TRI-PP-82-52
Dec 1982

# NEUTRINO CHARGE IN THE NON-LINEAR $R_\xi$ GAUGE

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Abstract

We show that the electromagnetic Ward identity for the charged W boson is satisfied in the non-linear $R_\xi$ gauge. Consequently the one-loop contributions to the neutrino charge give zero, which they do not in the conventional $R_\xi$ gauge.

(Submitted to Physics Letters B)

December 1982

In the standard electroweak model in the $R_\xi$ gauge, it is conventional [1] to use the following gauge fixing term for the charged vector boson:

$$L_{fix}^{old} = -\xi \left| \partial_\mu W^{+\mu} + i \frac{M}{\xi} s^+ \right|^2 \tag{1}$$

where $\xi$ is the gauge parameter and $s^+$ the ghost scalar field which accompanies the $W^-$. Recently the computational and conceptual advantages of the non-linear $R_\xi$ gauge condition introduced by Fujikawa [2] and Bacé and Hari Dass [3] have been emphasized by Gavela *et al.* [4]. With this non-linear gauge condition the ordinary derivative in eq. (1) is simply replaced by an electromagnetic covariant derivative and the new gauge fixing term is:

$$L_{fix}^{new} = -\xi \left| (\partial_\mu - ieA_\mu) W^{+\mu} + i \frac{M}{\xi} s^+ \right|^2 \quad . \tag{2}$$

Here $A_\mu$ is the photon field.

Although this new term changes the Feynman rules (in fact those for $W^-$ photon couplings become explicitly $\xi$ dependent!), the great advantage is the preservation of the electromagnetic U(1) symmetry and hence of the electromagnetic Ward identities. The conventional gauge fixing term in eq. (1) breaks the U(1) symmetry and so the Ward identities are lost [3]. Gavela *et al.* [4] have discovered the great simplification achieved in the one-loop calculation of Higgs $\to 2\gamma$, and also the elegance of the charged Faddeev-Popov ghost scalar couplings with this new gauge condition.

In this letter we wish to point out a further amusing consequence of the non-linear gauge condition. We find that one-loop corrections give zero contribution to the neutrino charge, whereas using the conventional $R_\xi$ gauge the sum of such contributions is divergent and certainly not zero. Usually this divergence is absorbed by a neutrino charge counter term (see Sakakibara [5]). In the non-linear gauge this is unnecessary.

We show that the electromagnetic Ward identity for the $W^-$ is satisfied in the non-linear gauge, whereas it is not in the usual $R_\xi$ gauge. For an electron propagator the Ward identity is

$$-e \frac{\partial}{\partial p_\mu} (\not{p}-m)^{-1} = (\not{p}-m)^{-1}\, e\gamma_\mu (\not{p}-m)^{-1} \tag{3}$$

displaying the equivalence between zero momentum photon insertion, and the action of the derivative on the propagator. A similar identity holds for charged scalars. What happens for charged vector bosons?

In the $R_\xi$ gauge the $W^-$ propagator is $\xi$ dependent. It is therefore not surprising to discover that in order for the Ward identity to hold for arbitrary $\xi$, the $W^-$-photon couplings must also be $\xi$ dependent. In fact the required $\xi$ dependence is precisely of the form which results from imposition of the non-linear $R_\xi$ gauge. Only in the U gauge ($\xi = 0$), where the old and new couplings coincide, can the old coupling satisfy the Ward identity.

In the $R_\xi$ gauge the $W^-$ propagator is

$$P_{\alpha\beta} = -i\left[g_{\alpha\beta} + (\xi^{-1}-1)\frac{P_\alpha P_\beta}{(p^2-M^2\xi^{-1})}\right](p^2-M^2)^{-1} \; . \tag{4}$$

The $W^-W^-\gamma$ vertex (see fig. 1) in the non-linear $R_\xi$ gauge is [3,4]

$$+ie\Big[g_{\alpha\beta}(\ell+\ell')_\mu + (\xi-1)\ell_\beta\, g_{\alpha\mu} + (\xi-1)\ell'_\alpha\, g_{\beta\mu} + + (\xi+1)q_\beta\, g_{\alpha\mu} - (\xi+1)q_\alpha\, g_{\beta\mu}\Big] \; . \tag{5}$$

If we call the vertex resulting from zero momentum photon insertion in fig. 2 $V_{\mu\alpha\beta}$, then the required Ward identity analogous to eq. (3) is

$$-e \frac{\partial P_{\alpha\beta}}{\partial p_\mu} = V_{\mu\beta\alpha} \quad . \tag{6}$$

Taking the derivative of eq. (4) yields

$$\begin{aligned} -e \frac{\partial P_{\alpha\beta}}{\partial p_\mu} = & -ie(p^2-M^2)^{-2}\left[2g_{\alpha\beta}p_\mu + (\xi^{-1}-1)2p_\alpha p_\beta p_\mu (p^2-M^2\xi^{-1})\right] \\ & -ie(p^2-M^2)^{-1}(\xi^{-1}-1)^{-1}\Big[2p_\alpha p_\beta p_\mu (p^2-M^2\xi^{-1})^{-2} \\ & - (g_{\mu\alpha}p_\beta + g_{\mu\beta}p_\alpha)(p^2-M^2\xi^{-1})^{-1}\Big] \quad . \end{aligned} \tag{7}$$

On the other hand:

$$\begin{aligned} V_{\mu\alpha\beta} = & -ie\left[g_{\alpha\gamma} + (\xi^{-1}-1)p_\alpha p_\gamma (p^2-M^2\xi^{-1})^{-1}\right]\left[g_{\delta\beta} + (\xi^{-1}-1)p_\delta p_\beta (p^2-M^2\xi^{-1})^{-1}\right] \\ & \left[2p_\mu g_{\gamma\delta} + (\xi-1)(p_\gamma g_{\delta\mu} + p_\delta g_{\gamma\mu})\right] \times [p^2-M^2]^{-2} \quad . \end{aligned} \tag{8}$$

After some algebra, it can be shown that the right-hand sides of eqs. (7) and (8) are equivalent, thus establishing the Ward identity, eq. (6).

This identity can now be used to show the exact cancellation of the one-loop contributions to the neutrino charge.

There are two one-loop diagrams which contribute to the neutrino charge in the limit of zero photon momentum. These are given in fig. 3. The other possible one-loop diagrams with an internal Z can be shown to give zero contribution to the charge. Diagrams A and B in fig. 3 are derived from the neutrino self-energy diagrams of fig. 4 by the insertion of a zero momentum photon in the $W^-$ and e internal lines, respectively. In fig. 4 diagrams A and B are equivalent, the difference being merely the choice of internal loop momentum; these choices have been made to facilitate photon insertion via differentiation using eqs. (3) and (6). We shall handle divergences with dimensional regularization so that one may freely shift internal momenta. Let us denote the vertex terms of fig. 3 by $\Lambda$, and the self-energy terms of fig. 4 by $\Sigma$. Then the electromagnetic Ward identities give

$$\Lambda_A = +e \frac{\partial \Sigma_A}{\partial p_\mu} , \qquad \Lambda_B = -e \frac{\partial \Sigma_B}{\partial p_\mu} . \tag{9}$$

Hence

$$\Lambda_A + \Lambda_B = e \frac{\partial}{\partial p_\mu} (\Sigma_A - \Sigma_B) = 0$$

since $\Sigma_A = \Sigma_B$ under momentum shift, and so the contribution to the neutrino charge is zero. As long as eqs. (3) and (6) are satisfied, this result is valid equally for $W^-$ and $s^-$ diagrams. But the argument will break down when the Feynman rules of Fujikawa, Lee and Sanda [1] are used, for two reasons: (1) the Ward identity eq. (6) does not hold (except in the U gauge); (2) there now exists a $W^- s^- \gamma$ coupling, which did not exist in the non-linear gauge, giving rise to graphs that cannot be derived from self-energy loops.

To confirm these results, we have explicitly calculated the divergent part of the one-loop diagrams in the old $R_\xi$ gauge (for $\xi = 1$) and in the non-linear $R_\xi$ gauge. For the old $R_\xi$ gauge one gets for the divergent part of the coefficient of $\bar{\nu}\gamma_\mu \frac{(1-\gamma_5)}{2} \nu$, the following:

$$\begin{aligned} A_{div.} &= \frac{+ieg^2}{16\pi^2} \Gamma(2-n/2)(3/2) \\ B_{div.} &= \frac{-ieg^2}{16\pi^2} \Gamma(2-n/2)(1/2) \end{aligned} \tag{10}$$

and graphs with $W^- s^- \gamma$ coupling are finite. $\Gamma$ is the gamma function, divergent at $n = 4$, g is the semi-weak coupling constant. The sum of A and B is divergent and *not* zero. For the non-linear gauge one finds instead;

$$A_{div.} = \frac{+ieg^2}{16\pi^2}\,\Gamma(2-n/2)(2\xi)^{-1}$$
$$B_{div.} = \frac{-ieg^2}{16\pi^2}\,\Gamma(2-n/2)(2\xi)^{-1} \quad . \qquad (11)$$

Here the sum *is* zero for *all* values of the gauge parameter $\xi$. In the Landau gauge ($\xi \to \infty$) the divergence disappears. In the U gauge ($\xi \to 0$) the divergence becomes *more* singular, but still there is exact cancellation, as first observed by Bardeen *et al.* [6].

As usual, the finite parts of the divergent graphs are determined only up to an arbitrary constant. Therefore the Ward identity must be used to enforce exact cancellation of the finite parts at $q^2 = 0$.

Finally we remark on yet another amusing feature of the non-linear gauge. It was pointed out in [4] that, since the charged Faddeev-Popov ghost scalars now also must satisfy the electromagnetic Ward identity, they must couple in the conventional way for charged scalars (they did *not* in the old $R_\xi$ gauge). A direct consequence is that the charged scalar ghost ($s^-$) and the charged Faddeev-Popov ghost one-loop contributions to the photon self-energy precisely cancel, leaving only the $W^-$ loop contributions from the boson sector! (Recall that Faddeev-Popov loops acquire an extra minus sign.) A calculation is in progress of the $W^-$ loop contribution in the general non-linear $R_\xi$ gauge. It will be interesting to see if there are some special, simplifying features which emerge, on account of the U(1) symmetry of the non-linear $R_\xi$ gauge.

## References


[1] K. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D6 (1972) 2923.

[2] K. Fujikawa, Phys. Rev. D7 (1973) 393.

[3] M. Bacé and N.D. Hari Dass, Ann. Phys. 94 (1975) 349.

[4] M. Gavela, G. Girardi, C. Malleville and P. Sorba, Nucl. Phys. B193 (1981) 257.

[5] S. Sakakibara, Phys. Rev. D24 (1981) 1149.

[6] W. Bardeen, R. Gastmans and B. Lautrup, Nucl. Phys. B46 (1972) 319.


## Figure Captions

1. The $W^-W^-\gamma$ vertex.
2. The Ward identity for $W^-$ propagator
3. One-loop contributions to neutrino charge.
4. Neutrino self-energy diagram; derivatives of these yield the diagrams of fig. 3.

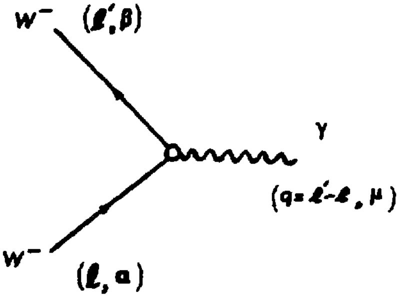


Fig. 1

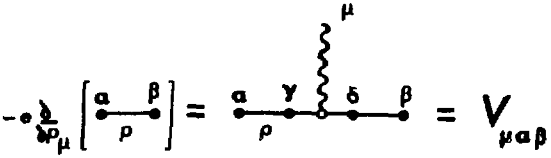


Fig. 2

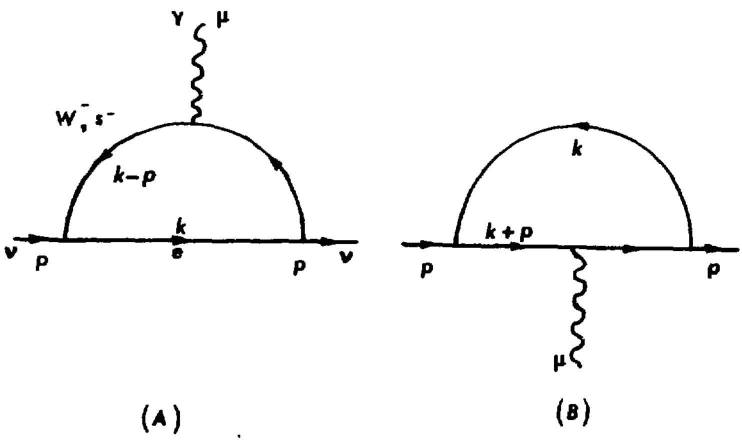


Fig. 3

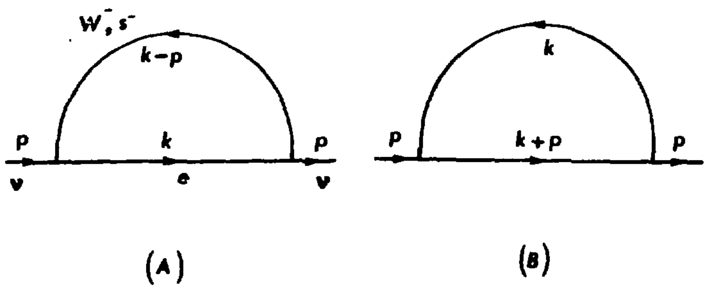


Fig. 4