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PERFORMING DYNAMIC TIME HISTORY ANALYSES BY EXTENSION OF THE RESPONSE SPECTRUM METHOD

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ABSTRACT

A method is presented to calculate the dynamic time history response of finite-element models using results from response spectrum analyses. The proposed "modified" time history method does not represent a new mathematical approach to dynamic analysis but suggests a more efficient ordering of the analytical equations and procedures. The modified time history method is considerably faster and less expensive to use than normal time history methods. This paper presents the theory and implementation of the modified time history approach along with comparisons of the modified and normal time history methods for a prototypic seismic piping design problem.

NOMENCLATURE

$a_i(t)$	=	acceleration time history response of single d.o.f. oscillator with frequency ω_i and excitation Z(t)
A_i	=	acceleration amplitude from the response spectru at frequency ω_i
b_i	=	arbitrary acceleration amplitude for frequency ω_i
$\{D\}_i$	=	nodal displacement vector contributed by the ith mode for the acceleration amplitude b_i
$\{D(t)\}$	=	complete displacement response time history
N	=	number of modes of vibration
p_i	=	$(1 - \beta_i^2)^{1/2}$

P_i	=	participation factor for ith mode
t	=	time
$\{X\}_i$	=	vector of structural displacements contributed by ith mode
$\ddot{Z}(t)$	=	input acceleration time history
β_i	=	fraction of critical damping for ith mode
$\{\phi\}_i$	=	ith mode of vibration (eigenvector)
ω_i	=	natural frequency of ith mode (eigenvalue)

INTRODUCTION

When designing structures to withstand seismic events, finite element idealizations of the structure are often developed and dynamic analyses performed to verify structural adequacy. The analyst can typically perform the dynamic analysis using the response spectrum method or one of two time history methods. Response spectrum methods are relatively inexpensive, conservative techniques for calculating dynamic structural response. Time history methods provide a more mathematically exact structural response solution but the computational costs are significantly greater than response spectrum methods. Structural design is often an iterative process, requiring numerous design modifications and dynamic analyses until a satisfactory design is achieved. In such an environment, the response spectrum method is most often used to analyze structural design iterations to minimize computational costs. However, trading the more exacting, less conservative time history approach for the less expensive, more conservative response spectrum method may not be clearly advantageous in the design iteration process. It is relatively obvious that a particular structural design could meet the appropriate design limits if its dynamic response was calculated using the time history method, yet have its structural response exceed those design limits if calculated using the response spectrum approach. Unfortunately, due to the complicated nature of structural dynamics, there is no general rule which indicates when such a situation would occur in the design iteration process.

To resolve the difficulty in choosing an appropriate dynamic analysis method in the design iteration process, one approach is to first perform a response spectrum analysis of the structure. If the structural response exceeds the allowable design limits by a factor of 10 or greater, analyzing the structural design using the time history method will probably not result in an acceptable design. In this case, the design should be modified and the new design analyzed using the response spectrum method. On the other hand, if the response spectrum analysis results exceed the allowable limits but do not exceed these limits by a factor of 10, then analyzing the structural design using the time history method may show the design to be acceptable. This combination of analysis techniques can speed the convergence towards an acceptable structural design. The penalty for utilizing the response spectrum analysis - time history analysis approach is the increased computational costs for performing the time history analyses. Thus, the above analytical approach would be particularly useful if a more efficient time history method were utilized, compared to the normal time history methods.

This paper describes a technique developed to calculate the dynamic time history response of a structural model using results obtained from a response spectrum analysis of the finite element idealization. The described analytical technique, which is called the modified time history method throughout the remainder of this paper, does not represent a new mathematical solution to the classical dynamic equations of motion. Instead, the modified time history method represents a re-ordering of the mathematical procedures used for modal superposition time history analyses. This re-ordering permits the modified time history method to use response spectrum analysis results from most general purpose finite element programs and requires no modifications to these programs. The modified time history method eliminates much of the effort required to perform dynamic time history analyses and is ideally suited to the design iteration process. As described above, a response spectrum analysis of a structural design iteration is first performed. If the design does not meet allowable limits, the modified time history analysis can be performed using results from the prior response spectrum analysis. As will be shown, performing the modified time history analysis, once the necessary input is available (the prior response spectrum analysis results), can result in less computational expense than performing another response spectrum analysis. In other words, using the modified time history method, it is more economical to perform the response spectrum analysis- time history analysis approach than it is to perform two response spectrum analyses where the second response spectrum analysis is performed for a new design iteration. The modified time history method can therefore reduce the costs of iterative structural design both in terms of computational expense and speeding convergence towards an acceptable design.

DERIVATION OF THE MODIFIED TIME HISTORY METHOD

To show how to perform time history analyses using results from response spectrum analyses, the response spectrum method should be reviewed. The basic principles behind the response spectrum method are:

A. decouple the structural dynamic equations of motion into normal modes of vibration,

B. calculate the maximum response of the structure contributed by each normal mode, and

C. combine the maximum modal responses to obtain the complete dynamic response.

It should be noted that the terms "structural response" or "response" refer to either elemental stresses, elemental forces or nodal displacements of the finite element idealization. The equations in this section are specifically derived for nodal displacements. However, the modified time history method can calculate the dynamic transient response of any structural model parameter provided that the finite element code, from which the response spectrum analysis results are obtained, provides the structural response solution in the required form. This point will be expanded later.

In the response spectrum method, after the eigenvalue problem is solved, the structural response is calculated in a mode-by-mode-fashion. From [1], the nodal displacements contributed by each mode are calculated using,

$$\{X\}_i = P_i\{\Phi\}_i \frac{A_i}{\omega_i^2} \tag{1}$$

The finite element code calculates ω_i, $\{\Phi\}_i$ and P_i. The input excitation term, A_i, is specified by the analyst using a response spectrum curve. A response spectrum represents a plot of the maximum dynamic responses of a series of single degree-of-freedom (d.o.f.) oscillators which are excited by an acceleration (or velocity or displacement) time history applied as a base (ground) motion. Each oscillator is tuned to a different frequency. It is important to note that each point on the response spectrum represents only the maximum response of a single d.o.f. oscillator. From [2], this is expressed as,

$$a_i(t) = \frac{-1}{\omega_i p_i} \int_o^t \ddot{Z}(t)\, e^{-\beta\omega_i(t-\tau)} \sin \omega_i p(t-\tau)\, d\tau \qquad (2)$$

$$A_i = MAX\,(a_i(t))$$

The nodal displacements calculated using Equation 1 are dependent on the magnitude of A_i and this dependence is clearly linear. Thus, knowing the magnitude of nodal displacements for the acceleration amplitude A_i, the nodal displacements for another value of acceleration amplitude, say b_i, can be calculated using,

$$\{D\}_i = \{X\}_i \frac{b_i}{A_i} \qquad (3)$$

Because linear analyses are used, Equation 3 can be easily written as a function of time. Thus,

$$\{D(t)\}_i = \frac{\{X\}_i}{A_i} b_i(t) \qquad (4)$$

Equation 4 states that it is possible to obtain the nodal displacement time history response of a structure, for the ith mode, from response spectrum analysis results and the time history response, $b_i(t)$, of a single d.o.f. oscillator tuned to ω_i and excited by base motion $\ddot{Z}(t)$. In other words, equation 4 can be written,

$$\{D(t)\}_i = \frac{\{X\}_i}{A_i} a_i(t) \qquad (5)$$

where $a_i(t)$ replaces $b_i(t)$ and is calculated using Equation 2. Equation 5 represents the primary step in the modified time history method. The nodal displacement vector calculated using the modified time history method is identical to the nodal displacement vector calculated using the standard modal superposition time history method. Equivalence of the two methods can be readily proven.

The equations used to derive the modified time history method specifically referred to the ith mode. The modified time history method is actually a slight variation of the modal superposition time history technique. As such, the modified method uses the response spectrum analysis results to independently obtain the nodal displacement solution vectors contributed by each mode. Like the normal modal superposition methods, the complete structural response solution is calculated using,

$$\{D(t)\} = \sum_{i=1}^{N} \{D(t)\}_i \qquad (6)$$

Because the modified time history method is a modal superposition approach, it is essential that the finite element program enables access to the individual modal contribution vectors to the structural response solution which are calculated during a response spectrum analysis. The modified time history method cannot be used if the only available response spectrum analysis results consist of the complete dynamic response solution (typically obtained using a square root of the sum of the squares of the modal contribution vectors).

IMPLEMENTATION

The previous section described the mathematical formulation of the modified time history method. The most efficient method of implementing this dynamic analysis technique requires a computer program which performs the calculations given by Equations 2, 5, and 6. The basic elements of this computer program are presented in this section. Also presented are the reasons for the increase in performance over normal time history methods.

As mentioned in the previous section, the first step in the modified time history method is to obtain the response spectrum analysis results. These results may be nodal displacements, elemental stresses or elemental loads. The key point is that the response spectrum analysis results must be available on a mode-by-mode basis and not as the total combination of modal contributions. The modified time history method is applicable to any structural response parameter provided that the above restriction is met. The additional data required from the response spectrum analysis are the structural model natural frequencies of vibration and the response spectrum curve used as the input excitation.

A flowchart of the modified time history computer program is shown in Figure 1. The first step is to read the necessary run parameters, which are defined as follows:

NMODES	=	number of modes of vibration included in the response spectrum analysis
NTIME	=	number of time steps in the excitation time history
DELT	=	length of time step
BETA	=	fraction of critical damping for Equation 2

The excitation time history is input to the program. Then, the response spectrum analysis data are read on a mode-by-mode basis. This part of the program completes the input of the necessary information to perform the modified time history analysis. The second part of the program calculates the structural response time histories. Two loops over Equations 2, 5, and 6 comprise this second program segment. The outermost loop sequences through the excitation time history time steps while the innermost loop iterates over the number of structural modes. The time history response of the single d.o.f. oscillator is solved for the current natural frequency, ω_M, and at the current point in time, T. This solution is performed by numerical integration of Equation 2. The computer code used to perform the numerical integration of Equation 2 is an adaptation of the numerical integration scheme in [3].

Having solved Equation 2 for the current time T, and structural mode M, the structural response solution is calculated using Equation 5. The result from Equation 2 is divided by the response spectrum acceleration amplitude, A_M, which was used in the response spectrum analysis. This result is then multiplied by the structural response vector for mode M, designated as $\{X\}_i$ in Equation 5. The final result from this step represents the dynamic structural response contributed by structural mode M at time T.

Solution of Equation 6 completes the second part of the modified time history program. The structural responses contributed by mode M are added to the structural responses contributed by the previous M-1 modes. Equations 2, 5, and 6 are solved for the current time T and for all M structural modes. Time is then incremented and the loop over Equations 2, 5, and 6 repeated. The final result from this program segment consists of the complete structural response time history solution. These results are written to tape for permanent storage, if desired. The computer program then checks each response time history and obtains the maximum and minimum values for the particular structural response parameter. These values are printed along with the times corresponding to their occurence in the dynamic event.

One advantage of the modified time history method, compared to normal time history methods, is that the analyst can select a subset of structural response parameters for which the time history responses are desired. In direct integration time history analyses, the dynamic transient responses of all structural parameters are calculated, which results in a large number of computations. In the modified time history method, only those structural response parameters of interest are retrieved from the response spectrum analysis. Because the transient response of the complete set of parameters is not calculated, much computational effort is eliminated. In the example that follows, only the dynamic nozzle loads were of interest, so the remaining stresses, loads, and displacements of the structure were not included in the modified time history analysis.

However, the primary advantage of the modified time history method does not depend on the number of dynamic structural response time histories being calculated. The increased performance of the modified time history method is due to the order in which the dynamic equations of motion are solved. Finite element idealizations of nuclear power plant systems are typically large models, consisting of many d.o.f.s and elements. Dynamic analysis of such models requires large matrix and vector manipulations. These manipulations consume a great deal of memory and necessitate numerous Input/Output (I/O) operations between main core memory and auxiliary storage, even on large mainframe computers. The computational time and cost involved with the I/O operations are significantly greater than those associated with calculations performed in-core. The modified time history method reduces the quantity of I/O operations required in a time history analysis by restricting these operations to the response spectrum analysis. In other words, all of the large matrix and vector manipulations are completed in the response spectrum analysis. The computer program described above requires no large matrix manipulations; the majority of computational effort being spent in solving Equations 2 and 5. If the response spectrum analysis results are available, as would be likely in the design iteration environment, the computational effort consumed by the modified time history method computer program is minimal. As will be shown in the following example, even if the response spectrum analysis results are not available, the modified time history analysis costs plus the response spectrum analysis costs are still substantially less than the costs associated with the standard modal superposition time history method. This suggests that the modified time history method is less expensive to use than normal time history methods even if structural design iterations are not performed.

EXAMPLE

To illustrate the benefits of the modified time history method, seismic analyses were performed for the finite element idealization of a prototypic Liquid Metal Fast Breeder Reactor (LMFBR) piping loop shown in Figure 2. This structural model contains 177 elements and 147 nodes. Fifty modes of vibration and 117 dynamic d.o.f.s were retained in the analyses. The dynamic excitation was an Operating Basis Earthquake (OBE) event. This excitation is specified both in terms of acceleration response spectra and acceleration time histories. Three independent time histories define the OBE event, one for each orthogonal direction (E-W, N-S, and vertical). These time histories have twenty second durations and significant energy content between one Hz and 33 Hz. To ensure sufficient numerical accuracy across this frequency bandwidth, an integration time step of 0.003 seconds was selected. This resulted in 6667 time steps to completely define the OBE event. The length of the excitation time histories and the size of the finite element model are mentioned primarily to indicate the large size of this particular dynamic analysis.

The response spectrum, modal superposition time history, and direct integration time history analyses were conducted using a commercially available, general purpose finite element program [1]. This program is a well designed code and, as such, is felt to be representative of other general purpose finite element programs. All of the dynamic analyses, including the modified time history analyses, were performed on a CDC 7600 computer. Job execution time is given in seconds while computational costs are proportional to Computer Resource Units (CRUs). In accordance with [4], three separate time history analyses should be performed, one analysis for each independent excitation time history. Due to the costs of the direct integration time history analysis, the three independent excitation time histories were input simultaneously. The cost of performing the direct integration time history analysis became prohibitive and the analysis was stopped after 13 seconds of the dynamic structural response was computed.

Three separate modal superposition time history analyses were performed in accordance with [4]. The total structural response was obtained by combining the maximum responses from the three separate analyses using the square root of the sum of the squares (SRSS) approach. It should be noted that the modal superposition time history method used enables the analyst to select only the structural response parameters of interest for subsequently calculating their dynamic response time histories. As with the modified time history method, this feature helps reduce the computational effort associated with dynamic analyses. Of particular interest in this study were the dynamic transient responses of the reactor vessel nozzle loads (elemental loads).

The first step in the modified time history method requires results from response spectrum analyses. Three separate response spectrum analyses were performed, one analysis for each orthogonal direction of seismic excitation. For these analyses, the seismic excitation was specified using the response spectra. The response spectra, response spectrum analysis results, and the OBE excitation time histories were input into the modified time history computer program. As with the modal superposition time history analyses, three separate modified time history analyses were performed and their maximum responses combined using the SRSS procedure.

Table 1 lists the maximum dynamic loads for the reactor vessel nozzle from the modal superposition and modified time history analyses. The small difference (less than 5%) between the results was due to a difference in the selection of time steps between the two analytical methods. The first three discrete time points in the modal superposition time history analyses were 0.0, 0.001, and 0.004 seconds. In the modified time history analyses, the first three discrete time points were 0.0, 0.003, and 0.007 seconds. Both sets of analyses used a 0.003 second time step after the first two time points. The 0.001 second difference between the two sets of time histories resulted in the slight difference in the calculated structural loads listed in Table 1.

Table 2 summarizes the computational times and costs for the direct integration, modal superposition, and modified time history methods. The computational parameters for the direct integration time history analyses were extrapolated from the aborted analysis. It is important to note that the costs of the response spectrum analyses were included in the modified time history analysis cost summary. Comparing the direct integration and modified time history method, the modified approach required 40 times less computational time and was 65 times less expensive. Compared to the modal superposition method, the modified time history approach was 2.3 times faster and resulted in a cost savings factor of 3.9. Thus, the modified time history approach is more advantageous to use than either the direct integration or modal superposition time history methods.

As noted, the above comparisons included the costs of the response spectrum analyses in the modified time history analysis costs. These comparisons assume that the results from the response spectrum analyses are not available for the modified time history calculations. Such a situation would occur outside of a design iteration environment where the analyst has chosen to perform a dynamic time history analysis. The comparisons in Table 2 show that the modified time history is the best choice of dynamic time history analyses, in terms of computational time and cost.

In the design iteration environment, the response spectrum analyses would already be performed, using the analytical approach suggested in the introduction to this report. Table 3 shows a division of the modified time history analysis costs into response spectrum and time history calculation segments. For the example problem, the response spectrum analyses required 128 seconds execution time and cost 0.34 CRUs. Thus, the modified time history analyses cost 0.26 CRUs and consumed 299 seconds execution time.

Comparing these modified time history costs (without the response spectrum analysis costs) to the modal superposition time history costs, the modified approach was 3.3 times faster and required 9.0 times fewer CRUs. Thus, in the design iteration environment, where the response spectrum analyses have already been performed, the modified time history method is clearly faster and less expensive than the normal time history methods.

Table 4 shows a division of the modal superposition time history analysis costs into eigenvector extraction /modal contribution, time history calculation, and post processing segments. The costs of performing the eigenvector extraction/modal contribution segment is nearly equal to the costs of performing the response spectrum analyses. Thus, the significant difference between the modified and modal superposition time history methods is the computational time and costs associated with calculating the time history responses (numerical integration of equations of motion). As discussed in the previous section, these differences are due to the extensive I/O operations associated with the normal time history methods.

There is an additional advantage to performing modified time history analyses in a design iteration environment. The example in this paper shows the modified time history analysis was actually less expensive to perform than the response spectrum analysis. Thus, for this particular problem, it would be less expensive to perform a modified time history analysis, subsequent to an initial response spectrum analysis, than to modify the structural model and perform another response spectrum analysis.

The modified time history method can therefore reduce the costs of dynamic analysis in a design iteration environment in two ways. First, the modified time history method calculates structural dynamic responses which are more mathematically exact, thereby eliminating some of the conservatism included in response spectrum analyses to account for the mathematical approximations. These more exact results will enable an acceptable structural design to be realized with fewer design iterations. Fewer design iterations naturally translates into reduced computational time and expense in addition to reduced engineering effort and time. Of course, this cost benefit is acheivable by any time history method. However, the above comparisons showed the modified time history method to be the most advantageous dynamic time history analysis method to be used in the design iteration environment. The second cost reduction benefit of the modified time history method is mostly problem dependent. For the given example, the modified time history method required more computational time but resulted in lower costs when compared to the previous response spectrum analysis. This suggests that the modified time history method is indeed less expensive to use than the response spectrum method. If however, more time steps were needed in the time history analysis, the mentioned cost savings would diminish or perhaps result in higher costs for the modified time history analysis. On the other hand, fewer time steps would result in an increased cost savings when compared to the response spectrum analysis. In either case, the costs of performing the modified time history analysis are small enough to justify its use in the design iteration environment.

CONCLUSIONS

A method was presented to perform linear dynamic time history analyses using results from response spectrum analyses. This "modified" time history procedure is easy to implement using response spectrum analysis results and a response spectrum generation computer program. When compared to normal time history methods, the modified time history approach was shown to be faster and substantially less expensive to use. It is particularly useful in a design iteration environment in which response spectrum analyses are first performed to assess the adequacy of a structural design.

In a design iteration environment which utilizes the response spectrum method, use of the modified time history approach can speed convergence towards an acceptable structural design more quickly and with less computational expense than additional design iterations and response spectrum analyses. The major cost in the modified time history method is performing the response spectrum analyses. For a given design iteration, subsequent to the response spectrum analysis, a modified time history analysis can be performed before performing additional response spectrum analyses. Because the time history analysis results are mathematically more exact than response spectrum analysis results, convergence towards an acceptable structural design will be improved. Thus, the modified time history method is a valuable analytical tool in a production analysis environment as well as for most other uses of linear time history methods.

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TABLE 1
COMPARISON OF REACTOR VESSEL NOZZLE LOAD CALCULATIONS

Load Description	Modal Superposition TH Analysis	Modified TH Analysis
F_x, N	2120	2130
F_y, N	4360	4360
F_z, N	600	610
M_x, N-m	333	346
M_y, N-m	568	545
M_z, N-m	4380	4390

TABLE 2
TIME HISTORY ANALYSIS COST SUMMARY

Analysis Type	CP Seconds	CRUs
Direct Integration	16980	39
Modal Superposition	990	2.38
Modified	427	0.60

TABLE 3
MODIFIED TIME HISTORY ANALYSIS COST EVALUATION

Analysis Description	CP Seconds	CRUs
Response Spectrum Analysis and post process results	134	0.36
Run program MODTH	293	0.24

TABLE 4
MODAL SUPERPOSITION TIME HISTORY ANALYSIS COST EVALUATION

Analysis Description	CP Seconds	CRUs
Eigenvector Extraction and Modal Contribution Calculation	184	0.40
Calculate modal response time histories	743	1.86
Post process results	63	0.12

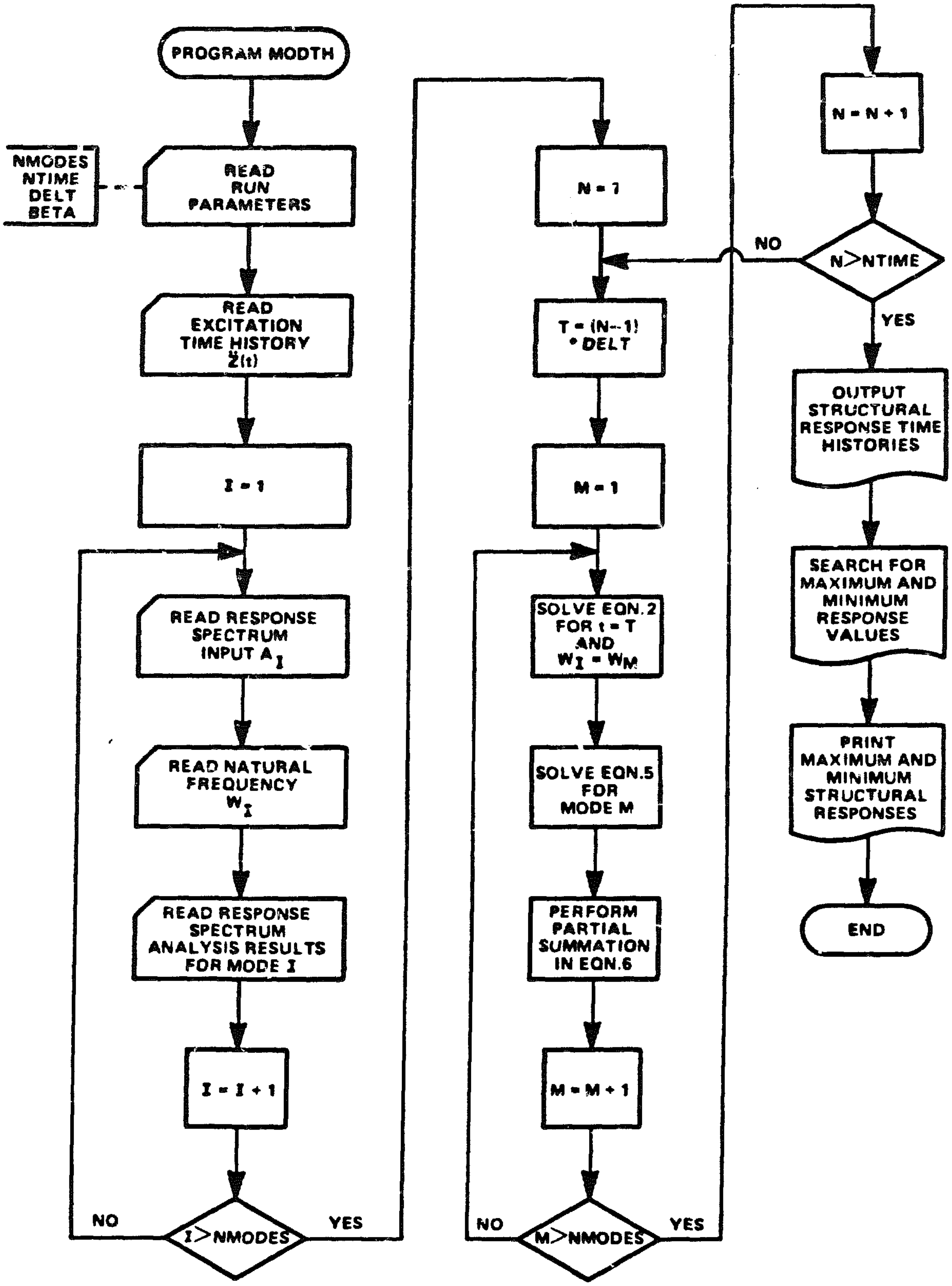

Figure 1. Modified Time History Flowchart

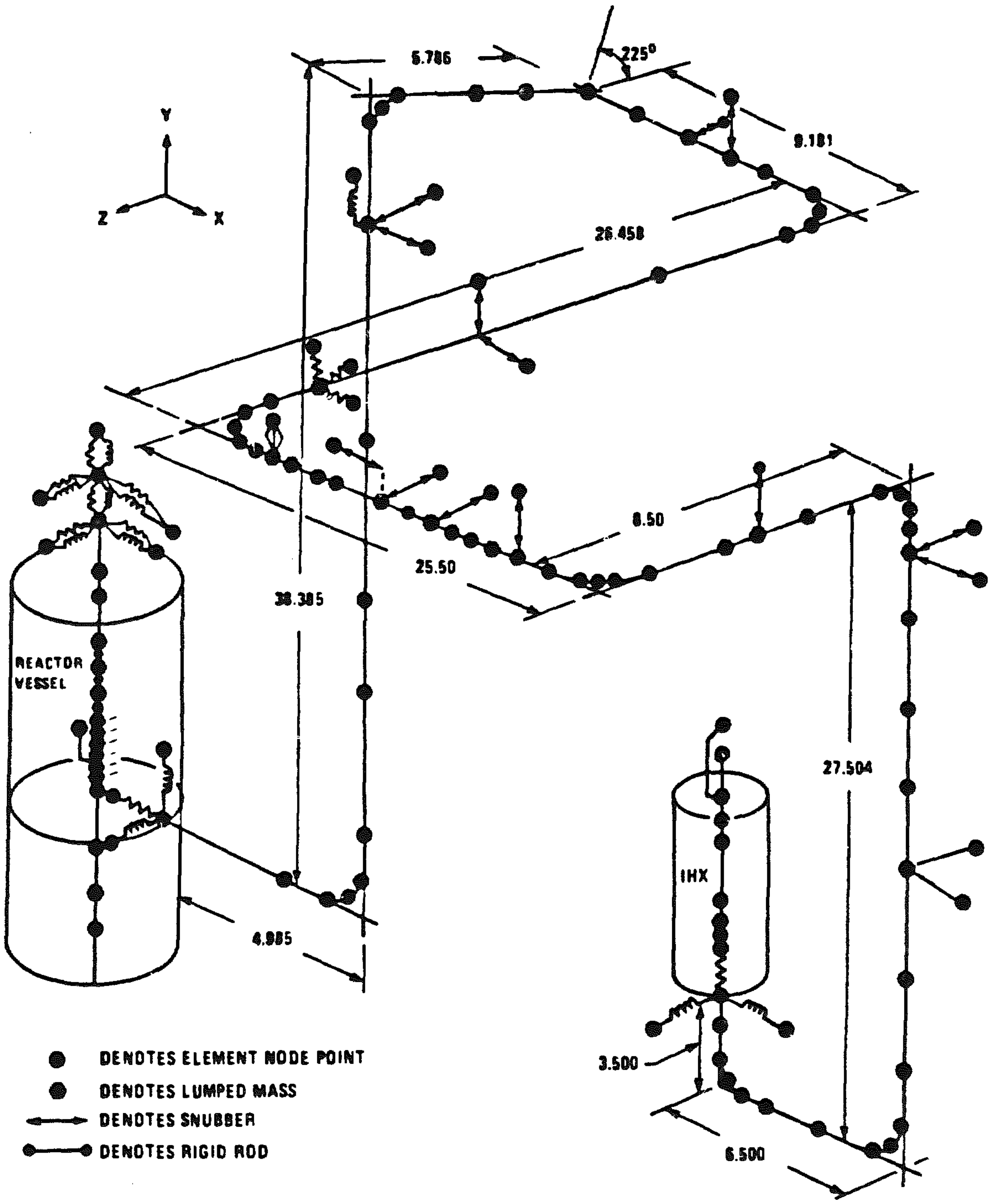

Figure 2. Finite element idealization of Prototypic Piping System