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μ^- Conversion Via Doubly Charged Higgs Scalar

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ABSTRACT

A new mechanism is used to calculate $\mu^- \to e^+$ conversion in nuclei, based on the existence of a doubly charged Higgs scalar. The scalar is part of a triplet which generates the spontaneous breakdown of B-L symmetry in an extension of the standard model, as proposed by Gelmini and Roncadelli. We find a limit for conversion rates which is comparable to those of earlier calculations.

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Lepton number conservation has received much attention in the last few years. A feature of gauge theories which is not well understood is the existence of several generations of leptons, with the theory repeating itself for each one. A spontaneous breakdown of B-L symmetry in an extension of the standard $SU(2)_L \times U(1)$ gauge model was recently proposed by Gelmini and Roncadelli (1). With this approach, Majorana neutrino masses result from the expectation value of a Higgs scalar triplet. The triplet contains doubly charged scalars which couple to leptons, quarks, gauge bosons and other scalars. Its neutral component is a massless Nambu-Goldstone boson, called the Majoron. Several calculations employing the features of this model have appeared in the literature; they include Majoron emission in $\mu^- \to e^+$ conversion (2) and in particle decays from external (3,4) and internal (5) lines, problems in cosmology (4), and effects on supernovae (6). More recently (7) the doubly-charged Higgs scalar has been studied in the context of weak vector boson decays, with the conclusion that their effect could be substantially higher than previously recognized.

The conversion $\mu^- + A(z) \to A(z-2) + e^+$ is presently under experimental investigation at TRIUMF. It has been studied theoretically using various approaches, including the exchange of heavy Majorana neutrinos (8,9) and the possibility of Majoron emission during the conversion process (2). In this paper we present a calculation of $\mu^- \to e^+$ conversion via the doubly-charged Higgs scalar of the Gelmini-Roncadelli model. Using limits on the masses and coupling parameters obtained in the earlier studies, we find a limit for conversion rates which is comparable to those found in previous $\mu^- \to e^+$ calculations.

The process proceeds as shown in Fig. 1. The Lagrangian terms which include the Yukawa couplings of the doubly charged Higgs boson to leptons are (1)

$$[1] \quad \mathcal{L} = \frac{1}{2\sqrt{2}} \sum_{\ell,\ell'} g_{\ell\ell'} \left[\bar{\ell}^c (1+\gamma_5) \ell' \chi^{++} + \bar{\ell}' (1-\gamma_5) \ell^c \chi^{--}\right] .$$

The Higgs scalar triplet can be written in terms of the expectation values as

$$[2] \quad \begin{bmatrix} \chi^{++} \\ \chi^{+} \\ \chi^{0} \end{bmatrix} = \begin{bmatrix} \chi^{++} \\ \chi^{+} \\ (v_T+\rho_T+i\eta_T)/\sqrt{2} \end{bmatrix}$$

with

$$\langle\chi\rangle = \begin{bmatrix} 0 \\ 0 \\ v_T \end{bmatrix} .$$

Limits for the values of the coupling constants appearing in [1] have been obtained by studying various processes. In particular, the decay $K \to \ell\nu$ has yielded (3) a limit $g_{\mu e} < 7 \times 10^{-3}$.

We assume that the reaction that we are calculating occurs via the exchange of a W boson between two nucleons, as shown in Fig. 1. The invariant amplitude for the process can be written as

$$[3]\quad M = i\, g^2 \frac{g_{\mu e}}{2\sqrt{2}} (g M_W) \int d^4x \int d^4y \int \frac{d^4k}{(2\pi)^4}\, e^{ik_e\cdot x}\, e^{-iE_\mu y_o}\, e^{-ik\cdot(x-y)}$$

$$\cdot \frac{F(k^2)}{k^2-M_W^2} \cdot \frac{F\left[(k-q)^2\right]}{(k-q)^2-m_W^2} \cdot \frac{1}{q^2-M^2} \cdot u_e^T C(1+\gamma_5) u_\mu$$

$$\cdot \sum_j e^{i(E_i-E_j)y_o}\, e^{-i(E_j-E_f)x_o}\, \Phi_\mu(\vec{y})$$
$$\cdot \langle f|J_\mu(\vec{x})|j\rangle\langle j|J_\mu(\vec{y})|i\rangle$$

where M_W and M are the masses of the W and χ bosons, respectively, k_e, k_μ and k are the positron, muon and W momenta, respectively, $\Phi_\mu(\vec{y})$ is the muon wave function, $|i\rangle$, $|f\rangle$ and $|j\rangle$ are the initial, final and intermediate nuclear states with energies E_i, E_f, and E_j respectively, and J_μ is the usual weak hadronic current. For the vector form factors we use the dipole parametrization

$$[4]\qquad F(k^2) = \frac{1}{(1-k^2/M_A^2)^2}\,,$$

where (10) $M_A = 0.85$ GeV/c. As in earlier calculations the form factor is necessary, otherwise the transition rate would be strongly suppressed and short-range correlations would be significant.

We follow the usual procedure by invoking the closure approximation for the sum over intermediate states, with $E_j = \langle E_j\rangle$. We also use the approximation $\langle E_j\rangle - E_j \ll M_A, M_W$ and neglect all external momentum dependence in the form factors and propagators. The integrations over x_o, y_o and k_o yield

$$[5] \qquad M = ig^2 \frac{g_{\mu e}}{2\sqrt{2}} \frac{(gM_w)M_A{}^8}{M^2M^4{}_w} \left[\langle\phi_\mu^2\rangle\right]^{1/2} u_e^T C(1+\gamma_5)u_\mu$$

$$\cdot \int d^3x d^3y \; G \; \langle f|\; J_\mu(\vec{x})J^\mu(\vec{y})\,|i\rangle .$$

The muon wave function is approximately constant inside the nucleus and has been taken out of the integral. The quantity G is the same one which appears in equation [13] of reference (11). Taking into accunt that $M_W >> M_A$, the leading term that dominates is

$$[6] \qquad G = \frac{(2\pi)^2 \, e^{-M_A r}}{16 \; M_A{}^5 \; M_W{}^4} \left(3 + 3\, M_A r + M_A{}^2 \, r^2\right)$$

where $\vec{r} = \vec{x} - \vec{y}$.

The hadronic current is

$$[7] \qquad J_\mu(\vec{x}) = \frac{1}{2} \sum_k \delta(\vec{x}-\vec{x}_k)\mathcal{J}_\mu(k)$$

where the index k denotes the kth nucleon. Inside the nucleus we use the nonrelativistic form

$$\mathcal{J}_\mu(k) = \begin{cases} C_V \tau_k^- & \mu = 0 \\ -C_A \tau_k^- \vec{\sigma}_k & \mu \neq 0 \end{cases}$$

with $C_A = 1.24\; C_V$. We thus obtain

[8] $$J_\mu(\vec{x})J^\mu(\vec{y}) = \frac{1}{4}\sum_{k\neq \ell}\left(C_V{}^2 - C_A{}^2\,\vec{\sigma}_k\cdot\vec{\sigma}_\ell\right)\tau_k^-\tau_\ell^-\delta(\vec{x}-\vec{x}_k)\delta(\vec{y}-\vec{x}_\ell)$$

and

[9] $$M = ig^2\,\frac{g_{\mu e}}{2\sqrt{2}}\,\left(gM_W\right) M_A{}^3\, u_e^T C(1+\gamma_5)\, u_\mu\, \left[\langle\phi_\mu^2\rangle\right]^{1/2}$$

$$\cdot \int d^3\vec{x}\; d^3\vec{y}\; \langle f|\,\frac{1}{4}\sum_{k\neq 1}\left(c_V{}^2 - c_A{}^2\,\vec{\sigma}_k\cdot\vec{\sigma}_\ell\right)\tau_k{}^-\tau_\ell{}^- F\left(r_{k\ell}\right)|i\rangle$$

where $F\left(r_{k\ell}\right) = \left(3+3M_A\, r_{k\ell} + M_A{}^2\, r_{k\ell}{}^2\right)e^{-M_A r_{k\ell}}$

and $r_{k\ell} = |\vec{x}_k - \vec{x}_\ell|$.

With the assumption that the two nucleons are in spin singlets, the amplitude can be put in the form

[10] $$M = ig^2\left(\frac{g_{\mu e}}{2\sqrt{2}}\right)\left(gM_W\right) M_A{}^3\, u_e{}^T C\,\left(1+\gamma_5\right) u_\mu\, \left[\langle\phi_\mu^2\rangle\right]^{1/2}$$

$$\cdot\,\frac{C_V{}^2+3C_A{}^2}{4}\,\langle f|\sum_{k\neq \ell}\tau_K^-\,\tau_\ell^-\,F\left(r_{k\ell}\right)|i\rangle\ .$$

To calculate the conversion rate we square the amplitude, sum over electron and average over muon spins, and integrate over the final phase space. We assume that the two-body operator dominates, and we calculate conversion rates to all final states; this approach allows us to use the closure approximation

$$[11] \qquad \sum_f \left|\langle f| \sum_{k\neq \ell} \tau_k^- \tau_\ell^- F|i\rangle\right|^2 = \langle i|\tau_k^+ \tau_k^- \tau_\ell^+ \tau_\ell^- |F|^2|i\rangle \ .$$

To carry out a calculation of the right hand side of [11] we need to know the nuclear density and correlation function. For our purposes it is adequate to employ the approximation used in earlier calculations (3, 8, 9). We let

$$[12] \qquad \langle i| \sum_{k\neq \ell} \tau_k^+ \tau_k^- \tau_\ell^+ \tau_\ell^- \, |F|^2 \, |i\rangle = \frac{Z(Z-1)}{2} \langle |F|^2\rangle$$

with $\langle |F|^2\rangle = \int |F(r)|^2 P(r) d^3r$

and $P(r) = V_N^{-1} = \left\{\frac{4}{3}\pi\,(2R)^3 - r_c^{\,3}]\right\}^{-1}$ for $r_c < r < 2R$, $= 0$ for $r < r_c$.

Here the nuclear radius is $R = 1.1A^{1/3}$ fermi and we use a nucleon-nucleon hard-core radius $r_c = 0.4$ fermi.

Following these procedures, we obtain a conversion rate given by

$$[13] \qquad \Gamma = g^4\left(\frac{g_{\mu e}^2}{8}\right)(gM_w)^2\left(\frac{C_V^{\,2}+3C_A^2}{16}\right) \frac{M_A^{\,6}\langle\phi_\mu^2\rangle}{(192M_w^{\,4}M^2)^2}$$

$$\cdot \frac{4m_\mu^{\,2}}{(2\pi)^3} \frac{Z(Z-1)}{2} \langle |F|^2\rangle$$

where each factor results from each of the steps outlined above. To simplify this expression further, we note that $g^2 = 32\, G_F M_W^2$, and the average of the muon wave function is given by (12)

[14] $$\langle \phi_\mu^2 \rangle = \frac{m_\mu^3 \alpha^3 Z_{eff}^4}{\pi Z} .$$

Our final form for the conversion rate becomes

[15] $$\Gamma = \frac{G_F^3 g_{\mu e}^2}{72(2\pi)^3} \frac{M_A^6}{M^4} m_\mu^5 \frac{\alpha^3 Z_{eff}^4}{\pi Z} \left(c_V^2 + 3C_A^2\right)^2 Z(Z+1) \langle |F|^2 \rangle .$$

For the case A = 60, Z = 30 the procedure outlined in [12] yields $\langle |F|^2 \rangle = 1.5 \times 10^{-3}$. Taking $g_{\mu e} = 10^{-3}$, the analysis of (7) yields $M > 22$ GeV. The value of the other constants appearing in [15] were given earlier in the text. The conversion rate can now be compared to the ordinary muon capture rate, which is given by (12)

[16] $$\Gamma_o = \frac{3G_F^2 \alpha^3}{\pi^2} m_\mu^5 Z_{eff}^4 f(A,Z) .$$

f(A,Z) is a correction due to the two-nucleon correlation, and is ~ 0.13 for the case A = 2Z. A comparison of [15] and [16] yields

[17] $$\Gamma/\Gamma_o \lesssim 4 \times 10^{-20}.$$

To put this result in perspective, we examine the rates predicted by earlier calculations: In (2) the assumption was made that a Majoron could be emitted simultaneously with the e^+. The calculation assumed the exchange of a Majorana neutrino of mass 5 MeV and a coupling constant between neutrino and Majoron of order ~ 1. The calculated rate was $\Gamma_1/\Gamma_0 = 2 \times 10^{-21}$. Using a smaller coupling constant (as suggested by (3, 5)) would lower the rate by three orders of magnitude. (2) also presents a result for the rate due to the exchange of light neutrino exchange without Majoron emission. In that case $\Gamma_2/\Gamma_0 = 5 \times 10^{-21}$. In (9) the approach was based on the exchange of a very heavy Majorana neutrino (cosmological arguments put a lower bound of ~ 10 GeV). In that case, the predicted branching ratio is $\Gamma_3/\Gamma_0 = 3 \times 10^{-21}$.

By comparing [17] to the rates discussed in the pargraph above we conclude that, even though the prediction is substantially lower than present experimental limits, $\mu^- \to e^+$ conversion via a Gelmini-Roncadelli doubly charged Higgs boson cannot be ruled out as a possible contributor to the total rate.

A final comments on the model used for our calculation is in order. Doubly charged Higgs scalars also appear naturally in left-right symmetric models (13) which are based on a $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group, rather than the standard $SU(2) \times U(1)$ considered here. Several calculations employing the features of the left-right symmetric models have appeared in the literature (11, 14, 15, 16). Characteristic of these models is that the right-handed gauge boson masses are considerably larger, thereby resulting in lower rates in spite of the possibly stronger couplings. We have redone our calculation using the doubly charged Higgs scalar and the parameters of the left-right symmetric models, and have obtained $\Gamma'/\Gamma_0 = 2 \times 10^{-22}$, two orders of magnitude smaller than [17].

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FIGURE CAPTION

1. Mechanism for $\mu^- \to e^+$ conversion via doubly charged Higgs scalar.