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The (μ^-, e^+) Conversion with Majoron Emission

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ABSTRACT

We study the process of $\mu^- \to e^+$ conversion in nuclei with the emission of a Majoron in a model proposed by Gelmini and Roncadelli, with B-L symmetry spontaneously broken by the vacuum expectation value of a Higgs triplet. We find that this mechanism may contribute to the $\mu^- \to e^+$ conversion rate at the same level as predicted by earlier models.

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INTRODUCTION

Gauge theories assume that the basic constituents of matter, namely leptons and quarks, transform as particular representations of the gauge groups. Perhaps the least understood feature of these theories is the existence of several generations of fermions, with the theory repeating itself for each one. Generation-number is known not to be conserved in the quark sector, and these violations are described in terms of Cabibbo-like rotations. Since there is an underlying assumption that quark and lepton representations should behave in a similar fashion, the concept of lepton-number violation, including the mixing of different lepton generations, has naturally received much attention [1].

Some lepton-violating processes which have been studied theoretically and experimentally are $K^{\pm} \rightarrow e^{\pm} e^{\pm} \pi^{\mp}$ [2], double-beta decay [3], the decay $\mu \rightarrow e\gamma$ [4], and the conversions $\mu^- + A(Z) \rightarrow A(Z\pm2) + e^{\mp}$ [5]. In particular, the conversion $\mu^- + A(Z) \rightarrow A(Z-2) + e^+$ violates total lepton number, and its observation would mean that the other processes mentioned should also occur at some level.

The conversion $\mu^- + A(Z) \rightarrow A(Z-2) + e^+$ is presently under experimental investigation. It has been studied by Kamal and Ng [6] and later by Vergados and Ericson [7] in a model which relies on the exchange of a heavy Majorana neutrino, and also by Vergados [8] in a process mediated by Higgs particles. In this paper we present a calculation of the process in a model which was developed by Gelmini and Roncadelli [9], whereby neutrino masses are incorporated into the Weinberg-Salam model as a manifestation of the spontaneous breaking of the global symmetry of lepton number by extending the Higgs sector. The symmetry breaking is accompanied by the appearance of a massless Goldstone boson, the Majoron, and a salient feature is that, unlike earlier similar models, this model does not introduce heavy unobserved neutrinos; that is, the fermion sector is not extended by an *ad hoc* assumption. The coupling of the Majoron to electron, muon and tau neutrinos allows for the breaking of lepton number in a natural way, and this mechanism was used by Georgi, Glashow and Nussinov [10] to show that the double-beta decay process $A(Z) \rightarrow A(Z+2) + e^- + e^- + \chi$, in which a Majoron χ appears with the two electrons in neutrinoless double beta decay, can contribute a significant fraction of the total $\beta\beta$ decay rate. In the same spirit, in the present paper we calculate the rate for the process $\mu^- + A(Z) \rightarrow A(Z-2) + e^+ + \chi$ by having the Majoron couple to a mixture of electron, muon, and tau neutrinos. Similar to the case of $\beta\beta$ decay, we find conversion rates which are comparable to those found in earlier calculations. Specifically, for a coupling constant of the order ~ 1, mixing parameters $(\beta\gamma)^2 \sim 10^{-3}$ and $m_3 \sim 5$ MeV, we find a conversion rate (compared to the standard μ capture rate) of the order $\sim 10^{-21}$.

For completeness we present an outline of the Gelmini-Roncadelli model in the next section, in particular those features which are used in our calculation. Section 3 deals with the $\mu^- \rightarrow e^+$ conversion calculation, and finally the results and conclusion are presented in section 4.

2. THE MODEL

In the Gelmini-Roncadelli model the neutrinos are Majorana particles. They obtain a mass under symmetry breaking through a Yukawa coupling of the leptons with a new scalar field Φ, which is a triplet under $SU(2)_L$. The triplet is assumed to have a nonzero lepton number; thus, lepton number symmetry is broken spontaneously at a new mass scale set by $\langle\Phi\rangle$. The symmetry breaking is associated with a Goldstone boson, the Majoron, which couples to all leptons and quarks.

Gelmini and Roncadelli restricted their discussion to a single lepton family, but the development is easily generalized to several generations [11]. The term which appears in the Lagrangian in addition to those of the standard Weinberg-Salam model is

$$\mathcal{L} = -\sum_a f_a \overline{\Psi}_{aL} \phi \ell_{aR} - \sum_{ab} f_{ab} \overline{\Psi}^c_{aR} \, i\tau_2 \, \vec{\tau} \cdot \vec{\Phi} \, \psi_{bL} + \text{h.c.} \tag{1}$$

where ϕ is the usual Higgs doublet, the fermion content is

$$\psi_{aL} = \begin{bmatrix} \nu_a \\ \ell_a \end{bmatrix}_L , \quad \ell_{aR}$$

and the indices a and b refer to different generations. The field $\vec{\Phi}$ is related to the Higgs triplet by

$$\begin{aligned} \Phi_1 &= (\phi^0 + \phi^{++}) / \sqrt{2} \\ \Phi_2 &= i(\phi^{++} - \phi^0) / \sqrt{2} \\ \Phi_3 &= \phi^+ \end{aligned} \tag{2}$$

The fields can be written in terms of the expectation values as

$$\begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \begin{bmatrix} \phi^+ \\ (v_D + \rho_D + i\,\eta_D)/\sqrt{2} \end{bmatrix} \qquad \begin{bmatrix} \phi^{++} \\ \phi^+ \\ \phi^0 \end{bmatrix} = \begin{bmatrix} \phi^{++} \\ \phi^+ \\ (v_T + \rho_T + i\,\eta_T)/\sqrt{2} \end{bmatrix} \tag{3}$$

such that $\langle \phi \rangle = \begin{bmatrix} 0 \\ v_D \end{bmatrix}$, $\langle \Phi \rangle = \begin{bmatrix} 0 \\ 0 \\ v_T \end{bmatrix}$.

The charged leptons acquire mass through the vacuum expectation value of the doublet, while the neutrinos do so through $\langle \Phi \rangle$:

$$-\mathcal{L}_m = \sum_{ab} m_{ab}\, \overline{\nu}_{aL}\, \nu^c_{bR} + h.c. \tag{4}$$

where $m_{ab} = v_T f_{ab}$. The matrix m_{ab} can be diagonalized with the orthogonal matrix O assuming CP conservation:

$$\sum_{ab} O_{ia}\, m_{ab}\, (O^T)_{bj} = \delta_{ij}\, \beta_i\, m_i \tag{5}$$

where $\beta_i = \pm 1$ and $m_i > 0$. We can now define the mass eigenstates by the index i as follows:

$$\begin{aligned} \nu_{iL} &= \sum_a O_{ia}\, \nu_{aL} \\ \nu^c_{iR} &= \sum_a O_{ia}\, \nu^c_{aR} \end{aligned} \tag{6}$$

The Majorana fields are

$$\nu_i = \nu_{iL} + \beta_i \nu_{iR}^c \tag{7}$$

and eq. (4) can now be rewritten as

$$-\mathcal{L}_m = \sum_i m_i \overline{\nu}_i \nu_i \,. \tag{8}$$

ν_i is a CP eigenstate of eigenvalue β_i. β_i must be included in the definition eq. (7) so that ν_i satisfy field equations for positive mass [11]. The coupling of the leptons to the W bosons can be expressed in an explicitly CP-invariant form as

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \sum_{ia} O_{ia} \overline{\nu}_i (\gamma^\mu \ell_{iL} W_\mu^+ + \beta_i \gamma^\mu \ell_{iR}^c W_\mu^-) \quad . \tag{9}$$

To obtain the coupling between the neutrino and the Majoron, note that the total contribution to the Lagrangian from $< \Phi^0 >$ is given by

$$-\mathcal{L}_\Phi = \sum_{ab} f_{ab} \, v_T \, \overline{\nu}_{aL} \nu_{bR}^c + \sum_{ab} f_{ab} (\rho_T + i \, \eta_T) \overline{\nu}_{aL} \nu_{bR}^c + \text{h.c.} \tag{10}$$

The fields eaten up by Z_μ^0 and W_μ^-, respectively, are:

$$\begin{aligned} \theta &= \eta_D + 2(v_T / v_D)\, \eta_T \\ \sigma^+ &= i \, [\phi^+ + (v_T / v_D) \sqrt{2} \, \Phi^+] \end{aligned} \tag{11}$$

whereas the Majoron and the physical singly charged Higgs are perpendicular to those:

$$\begin{aligned} \chi &= 2 \, (v_T / v_D) \, \eta_D - \eta_T \\ \omega^+ &= i \, [(v_T / v_D) \sqrt{2} \, \phi^+ - \Phi^+] \, . \end{aligned} \tag{12}$$

With the approximation $v_T \ll v_D$ [9], it is straightforward to show that eq. 10 leads to a coupling between neutrinos and the Majoron χ of the form $i \sum_i f_i \overline{\nu}_i \gamma_5 \nu_i \chi$, where $f_i = m_i / v_T$.

Although there are many other interesting characteristics of this model, the foregoing discussion suffices in introducing the important features with respect to our $\mu^- \to e^+$ conversion calculation.

$\mu^- \to e^+$ CONVERSION CALCULATION

The process that we are calculating, $\mu^- + A(Z) \to A(Z-2) + e^+ + \chi$, is shown in fig. 1, wherein the relevant parameters are defined. We assume that the reaction occurs via neutrino exchange between two nucleons in the nucleus. As explained in the previous section, the neutrinos are a mixture of the usual electron, muon and tau neutrinos. The invariant amplitude for the process can be written as

$$M = \frac{i g^4}{16} \sum_i \int d^4x \int d^4y \int \frac{d^4 k_n}{(2\pi)^4}\, e^{i(k_e+k)\cdot x}\, e^{-iE_\mu y_0}\, e^{-ik_n\cdot(x-y)}$$

$$\cdot \frac{1}{(k_\mu - k_n)^2 - M_W^2} \cdot \frac{1}{(k_n - k - k_e)^2 - M_W^2} \cdot \frac{1}{k_n^2 - m_i^2} \cdot \frac{1}{(k_n - k)^2 - m_i^2}$$

$$\cdot \left[u_e^T C \gamma^\alpha (1-\gamma_5)(\not K_n - \not K + m_i)\gamma_5 (1-\gamma_5)(\not K_n + m_i)\gamma_\beta (1-\gamma_5) u_\mu\right] \qquad (13)$$

$$\cdot\, g_i\, O_{\mu i}\, O_{ei} \cdot \sum_j e^{-i(E_i - E_j)y_0}\, e^{-i(E_j - E_f)x_0}\, \phi_\mu(\vec{y})$$

$$\cdot \langle f | J_\alpha(\vec{x}) | j \rangle \langle j | J_\beta(\vec{y}) | i \rangle$$

where $g_i = \beta_i m_i / v_T$, k_e, k_μ and k are the positron, muon and Majoron momenta respectively, $\phi_\mu(\vec{y})$ is the muon wave junction, and $|i\rangle$, $|f\rangle$ and $|j\rangle$ are the initial, final and intermediate nuclear states with energies E_i, E_f and E_j, respectively. J_α and J_β are the usual weak hadronic currents.

Some manipulation of the gamma matrices reduces the term sandwiched between the electron and muon spinors to

$$f(k_n) = -4m_i \left[\Gamma_1 k_{n0} - \vec{\Gamma}_2 \cdot \vec{k}_n\right] \qquad (14)$$

where $\Gamma_1 = \gamma_\alpha \gamma_0 \gamma_\beta (1-\gamma_5)$ and $\vec{\Gamma}_2 = \gamma_\alpha \vec{\gamma}\, \gamma_\beta (1-\gamma_5)$. We neglect all external momenta compared to M_W, and we also take advantage of $m_i^2 \ll M_W^2$. With these approximations, the integration over k_{n0} yields

$$\int dk_{n0} \frac{e^{-ik_{n0}(x_0 - y_0)} f(k_n)}{(k_n^2 - M_W^2)^2 \, (k_n^2 - m_i^2) \, [(k_n - k)^2 - m_i^2]} =$$

$$- \frac{i\,8\pi m_i}{4M_W^4} \Bigg[\Theta(x_0 - y_0) \Bigg\{ \frac{e^{-i\omega_W(x_0 - y_0)}}{\omega_W^2} \Bigg[\left(ix_0 - iy_0 + \frac{4\omega_W}{M_W^2} \right) \omega_W \Gamma_1$$

$$- \left(ix_0 - iy_0 + \frac{1}{\omega_W} + \frac{4\omega_W}{M_W^2} \right) \vec{\Gamma}_2 \cdot \vec{k}_n \Bigg] + \frac{e^{-i\omega_i(x_0 - y_0)}}{\omega_i (k\omega_i - \vec{k} \cdot \vec{k}_n)} [\Gamma_1 \omega_i - \vec{\Gamma}_2 \cdot \vec{k}_n]$$

$$+ \frac{e^{-i(k + \omega_q)(x_0 - y_0)}}{\omega_q (\vec{k} \cdot \vec{k}_n - k^2 - k\omega_q)} [\Gamma_1 (\omega_q + k) - \vec{\Gamma}_2 \cdot \vec{k}_n] \Bigg\} \tag{15}$$

$$+ \Theta(y_0 - x_0) \Bigg\{ \frac{e^{i\omega_W(x_0 - y_0)}}{\omega_W^2} \Bigg[\left(ix_0 - iy_0 - \frac{4\omega_W}{M_W^2} \right) \omega_W \Gamma_1$$

$$+ \left(ix_0 - iy_0 - \frac{1}{\omega_W} - \frac{4\omega_W}{M_W^2} \right) \vec{\Gamma}_2 \cdot \vec{k}_n \Bigg]$$

$$+ \frac{e^{i\omega_i(x_0 - y_0)}}{\omega_i (k\omega_i + \vec{k} \cdot \vec{k}_n)} [\Gamma_1 \omega_i + \vec{\Gamma}_2 \cdot \vec{k}_n]$$

$$+ \frac{e^{-i(k - \omega_q)(x_0 - y_0)}}{\omega_q (\vec{k} \cdot \vec{k}_n - k^2 + k\omega_q)} [\Gamma_1 (k - \omega_q) - \vec{\Gamma}_2 \cdot \vec{k}_n] \Bigg\} \Bigg] ,$$

where $\omega_i^2 = \vec{k}_n^2 + m_i^2$, $\omega_W^2 = \vec{k}_n^2 + M_W^2$, and $\omega_q^2 = (\vec{k} - \vec{k}_n)^2 + m_i^2$.

Next we use the usual approximation $E_j \approx \langle E_j \rangle$, where $\langle E_j \rangle$ is some appropriate average energy of the intermediate states, and use the closure approximation, so that

$$\sum_j e^{-i(E_i - E_j)y_0} e^{-i(E_j - E_f)x_0} \langle f | J_\alpha(\vec{x}) | j \rangle \langle j | J_\beta(\vec{y}) | i \rangle$$
$$\approx e^{-i(E_i - \langle E_j \rangle)y_0} e^{-i(\langle E_j \rangle - E_f)x_0} \langle f | J_\alpha(\vec{x}) J_\beta(\vec{y}) | i \rangle \quad . \tag{16}$$

Integrations over x_0, y_0 yield

$$M = \frac{i g^4}{64 \pi^2 M_W^4} \int d^3\vec{x}\, d^3\vec{y}\, \delta(E_\mu + E_i - E_e - E_k - E_f) \sum_i g_i\, O_{\mu i}\, O_{ei}\, m_i\, \Phi_\mu(\vec{y})$$
$$\cdot \langle f | J_\alpha(\vec{x})\, J_\beta(\vec{y}) | i \rangle\, e^{-i(\vec{k}_e + \vec{k}) \cdot \vec{x}} \tag{17}$$
$$\cdot\, u^T C \int d^3\vec{k}_n\, [Q\,\Gamma_1 - \frac{1}{m_\mu}\, Q\vec{\Gamma}_2 \cdot \vec{k}_n]\, u_\mu\, e^{-i\,\vec{k}_n \cdot (\vec{x} - \vec{y})}$$

where

$$Q = \frac{4m_\mu}{(k_n^2 + M_W^2)^2} + \frac{8m_\mu}{M_W^2\,(k_n^2 + M_W^2)} + \frac{4m_\mu}{(k_n^2 - m_\mu^2)(k_n^2 - m_\mu^2 + 2m_\mu k - 2\vec{k} \cdot \vec{k}_n)} \quad .$$

These results were obtained by taking $m_i^2 \ll m_\mu^2 \ll M_W^2$. These spproximations are responsible for the signs appearing in the denominators of Q, which have a strong influence on the results that follow. The next step is to perform the $\vec{k}_n$ integration. Although it is possible to do the angular part exactly, the procedure leads to unnecessary grave complications subsequently. Instead, we will approximate $\vec{k} \cdot \vec{k}_n \cong \langle \vec{k} \cdot \vec{k}_n \rangle = 0$; this will shift the poles in the k_n integral by a small amount (at least for the experimental geometry that we will be interested in), and has a small effect on the final answer.

After the angular integration the $\vec{k}_n$ integral in eq. (17) becomes

$$\int d^3\vec{k}_n [Q\Gamma_1 - \frac{1}{m_\mu} Q\vec{\Gamma}_2 \cdot \vec{k}_n] e^{-i\vec{k}_n\cdot\vec{r}} = 4\pi\int dk_n k_n^2 [Q\Gamma_1 j_0(k_n r) + \frac{k_n}{m_\mu} Q\vec{\Gamma}_2 \cdot \hat{r} j_1(k_n r)] \tag{18}$$

where $\vec{r} = \vec{x} - \vec{y}$, and the integration over k_n yields

$$\int dk_n k_n^2 Q j_0(k_n r) = \pi\left\{\left(\frac{m_\mu}{M_W} + \frac{4m_\mu}{M_W^2 r}\right)e^{-M_W r} + \frac{1}{kr}(\cos m_\mu r - \cos m_k r)\right\}$$

$$\int dk_n k_n^3 \frac{Q}{m_\mu} j_1(k_n r) = \pi\left\{\left(1 + \frac{4}{M_W^2 r^2} + \frac{4}{M_W r}\right)e^{-M_W r} + \frac{1}{m_\mu k r^2}(\cos m_\mu r + m_\mu r \sin m_\mu r - \cos m_k r - m_k r \sin m_k r)\right\} \tag{19}$$

where $m_k = \sqrt{m_\mu^2 - 2m_\mu k}$ and $k \le m_\mu/2$. This last condition is consistent with the ongoing experiment [12]. The terms which contain $e^{-M_W r}$ are very highly suppressed and will be ignored from now on. Note that in calculations based on the exchange of a heavy neutrino, the presence of the neutrino mass could not be ignored relative to m_μ in the third term of the expression for Q in eq. (17). In that case the third term would be similar to the first two, and a similar suppression of the form e^{-Mr}, where M is the neutrino mass, would occur. In such situations one may have to consider the momentum dependence of the nucleon form factors, which we have safely ignored in our case.

For the hadronic current we use

$$J_\mu(\vec{x}) = \frac{1}{2}\sum_k \delta(\vec{x} - \vec{r}_k)\mathcal{L}_{\mu(k)} \tag{20}$$

where the index k denotes the k^{th} nucleon, with the non-relativistic form given by

$$\mathcal{L}_\mu(k) = \begin{cases} f_V \tau_k^- & \mu = 0 \\ -f_A \tau_k^- \vec{\sigma}_k & \mu \neq 0 \end{cases} \tag{21}$$

with $f_A = 1.24\, f_V$.

Inserting all of these results into the expression for the amplitude in eq. (17), after some Dirac matrix algebra it is possible to write down the amplitude using notation similar to that of ref. [7]:

$$M = i G_F^2 \sum_i (g_i\, m_i\, O_{\mu i}\, O_{ei}) [\langle \phi_\mu^2 \rangle]^{\frac{1}{2}}\ \delta(E_\mu + E_i - E_\ell - E_k - E_f) \langle f | \Omega \cdot \ell | i \rangle \tag{22}$$

where

$$\Omega \cdot \ell = \ell_0 \Omega_0 + \vec{\ell} \cdot \vec{\Omega}$$

$$\Omega_0 = \sum_{k \neq \ell} e^{-i(\vec{k}_e + \vec{k}) \cdot \vec{x}_k}\, \tau_-^k \tau_-^\ell \omega_{k\ell}^0$$

$$\vec{\Omega} = \sum_{k \neq \ell} e^{-i(\vec{k}_e + \vec{k}) \cdot \vec{x}_k}\, \tau_-^k \tau_-^\ell \vec{\omega}_{k\ell}$$

(23)

$$\omega_{k\ell}^0 = -F_1 (f_V^2 + f_A^2\, \vec{\sigma}_k \cdot \vec{\sigma}_\ell) - F_2 [f_A^2\, \vec{\sigma}_\ell \cdot \frac{\hat{r}}{i} + f_V f_A (\vec{\sigma}_k + \vec{\sigma}_\ell) \cdot \frac{\hat{r}}{i}]$$

$$\vec{\omega}_{k\ell} = F_1 [f_V f_A (\vec{\sigma}_k + \vec{\sigma}_\ell) + f_A^2\, i\, \vec{\sigma}_k \times \vec{\sigma}_\ell] +$$
$$F_2 [f_A^2 (\vec{\sigma}_\ell - \vec{\sigma}_k) \times \hat{r} + (f_V^2 - f_A^2\, \vec{\sigma}_k \cdot \sigma_\ell) \frac{\hat{r}}{i} + f_A^2 (\frac{\vec{\sigma}_k \cdot \hat{r}}{i}\, \vec{\sigma}_\ell + \vec{\sigma}_k\ \frac{\vec{\sigma}_\ell \cdot \hat{r}}{i})]$$

$$\ell^\lambda = u_e^T C \gamma^\lambda (1 - \gamma_5)\, u_\mu\ .$$

$$\vec{r} = \vec{x}_k - \vec{x}_\ell$$

$$F_1(r) = \frac{1}{2kr} (\cos m_\mu r - \cos m_k r)$$

$$F_2(r) = \frac{1}{2m_\mu k r^2} (\cos m_\mu r + m_\mu r \sin m_\mu r - \cos m_k r - m_k r \sin m_k r)\ .$$

The muon wave function is approximately constant inside the nucleus and so has been taken out of the integrals [13]:

$$< \phi_\mu^2 > \cong \frac{m_\mu^3 \alpha^3 Z_{eff}^4}{\pi Z} .$$

To calculate the conversion rate we square the amplitude and, after summing over electron and averaging over muon spins and integrating over the final phase space, we obtain the following electron energy spectrum, valid for $E_e \gtrsim m_\mu / 2$:

$$\frac{d\Gamma}{dE_e} = \frac{G_F^4}{16\pi^5} \left| \sum g_i \, m_i \, O_{\mu i} \, O_{ei} \right|^2 < \phi_\mu^2 > (m_\mu - E_e) \, E_e^2 \cdot \left[\left| <f|\Omega_0|i> \right|^2 + \left| <f|\vec{\Omega}|i> \right|^2 \right] . \qquad (24)$$

Eq. (25) gives the conversion rate as a function of the electron energy for the process $\mu^- \to e^+ \chi$ via the exchange of a light neutrino, as described in fig. 1. The calculation of the rate for the process $\mu^- \to e^+$ via light Majorana neutrino exchange, as shown in fig. 2, has never appeared explicitly in the literature. Reference has been made to the expectation that the rate is much smaller than that via heavy Majorana-neutrino exchange, due to the helicity suppression at the lepton vertices. Our motivation for calculating $\mu^- \to e^+ \chi$ via light neutrino exchange is that the Majoron emission allows the necessary helicity flip, and therefore we expect larger rates which are comparable to those predicted by the other calculations; this effect has been exploited in other processes, such as double-beta decay as calculated by Georgi, Glashow and Nussinov [10]. It will be useful to compare our result to the $\mu^- \to e^+$ rate via light neutrino exchange. To that end we present the rate for that process below. Following steps similar to those leading to eq. 24, we obtain the following rate for $\mu^+ \to e$ via light neutrino exchange:

$$\Gamma' = \frac{G_F^4}{4\pi^3} \left| \sum_i m_i^2 \, 0_{\mu i} \, 0_{ei} \right|^2 < \phi_\mu^2 > E_e \, p_e \cdot \left[\left| <f|\Omega_0'|i> \right|^2 + \left| <f|\vec{\Omega}'|\, i> \right|^2 \right] \tag{25}$$

where in this case the nuclear operators are given by

$$\Omega_0' = \sum_{k\neq\ell} e^{-i\vec{k}_e \cdot \vec{x}_k} \tau_-^k \tau_-^\ell \omega_{k\ell}^0$$
$$\vec{\Omega}' = \sum_{k\neq\ell} e^{-i\vec{k}_e \cdot \vec{x}_k} \tau_-^k \tau_-^\ell \vec{\omega}_{k\ell} \,. \tag{26}$$

$\omega_{k\ell}^0$ and $\vec{\omega}_{k\ell}$ are given by eq. 23, but with different expressions for $F_1'(r)$ and $F_2'(r)$:

$$F_1'(r) \simeq \frac{1}{2}\left[\sin m_\mu r + \left(\frac{2m_\mu}{M_W}\right)^2 \frac{\cos m_\mu r}{m_\mu r}\right] \sim \frac{1}{2} \sin m_\mu r$$

$$F_2'(r) \simeq \frac{1}{2}\left[\cos m_\mu r + \left(\frac{2m_\mu}{M_W}\right)^2 \frac{\cos m_\mu r}{(m_\mu r)^2} + \left(\frac{2m_\mu}{M_W}\right)^2 \frac{\sin m_\mu r}{m_\mu r}\right] \sim \frac{1}{2} \cos m_\mu r \,.$$

To carry out an explicit evaluation of the matrix elements we use approximations similar to those used in earlier calculations. We calculate conversion rates to all final nuclear states. Then, using closure we obtain

$$\sum_f \left[\left|<f|\Omega_0|i>\right|^2 + \left|<f|\vec{\Omega}|i>\right|^2 \right] \simeq <i|\Omega_0^\dagger \Omega_0|i> + <i|\vec{\Omega}^\dagger \cdot \vec{\Omega}|i> \,. \tag{27}$$

We assume that the two-body operator dominates, so that

$$\Omega^\dagger \Omega \sim \sum_{k\neq\ell} \omega_{k\ell}^\dagger \omega_{k\ell} \tag{28}$$

and also that the two nucleons are spin singlets. Then we obtain the following generalization of the result in eqs. 54 - 56 of ref. [7]:

$$\langle i|\Omega_0^\dagger \Omega_0|i\rangle + \langle i|\vec{\Omega}^\dagger\cdot\vec{\Omega}|i\rangle = \sum_{k\neq\ell} \tau_+^k \tau_-^k \tau_+^\ell \tau_-^\ell \left\{ |F_1|^2[(f_V^2 - 3f_A^2)^2 + 12f_A^4] + |F_2|^2[(f_V^2 + f_A^2)2 + 28f_A^4] \right\} \quad (29)$$

We need to know the two-body nuclear density and correlation function. For our purposes it is sufficient to use the following expressions [13]:

$$\langle i|\Omega^\dagger \Omega|i\rangle \simeq \frac{Z(Z-1)}{2} \langle F \rangle$$

$$\langle F \rangle = \int f(r)\, P(r)\, d^3 r \quad (30)$$

$$P(r) = \begin{cases} V_N^{-1} = \left\{\frac{4}{3}\pi\,[(2R_A)^3 - r_c^3]\right\}^{-1} & r_c \le r < 2R_A \\ 0 & r < r_c \end{cases}$$

where $R_A = 1.1\, A^{1/3}$ fermi, $r_c = 0.4$ fermi, and $f(r)$ stands for the radial functions associated with the operators in eq. (29). Following this approach we obtain

$$\langle F \rangle = 4\pi[F(2R_A) - F(r_c)]\,/\,V_N \quad (31)$$

where

$$F(r) = \frac{1}{4(m_\mu - E_e)^2}\,[(f_V^2 - 3f_A^2)^2 + 12f_A^4][r + \frac{\sin 2m_\mu r}{4m_\mu} + \frac{\sin 2m_k r}{4m_k} - \frac{\sin(m_\mu + m_k)r}{m_\mu + m_k} - \frac{\sin(m_\mu - m_k)r}{m_\mu - m_k}]$$

$$+ \frac{1}{4(m_\mu - E_e)^2}\,[(f_V^2 + f_A^2)^2 + 28f_A^4][- \frac{1}{2m_\mu^2 r}(\cos 2m_\mu r + \cos 2m_k r) - \frac{1}{m_\mu^2 r} + \frac{r}{2}(1 + \frac{m_k^2}{m_\mu^2}) - \frac{1}{2m_\mu}(\sin m_\mu r \cos m_\mu r + \frac{m_k}{m_\mu}\cos m_k r \sin m_k r) + \frac{2}{m_\mu^2 r}\cos m_\mu r \cos m_k r - \frac{m_k}{m_\mu}\frac{\sin(m_\mu - m_k)r}{m_\mu - m_k} + \frac{m_k}{m_\mu}\frac{\sin(m_\mu + m_k)r}{m_\mu + m_k}\,] \quad (32)$$

and $m_k = \sqrt{2E_e m_\mu - m_\mu^2}$.

Inserting this result into eq. (25), we finally obtain

$$\frac{d\Gamma}{dE_e} = \frac{G_F^4}{(2\pi)^5} \left| \sum_i g_i m_i O_{\mu i} O_{ei} \right|^2 \langle\phi_\mu^2\rangle (m_\mu - E_e) E_e^2 Z(Z-1) \langle F \rangle \ . \tag{33}$$

With the same approximations and assumptions we can derive a similar result for the $\mu^- \to e^+$ rate with no Majoron emission, via light Majorana neutrino exchange:

$$\Gamma' = \frac{G_F^4}{(2\pi)^3} \left| \sum_i m_i^2 O_{\mu i} O_{ei} \right|^2 \langle\phi_\mu^2\rangle E_e p_e \langle F' \rangle Z(Z-1) \tag{34}$$

where

$$\langle F' \rangle = 4\pi \left[F'(2R_A) - F'(r_c)\right] / V_N$$

$$F'(r) = \frac{1}{96}\left[(f_V^2 - 3f_A^2)^2 + 12 f_A^4\right] \cdot \left[4r^3 - \frac{6r\cos 2m_\mu r}{m_\mu^2} - \frac{6r^2 \sin 2m_\mu r}{m_\mu} + \frac{3\sin 2m_\mu r}{m_\mu^3}\right]$$

$$+ \frac{1}{96}\left[(f_V^2 + f_A^2)^2 + 28 f_A^4\right]\cdot\left[4r^3 + \frac{6r\cos 2m_\mu r}{m_\mu^2} + \frac{6r^2 \sin 2m_\mu r}{m_\mu} - \frac{3\sin 2m_\mu}{m_\mu^3}\right] .$$

RESULTS AND DISCUSSION

We have calculated the rate for the process $\mu^- \to e^+ \chi$, i.e. a process which includes the emission of a massless Majoron. In order to obtain the result in eq. (33) we have made use of some approximations used in earlier calculations. They are: neglect of external momenta when possible, closure in sums over intermediate or final states, two interacting nucleons in singlet state, constant nuclear density, and neglect of four-body operators in nuclear matrix elements. In addition we have also assumed $\vec{k}\cdot\vec{k}_n \equiv \langle\vec{k}\cdot\vec{k}_n\rangle = 0$ to obtain eqs. (18) and (19). This last approximation simplified the integrations substantially. We have carried the calculation further without making that approximation and we found that the results we obtained do not differ substantially from those given in the previous section.

In order to give explicit numerical results, it is necessary to make some assumptions about the parameters of the gauge model. Before doing that we can still present the general behaviour of the energy spectrum $d\Gamma/dE_e$ in eq. (33), which is shown in arbitrary units in fig. 3. We show the spectrum for $E_e \geq 70$ MeV, which represents the energy acceptance of the experiment presently in preparation [12].

We now turn our attention to the rate Γ defined by

$$\Gamma = \int_{70\ \mathrm{MeV}}^{m_\mu} \left(\frac{d\Gamma}{dE_e} \right) dE_e \quad . \tag{35}$$

We obtained this result by numerical integration of $d\Gamma/dE_e$ in eq. (33), and we will be discussing it in terms of the ordinary muon capture rate. The ordinary capture rate is given by [13]

$$\Gamma_0 = \frac{G_F^2 \alpha^3}{2\pi^2} m_\mu^5 Z_{eff}^4 \left[f_V^2 + 3f_A^2 + f_P^2 - 2f_P f_A \right] f(A,Z) \tag{36}$$

where $f_V^2 + 3f_A^2 + f_P^2 - 2f_P f_A \simeq 5.9$, and the function $f(A,Z)$ is a correction due to the two-nucleon correlation:

$$f(A,Z) \simeq 1 - 0.03 \frac{A}{Z} + 0.25 \left(\frac{A}{2Z} - 1 \right) + 3.24 \left(\frac{Z}{2A} - \frac{1}{2} - \left| \frac{1}{8Z} - \frac{1}{4A} \right| \right) \quad . \tag{37}$$

As seen in eq. (33), the branching ratio Γ/Γ_0 depends on the quantity $\sum_i g_i m_i O_{\mu i} O_{ei}$. If we assume that there are only three lepton generations, from section 2 we can write this sum as

$$\sum_i g_i m_i O_{\mu i} O_{ei} = g_3 m_3 \left[\left(\frac{m_1}{m_3} \right)^2 O_{\mu i} O_{ei} + \left(\frac{m_2}{m_3} \right)^2 O_{\mu 2} O_{e2} + O_{\mu 3} O_{e3} \right] \quad . \tag{38}$$

Although we cannot say it with certainty, any reasonable expectation about the neutrino masses and mixing parameters would lead to the sum being dominated by the 3rd neutrino term, so we assume $\sum_i g_i m_i O_{\mu i} O_{ei} \simeq g_3 m_3 O_{\mu 3} O_{e3}$. The numerical integration of eq. (35) yields the following result:

$$\frac{\Gamma}{\Gamma_0} = (g_3 m_3 O_{\mu 3} O_{e3})^2 \, 2.55 \times 10^{-20} \, \frac{1}{\mathrm{MeV}^2} \quad . \tag{39}$$

In order to compare our results later with those of ref. [7], we take $(O_{\mu 3}\, O_{e3})^2 = 3\times 10^{-3}$, this being the same value used in that work for the mixing parameters $(\beta\gamma)^2$. Taking $m_3 = 5$ MeV and $g_3 = 1$ yields $\Gamma/\Gamma_0 \simeq 1.9\times 10^{-21}$ for the nucleus A = 64, Z = 30.

In contrast to calculations for conversion rates via heavy Majorana neutrino exchange, our approach involves only long-range operators and nuclear uncertainties are minimized. Thus, our results are not very sensitive to the parameter r_c, and the uncertainty in our results comes primarily from the parameters of the gauge model and the neutrino masses. To compare our result to those of ref. [7], we note that their approach is based on the exchange of a heavy Majorana neutrino whose mass is expected to be very large, according to recent gauge and grand unification theories. In fact, cosmological arguments put a lower bound of $\sim$ 10 GeV. Referring to table 1 of ref. [7], we find that for $m_\nu = 10$ GeV and reasonable values of their nuclear parameters, $r_c = 0.4$ fermi and $m_A = 0.85$ GeV, the predicted branching ratio is approximately 3×10^{-21}. We see that our rates with Majoron emission are of the same order of magnitude.

Comparing now our results with Majoron emission (fig. 1) to those with light neutrino exchange without Majoron emission (fig. 2), we note the following differences: 1) the former is proportional to $(g_3\, m_3)^2$, the latter to m_3^4 (as seen in eqs. (24) and (25)); 2) the former has a three-particle final phase space; 3) the nuclear matrix elements are different (eqs. (23) and (26)); the former relies on a neutrino mass to obtain the necessary helicity states in the lepton-W coupling, the latter obtains the necessary helicity flip in the emission of the Majoron. This last point was exploited by Georgi, Glashow and Nussinov [10] to show that double-beta decay with Majoron emission may have a rate comparable to that with light neutrino exchange. It was also used by Kolb *et al.* [14] to show that lepton-violating reactions can drastically affect the gravitational collapse of massive stars. In our case, we can obtain the branching ratio for $\mu^- \to e^+$ via light neutrino emission from eq. (34). For

$m_3 = 5$ MeV and the same mixing parameters as before, we find $\Gamma'/\Gamma_0 = 5.0 \times 10^{-21}$. Thus we see that, here also, the rates with and without Majoron emission are comparable.

With regard to the coupling $g_3 \sim 1$ that was used for our calculation, a final comment is in order. Until now no explicit bounds have been put on g_3 from experimental limits. Recently [15] π and K leptonic decays have been analysed in the context of the Gelmini-Roncadelli model in terms of neutrino weak eigenstates, and bounds were put on the couplings $g_{\ell\ell'}$, where ℓ,ℓ' run through e,μ,τ. The limits obtained are $|g_{ee}|^2 + |g_{\mu e}|^2 + |g_{\tau e}|^2 < 4.5 \times 10^{-5}$ and $|g_{e\mu}|^2 + |g_{\mu\mu}|^2 + |g_{\tau\mu}|^2 < 2.4 \times 10^{-4}$. If we reinterpret these results in terms of mass eigenstates, we obtain $|g_3^2\, 0_{\mu 3}\, 0_{e3}|^2 + \ldots < 10^{-8}$. Taking $|0_{\mu 3}\, 0_{e3}|^2 \sim 3 \times 10^{-3}$ as before, we obtain $g_3 < .06$. Using $g_3 \sim .06$ would lower our result for the $\mu^- \to e^+ \chi$ rate by three orders of magnitude. The rate would then be comparable to that for $\mu^- \to e^+$ conversion (without Majoron emission) via the exchange of a light (~ 10 eV) Majorana neutrino.

In conclusion, we have shown that the Gelmini-Roncadelli model of spontaneously broken B-L symmetry predicts rates for $\mu^- \to e^+ \chi$ conversion with Majoron emission which, even though substantially lower than present experimental upper limits, are of the same order as rates for conversions via other mechanisms.

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FIGURE CAPTIONS

Fig. 1 Mechanism for $\mu^- \to e^+$ conversion with Majoron emission.

Fig. 2 The $\mu^- \to e^+$ conversion via Majorana neutrino exchange.

Fig. 3 Shape of the electron energy spectrum for the process $\mu^- \to e^+ \chi$. The energy range represents the acceptance of the experiment presently in preparation [12]. Γ_0 is the ordinary capture rate, and the quantity $C_0 = (\sum_i g_i m_i O_{\mu i} O_{ei})^2$ is discussed in the test.

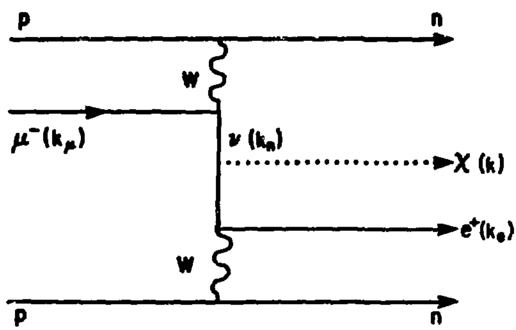


Fig. 1

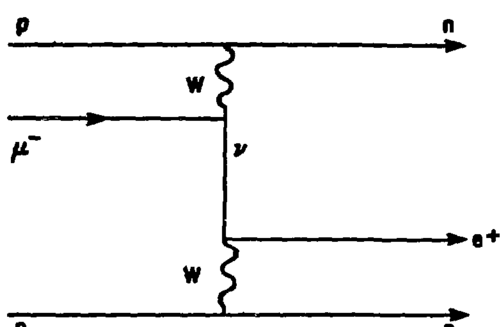
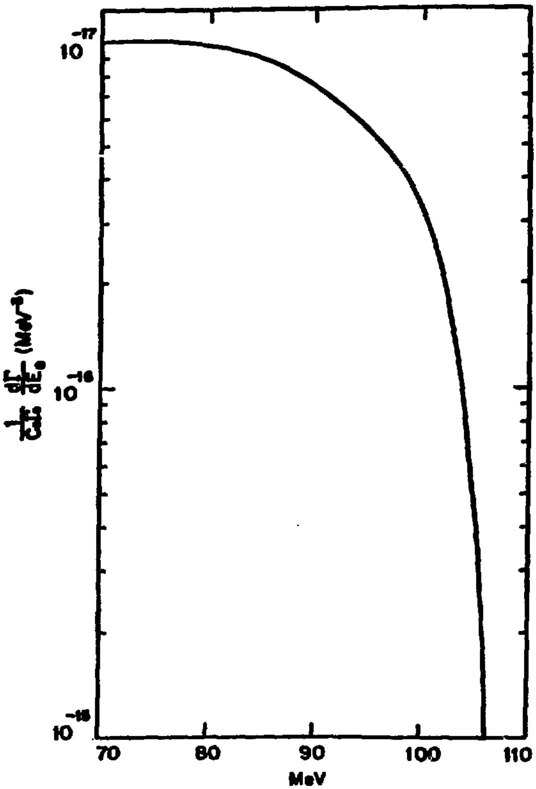


Fig. 2



$\frac{1}{\Gamma_{\mu}} \frac{d\Gamma}{dE_e}$ (MeV^{-1})
10^{-17}
10^{-18}
10^{-19}
70
80
90
100
110
MeV

Fig. 3