reactions. However, from a detailed analysis[3] it has been shown that in the standard model the coupling of doubly charged Higgs to fermions is too small to be detected and therefore not very exciting. For a study of $\beta\beta$ decay in $SU(2) \times U(1)$ theories, see Ref. 4. In the left-right symmetric model, heavy doubly charged Higgs scalars can occur in association with heavy Majorana neutrinos and, because of their large mass, their coupling to the fermions is stronger. These effects may be observable at future accelerators, but clearly predictions should also be consistent with existing experimental constraints on reactions such as $\mu \to e\gamma$ and double-β decay.[5] The purpose of this paper is to examine in detail the neutrinoless double-β decay rates predicted by the left-right symmetric model with doubly charged Higgs. We find that the values of the model's phenomenological parameters which are necessary to predict a neutrinoless double-beta decay rate equal to a small percentage of the total $\beta\beta$ decay rate are consistent with the values obtained previously from earlier analyses[1,6] of neutral-current data. From this we can conclude that there may be a contribution to neutrinoless $\beta\beta$ decay as predicted by the left-right symmetric model, and without the exchange of a Majorana neutrino. At the same time, setting the right-handed Higgs and Majorana neutrino masses approximately equal, we find that the predicted neutrinoless decay rate is proportional to η_R^5, where $\sqrt{\eta_R}$ is approximately equal to the ratio of left-handed to right-handed gauge-boson masses. Thus, this ratio is not very sensitive to the proportion of $\beta\beta$ decay which may occur in the neutrinoless mode; for example, a reduction of this proportion by an order of magnitude would imply a decrease in the mass ratio by only a factor of ~1.2. On the other hand, because of this large power dependence we can also conclude that a value of η_R somewhat greater than the limit that we obtain ($\eta_R < 0.12$) would predict neutrinoless decay ratios that are much too large.

To incorporate doubly charged scalars in the left-right symmetric model, the symmetry is broken by Higgs with the following representation[3,7]:

$$\Delta_{L,R} = \begin{vmatrix} \Delta^{--} \\ \Delta^{-} \\ \Delta^{0} \end{vmatrix}_{L,R} \quad (1,0,2) \text{ and } (0,1,2), \qquad H = \begin{vmatrix} \chi^{0\prime} & \chi^{+} \\ \chi^{-} & \chi^{0} \end{vmatrix} \ (1/2,1/2,0). \quad (1)$$

The vacuum expectation values are

$$\langle\Delta_{L,R}\rangle = \begin{vmatrix} 0 \\ 0 \\ v_{L,R} \end{vmatrix} , \quad \langle H\rangle = \begin{vmatrix} h' & 0 \\ 0 & h \end{vmatrix} , \qquad (2)$$

where v_L^2, $h'^2 \ll h^2$, v_R^2.

The neutrinos obtain Majorana masses by coupling to Δ_L and Δ_R. The interaction terms which are allowed in the Lagrangian by the gauge symmetry and which are of interest to us are the following:

1) Coupling between doubly charged Higgs and leptons:

$$L_\ell = \kappa_1 \ell_1^T C(1 + \gamma_5)\ell_1 \ \Delta_R^{--} , \qquad (3)$$

where κ_1 can be fixed from the fact that the doubly charged Higgs also generate a Majorana mass in the heavy neutrinos N. That is

$$\kappa_1 \cong \frac{M_N}{2v_R}$$

$$\sim \frac{M_N}{4M_R}\, g , \qquad (4)$$

where M_R^2 $(\sim 1/4\ g^2 v_R^2)$ is the mass of the right-handed gauge boson W_R and M_N the mass of the heavy Majorana neutrino.

2) a) Coupling between doubly and singly charged Higgs:

$$L_{HH} \sim (\lambda_0 M_0) H^\dagger H \Delta \ . \qquad (5)$$

b) Coupling between doubly and singly charged Higgs (of mass M_H) and right-handed gauge bosons:

$$L_{HW} \sim (\lambda_0 M_0)(\partial_\mu H) W^\mu \ \Delta \ . \qquad (6)$$

c) Coupling between doubly charged Higgs and two gauge bosons:

$$L_{WW} \sim (\lambda_0 M_0) \ W_R W_R \Delta_R \ .$$

Values of the coupling strength $\lambda_0 M_0$ will differ for the three cases above; they are[3,8]

a) $\lambda_0 M_0 \sim M_H$, where $\lambda_0 \sim 1$ can be chosen

b) $\sim (g/M_R) M_R$

c) $\sim (g/M_R) M_R^2$. (7)

Neutrinoless double-beta decay can proceed through coupling 1) and any of the three couplings in 2) (a,b,c) above. The processes are described in Fig. 1. The strength of each interaction can be estimated by considering the nuclear matrix elements as follows (we show only the useful terms in each expression):

a) $$\sim \frac{g_q^2(\lambda_0 M_0)}{M_H^4 \ M_\Delta^2} \langle f | J_S | J \rangle \langle J | J_S | i \rangle$$

b) $$\sim \frac{g_q g(\lambda_0 M_0)}{M_H^2 \ M_R^2 \ M_\Delta^2} \langle f | J_S (2k-q)_\mu | J \rangle \langle J | J_{VR}^\mu | i \rangle$$

c) $$\sim \frac{g^2(\lambda_0 M_0)}{M_R^4 \ M_\Delta^2} \langle f | J_{VR}^\mu | J \rangle \langle J | J_{VR\mu} | i \rangle \ , \qquad (8)$$

where g_q is the coupling of the Higgs scalars with quarks and can be assumed to be small since it is related to m_q and the masses of the quarks;[9] that is, $g_q \sim m_q/M_H$. Here M_Δ is the mass of the doubly charged Higgs, J_S is the scalar part and J_{VR}^μ is the right-handed vector part of the nuclear current. Near the completion of our work we became aware of a $\beta\beta$ decay calculation[10] based on the process shown in Fig. 1a. However , by comparing the relative values of $(\lambda_0 M_0)$ in Eqs. (7) and (8), we can infer that the process described in Fig. 1c dominates in neutrinoless double-beta decay provided the nuclear matrix elements do not differ from each other considerably, even with the assumption $M_H \sim M_\Delta \sim 100$ GeV that was made in Ref. 10. With the likely possibility that $M_H \gg M_L$ (the left-handed gauge-boson mass) process 1a is suppressed even further by the Higgs propagators. Therefore, from now on we will concentrate on the reaction in Fig. 1c.

In terms of the couplings described above, the amplitude for the process can be written as [4,11]

$$M = \frac{g^2}{2}\,\kappa\,(\lambda_0 M_0)\int d^4x\int d^4y\;\frac{d^4k}{(2\pi)^4}\,\frac{iF(k^2)}{k^2-M_R^2}\cdot\frac{iF((k-q))}{(k-q)^2-M_R^2}$$

$$\cdot\,\frac{i}{q^2-M_\Delta^2}\,(1-P_{12})\,L\sum_j e^{-i(E_i-E_j)y_0}\,e^{-i(E_J-E_f)x_0}\,e^{-i(x-y)\cdot k}\,e^{i(k_{1e}+k_{2e})\cdot x}$$

$$\cdot\,\langle f|J^R{}_\mu(\vec{x})|J\rangle\langle J|J^{R\mu}(\vec{y})|i\rangle\,, \tag{9}$$

where

$$L = u_e^T(k_{2e},s_2)\;C\;(1+\gamma_5)u_e(k_{1e},s_1)$$

and other various symbols are defined in Fig. 1. P_{12} is the electron exchange operator. From a detailed analysis an expression for $(\lambda_0 M_0)$ has been given by Rizzo[3] as follows:

$$\lambda_0 M_0 = \lambda M_R^2$$

where

$$\lambda = \left\{\frac{4\sqrt{2}G_F\eta_R}{(1+\eta_R)^2}\right\}^{1/2}$$

$$\eta_R \cong \frac{h^2 + h'^2}{v_R^2} \sim M_L^2/M_R^2 \tag{10}$$

and M_L is the mass of the left-handed gauge boson, which is expected to be much smaller than M_R.

For the vector form factors we use the dipole parametrization

$$F(k^2) = \frac{1}{(1-k^2/M_A^2)^2} \quad , \tag{11}$$

where[12] $M_A = 0.85$ GeV/c. The use of the form factor is rather important; without it the transition rate would be greatly suppressed, and also poorly understood nuclear short-range correlations would become significant.[13]

Following the usual procedure in earlier ββ decay calculations, we invoke the closure approximation for the sum over intermediate nuclear states, taking $E_j \approx \langle E_j \rangle$ to be some appropriate average energy. We also neglect all external momentum dependence in the form factors and propagators; this is a very good approximation and it simplifies the calculation substantially. We also take into account the fact that $\langle E_j \rangle - E_j$ is much smaller than M_A and M_R.

With these approximations it is now possible to do the integrations over y_0, x_0 and k_0. After a tedious but straightforward calculation we obtain

$$M = \frac{ig^2\kappa(\lambda_0 M_0)M_A^8(1-P_{12})L\delta(E_i-E_f-\varepsilon_1-\varepsilon_2)}{12(2\pi)^2 M_A^2 M_R^4}$$

$$\cdot \int d^3\vec{x}\, d^3\vec{y}\; G \,\langle f|J^R{}_\mu(\vec{x})J^{R\mu}(\vec{y})|i\rangle\; e^{-i(\vec{k}_{1e}+\vec{k}_{2e})\cdot\vec{x}} \quad , \tag{12}$$

where
$$G = \int d^3\vec{k}\; e^{i\vec{k}\cdot\vec{r}} \left(\frac{6}{\omega_A^8} - \frac{11}{M_R^2\,\omega_A^6} + \frac{24}{\omega_R^4\,M_R^4} - \frac{22}{M_R^6\,\omega_A^2} + \frac{24}{M_R^6\,\omega_R^2}\right)$$

and
$$\omega_R^2 = \vec{k}\cdot\vec{k} + M_R^2,\quad \omega_A^2 = \vec{k}\cdot\vec{k} + M_A^2,\quad \vec{r} = \vec{x} - \vec{y}\ .$$

The integral G can now be evaluated exactly to obtain

$$G = \frac{(2\pi)^2\, e^{-M_A r}}{16\, M_R^4\, M_A^5} \left\{\left(3 + 3M_A r + M_A^2 r^2\right) - 11\left(1 + M_A r\right)\left(\frac{M_A}{M_R}\right)^2\right.$$

$$\left. + 72\left(\frac{M_A}{M_R}\right)^4 + \left[\left(\frac{192}{M_R r} + 24\right) e^{(M_A-M_R)r} - \frac{176}{M_A r}\right]\left(\frac{M_A}{M_R}\right)^5\right\} . \tag{13}$$

The form of this result is similar to the one obtained for a process mediated by a heavy Majorana neutrino;[14] in that case the exchange of a heavy lepton gives rise to an effective Yukawa interaction ~ $(e^{-Mr})/r$, where M is the Majorana neutrino mass. In our case the nucleon form factor takes the formal role that the Majorana neutrino would have taken, and the factors $e^{-M_A r}$ and $e^{-M_R r}$ appear instead. Note also that we are dealing in a mass range $M_R \gg M_A$; therefore the leading term dominates and

$$G = \frac{(2\pi)^2 e^{-M_A r}}{16\ M_A^5\ M_R^4} \left(3 + 3\ M_A r + M_A^2 r^2\right) . \tag{14}$$

This is the result that we would have obtained if we had ignored all the momentum dependence in the right-handed gauge particle propagator from the beginning, say by treating the interaction in terms of an effective Hamiltonian with an effective coupling between the leptons and the nucleons, as was done in Ref. 10.

To simplify the subsequent steps we retain the leading nonvanishing terms in the sum of the electron momenta, $\vec{k}_{1e} + \vec{k}_{2e} = -\vec{P}_f$. Then our amplitude becomes

$$M = \frac{i g^2 \kappa(\lambda_0 M_0) M_A^3 (1-P_{12}) L \delta(E_i - E_f - \varepsilon_1 - \varepsilon_2)}{192\ M_R^4\ M_\Delta^2}$$

$$\cdot \int d^3\vec{x}\, d^3\vec{y}\ \langle f | J^R{}_\mu(\vec{x}) J^{R\mu}(\vec{y}) F(r) | i \rangle \tag{15}$$

with $F(r) = \left(3 + 3\ M_A r + M_A^2 r^2\right) e^{-M_A r}$.

The right-handed weak hadronic current is

$$J_\mu(\vec{x}) = \frac{1}{2}\ \psi_p(\vec{x}) \gamma_\mu (c_V + c_A \gamma_5) \psi_n(\vec{x}) \ . \tag{16}$$

We assume that $c_A/c_V = 1.2$, and inside the nucleus we use the nonrelativistic expressions

$$\mathcal{L}_\mu(i) = \begin{cases} c_V \tau_i^+ & \mu = 0 \\ c_A \tau_i^+ \vec{\sigma}_i & \mu \neq 0 \end{cases} . \tag{17}$$

Thus, inside the nucleus the hadronic current density is

$$J^R{}_\mu(\vec{x}) = \frac{1}{2} \sum_i \delta(\vec{x} - \vec{x}_i)\, \mathcal{L}_\mu(i),$$ and we obtain

$$J^R{}_\mu(\vec{x}) J^{R\mu}(\vec{y}) = \frac{1}{4} \sum_{i \neq j} \left(c_V^2 - c_A^2 \vec{\sigma}_i \cdot \vec{\sigma}_j\right) \tau_i^+ \tau_j^+ \delta(\vec{x} - \vec{x}_i) \delta(\vec{x} - \vec{x}_j) \ . \tag{18}$$

To proceed we must now evaluate the integral in Eq. 15. Nuclear matrix elements similar to the one which appears in the integrand have been calculated in some of the earlier $\beta\beta$ calculations by using shell-model wave functions. The calculations are complex and different authors have obtained substantially different results for the same transition.[15] In our case, we will ultimately extract a value of the ratio $\sqrt{\eta_R} \simeq M_L/M_R$ from a comparison to experimental half-lives. This parameter will appear in our result as (M_L/M_R),[10] so that a moderate uncertainty in the matrix element will have a very small effect on our final value of $\sqrt{\eta_R}$. Thus it is quite adequate to use the following approximation[14]

$$\int d^3\vec{x}\, d^3\vec{y}\, \langle f|J^R{}_\mu(\vec{x})J^{R\mu}(\vec{y})F(r)|i\rangle \simeq \langle F\rangle \mathcal{M}_R$$

$$\mathcal{M}_R = \int d^3\vec{x}\, d^3\vec{y}\, \langle f|J^R{}_\mu(\vec{x})J^{R\mu}(\vec{y})|i\rangle \ , \quad \langle F\rangle = \int F(r)P(\vec{r})d^3\vec{r}$$

$$P(r) \simeq \begin{cases} \left\{\frac{4}{3}\pi\left[(2R)^3 - r_c^3\right]\right\}^{-1} & r_c < r < 2R \\ 0 & r < r_c \end{cases} \tag{19}$$

with nuclear radius $R \simeq 1.1A^{1/3}$ fermi and a nucleon-nucleon hard-core radius $r_c = 0.4$ fermi. Straightforward integration yields

$$\begin{aligned}\langle F\rangle = \Bigg[&\frac{48}{M_A^3}\left(e^{-M_A r_c} - e^{-2M_A R}\right) + \frac{48}{M_A^2}\left(r_c e^{-M_A r_c} - 2R\, e^{-2M_A R}\right) \\ &+ \frac{24}{M_A}\left(r_c^2\, e^{-M_A r_c} - 4R^2 e^{-2M_A R}\right) + 7\left(r_c^3\, e^{-M_A r_c} - 8R^3 e^{-2M_A R}\right) \\ &+ M_A\left(r_c^4\, e^{-M_A r_c} - 16R^4 e^{-2M_A R}\right)\Bigg]\ 4\pi\left\{\frac{4}{3}\pi\left[(2R)^3 - r_c^3\right]\right\}^{-1} .\end{aligned} \tag{20}$$

Note that the exponential damping which would strongly suppress the rate of decay via heavy Majorana neutrino exchange is moderated in our case due to the momentum dependence of the form factors.[13]

Squaring the total amplitude, summing over electron spins and integrating over phase space yields the following expression for the decay rate:

$$\lambda^{0\nu} = \frac{G_F^4}{72(2\pi)^5}\,\frac{\eta_R^5}{(1+\eta_R)^2}\,\frac{M_A^6 m_e^5}{M_L^2}\,|F_e(Z)|^2|\mathcal{M}_R|^2|\langle F\rangle|^2 F_{0\nu}(T) \ , \tag{21}$$

where
$$m_e T = M_i(A,Z) - M_f(A,Z+2) - 2m_e$$
$$F_e(Z) = 2\pi\alpha Z\left(1 - e^{-2\pi\alpha Z}\right)^{-1}$$
$$F_{o\nu}(T) = \frac{2T}{15}\left(T^4 + 10T^3 + 40T^2 + 60T + 30\right) .$$

The Fermi factor $F_e(Z)$ takes into account the distortion of the electron wave functions in the Coulomb field of the nucleus. This expression for the decay rate was obtained with the assumption $M_N \sim M_R \sim M_\Delta$. From Eq. (4), the value used for κ is ~0.16.

In order to obtain estimates of η_R we compare our expression to the rate predicted by the standard two-neutrino decay calculations. Those calculations give reasonable agreement with the experimental data, and by taking this approach we do not need to calculate $\mathcal{M}_R$ explicitly. The two-neutrino decay rate is given by[16]

$$\lambda^{2\nu} = \frac{G_F^4\, m_e^9 |F_e(Z)|^2}{\mu_o^2\, (2\pi)^7}\ |\mathcal{M}_L|^2\ F_{2\nu}(T) \qquad (22)$$

where
$$F_{2\nu}(T) = \frac{2^{12}}{11!}\, T^7(T^4 + 22\ T^3 + 220\ T^2 + 990\ T + 1980)$$
$$\mathcal{M}_L = \int d^3\vec{x}\, d^3\vec{y}\ \langle f|J^L{}_\mu(\vec{x})J^{L\mu}(\vec{y})|i\rangle$$

and $\mu = \mu_o m_e$ is an average of the energy difference between intermediate and initial nuclear states. Taking[16,17] $\mu_o \sim 20$ yields the following ratio for $^{130}T_e$ (T = 5 MeV):

$$R = \frac{\lambda^{o\nu}}{\lambda^{2\nu}} = 4.5 \times 10^3\ \eta_R^5 . \qquad (23)$$

In obtaining this result we made the reasonable assumption $\mathcal{M}_R \sim \mathcal{M}_L$. Although at present there is no convincing evidence that the neutrinoless mode contributes significantly to the double-beta decay of $^{130}T_e$, earlier analyses[5] indicated a maximum contribution of ~10%. If we assume this value for the ratio R, we obtain

$$\eta_R < 0.12$$
$$M_R \approx M_L/\sqrt{\eta_R} \gtrsim 240\ \text{GeV} . \qquad (24)$$

Note that the value of η_R is not very sensitive to the value of R, and a decrease of an order of magnitude in R would only change η_R by a factor of ~1.6. At the same time, however, a small increase in η_R would predict neutrinoless rates that are much too large.

In previous analyses[18] of double-beta decay there have been two approaches in trying to justify a possible contribution from the neutrinoless mode to the total decay rate. One approach relied on a small ($\sim 10^{-5}$) right-handed component in the weak interaction, and the other one assumed that lepton number violation is due to an electron neutrino with a small Majorana mass (~30 eV). More recently,[1] an approach to spontaneous parity nonconservation based on left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ has been shown to bring out explicitly the connection between the neutrino mass and the suppression of the right-handed weak interaction. The resulting estimate relates the neutrino mass to the mass of the right-handed bosons:

$$m_{\nu_\ell} \simeq m_\ell^2/gM_R \ . \tag{25}$$

Thus, using our result for M_R it is possible to put a limit on the neutrino masses: $m_{\nu_e} < 1.5$ eV, $m_{\nu_\mu} < 0.05$ MeV and $m_{\nu_\tau} < 18$ MeV. Although in general there could be mixing among the neutrinos of different families, between W_L and W_R, and between Δ_L^{--} and Δ_R^{--}, we have neglected[3] any such mixing for the purpose of our conclusions.

In summary, we have shown that experimental information about neutrinoless double-beta decay puts a stringent lower bound on the masses of possible right-handed bosons in left-right symmetric models. Moreover, these limits ($\eta_R < 0.12$ and $M_R \gtrsim 3M_L$) are consistent with the results of a comprehensive analysis of neutral-current data.[1,6]

We thank D.A. Bryman for bringing some of the earlier work on this model to the authors' attention. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under grant No. A5926.

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Figure Caption

1. Double-beta decay via coupling of the Δ^{--} to an electron pair and to a) two singly charged Higgs, b) singly charged Higgs and right-handed gauge boson, and c) two right-handed gauge bosons.

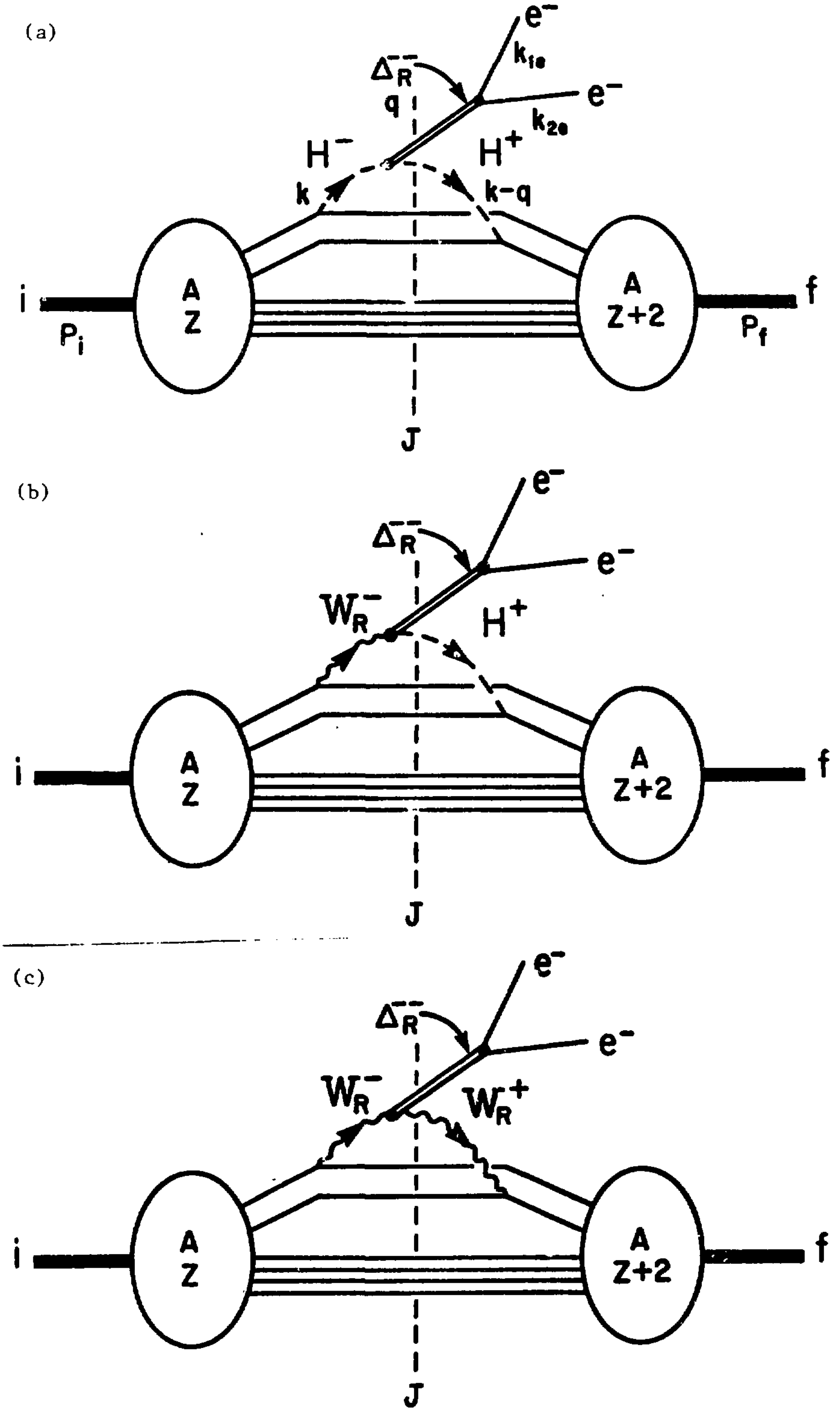
(a)
e⁻
k1e
Δ_R⁻⁻
q
e⁻
k2e
H⁻
H⁺
k
k-q
A
Z
A
Z+2
i
f
Pi
Pf
J
(b)
e⁻
Δ_R⁻⁻
e⁻
W_R⁻
H⁺
A
Z
A
Z+2
i
f
J
(c)
e⁻
Δ_R⁻⁻
e⁻
W_R⁻
W_R⁺
A
Z
A
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Fig. 1