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ATOMIC ENERGY OF CANADA LIMITED

L'ÉNERGIE ATOMIQUE DU CANADA LIMITÉE

MULTIVARIABLE CONTROL IN NUCLEAR POWER STATIONS: OPTIMAL CONTROL

Contrôle multivariable dans les centrales nucléaires: Contrôle optimal

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Chalk River Nuclear Laboratories — Laboratoires nucléaires de Chalk River

Chalk River, Ontario

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Résumé

Les méthodes multivariables offrent la possibilité d'améliorer le contrôle des centrales nucléaires et autres grandes installations. Le contrôle optimal linéaire-quadratique est une méthode multivariable fondée sur la minimisation d'une fonction de coût. Une technique connexe fait appel au filtre Kalman pour l'estimation de l'état d'une centrale à partir des mesures de bruit.

On a développé un programme conceptuel pour le contrôle optimal et le filtrage Kalman faisant partie d'un ensemble de conception aidée par ordinateur pour les systèmes de contrôle multivariable. On décrit la démonstration de la méthode faite sur une maquette de générateur de vapeur de centrale nucléaire et l'on donne les résultats simulés.

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ABSTRACT

Multivariable methods have the potential to improve the control of large systems such as nuclear power stations. Linear-quadratic optimal control is a multivariable method based on the minimization of a cost function. A related technique leads to the Kalman filter for estimation of plant state from noisy measurements.

A design program for optimal control and Kalman filtering has been developed as part of a computer-aided design package for multivariable control systems. The method is demonstrated on a model of a nuclear steam generator, and simulated results are presented.

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NOMENCLATURE

In general, capital letters represent matrices, while lower-case letters are used for vectors and scalars. Following modern practice, no additional marking is used to identify vectors or matrices. Such distinction will be clear from the context.

Symbol	Description	Defining Equation
A	Plant dynamics matrix	(1)
A_r	Reduced model dynamics matrix	(27)
B	Plant input matrix	(1)
B_r	Reduced model input matrix	(27)
C	Plant output matrix	(2)
C_m	Augmented system output matrix	(32)
C_r	Reduced model output matrix	(27)
D_r	Reduced model input-output direct coupling matrix	(27)
F	Output terminal cost matrix	(3)
H	Kalman filter gain matrix	(19)
H_r	Reduced Kalman filter gain matrix	(27)
I_r	Identity matrix (rxr)	(32)
J	Quadratic cost functional	(3)
K	Optimal controller state-feedback matrix	(4)
M	State-costate equation matrix	(8)
P	Riccati matrix-differential equation solution	(5)

NOMENCLATURE (continued)

Symbol	Description	Defining Equation
P_o	Initial state covariance matrix	(21)
Q	Output cost matrix	(3)
r	Order of the reduced model	(27)
R	Input cost matrix	(3)
t	Time	
t_o	Initial time of observation	(18)
T	Terminal time	(3)
u	Plant input vector	(1)
$V_{11}, V_{12}, V_{21}, V_{22}$	Partitions of the eigenvector matrix of M	(12)
x	Plant state vector	(1)
$\bar{x}_o$	Initial state mean	
$\hat{x}$	State vector estimate	(18)
$\hat{x}_r$	Reduced state vector estimate	(27)
x^*	Costate vector	(7)
y	Plant output vector	(2)
δ	Small perturbation from nominal value	
θ	Output noise vector	(17)
Θ	Output noise covariance matrix	
Λ	Positive block of the diagonal eigenvalue matrix of M	(12)

NOMENCLATURE (continued)

Symbol	Description	Defining Equation
ξ	Input noise vector	(16)
Ξ	Input noise covariance matrix	
τ	Reversed time (T-t)	(9)

SUPERSCRIPTS

-1	Matrix inverse	
T	Matrix transpose	

1. INTRODUCTION

Multivariable methods have the potential to improve the control of large systems such as nuclear power stations. The complexity of nuclear power stations requires careful analysis of the roles and interactions of plant variables to determine the appropriate control strategy and to meet engineering specifications. Conventional single-variable methods are customarily used, but they lead to a difficult and empirical design procedure. Previous developments have shown that multivariable techniques can deal with such complex problems. For this reason, MVPACK [1], a computer-aided design package for multivariable controllers, is being developed in the Dynamic Analysis Laboratory at the Chalk River Nuclear Laboratories. This report presents the theory and computer implementation of linear-quadratic optimal control. Extensive theoretical development and successful application in the aerospace industry proves the maturity of this technique and motivates this study.

Optimal control is a state-space multivariable method based on the time response of the system. Its objective is to optimize the trajectory of the plant according to a performance index given by the designer. This index must take into account engineering and economic constraints to ensure safe and efficient operation of the plant. Optimal control theory handles two families of problems: the deterministic optimal controller, and the stochastic state estimator or Kalman-Bucy filter. Both are part of the optimal control problem encountered in practical situations. There are several forms of optimal controller, of which the most important is the optimal regulator that tries to keep the plant close to a constant reference.

This report presents the theory of the optimal deterministic regulator and the steady-state Kalman-Bucy filter. It describes the implementation of the corresponding module MVOPT within MVPACK. The technique is illustrated on the control of a steam generator.

2. OPTIMAL CONTROL THEORY

This section presents the mathematical theory used for the development of the optimal control design module MVOPT. We do not attempt to give any new results in optimal control but simply define the problem and present the solution method used. The notation is that adopted earlier [2] for the development of MVPACK.

The general problem of optimal control is to find an input signal such that the response of a given plant minimizes a specified criterion, or cost function. A wide range of problems fit this definition, some of which have been covered by Athans and Falb [3]. However, in this report, we present only the special case of a linear system with a quadratic criterion, subjected to Gaussian noise [4]. In this case, the optimal input is a linear function of the process state, and the result is a state-feedback controller. This special problem is of importance because it applies to the regulation of any system to a constant reference condition.

2.1 Optimal Regulator Concept

The linear-quadratic Gaussian optimal control problem can be decomposed into two associated problems using the separation theorem [4]. These are:

- deterministic ideal response analysis and design,
- stochastic estimation analysis and design.

Then, the complete solution of the stochastic feedback control problem is obtained by combining the results of the first two steps. This section deals with the deterministic optimal control case where input, state, and output variables are assumed to be measured exactly.

The first step in optimal control strategy is to determine the desired trajectory the plant must follow. This ideal trajectory can be obtained by experience, coupled with computer simulation or as the result of a preliminary optimization. It is chosen to meet economic, safety and engineering requirements on plant operation. However, due to disturbances and modeling approximations, the corresponding ideal input cannot drive the actual plant along the ideal trajectory. Thus, a feedback structure is required as shown in Figure 1. The resulting input causes the plant to follow the ideal trajectory with acceptable accuracy during the time interval of interest. The deterministic optimal control problem is to find the feedback controller such that a quadratic functional of input and state deviations is minimized. This insures that the plant follows the desired trajectory within a certain margin and with limited control action.

The solution is analytically provided by the minimum principle of Pontryagin [3] and is given as a state feedback gain matrix. Provided that the trajectory is known in advance, the general optimal controller is a time-varying feedback matrix to be precomputed. So the resulting controller can be very complex. However, this theory provides simple solutions in certain practical cases such as the optimal regulator.

The optimal regulator is defined as a restriction of the optimal controller in which the reference trajectory is constant. In normal operation, a power station is characterized by:

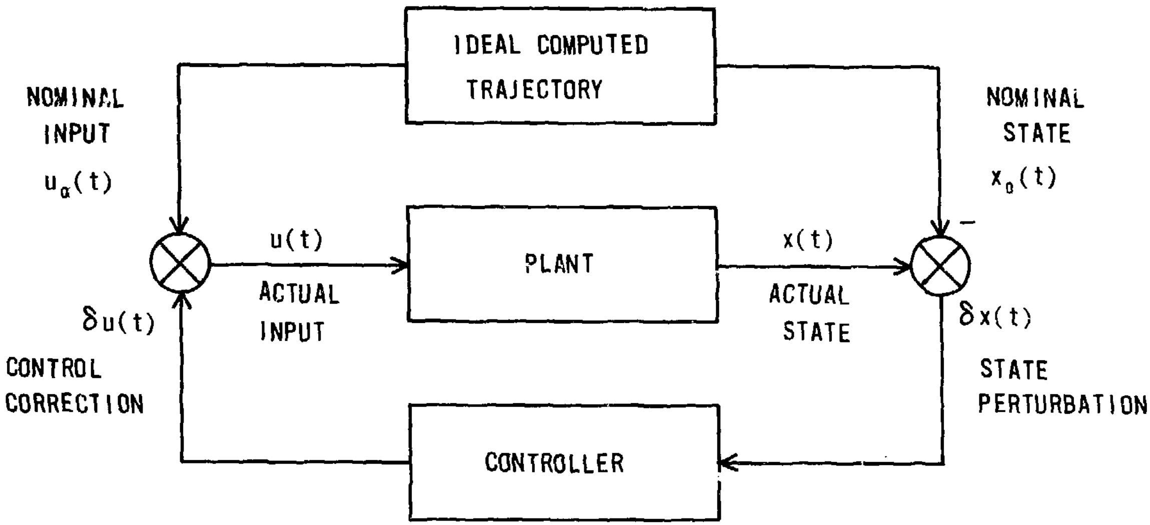

FIGURE 1: OPTIMAL CONTROLLER STRUCTURE

- constant or slowly varying reference compared to the dynamics of the plant,

- time interval of interest large compared to the time-response of the plant.

Then, a regulator can be used to keep the plant close to the reference given by the overall power control mechanisms. Specifically, if we assume that

- the reference is constant, and

- the time interval of interest is infinite,

the resulting optimal controller is a constant feedback-gain matrix. This is the deterministic, time-invariant optimal regulator presented in the next section.

2.2 The Deterministic Optimal Regulator

The optimal regulator structure is derived from Figure 1 but with constant ideal references obtained from the complete model of the plant. The linearized plant is time-invariant while its state remains near the operating point. For each specific operating point, the optimal regulator can be scheduled to be a time-invariant regulator as shown in Figure 2. All variables are now represented as perturbations from the reference values, so the regulator always attempts to drive the state to zero.

The plant is described by the linearized perturbation model [4]

$$\frac{d}{dt} x(t) = Ax(t) + Bu(t) \qquad (1)$$

$$y(t) = Cx(t) \qquad (2)$$

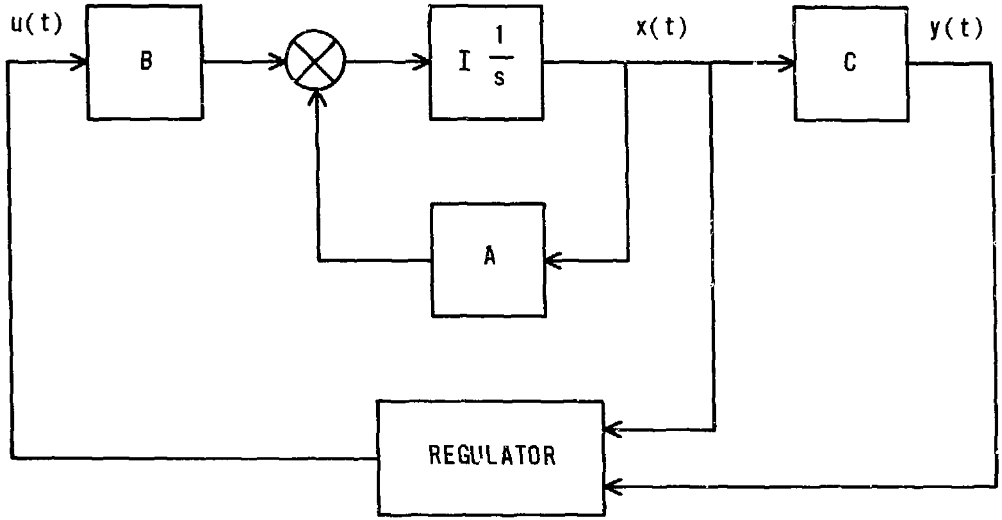

FIGURE 2: TIME - INVARIANT OPTIMAL REGULATOR

where

x is the state vector, of dimension n,

u is the input vector, of dimension p,

y is the output vector, of dimension m,

A,B and C are constant matrices of appropriate dimensions.

The matrices A, B and C are obtained by linearizing about the operating point, and x, u and y represent deviations from the operating point. The optimal regulator is designed to minimize the quadratic cost functional

$$J = y^T(t)Fy(t) + \int_0^T [y^T(t)Qy(t) + u^T(t)Ru(t)]dt \qquad (3)$$

where

F is a positive-semidefinite, symmetric-output terminal cost matrix,

Q is a positive-semidefinite, symmetric-output cost matrix,

R is a positive-definite, symmetric-input cost matrix,

T is the terminal time.

J is such that F and Q become state cost matrices if the plant output matrix C is replaced by the identity matrix.

The solution to this central problem is the state-feedback controller shown in Figure 3, with

$$K(t) = R^{-1}B^TP(t) \qquad (4)$$

where P(t) is the symmetric, positive-definite solution of the Riccati matrix-differential equation

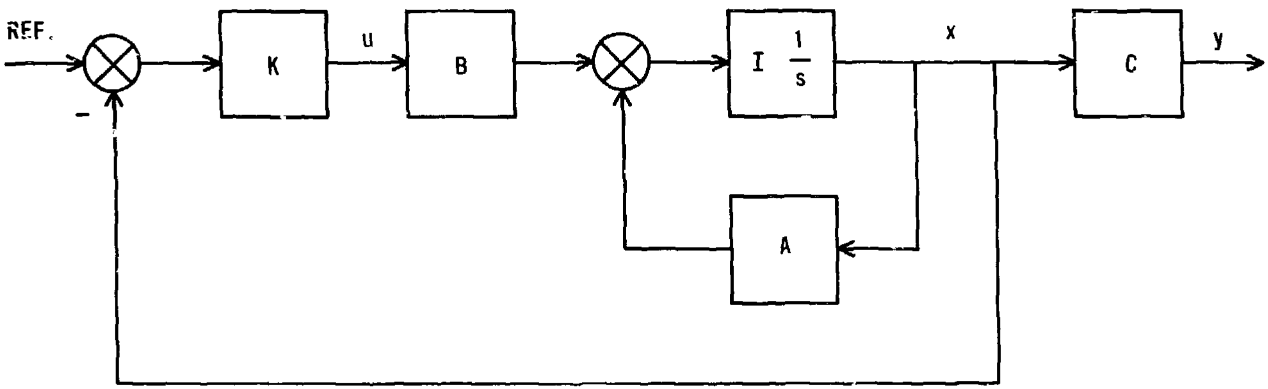

FIGURE 3: STATE - FEEDBACK CONTROLLER

$$\frac{d}{dt} P = -PA - A^T P - C^T QC + PBR^{-1}B^T P \tag{5}$$

with the terminal condition

$$P(T) = C^T FC \tag{6}$$

However, the matrix P can be also obtained by solution of the reversed-time state-costate equation [5]

$$\frac{d}{d\tau}\begin{pmatrix} x \\ x^* \end{pmatrix} = M\begin{pmatrix} x \\ x^* \end{pmatrix} \tag{7}$$

where

$$M = \begin{pmatrix} -A & BR^{-1}B^T \\ C^T QC & A^T \end{pmatrix} \tag{8}$$

and

$$\tau = T - t \tag{9}$$

The terminal condition of equation (6) becomes the initial condition

$$x^*(0) = C^T FCx(0) \tag{10}$$

Then, $P(\tau)$ is such that

$$x^*(\tau) = P(\tau)x(\tau) \tag{11}$$

for all τ.

The matrix $P(\tau)$ can then be found from a modal analysis of M [5,6,7]. M is a Hamiltonian matrix [6] so the eigenvalues are symmetric with respect to the imaginary axis, and this modal decomposition can be written

$$M \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} \Lambda & 0 \\ 0 & -\Lambda \end{pmatrix} \qquad (12)$$

The eigenvector matrix is partitioned into four nxn blocks such that the first n columns are the eigenvectors corresponding to Λ, the diagonal matrix of right half-plane eigenvalues. If the matrix M does not have 2n independent eigenvectors, the modal decomposition can be replaced by the Jordan canonical form of M [7, pg. 323]. The resulting solution can be obtained in a form such that it involves only real matrices and negative exponentials, so it is stable for large values of τ.

As discussed in Section 2.1, for a regulator it is appropriate to let the time interval T be infinite. Then, the cost functional becomes

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \qquad (13)$$

and the matrix P appearing in the feedback matrix K is constant. P is obtained as the infinite-time limit of the negative exponential solution of the Riccati matrix differential equation [5]

$$P = V_{21}V_{11}^{-1} \qquad (14)$$

Then, the solution is a time-invariant regulator with the state-feedback matrix

$$K = R^{-1}B^TV_{21}V_{11}^{-1} \qquad (15)$$

Two properties of the optimal regulator obtained can be pointed out:

(i) The modes of the closed-loop optimal regulator are the left half-plane eigenvalues of the matrix M and the eigenvector matrix is V_{11} [7].

(ii) The quadratic criterion reduces the higher order terms [4] in the Taylor series expansion used to linearize the plant. Thus, the linear model approximation is kept "honest".

2.3 Stochastic Estimation of the State Variables

The stochastic estimation problem is approached in this report because of its complementarity and duality with deterministic optimal control. As shown in the previous sections, optimal control theory generates state-feedback controllers. However, with the high-order models required to describe complex systems, the state vector cannot be measured completely and exactly. Indeed, some of the state variables are not directly measurable and noise often corrupts sensor information. Then, the state vector must be optimally estimated to complete the optimal controller.

An estimator produces a state vector estimate using plant input and output information. To design this stochastic estimator, the plant is modeled as shown in Figure 4. The vector ξ can be used to model parameter uncertainty while the vector θ models instrumentation noise. Once the plant model is linearized at the current setpoint, it is described by the linear equations

$$\frac{d}{dt} x(t) = A(t)x(t) + B(t)(u(t) + \xi(t)) \tag{16}$$

$$y(t) = C(t)x(t) + \theta(t) \tag{17}$$

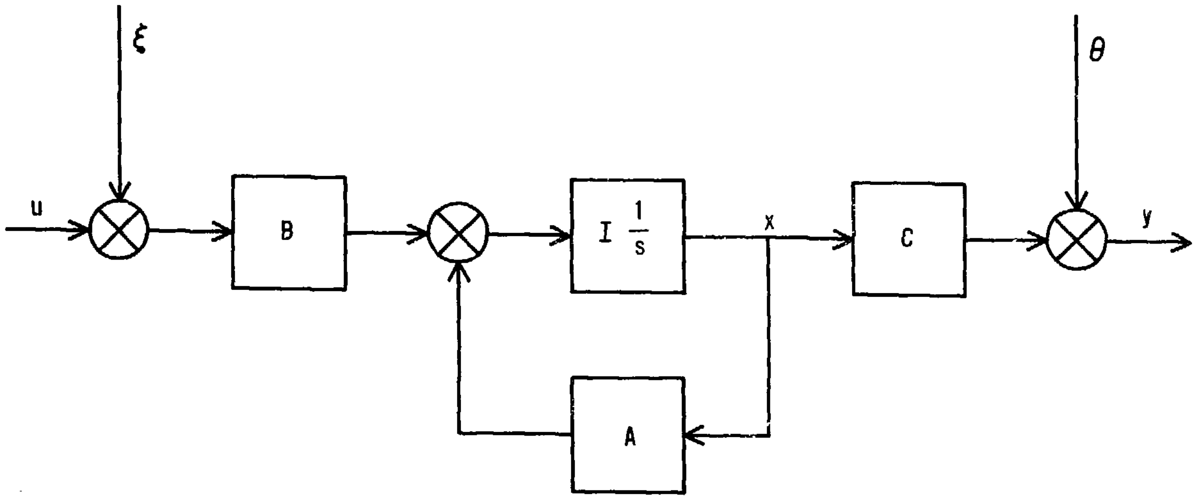

FIGURE 4: NOISY PLANT MODEL

For the development of the theory, the following assumptions are made:

- The initial state vector is Gaussian with known mean $\bar{x}_o$ and covariance P_o.

- ξ and θ are independent white Gaussian noise vectors with zero mean and known covariance matrices $\Xi(t)$ and $\theta(t)$, respectively. $\Xi(t)$ is symmetric positive semi-definite, and $\theta(t)$ is symmetric positive definite.

- The plant matrices A, B and C are deterministic and known.

The stochastic estimation problem is to find an estimate $\hat{x}(t)$ of the true state vector $x(t)$ which is optimal in a given statistical sense. The Kalman-Bucy filter provides such an estimate satisfying simultaneously various criteria such as least squares, minimum variance and maximum likelihood. This filter is presented in the next section and is used in steady-state form to complete the optimal regulator.

2.4 Kalman-Bucy Filter

The structure of the optimal filter is shown in Figure 5. The optimal estimate $\hat{x}$ is generated by

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu + H(y - C\hat{x});\hat{x}(t_o) = \bar{x}_o \tag{18}$$

where t_o is the initial time, and the filter gain matrix H is given by

$$H(t) = P(t)C^T(t)\ \theta^{-1}(t) \tag{19}$$

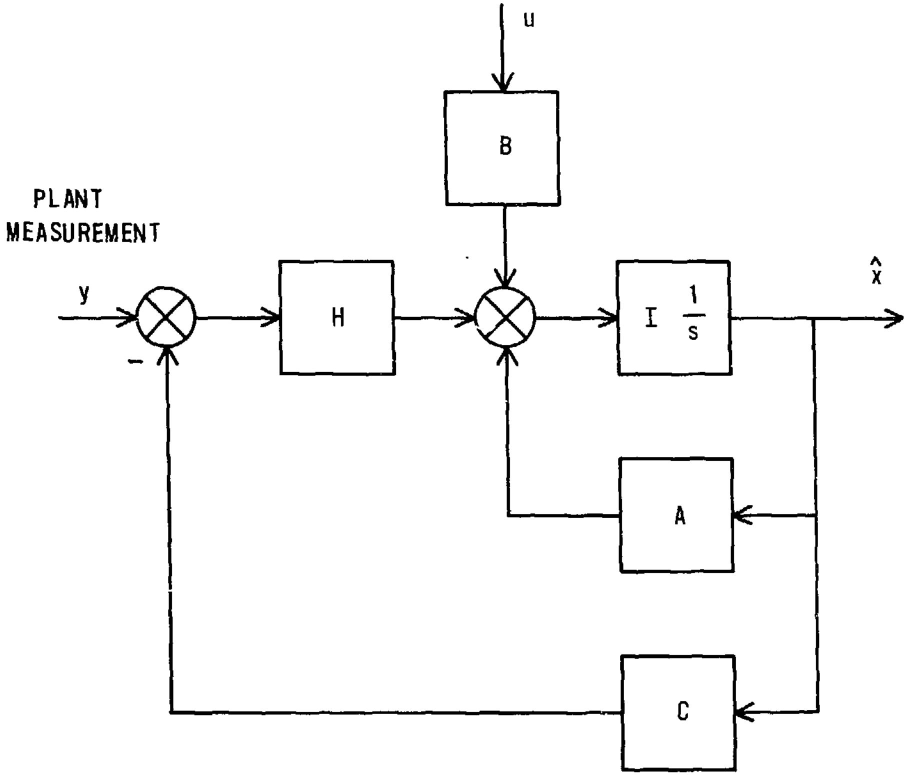

FIGURE 5: KALMAN - BUCY FILTER

P(t) is the positive definite solution of the Riccati matrix differential equation

$$\frac{d}{dt} P(t) = AP + PA^T + B\Xi B^T - PC^T \theta^{-1} CP \tag{20}$$

with the initial condition

$$P(t_o) = P_o \tag{21}$$

where P_o is the covariance matrix of $x(t_o)$.

The Riccati equation (20) may be solved by exploiting duality with the optimal regulator. If the Riccati equation (5) for the regulator is rewritten for $\tau = T - t$, it becomes

$$\frac{d}{d\tau} P = A^T P + PA + C^T QC - PBR^{-1} B^T P \tag{22}$$

and comparison of equations (20) and (22) shows that

$$M = \begin{pmatrix} -A^T & C^T \Theta^{-1} C \\ B\Xi B^T & A \end{pmatrix} \tag{23}$$

corresponds to the regulator state-costate matrix. The initial condition is

$$x^*(t)_o = P_o x(t_o) \tag{24}$$

and, as before, P(t) satisfies

$$x^*(t) = P(t)x(t) \tag{25}$$

This analogy may be verified by substituting equations (25) and (23) into equation (7), with τ replaced by t, to obtain the filter Riccati equation (20). Thus, the Kalman-Bucy filter in forward time is analogous to the optimal controller in reversed time.

As in the case of the optimal controller, the general solution of this problem is complex and time-varying. However, in practical applications it is possible to use a steady-state Kalman-Bucy filter. In this case, the following assumptions are made:

- The linear model of the plant is time-invariant, so A, B, C are constant matrices.
- Noise statistics are stationary, so Ξ and θ are constant matrices.
- The initial time of observation t_o is far in the past, so $t_o \to -\infty$.

These assumptions are valid for a plant that follows a constant or slowly varying reference as in the regulator case. Then the matrix P of the filter is obtained as the infinite-time limit of the negative exponential solution of equation (20). This solution has been presented for the time-invariant optimal regulator. So, the steady-state Kalman-Bucy filter is time-invariant and precomputable with a filter gain matrix

$$H = V_{21} V_{11}^{-1} C^T \theta^{-1} \tag{26}$$

The important properties of the Kalman-Bucy filter are:

- The deterministic optimal controller, combined with the Kalman-Bucy filter, produces the optimal control correction in the mean-square sense.

- The modes of this filter are only excited by the noise [1, pg. 59].

- The modes of the combined closed-loop system, including the Kalman-Bucy filter and the deterministic optimal controller, are those of the plant with state feedback (eigenvalues of A-BK) and those of the open-loop filter (eigenvalues of A-HC) [4]. Thus, the closed-loop system is stable.

2.5 Determination of Cost or Noise Covariance Matrices

In the optimal controller the role of the cost matrices is to weight the deviations of the variables from the ideal trajectory. However, it is usually impossible to translate engineering specifications into a quadratic form. Moreover, no rule can be defined to choose these costs. So, their determination relies mainly on experience and is based on a trial procedure that aims to achieve acceptable dynamic response. One means of assessing the dynamics is to examine the positions of the closed-loop poles.

In the Kalman-Bucy filter, the noise covariance matrices model plant uncertainties. Θ can be associated with output measurement noise and Ξ with parameter uncertainties. Again, determination of these matrices is a difficult problem which the designer must solve empirically.

For given cost and covariance matrices, the separation theorem states that the complete controller-estimator is optimal in the least-square sense. However, it has been shown that the deterministic optimal controller and the Kalman-Bucy filter should not be designed independently [8]. In any case, the design will be initiated with arbitrary cost and noise covariance matrices. A common way to begin the regulator design is to fix deviations δx_i or δu_j for each variable involved in the cost. Then the corresponding diagonal cost term will be $1/\delta \hat{x}_i^2$ or $1/\delta \hat{u}_j^2$. Further adjustment of the cost can be made using the result of a modal analysis or simulation. If some engineering specifications are given, such as the response time or overshoot percentage of a variable, the factor weighting that variable can be used to meet the requirements.

3. IMPLEMENTATION IN MVPACK

MVPACK [1] is a multivariable control design package being developed in the Dynamic Analysis Laboratory at CRNL. It is an interactive system of computer programs running on a PDP-11/45 minicomputer. MVOPT is the optimal controller and Kalman-Bucy filter design program, based on the theory presented in this report.

MVOPT accepts a linear state-space model written in standard form. Cost or noise covariance matrices may be read from a file, or entered interactively. MVOPT solves the steady-state matrix Riccati equation using the eigenvectors of the state-costate matrix M, as presented above. The necessary eigenanalysis is performed by the MVEIG module of MVPACK. The controller or filter generated is the steady-state time-infinite version.

A second module, MVOPTA, is also available as an aid to optimal control design. It is a fast and interactive module that simply computes the modes of the closed-loop controller corresponding to the cost or noise covariance matrices chosen. It is used to accelerate the iterative process of choosing weights using the eigenvalues of the controlled plant as a decision criterion.

Finally, it is desirable to simulate the combined controller-estimator. The MVSIM module in MVPACK allows for state feedback, but does not accept controller dynamics. This can be overcome by creating an augmented system, as follows. The complete system is described by its three components:

- the plant model, equations (1) and (2),
- the Kalman filter, described by

$$\frac{d}{dt}\hat{x}_r = A_r\hat{x}_r + B_r u + H_r(y - C_r\hat{x}_r - D_r u) \tag{27}$$

and based on either the full model, or on an equivalent reduced model of order r,

- the optimal controller designed on the same model used for the filter

$$u = -K\hat{x}_r \tag{28}$$

Then, introducing a combined state vector, a new set of state-space equations results

$$\frac{d}{dt}\begin{pmatrix} x \\ \hat{x}_r \end{pmatrix} = \begin{pmatrix} A & 0 \\ H_r C & A_r - H_r C_r \end{pmatrix}\begin{pmatrix} x \\ \hat{x}_r \end{pmatrix} + \begin{pmatrix} B \\ B_r - H_r D_r \end{pmatrix} u \tag{29}$$

$$y = (C,\ 0)\begin{pmatrix} x \\ \hat{x}_r \end{pmatrix} \tag{30}$$

$$u = -KC_m\begin{pmatrix} x \\ \hat{x}_r \end{pmatrix} \tag{31}$$

with

$$C_m = (0, I_r) \tag{32}$$

The $D_r u$ term in equations (27) and (29) is required if such a term appears in the output equation of the reduced model. The MVOSIM module is provided to generate these matrices, permitting simulation using MVSIM.

4. EXAMPLE OF A NUCLEAR STEAM GENERATOR

Optimal control of a nuclear steam generator is used as an example to demonstrate the application of the design tool developed in the preceding sections. This steam generator [9] is modeled as a 15th-order linear system. No attempt has been made to produce a 15th-order optimal controller. Instead, a 6th-order reduced model [10] was used for filter and controller design. Then, the controller was tested on the complete model to establish both the accuracy of the reduced model and the efficiency of the optimal controller.

The 6th-order reduced model involves the following state variables:

x_1 - Downcomer level

x_2 - Steam pressure

x_3 - Downcomer temperature

x_4 - Steam quality in the riser

x_5 - Length of subcooled region on the secondary side

x_6 - Tube metal temperature of 4th lump, in 4-lump model

where only x_1 and x_2 are accessible to measurement. The state matrices of the reduced model are [10]

$$A = \begin{pmatrix} -0.053 & -0.0026 & 0.0067 & -15.88 & 0.034 & 0.018 \\ -0.143 & -0.293 & 0.332 & -0.258 & -0.089 & 0.887 \\ 0.0019 & 0.0084 & -0.080 & -5.84 & -0.0029 & 0.0072 \\ 4.6\times10^{-4} & 6.1\times10^{-4} & -0.005 & -0.309 & -0.022 & 4.7\times10^{-4} \\ 0.055 & 0.027 & -0.188 & 0.113 & -0.813 & -0.008 \\ -0.108 & 0.009 & 0.549 & -0.588 & 1.44 & -0.847 \end{pmatrix} \quad (33)$$

$$B = \begin{pmatrix} 1.27 & 0.672 \\ -62.2 & 1.92 \\ 0.518 & -1.085 \\ -0.030 & -0.0013 \\ -1.59 & -0.059 \\ 4.94 & 1.63 \end{pmatrix} \quad (34)$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (35)$$

$$D = \begin{pmatrix} 0.413 & 0.247 \\ -0.132 & -0.036 \end{pmatrix} \quad (36)$$

4.1 Kalman Filter Design

The Kalman-Bucy filter was designed on the 6th-order reduced model. For simulation, the plant was considered free of output noise. So, the actual role of the filter, once implemented on the complete model, is to produce an estimate of the 6 desired state variables, filtering the inaccuracies of the reduced model. The noise covariance factors were chosen using MVOPTA with the aim of obtaining filter dynamics that are faster than the expected dynamics of the optimal controller. Since θ must be nonsingular, a small amount of output noise was assumed.

The filter parameters finally chosen were

$$\Xi = \mathrm{diag}(10^3,\ 10^3,\ 10^6,\ 10^6,\ 10^6,\ 10^6) \tag{37}$$

$$\Theta = \mathrm{diag}(1,\ 1) \tag{38}$$

The resulting gain matrix is

$$H = \begin{pmatrix} 2.0978 & 180.63 \\ 53.823 & 2.0978 \\ 850.10 & 48.931 \\ -13.659 & -996.37 \\ 289.92 & 8.5383 \\ 797.08 & -0.78672 \end{pmatrix} \tag{39}$$

and the filter eigenvalues are -0.2511, -1.743, $-26.92 \pm j15.01$, $-90.51 \pm j87.70$.

A simulation of this filter for estimating the 6 desired state variables from the output of the 15th-order model is shown in Figure 6, where each variable is plotted with its estimate. The test disturbance is a trapezoidal pulse in steam valve position.

4.2 Optimal Controller Design

The controller was also designed on the 6th-order reduced model. The value of MVOPTA for controller design was found very limited, at least in this case. Indeed, MVOPTA gives no indication of the gains required, and one must know the desired poles a priori. So, the main tool used in selecting the cost factors was simulation of the reduced model with its deterministic controller.

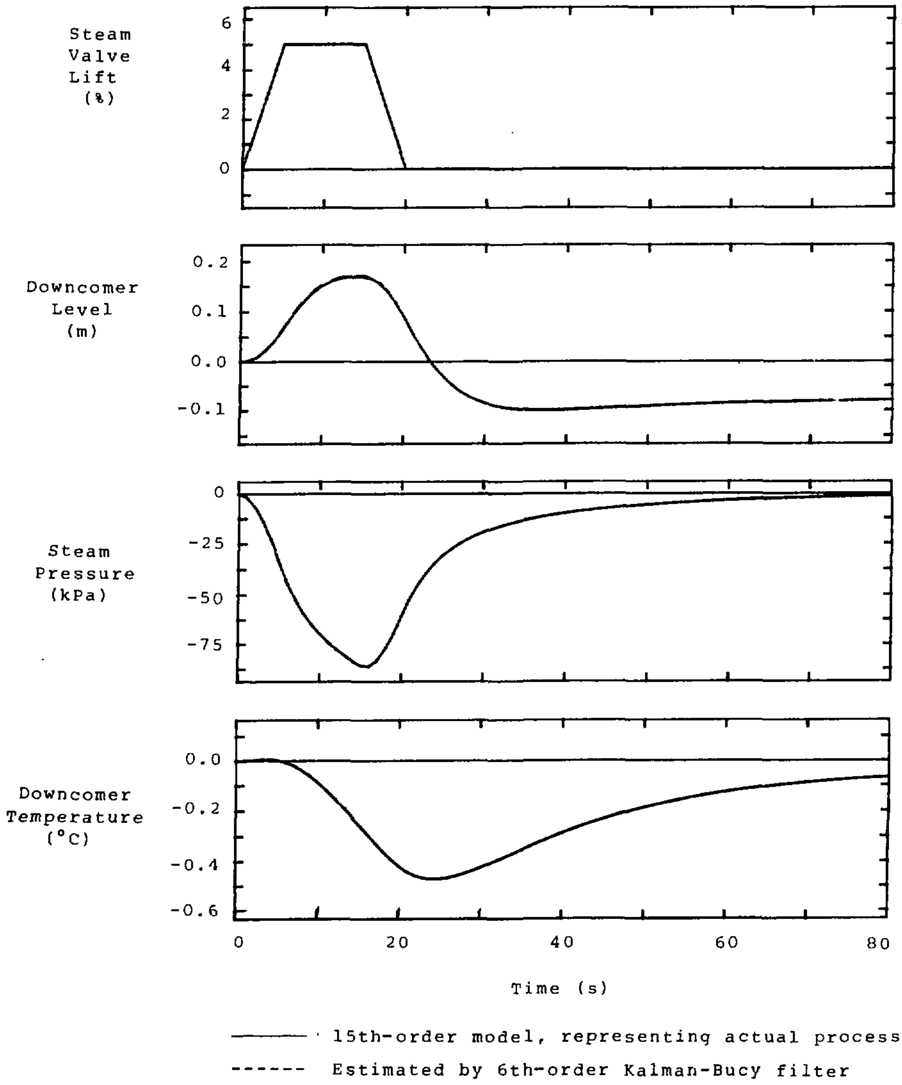

FIGURE 6a COMPARISON OF THE RESPONSES OF THE STEAM GENERATOR MODEL AND THE KALMAN-BUCY FILTER TO A STEAM VALVE DISTURBANCE

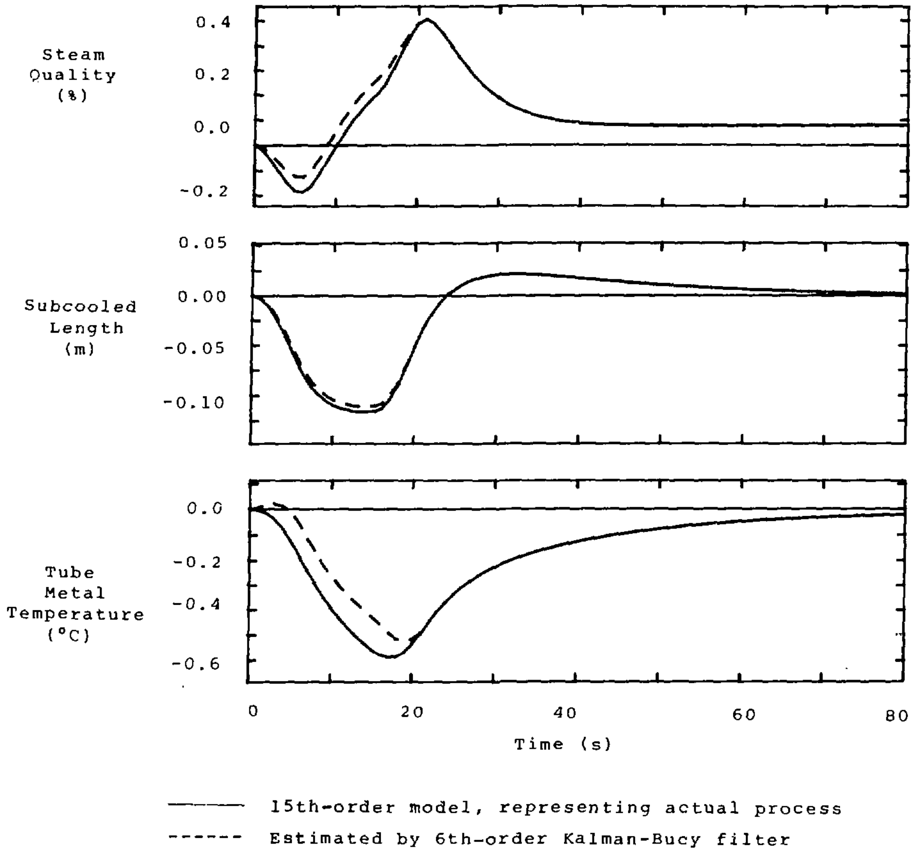

FIGURE 6b COMPARISON OF THE RESPONSES OF THE STEAM GENERATOR MODEL AND THE KALMAN-BUCY FILTER TO A STEAM VALVE DISTURBANCE

The weighting factors finally selected are a reasonable compromise between the dynamics of the states and the control actions. The chosen cost matrices are

$$Q = \mathrm{diag}(6,\ 0.4,\ 0.04,\ 0.04,\ 0.04,\ 0.04) \tag{40}$$

$$R = \mathrm{diag}(10^4,\ 10^4) \tag{41}$$

and the resulting state-feedback matrix is

$$K = \begin{pmatrix} -9.92\times10^{-4} & -3.43\times10^{-3} & -3.82\times10^{-3} & 0.212 & -8.95\times10^{-3} & -2.22\times10^{-3} \\ 2.12\times10^{-2} & 2.98\times10^{-4} & -6.41\times10^{-4} & -1.026 & 2.81\times10^{-2} & -1.36\times10^{-4} \end{pmatrix} \tag{42}$$

The closed-loop poles of the reduced model with this controller are -1.63×10^{-2}, -7.06×10^{-2}, $-0.363\pm j0.044$, $-0.902\pm j0.249$. When this controller is applied to the full model, the first 8 poles are -1.63×10^{-2}, -7.05×10^{-2}, $-0.362\pm j0.044$, $-0.901\pm j0.249$, -1.32, -1.43. The effect of the controller on the full model is almost identical to its effect on the reduced model. The remaining plant poles remain to the left of the controlled poles.

The controller was simulated with constant setpoints and an initial pressure deviation of 480 kPa (70 psi). The controller must then act to restore the pressure. Figure 7 shows the inputs and outputs for the deterministic case, in which the 6 states required are known exactly. The same transient is shown in Figure 8, but the states are obtained from the 6th-order Kalman filter. The response is close to the deterministic case, though the feedwater flow (u_2) shows the effect of inaccurate estimation. For this run, the filter was initialized with the actual initial state, including the pressure deviation. In practice, such a deviation would result from an unknown disturbance, and the filter initial state would be 0. This case is shown in Figure 9. The output response

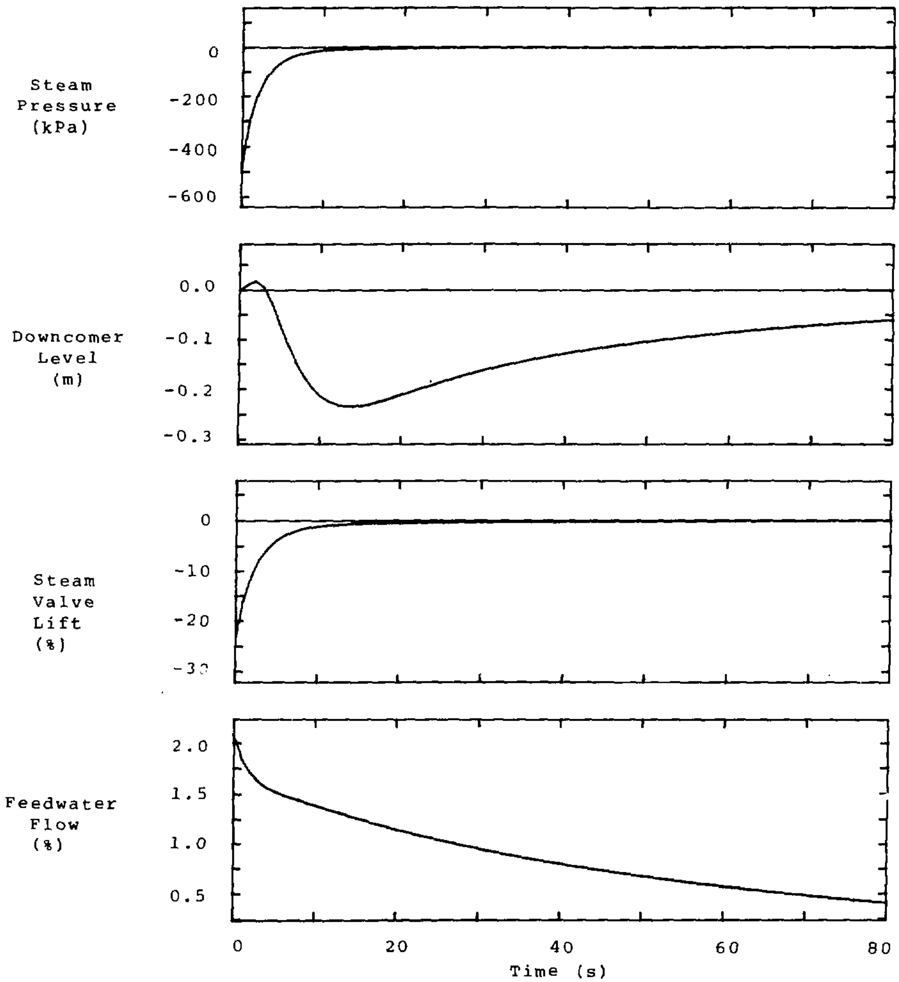

FIGURE 7 RESPONSE OF THE OPTIMAL CONTROLLER TO AN INITIAL PRESSURE DEVIATION, WITH THE 6 STATES PROVIDED BY THE MODEL

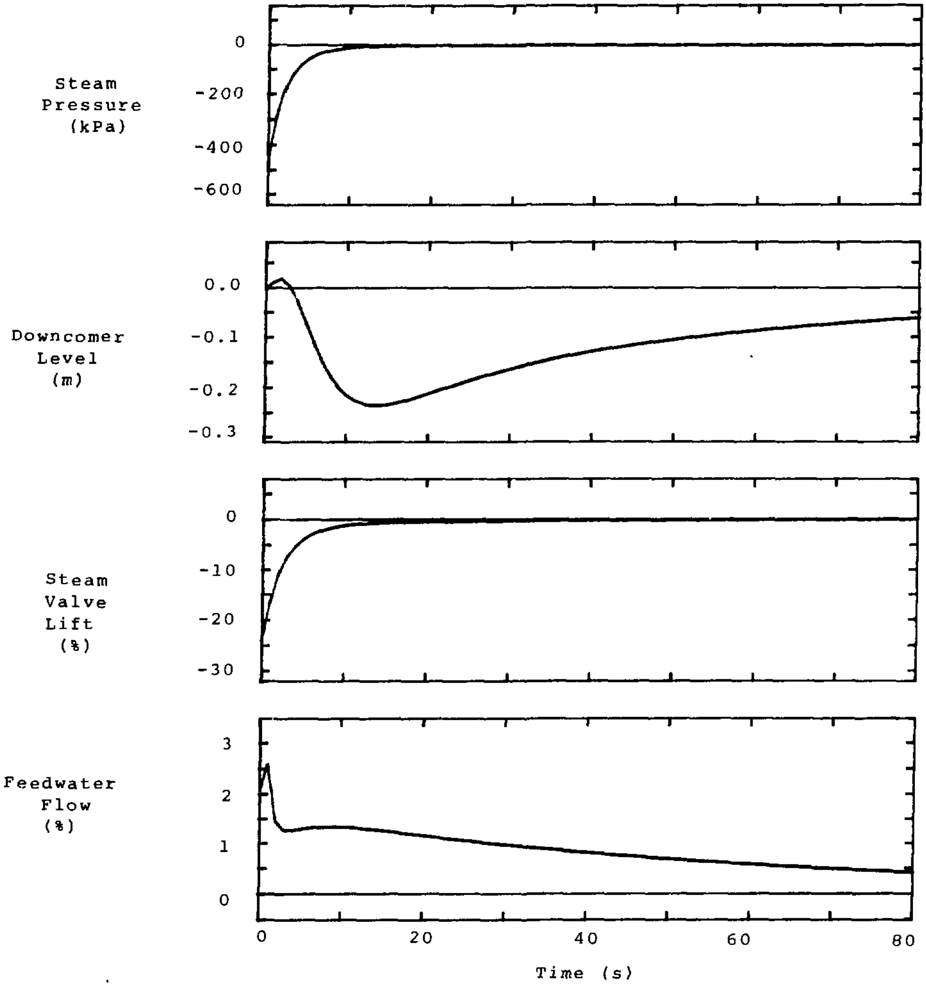

FIGURE 8 RESPONSE OF THE OPTIMAL CONTROLLER TO AN INITIAL PRESSURE DEVIATION WITH THE 6 STATES PROVIDED BY A KALMAN FILTER HAVING THE CORRECT INITIAL CONDITION

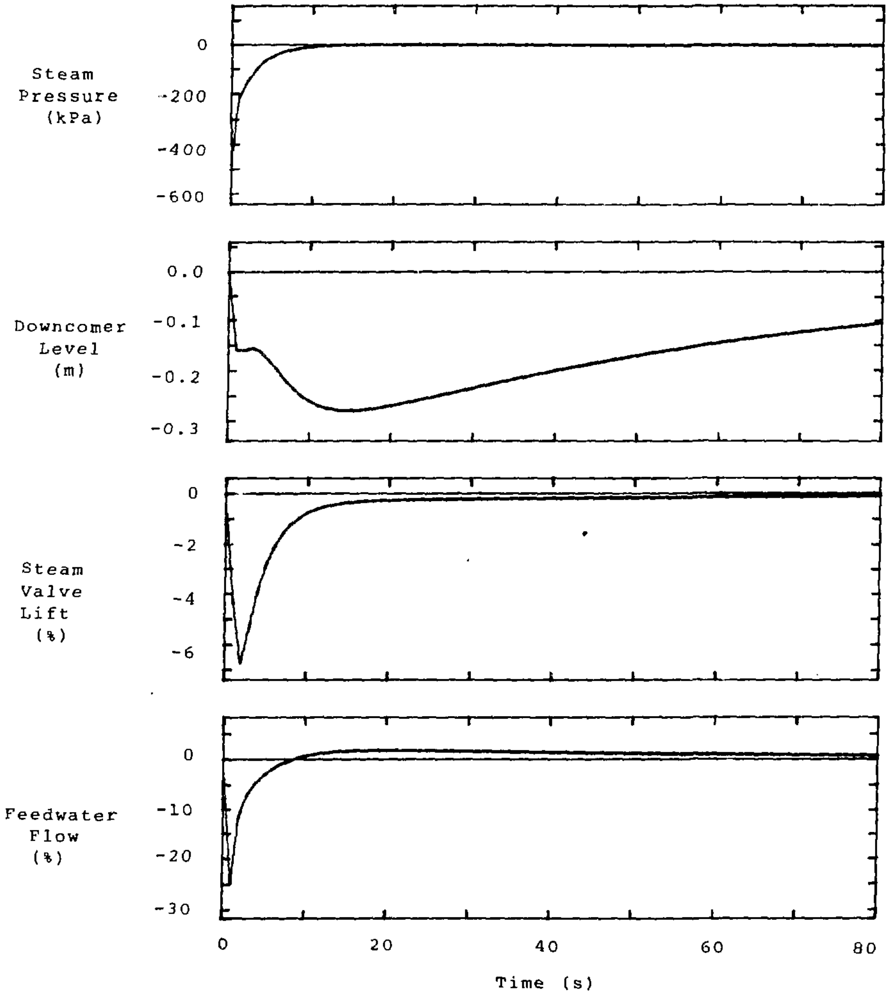

FIGURE 9 RESPONSE OF THE OPTIMAL CONTROLLER TO AN INITIAL PRESSURE DEVIATION WITH THE 6 STATES PROVIDED BY A KALMAN FILTER HAVING INCORRECT INITIAL CONDITIONS

is essentially unchanged, but the feedwater flow now goes sharply negative initially, as it is acting on incorrect information. This transient is governed by the 4 s dominant filter time constant.

These results demonstrate the potential of state-space methods to generate practical controllers having new structures.

5. SUMMARY AND CONCLUSIONS

The state-of-the-art of linear-quadratic optimal control theory is summarized in this report, and this method has been developed into an efficient program module for the design of optimal controllers. The module, called MVOPT, is now an operational part of the overall MVPACK program. MVOPT, in conjunction with model reduction, is used iteratively as a controller design tool. However, the method is limited by sensitivity to model accuracy, and there is no guarantee on the final result if the model is not accurate enough.

6. REFERENCES

[1] P.D. McMorran, "Development of Multivariable Methods for Plant Control", paper presented at the Seventh Symposium on Reactor Dynamic Simulation and Nuclear Power Station Control, Montreal, 1980 May.

[2] P.D. McMorran, "Multivariable Control in Nuclear Power Stations - Survey of Design Methods", Atomic Energy of Canada Limited, AECL-6583, 1979 December.

[3] M. Athans and P.L. Falb, "Optimal Control", McGraw-Hill, New York, 1966.

[4] M. Athans, "The Role and Use of the Stochastic Linear-Quadratic-Gaussian Problem in Control System Design", IEEE Transactions on Automatic Control, AC-16, pp. 529-552, 1971 December.

[5] D.R. Vaughan, "A Negative Exponential Solution for the Matrix Riccati Equation", IEEE Transactions on Automatic Control, AC-14, pp. 72-75, 1969 February.

[6] O.H.D. Walter, "Real-matrix Solutions for the Linear Optimal Regulator", Proc. IEE, 119, pp. 621-624, 1972 May.

[7] H. Kwakernaak and R. Sivan, "Linear Optimal Control Systems", Wiley-Interscience, New York, 1972.

[8] J.M. Mendel, "On the Need for and Use of a Measure of State Estimation Errors in the Design of Quadratic Optimal Control Gains", IEEE Trans. Auto. Control, AC-16, pp. 500-503, 1971 October.

[9] P.D. McMorran and T.A. Cole, "Multivariable Control in Nuclear Power Stations: Modal Control", Atomic Energy of Canada Limited, AECL-6690, 1979 December.

[10] M. Parent and P.D. McMorran, "Multivariable Control in Nuclear Power Stations: Model Order Reduction", Atomic Energy of Canada Limited, AECL-7245, report in preparation.

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