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The pion pole term in electroproduction of off-mass-shell pions

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Abstract

The dependence of the invariant amplitudes for electroproduction of off-mass-shell pions on the pion Born term is investigated when Current Algebra Ward identities and PCAC are used to determine pion electroproduction invariant amplitudes. We show that an amplitude satisfying the Ward identities can be constructed starting from the usual Born terms which do not satisfy them and that this same amplitude will be obtained for a large class of input Born terms.

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I. INTRODUCTION

In deriving any amplitude by the method of Ward identities, for example the πN analysis of Scadron and Thebaud[1,2] or the pion electroproduction analyses of MacMullen and Scadron (MS),[3] Weisberger,[4] and Dombey and Reid,[5] the Born pole terms are the basic ingredient of the analysis. Indeed they are expected to dominate the amplitude with background contributions coming from the Ward identity constraints. The physical origin of the background terms from Ward identity constraints is the summation of isobar and other resonance terms contributing to the process. That this is so can be verified by observing that the $\Delta(1230)$ contributes about 110% of the Fubini-Furlan-Rossetti[6] background term in pion photoproduction as shown by Adler and Gilman[7] and MS.[8] Nevertheless, the important feature of the Ward identity analysis is that the leading order background terms are obtained in a model independent way and the explicit isobar models are required only for the construction of the next to leading order terms.

In the pion electroproduction analysis, gauge invariance provides a constraint on the amplitude, however, as noted by several authors — Fubini, Nambu and Wataghin[9] were the first — the Born pole terms are not gauge invariant. This led various authors to modify the Born term to make them satisfy the gauge condition. One such modification is used in the MS analysis. In this work we study in detail the effect of this modification of the Born terms on the electroproduction amplitude. We are especially interested in extrapolations off the pion mass shell where modifications of the type made by MS may be important.

In Section II we define our notation which follows MS as much as possible for easy comparison. In Section III we derive the complete off-shell electroproduction amplitude using the standard Born terms, that is, without the modification made by MS. In Section IV we discuss the effect of modifying the Born terms.

From our analysis we find that for a certain class of modifications of the Born terms the complete electroproduction amplitude remains unchanged, although the decomposition into the pole amplitude and non pole amplitude is different. The MS modification is a member of this class of transformations so that the complete amplitude derived by MS is in agreement with our results based on the unmodified Born term.

II. NOTATION, KINEMATICS AND BORN TERMS

The S-matrix element for the electroproduction process illustrated in Fig. 1:

$$S_{fi} = \delta_{fi} + ei(2\pi)^4\delta^4(p'+q-p-k)\pi^i(q)T^i_\nu\ \varepsilon^\nu(\vec{k}) \ . \qquad (2.1)$$

Using the standard reduction technique,

$$T^i_\nu = -i\int dx^4\ e^{iq\cdot x}(\Box+\mu^2)\langle N'|T\{\phi^i_\pi(x),V^{em}_\nu(o)\}|N\rangle$$
$$= \overline{N}(\vec{p}\,')M_\nu N(\vec{p}) \ . \qquad (2.2)$$

The M-amplitude can be decomposed into various isospin components,

$$M^i_\nu = M^+_\nu\delta^{i3} + M^-_\nu i\varepsilon^{i3k}\tau^k + M^o_\nu\tau^i \ . \qquad (2.3)$$

For brevity we denote $I^i_+ = \delta^{i3}$, $I^i_- = i\varepsilon^{i3k}\tau^k$, $I^i_o = \tau^i$. Each of these can in turn be written in terms of amplitudes which are free of kinematic zeros and kinematic singularities,

$$M^i_\nu = \sum_{j=1}^{6} A^i_j(\nu,t,k^2,q^2)K^j_\nu \qquad (2.4)$$

where K^j_ν are the six gauge invariant covariants of Fubini, Nambu and Wataghin (FNW) with

$$K^1_\nu = \frac{1}{2}[\not{K}\ ,\gamma_\nu]\gamma_5 \qquad (2.5a)$$

$$K^2_\nu = 2(k\cdot qP_\nu - k\cdot Pq_\nu)\gamma_5 \qquad (2.5b)$$

$$K_\nu^3 = (k\cdot q\gamma_\nu - \not{k}\, q_\nu)\gamma_5 \quad (2.5c)$$

$$K_\nu^4 = 2(k\, P\gamma_\nu - \not{k}\, P_\nu)\gamma_5 - 2mK_\nu^1 \quad (2.5d)$$

$$K_\nu^5 = (k\cdot q k_\nu - k^2 q_\nu)\gamma_5 \quad (2.5e)$$

$$K_\nu^6 = (\not{k}\, k_\nu - k^2\gamma_\nu)\gamma_5 \ . \quad (2.5f)$$

We use the notation

$$P = \frac{1}{2}(p'+p), \quad \Delta = p'-p = k-q, \quad t = \Delta^2, \quad \nu = k\cdot P = q\cdot P = \frac{s-u}{4}$$

and m = nucleon mass with μ = pion mass.

The πNN, γNN and $\gamma\pi\pi$ vertices and form factors are defined by

$$\Gamma_{\pi NN} = -ig(q^2)\gamma_5\tau^i \ , \quad (2.6)$$

$$\Gamma_\nu^{NN\gamma} = \frac{1}{2}[F_1^S(k^2)+\tau^3F_1^V(k^2)]\gamma_\nu + \frac{1}{2}[F_2^S(k^2)+\tau^3F_2^V(k^2)]\frac{i\sigma_{\nu\lambda}k^\lambda}{2m} \quad (2.7)$$

and

$$\Gamma_\nu^{\pi\pi\gamma} = i\epsilon^{i3k}(2q_\nu - k_\nu)F_\pi(k^2)\tau^k \ . \quad (2.8)$$

The axial-vector nucleon vertex is given by

$$\Gamma_\mu^{iANN} = \frac{1}{2}\ \tau^i i\,[g_A(q^2)\gamma_\mu\gamma_5 - h_A(q^2)q_\mu\gamma_5] \ . \quad (2.9)$$

Explicity isolating the pion-pole, (2.9) may be written as

$$\Gamma_\mu^{iANN} = \frac{1}{2}\ \tau^i i\left[f_\pi g(q^2)\left(\frac{\gamma_\mu}{m} + \frac{2q_\mu}{q^2-\mu^2}\right) - \frac{\bar{h}_A(q^2)}{2m}(\gamma_\mu q^2 - \not{q} q_\mu)\right]\gamma_5 \quad (2.10)$$

with

$$h_A(q^2) = -2f_\pi g(q^2)(q^2-\mu^2)^{-1} + \bar{h}_A(q^2) \ . \quad (2.11)$$

The generalized Goldberger-Treiman relation is then

$$mg_A(q^2) + \frac{1}{2}\, q^2\bar{h}_A(q^2) = f_\pi g(q^2) \ . \quad (2.12)$$

With the vertices and form factors defined in (2.6), (2.7) and (2.8) the electroproduction Born terms can be written in terms of the gauge invariant FNW covariants as

$$M_\nu^{(+,0)P} = \frac{-g(q^2)}{(s-m^2)(u-m^2)}\left[F_1^{V,S}(k^2)(k\cdot q\ K_\nu^1 + K_\nu^2) + \frac{F_2^{V,S}(k^2)}{2m}(2\nu K_\nu^3 - k\cdot qK_\nu^4)\right] \quad (2.13)$$

$$M_\nu^{(-)P} = \frac{g(q^2)}{(s-m^2)(u-m^2)}\left[2\nu F_1^V(k^2)\left(K_\nu^1 + \frac{1}{k\cdot q}\ K_\nu^2\right) + \frac{F_2^V(k^2)}{2m}(k\cdot qK_\nu^3 - 2\nu K_\nu^4)\right] +$$
$$+ \frac{g(q^2)F_1^V(k^2)}{k\cdot q(t-q^2)}K_\nu^5 + \left[\frac{g(q^2)F_1^V(k^2)}{t-q^2} - \frac{F_\pi(k^2)g(t)}{t-\mu^2}\right](2q_\nu - k_\nu)\gamma_5 \ . \quad (2.14)$$

Notice that the last term is not gauge invariant. This lack of gauge invariance of the Born term is precisely what causes the difficulties we described in the Introduction—how to construct a complete amplitude which is gauge invariant and satisfies the Ward identities? In the next section we attack the problem directly, without introducing any modification to the Born terms of Eqs. (2.13) and (2.14). This contrasts with the approach of MacMullen and Scadron, who modified the final term in (2.14) replacing $g(t)(t-\mu^2)^{-1}$ with $g(q^2)(t-q^2)^{-1}$.

III. THE COMPLETE ELECTROPRODUCTION AMPLITUDE

We begin by defining the axial vector-vector two current amplitude

$$M^i_{\mu\nu} = i\int dx^4\ e^{iq\cdot x}\ T\{A^i_\mu(x), V^{em}_\nu(o)\} \tag{3.1}$$

and the divergence amplitude $\tilde{M}^i_\nu$, which is directly related to the π electroproduction amplitude M^i_ν, by PCAC

$$\tilde{M}^i_\nu = i\int dx^4\ e^{iq\cdot x}\ T\{\partial\cdot A^i(x), V^{em}_\nu(o)\} \tag{3.2a}$$

$$M^i_\nu = \frac{q^2-\mu^2}{f_\pi\mu^2}\,\tilde{M}^i_\nu\ . \tag{3.2b}$$

The Ward identities satisfied by the amplitude (3.1) are

$$M^i_{\mu\nu}\,k^\nu = -\frac{i}{2}\,iI^i_-[g_A(t)\gamma_\mu\gamma_5+h_A(t)\Delta_\mu\gamma_5] \tag{3.3}$$

$$q^\mu M^i_{\mu\nu} = -\frac{1}{2}\,iI^i_-[g_A(t)\gamma_\nu\gamma_5+h_A(t)\Delta_\nu\gamma_5] + i\tilde{M}^i_\nu \tag{3.4}$$

The full two current amplitude (3.1) can be decomposed into a nucleon pole contribution, $M^{Ni}_{\mu\nu}$ and the remainder $M^{i\,\prime}_{\mu\nu}$ which does not have nucleon poles.

$$M^i_{\mu\nu} = M^{iN}_{\mu\nu} + M^{i\,\prime}_{\mu\nu} \tag{3.5}$$

The substitution of (3.5) in the Ward identity (3.3) determines $M^{i\,\prime}_{\mu\nu}$ up to a gauge invariant term, $R^i_{\mu\nu}$ (see Fig. 2)

$$M^{i\,\prime}_{\mu\nu} = \frac{1}{2}\,iI^i_-[F^V_1(k^2)G(t,q^2)\gamma_\mu(2q_\nu-k_\nu)+g_A(t)\mathcal{F}_V(k^2)\gamma_\mu k_\nu-F^V_1(k^2)H(t,q^2)\ \times$$

$$\times\ q_\mu(2q_\nu-k_\nu)-h_A(t)\mathcal{F}_V(k^2)q_\mu k_\nu-h_A(t)g_{\mu\nu}]\gamma_5 + R^i_{\mu\nu} \tag{3.6}$$

where

$$G(t,q^2) = \frac{g_A(t)-g_A(q^2)}{t-q^2} \qquad (3.7)$$

$$H(t,q^2) = \frac{h_A(t)-h_A(q^2)}{t-q^2} \qquad (3.8)$$

and

$$\mathcal{F}_V(k^2) = \frac{F_1^V(k^2)-1}{k^2} . \qquad (3.9)$$

The gauge invariant term, $R^i_{\mu\nu}$, will contain the gauge invariant parts of the t-channel pion pole terms (a) and (b) in Fig. 3 which we parameterize in terms of an amplitude C^i_ν, as well as the background pion electroproduction amplitude $\overline{M}^i_\nu$ (c), and a background two current amplitude $\overline{M}_{\mu\nu}$, (d). This can be written

$$R^i_{\mu\nu} = \frac{if_\pi q_\mu}{q^2-\mu^2} (\overline{M}^i_\nu + C^i_\nu) + \overline{M}^i_{\mu\nu} , \qquad (3.10)$$

where

$$M^i_\nu = M^{Pi}_\nu + \overline{M}^i_\nu ,$$

and M^{Pi}_ν are the Born pole terms in the electroproduction amplitude (2.13-2.14). Note that $\overline{M}^i_{\mu\nu}k^\nu = 0$ and that $\overline{M}^i_{\mu\nu}$ contains neither Born u, s or t channel poles nor pion poles $(q^2-\mu^2)^{-1}$. The term depicted in Fig. 3(b) is incorporated in C^i_ν by allowing a term in C^i_ν which has a zero at $q^2=\mu^2$ but a pole at $t=\mu^2$.

Even with the aid of the decomposition of $R^i_{\mu\nu}$ shown in Fig. 3, C^i_ν cannot be determined completely because of the unknown axial vector vertex in (b) of Fig. 3. However, C^i_ν can be determined up to a gauge invariant term by (i) the gauge condition $R_{\mu\nu}k^\nu = 0$, which implies

$$k^\nu(M^i_\nu - M^{iP}_\nu + C^i_\nu) = 0 , \qquad (3.11)$$

and (ii) the Ward identity

$$k^\nu M^i_\nu = -\frac{1}{2} I^i_- \frac{q^2-\mu^2}{f_\pi \mu^2}[2mg_A(t)+th_A(t)]\gamma_5 . \qquad (3.12)$$

Thus we can write,

$$C^i_\nu = I^i_- \frac{(2q_\nu-k_\nu)}{t-q^2} g(q^2)F^V_1(k^2)\gamma_5 - I^i_- \frac{(2q_\nu-k_\nu)}{t-\mu^2}\gamma_5 g(t)F_\pi(k^2) - $$
$$- \frac{q^2-\mu^2}{f_\pi\mu^2} Y^i_\nu , \qquad (3.13)$$

where Y^i_ν is defined by its divergence,

$$Y^i_\nu k^\nu = \frac{1}{2} I^i_- [2mg_A(t)+th_A(t)]\gamma_5 \ . \qquad (3.14)$$

The factor $q^2-\mu^2$ in (3.13) multiplying Y^i_ν is just the statement that there is no pion pole in the axial current of Fig. 3(b). This is consistent with the pion pole being explicitly shown in Fig. 3(a) and with Fig. 3(a) having no gauge invariant contribution. We expect Y^i_ν to have a t-channel pion pole dependence.

After a little algebra, the background electroproduction amplitude can be extracted from the right hand side of (3.10) and becomes

$$\begin{aligned} i f_\pi \overline{M}^i_\nu = \frac{1}{2}\, i I^i_- \Big\{ & F^V_1(k^2) G(t,q^2)\,[K^6_\nu + 2K^3_\nu] - g_A(t)\mathcal{F}_V(k^2) K^6_\nu + \\ & + [F^V_1(k^2)(2mG(t,q^2)+q^2\overline{H}(t,q^2)) + \overline{h}_A(t)](2q_\nu - k_\nu)\gamma_5 + \\ & + [2mg_A(t) + q^2\overline{h}_A(t)]\mathcal{F}_V(k^2) k_\nu \gamma_5 - \\ & - \frac{2f_\pi g(t) F^V_1(k^2) q^2}{(t-q^2)(t-\mu^2)} (2q_\nu - k_\nu)\gamma_5 - \frac{2f_\pi g(t)}{(t-\mu^2)} (2q_\nu - k_\nu)\gamma_5 - \\ & - \frac{2f_\pi g(t)\mathcal{F}_V(k^2) q^2}{t-\mu^2} k_\nu\gamma_5 + \frac{2f_\pi g(t) F_\pi(k^2)}{t-\mu^2} (2q_\nu - k_\nu)\gamma_5 \Big\} - \\ & - \frac{i}{4m} g_A(q^2)\,[F^S_1(k^2) I^i_0 + F^V_2(k^2) I^i_+] K^1_\nu + \frac{iq^2}{\mu^2} Y^i_\nu - q^\mu \overline{M}^i_{\mu\nu} \ . \end{aligned} \qquad (3.15)$$

All that remains is to use the fact that the background amplitudes $\overline{M}^i_\nu$ and $\overline{M}^i_{\mu\nu}$ have no t-channel poles on the pion mass shell. This allows determination of the pole part of Y^i_ν, which we call Y^{Pi}_ν, since Y^{Pi}_ν must cancel the poles in $\overline{M}^i_\nu$.

Hence

$$\begin{aligned} Y^{Pi}_\nu = I^i_- \frac{f_\pi \mu^2}{q^2} \Big\{ & \Big[\frac{g(t) F^V_1(k^2) q^2}{(t-\mu^2)(t-q^2)} + \frac{g(t)}{t-\mu^2} - \frac{g(t) F_\pi(k^2)}{t-\mu^2} \Big] (2q_\nu - k_\nu) + \\ & + \frac{g(t)\mathcal{F}_V(k^2) q^2}{t-\mu^2} k_\nu \gamma_5 \Big\} . \end{aligned} \qquad (3.16)$$

Note that Y^{Pi}_ν has the t-channel pion pole which we expected.

Next we divide the background part of Y^i_ν into a gauge invariant part $\overline{Y}^i_{GI;\nu}$ and a non-gauge invariant part, $\overline{Y}^i_{NGI;\nu}$ so that

$$\gamma^i_\nu = \gamma^{Pi}_\nu + \overline{\gamma}^i_{GI;\nu} + \overline{\gamma}^i_{NGI;\nu} \tag{3.17}$$

and using Eq. (3.12) yields

$$\overline{\gamma}^i_{NGI;\nu}k^\nu = \frac{1}{2} I^i_- \frac{\mu^2}{q^2}(1-F_\pi(k^2))\Big\{[2mG(t,q^2)+q^2\overline{H}(t,q^2)+\overline{h}_A(t)](k^2-2q\cdot k)\gamma_5 + + 2f_\pi g(q^2)k^2\gamma_5 + \frac{q^2-\mu^2}{\mu^2}[2mg_A(t)+th_A(t)]\gamma_5\Big\} \tag{3.18}$$

Hence

$$\overline{\gamma}^i_{NGI;\nu} = -\frac{1}{2} I^i_- \frac{\mu^2}{q^2}(1-F_\pi(k^2))\Big\{[2mG(t,q^2)+q^2\overline{H}(t,q^2)+\overline{h}_A(t)](2q_\nu-k_\nu)\gamma_5 - - 2f_\pi g(q^2)k_\nu\gamma_5 + (2q_\nu-k_\nu)\frac{q^2-\mu^2}{\mu^2}\left[\frac{2mg_A(t)+th_A(t)}{t-q^2}\right]\gamma_5\Big\} . \tag{3.19}$$

Since by definition $\overline{\gamma}^i_{NGI;\nu}$ contains no gauge invariant terms, (3.19) is unique.

Finally, the background electroproduction amplitude is

$$\begin{aligned} f_\pi\overline{M}^i_\nu = \frac{1}{2} I^i_-\Big\{&2f_\pi g(q^2)\mathcal{F}(k^2)k_\nu\gamma_5 + F^V_1(k^2)G(t,q^2)[2K^3_\nu+K^6_\nu] - \\ &- g_A(t)\mathcal{F}_V(k^2)K^6_\nu - 2\mathcal{F}_V(k^2)[2mG(t,q^2)+q^2\overline{H}(t,q^2)]K^5_\nu\Big\} - \\ &- \frac{1}{4m} g_A(q^2)[F^S_2(k^2)I^i_0+F^V_2(k^2)I^i_+]K^1_\nu + \frac{q^2}{\mu^2}\overline{\gamma}^i_{GI\,\nu} + iq^\mu\overline{M}^i_{\mu\nu} - \\ &- \frac{1}{2} I^i_- \frac{(q^2-\mu^2)}{\mu^2(t-q^2)}[2mg_A(t)+th_A(t)](2q_\nu-k_\nu)\gamma_5(1-F_\pi(k^2)) + \\ &+ \frac{1}{2} I^i_- F_\pi(k^2)\Big\{2mG(t,q^2)+q^2\overline{H}(t,q^2)+\overline{h}_A(t)\Big\}(2q_\nu-k_\nu)\gamma_5 \end{aligned} \tag{3.20}$$

where

$$\mathcal{F}(k^2) = \frac{F_V(k^2)-F_\pi(k^2)}{k^2} .$$

Adding the background terms and the pole terms yields our complete electroproduction amplitude which satisfies the gauge condition.

$$\begin{aligned} M^i_\nu = \frac{g(q^2)}{(s-m^2)(u-m^2)}\Big\{&\Big[-(F^S_1(k^2)I^i_0+F^V_1(k^2)I^i_+) + \frac{2\nu}{k\cdot q} F^V_1(k^2)I^i_-\Big](k\cdot qK^1_\nu+K^2_\nu) - \\ &- \frac{1}{2m}[F^S_2(k^2)I^i_0+F^V_2(k^2)I^i_+](2\nu K^3_\nu-k\cdot qK^4_\nu) + \frac{1}{2m} F^V_2(k^2)I^i_-(k\cdot qK^3_\nu-2\nu K^4_\nu)\Big\} + \\ &+ \frac{g(q^2)I^i_-}{t-q^2}\Big[\frac{1}{k\cdot q} F^V_1(k^2)-2\mathcal{F}(k^2)\Big]K^5_\nu - \frac{g_A(q2)}{4mf_\pi}[F^S_2(k^2)I^i_0+F^V_2(k^2)I^i_+]K^1_\nu + \\ &+ f_\pi^{-1} I^i_-\Big\{F^V_1(k^2)G(t,q^2)K^3_\nu-\mathcal{F}_V(k^2)[2mG(t,q^2)+q^2\overline{H}(t,q^2)]K^5_\nu + \\ &+ \frac{1}{2}[F^V_1(k^2)G(t,q^2)-g_A(t)\mathcal{F}_V(k^2)]K^6_\nu\Big\} + \frac{q^2}{f_\pi\mu^2}\overline{\gamma}^i_{GI\,\nu} + f_\pi^{-1}\, iq^\mu\overline{M}^i_{\mu\nu} - \\ &- \frac{1}{2} I^i_- \frac{q^2-\mu^2}{f_\pi\mu^2}\, \frac{2mg_A(t)+th_A(t)}{t-q^2}\,(2q_\nu-k_\nu)\gamma_5 . \end{aligned} \tag{3.21}$$

It is remarkable that (3.21) contains *no* pole terms at $t=\mu^2$, although the Born term with which we began in (2.14) did contain such a pole. All of the explicit t-channel poles in (3.21) are at $t=q^2$ in spite of the fact that we did not continue any internal pions off their mass shell. In fact the total electroproduction amplitude derived here agrees with the previous analysis by MacMullen and Scadron even off the pion mass shell and even though MacMullen and Scadron began with different pole terms. The reason for this similarity is discussed in the next section. From (3.21) the standard 18 FNW electroproduction amplitudes can be read off. The chiral symmetry breaking terms $\overline{Y}^i_{GI;\nu}$ and the background amplitude $\overline{M}^i_{\mu\nu}$ are discussed in detail in Ref. 3, and we do not repeat their discussion here.

IV. DISCUSSION AND CONCLUSIONS

In spite of the fact that our Born amplitude (2.14) differs from that of MacMullen and Scadron, our final result (3.21) is identical with theirs. To understand how this happens we examine the difference between our pion pole term, $M^{i\pi}_\nu$, and that of MacMullen and Scadron $M^{i\pi}_{\nu,MS}$. From (2.14)

$$M^{i\pi}_\nu = -\frac{F_\pi(k^2)g(t)}{t-\mu^2}\,(2q_\nu-k_\nu)\gamma_5\; i\epsilon^{i3k}\tau_k\;, \tag{4.1}$$

while from Ref. 3,

$$M^{i\pi}_{\nu,MS} = -\frac{F_\pi(k^2)g(q^2)}{t-q^2}\,(2q_\nu-k_\nu)\gamma_5\; i\epsilon^{i3k}\tau_k\;. \tag{4.2}$$

A little algebra shows that

$$M^{i\pi}_{\nu,MS} - M^{i\pi}_\nu = +\left\{\frac{m}{f_\pi}\,G(t,q^2) - \frac{1}{2f_\pi}\,\overline{h}_A(q^2) - \frac{t}{2f_\pi}\,\overline{H}(t,q^2) + \frac{(q^2-\mu^2)g(t)}{(t-q^2)(t-\mu^2)}\right\}(2q_\nu-k_\nu)\gamma_5\;. \tag{4.3}$$

The essential feature of the right hand side of (4.3) is that it is *not gauge invariant* and has no poles on the pion mass shell. However, in Section III we were able to use the gauge condition (3.3) to uniquely fix the non-gauge invariant term in the amplitude. Adding the right hand side of (4.3)—or any non-gauge invariant amplitude—to the Born terms (2.13) and (2.14) simply generates a compensating term in the non-gauge invariant background amplitude to ensure that (3.3) is satisfied.

Thus we see that the final result (3.21) is invariant to transformations which alter the input Born terms (2.13) and (2.14) by the addition of a non-gauge invariant term.

However, adding a totally arbitrary non-gauge invariant term would permit us to alter the physical meaning of the expression "Born terms". While it is not necessary to preserve the final result it is appropriate that we at least demand that the terms added to (2.13) and (2.14) introduce no additional poles into the on-mass-shell amplitude. The right hand side of (4.3) satisfies this condition and hence does not introduce "unphysical poles" into the Born amplitudes. It is clear from our result that, when the external pion is continued off-mass-shell, it is no longer possible to uniquely specify the poles in the Born amplitude, but that the complete off-shell amplitude (3.21) is still unambiguously defined.

In conclusion, we have shown that the final result of MacMullen and and Scadron, our equation (3.21), does not depend on their particular choice of Born term. A Born term with a pion pole at $t=\mu^2$—a choice we would regard as more reasonable physically—also produces the same result. The final complete amplitude is independent of the initial choice of off-mass-shell extrapolation of the pion. It can therefore be used to extrapolate reliably off pion mass shell, and we would encourage its use to develop the electroproduction analog of the PCAC off-shell expansion which was successfully applied to πN scattering.[2]

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FIGURE CAPTIONS

Fig. 1. The electroproduction amplitude: N(p), N′(p′), $\pi^i(q)$, $V_\nu^{em}(k)$ represent the incoming nucleon, outgoing nucleon, the isotopic pion and the electromagnetic current respectively with 4-momenta p, p′, q and k.

Fig. 2. Diagrammatic representation of $R^i_{\mu\nu}$ in terms of the gauge in-invariant (GI) parts of the complete two current amplitude $M^i_{\mu\nu}$ and the nucleon contribution $M^{iN}_{\mu\nu}$.

Fig. 3. Diagrammatic decomposition of $R^i_{\mu\nu}$ in terms of the gauge invariant (GI) parts of the t-channel pion poles (a) and (b), the electroproduction background amplitude (c), and the background two current amplitude (d).

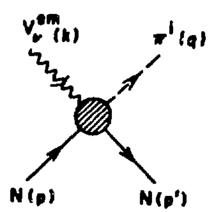

Fig. 1

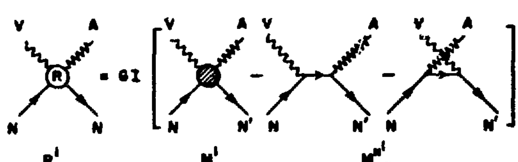

Fig. 2

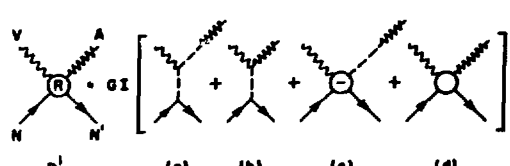

Fig. 3